## Your name is \_\_\_\_\_

Please circle your recitation:

$\alpha > 0$	OII OI	o your recitu	UIUIII		
M2	2-131	Darren Crowdy	crowdy@math	2 - 335	3-7905
M2	2 - 132	Yue Lei	yuelei@math	2 - 586	3-4102
M3	2-131	Darren Crowdy	crowdy@math	2 - 335	3-7905
T10	2-131	Sergiu Moroianu	bebe@math	2 - 491	3-4091
T10	2 - 132	Gabrielle Stoy	stoy@math	2 - 235	3-4984
T11	2-131	Sergiu Moroianu	bebe@math	2 - 491	3-4091
T11	2 - 132	Gabrielle Stoy	stoy@math	2 - 235	3-4984
T12	2 - 132	Anda Degeratu	anda@math	2-229	3-1589
T12	2-131	Edward Goldstein	$\operatorname{egold}$	2-092	3-6228
T1	2-131	Anda Degeratu	anda@math	2-229	3-1589
T2	2 - 132	Yue Lei	yuelei@math	2 - 586	3-4102
	M2 M3 T10 T10 T11 T11 T12 T12	M22-131M22-132M32-131T102-131T102-132T112-131T122-132T122-131T12-131T12-131	M2       2-131       Darren Crowdy         M2       2-132       Yue Lei         M3       2-131       Darren Crowdy         T10       2-131       Sergiu Moroianu         T10       2-132       Gabrielle Stoy         T11       2-131       Sergiu Moroianu         T11       2-132       Gabrielle Stoy         T12       2-132       Anda Degeratu         T12       2-131       Edward Goldstein         T1       2-131       Anda Degeratu	M2 2-132 Yue Lei yuelei@math M3 2-131 Darren Crowdy crowdy@math T10 2-131 Sergiu Moroianu bebe@math T10 2-132 Gabrielle Stoy stoy@math T11 2-131 Sergiu Moroianu bebe@math T11 2-132 Gabrielle Stoy stoy@math T12 2-132 Anda Degeratu anda@math T12 2-131 Edward Goldstein egold@math T1 2-131 Anda Degeratu anda@math	M2       2-131       Darren Crowdy       crowdy@math       2-335         M2       2-132       Yue Lei       yuelei@math       2-586         M3       2-131       Darren Crowdy       crowdy@math       2-335         T10       2-131       Sergiu Moroianu       bebe@math       2-491         T10       2-132       Gabrielle Stoy       stoy@math       2-235         T11       2-131       Sergiu Moroianu       bebe@math       2-491         T11       2-132       Gabrielle Stoy       stoy@math       2-235         T12       2-132       Anda Degeratu       anda@math       2-229         T12       2-131       Edward Goldstein       egold@math       2-092         T1       2-131       Anda Degeratu       anda@math       2-229

1. (a.) (10 pts) Find ALL the eigenvalues and ONE eigenvector of each of the matrices below:

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ -3 & 0 & -2 \end{bmatrix}$$

1. (b.) (10 pts) Find ONLY one eigenvalue of each of the matrices below: (This can be done with no arithmetic.)

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

- **2.** (20 pts) Let A have eigenvalues  $\lambda_1, \ldots, \lambda_n$  (all nonzero) and corresponding eigenvectors  $x_1, \ldots, x_n$  forming a basis for  $\mathbb{R}^n$ . Let C be its cofactor matrix. (The answers to the questions below should be in terms of the  $\lambda_i$ .)
  - (a) (5 pts) What is  $\operatorname{trace}(A^{-1})$ ?  $\det(A^{-1})$ ?
  - (b) (15 pts) What is  $\operatorname{trace}(C)$ ? What is  $\det(C)$ ? (Hint:  $A^{-1} = \frac{C^T}{\det A}$ )

**3.** (30 pts.) Suppose A is symmetric  $(n \times n)$  with rank r = 1 and one eigenvalue equal to 7. Let the general solution to

$$\frac{du}{dt} = -Au$$

be written as u(t) = M(t)u(0). (Note the minus sign!)

- (a) (5 pts.) Write down an expression for M(t) in terms of A and t.
- (b) (15 pts.) Is it true that for all t,  $\operatorname{trace}(M(t)) \ge \det(M(t))$ ? Explain your answer by finding all the eigenvalues of M(t).
- (c) (5 pts.) Can u(t) blow up when  $t \to \infty$ ? Explain.
- (d) (5 pts.) Can u(t) approach 0 when  $t \to \infty$ ? Explain.

**4.** (30pts.) (a). If B is invertible prove that AB has the same eigenvalues as BA. (Hint: Find a matrix M such that ABM = MBA.)

(b). Find a diagonalizable matrix  $A \neq 0$  that is similar to -A. Also find a nondiagonalizable matrix A that is similar to -A.