

Your name is: _____

Grading 1

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- 1 (a) (15) Find an orthonormal basis for the subspace S of \mathbf{R}^4 spanned by these three vectors:

$$a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad a_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 3 \end{bmatrix} \quad a_3 = a_1 + a_2$$

- (b) (15) Find the closest vector p in that subspace S to the vector

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

- 2** (a) **(15)** Start with the same subspace S . Find a basis (not necessarily orthonormal) for its orthogonal complement S^\perp (the space of all vectors perpendicular to S).
- (b) **(10)** Find the closest vector q in S^\perp to the same vector b .

- 3 (a) (10) Find the determinant of this matrix A_4 :

$$A_4 = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

- (b) (10) How many terms are *nonzero* out of the 24 terms in the big formula

$$\det A = \Sigma(\pm)a_{1\alpha}a_{2\beta}a_{3\gamma}a_{4\omega}$$

and what are those nonzero terms?

Suppose the matrices A_n all follow the same pattern as A_4 , with 2's on the main diagonal and 1's on the *second* diagonals above and below. Thus

$$A_1 = \begin{bmatrix} 2 \end{bmatrix} \quad A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad A_3 = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

- (c) (10) Use cofactors along row 1 of A_n to *find the relation between* $\det A_n$, $\det A_{n-1}$ and $\det M_{n-2}$.

That mysterious matrix M_{n-2} is not the same as A_{n-2} . Start with $n = 4$, and use cofactor to find M_{n-2} when this submatrix is 2 by 2. Describe M_{n-2} for larger n .

- 4 (a) (10) Give the formula for the projection matrix P onto the column space of a matrix A . Where does the formula assume that A has independent columns?
- (b) (5) The two properties of all these projection matrices are $P^2 = P$ and $P^T = P$. Suppose v^T is the first row of P and v_1 is the first entry in that row. Prove that $v^T v = v_1$.