Numerical and Experimental Methods for Determining the Strength and Motion of Trailing Vortices

by

William Durand Ramsey

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Author .................................................................  Department of Ocean Engineering

Department of Ocean Engineering

May 16, 1995

Certified by .............................................................. J.E. Kerwin

Professor of Naval Architecture

Thesis Supervisor

Accepted by .............................................................. A. Douglas Charmichael

Chairman, Departmental Committee on Graduate Students
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Abstract

As a wing produces lift it sheds vorticity into the wake in the form of a vortex sheet. Since the sheet must depart directly from the wing, it is initially flat and undeformed. This configuration is highly unstable, and the sheet tends to roll itself into a concentrated vortex core. Computing this motion using discrete vortices tends to be extremely difficult and numerous techniques have been attempted which remove the vortex singularities. A method will be presented to accurately predict the rollup of trailing vortices using discrete vortices in timescales orders of magnitude lower than any other present day techniques. In addition, tests were conducted at the Marine Hydrodynamics Water Tunnel to map out the vortex core behind a semi-span control surface using a Laser Doppler Velocimeter (LDV) to nonintrusively measure the velocities in the wake. Since the motion of the vortex core is chaotic, the velocity profiles obtained are effectively smeared due to this motion. A technique will be presented which utilizes standard deviations and velocities to determine the time averaged magnitude of Random Axial Meandering (RAM) in order to determine the actual velocity profiles moving with the vortex. The reconstructed profiles are then used to predict the strength of the vortex core and for validating numerical codes.

Thesis Supervisor: J.E. Kerwin
Title: Professor of Naval Architecture
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Chapter 1

Introduction

This work will be separated into two sections. The first section presents an experimental method for determining the magnitude of vortex meandering based on tests conducted in the MIT Marine Hydrodynamics Laboratory. The second section presents a new theory for discrete vortex motion using the example of predicting tip vortex formation to show its validity. Many of the concepts presented in this thesis are based on intuition and thus rigorous proofs are not included.

1.1 Trailing Vortices

Everyone living in the twentieth century has observed trailing vortices behind large airplanes. Probably only a very small percentage of them understand fully their existence and realize that what they are observing is water vapor caused by extremely low pressures in the vortex core due to very large velocity gradients proportional to the lift generated by the airplane. Cavitation inception is analogous to the airplane trailing vortices, the only difference is that the medium providing the lift is water and that low pressures cause the water to boil.

Predicting rollup has traditionally been very difficult, especially before computers became powerful enough to handle large computational jobs. Nonetheless, people have been attempting to compute the rollup by hand for almost a hundred years with some breakthroughs inspired by “approximations” and relaxation schemes. Thus a class of researchers called “vortex chasers” evolved as a frustrated group searching for robust algorithms to predict the strength and motion of tip vortices during the rollup process.
1.2 History of Vortex Chasing

Credit is usually given to Westwater in 1935 for being the first person to discretize the wake on an elliptically loaded wing and attempt to compute vortex rollup by simple hand calculations [4, 5, 9]. He used a small number of vortices and was able to obtain reasonable estimates since the number of vortices was small. However, when later researchers tried to better his results by increasing the number of vortices in the wake they discovered a chaotic behavior of the vortices. Since the initial failures to regularize the wake structure, many different approaches have been introduced which can keep the vortex rollup semi-stable.

One approach amalgamates vortices when they induce velocities of a certain magnitude on each other. Rather than moving the vortices in what would be a chaotic fashion, the two vortices are combined at the location of their center of vorticity. This approach can work well, although it has the disadvantage of losing resolution of the vortex core. However, it should be able to predict the final location and strength of the vortex core if properly implemented.

Another approach introduces finite cores around the discrete vortices. When the velocities are greater than a specified value, the method places a linear velocity profile about the center thus forming a Rankine vortex. Excellent results have been attained with this method, giving seemingly infinite spirals in the vortex core.

There are other approaches using panel methods and other clever techniques as well. In fact there are hundreds of papers on vortex chasing, almost saturating the field. The methods all rely on user inputs to keep the system stable introducing a problem that a "casual" user of rollup programs will not be able to sit down and obtain reasonable answers. Another problem with these approaches is that the computing time is extremely high due to the need for small timesteps in order to maintain stability unless small numbers of vortices are used in which case there are few problems with chaotic motion to be solved in the first place.

In the last couple of decades several techniques have been developed for nonintrusively measuring vorticity and vortex velocity profiles such as Laser Doppler Velocimetry (LDV) and Vorticity Optical Probes (VOP). These techniques utilize laser interference patterns to measure the flow and share the commonality that they are fixed in space while the structures they are measuring are moving in space with time. This causes the velocity
profiles measured to be altered due to this effect. D.H. Fruman [2] developed a technique for quantifying this motion by examining the standard deviations of the velocities. The first section of this thesis will present both his method and a new method for quantifying this motion.

1.3 A New Approach to Vortex Chasing

A new approach to vortex chasing will be presented in the last section of this thesis. The approach is based on the first principles of vortex motion, mainly that vortices tend to rotate about one another and thus their motion cannot be approximated as linear over large timescales. Using this technique the calculation of trailing vortex rollup is straightforward and requires no user inputs to regularize the solution with the exception that a reasonable time step be chosen, although even that can be modified greatly. As will be shown, this method has the potential to change the way vortex motion is calculated and implemented in codes.
Chapter 2

Experimental Setup

2.1 Marine Hydrodynamics Laboratory

The MIT Marine Hydrodynamics Water Tunnel is a two story closed loop tunnel with a square test section at the top driven by an impeller connected to a 75 horsepower motor which can attain a maximum water speed of 30 feet/second in the test section. A 5:1 contraction section fitted with a honeycomb mesh and a wake screen to promote flow uniformity is upstream of the test section. The contraction section contains a differential pressure cell measuring the freestream velocity, $v_{dp}$, which is used to normalize all of the measurements.

Freestream velocity is a term to describe the flow in an empty test section. The test section has removable plexiglass windows on four sides and measures 20 inches on a side and four feet in length.

The velocities of the flow are measured using a Laser Doppler Velocimeter (LDV) powered by a three watt argon-ion Lexel Model 95 laser mounted on a computer controlled three-axis table. While the table's position can be determined to within 5 microns, errors in the control system result in positional errors of up to ± 0.01mm. The location of the measurement volume can be moved to anywhere the laser beams are not blocked and the coordinates will be obtained from the positioning system on the table. The measurement volume is 53um x 610um for the axial velocity and 56um x 645um for the vertical velocity. The 0.5145nm wavelength is used to measure the streamwise velocity and the 0.488nm wavelength is used to measure the vertical velocity relative to the test section. At each data point the LDV will measure a specified number of data readings whose mean and standard deviation will be computed. Data readings outside of three standard deviations
will then be discarded and a new mean and standard deviation will be computed. Thus a typical data point will have $x,y,z,v_x,v_z,sdx,sdz,vdp$ where $x,y,$ and $z$ are the coordinates of the measurement volume obtained from the table, $v_x$ is the streamwise velocity, $v_z$ is the vertical velocity, $sdx$ and $sdz$ are their respective standard deviations, and $vdp$ is the differential pressure cell reading correlating to the empty test section velocity.

### 2.2 Tip Vortex Test Setup

<table>
<thead>
<tr>
<th>Sectional Profile</th>
<th>NACA0012-NACA0018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflow ($U_\infty$)</td>
<td>$7.24 \frac{m}{s}$</td>
</tr>
<tr>
<td>Angles of Attack ($\alpha$)</td>
<td>$5^\circ$ and $10^\circ$</td>
</tr>
<tr>
<td>Mean Chord</td>
<td>159mm</td>
</tr>
<tr>
<td>Span</td>
<td>203.2mm</td>
</tr>
<tr>
<td>Aspect Ratio ($A$)</td>
<td>2.56</td>
</tr>
<tr>
<td>$C_L$</td>
<td>0.244 and 0.496</td>
</tr>
<tr>
<td>$Re \left( \frac{v}{\nu} \right)$</td>
<td>$1.16 \times 10^6$</td>
</tr>
</tbody>
</table>

Table 2.1: Foil Geometry and Testing Conditions
The test setup consists of a control surface, from now on referred to as a foil, mounted to a halfbody profile which is mounted on a splitter plate bolted to the plexiglass tunnel window. The purpose of the splitter plate is to move the test apparatus outside the region of the tunnel wall boundary layer. A shaft attached to the foil at the quarter chord point goes through the half body, splitter plate, and the plexiglass tunnel window into a dynamometer which then measures the lift and drag forces and the moment about the quarter chord. The shaft is also attached to an encoder which is used to measure angle of attack. See Figure 2-1.

The half body profile is a half-section of an uncambered NACA 4-digit hydrofoil with a 16" chord and a thickness/chord ratio of 22.15%. The half-body span is 19.5" with corners rounded by a 1.75" radius. The shaft for the foil is mounted at 50% of the span and 28% of the chord of the half body.

The foil has a 8.06" span with a 9.23° leading edge sweep angle. The root chord is 7.5" with a sectional NACA0018 profile. The 4.905" tip is described by a NACA0012 profile [2]. The mean chord is 6.2". The shaft centerline is located at the quarter chord position of the foil. Turbulence inducers 0.008" high and spaced 0.1" apart were placed on the foil at a position of x/c=20% which was determined by a boundary layer code such that the displacement thickness at that point was 0.008".

The right handed coordinate system is defined as follows. The x axis is always aligned with the incoming flow. The y axis is oriented along the span, either outward or inward depending on whether the foil is on the starboard side. This experiment is a port-oriented foil so the spanwise axis is oriented inward. The z axis always points up when the foil is parallel to the ground. Thus the z axis is the cross product of the x and y axes. The corresponding velocity components measured by the LDV system are the streamwise and vertical velocities. The origin was located at the tip of the trailing edge by examining the diffraction pattern created on the rear window when the laser was positioned on the tip of the trailing edge. The origin can be determined to an accuracy of 0.5mm.

The experiments covered in this thesis used angles of attack 5.35° and 10°. An encoder was attached to the shaft connected to the foil. The water speed was set to 15 feet/second and lift measurements on the foil were taken. When the lift was zero, the encoder was zeroed. From the zero reference position the angle was adjusted. The error in this process is estimated at ±0.1°, but conservative bounds of ±0.5° will be placed on the level of accuracy of this
method.

2.3 Test Procedure

The freestream velocity of the tunnel was set to 24 feet/second. The vortex core was first mapped downstream by lowering the pressure in the tunnel section and determining the location of the core using the laser. Five grids of data 100mm on a side were taken to map out the location of the core using spacing of 5mm. The grids were located 9mm, 70mm, 136mm, and 263mm and 343mm behind the foil and will be referred to as coarse planes 1, 2, 3, 4 and 5 respectively. These grids were used to determine the location and size of the vortex core. Fine grids of 0.5mm spacing were then taken at each on the planes to obtain the detailed structure of the core. After data for the fine grids were collected, cuts of 0.1mm spacing were taken bisecting the center of the vortex core were taken to be used as the RAM data for this thesis. Examples of the three types of data are shown in Figure 2-2.
Chapter 3

Vortex Meandering

A laminar wake sheet will roll itself into a conical spiral shape corresponding to a concentrated tip vortex such that no section will cross another. With the addition of a little bit of turbulence this nice spiral wake sheet will form a concentrated tip vortex with a solid body core with inviscid velocity profiles outside that core and the spiral shape will be lost. Unlike the laminar case this vortex would also fluctuate in position as a consequence of the turbulence. Thus arises the phenomenon known as wandering or meandering, for the two can be used interchangeably. For the case of a wandering trailing vortex system it is convenient to use the term Random Axial Meandering (RAM) since the vortex is meandering about a mean line aligned with the axial velocity. A problem arises when measuring the velocities inside the core with a noninvasive technique such as the LDV system since the LDV system is fixed with respect to time while the velocity being measured is a function of the distance from the center of the core which is fluctuating with time. RAM causes a “smearing” of the actual velocity profiles and decreases the measured maximum velocities by up to 20% of their actual values. RAM is especially important to quantify since it can be used to extrapolate actual velocity profiles from measured profiles and thus the maximum velocities within the vortex core can be determined.

3.1 A Meandering Step Function

With many fluid “proofs”, the concept of a bug riding the fluid is especially useful in showing the validity on an assumption. Suppose there were a bug who was striding along the surface of a river containing an inviscid fluid. The river also had the property such that
Figure 3-1: Trailing Vortex Formation and Meandering

Figure 3-2: Bug on River
along the East half the river flowed North with a speed of $1 \text{ m/s}$ and along the West half the river flowed South with a speed of $1 \text{ m/s}$. Thus at the center of the river the velocity appeared to be a step function to the bug as it moved across the river. An earthquake began to shift the East-West position of the river while keeping a smooth surface while the bug remained fixed in space. The bug then noticed, after all it is a very intelligent bug, that at the center of the river the average velocity was zero and at small distances from the center the velocity of the river asymptotically reached $1 \text{ m/s}$ at one meter from the center. The bug also took the analysis a step further and realized that the standard deviation of the velocity at the center of the river was $1 \text{ m/s}$ while the standard deviation at one meter or more from the center of the river was zero. The bug's journey is shown in Figure 3-3.

This is useful is showing how the step-like profile can be smeared by the wandering effect. From now on the river analogy will be referred to in the from “riding”, as in the bug is “riding” a square wave.
3.2 A Meandering Sine Wave

Just as in the last section, suppose the bug were now riding a sine wave. As with the square wave, when the wandering starts the bug will begin to sense the velocities on either side of it. If the bug is riding a peak, it will still see a peak although the magnitude of the peak will be decreased depending on the amount of wandering taking place. Likewise if the bug happens to be situated on a point where the velocity is zero, the velocity of that point will still be zero, but there will be a high standard deviation of the velocity due to the wandering.

3.3 Effect of Meandering on Velocities

The results from the sine and square waves can be used to extrapolate the effect of wandering on arbitrary velocity profiles. As can be seen in the case of the sine wave, peaks will remain peaks although their magnitudes will be diminished. Smearing will take place if there are asymmetrical velocities on either side of a point. Inflection points will retain their values. In many cases the results can be obtained intuitively just by looking at the profile and
observing what would happen if the axis were shifted slightly to either side.

3.4 Effect of Meandering on Standard Deviations

More important than the effect of meandering on the velocities is the effect of meandering on standard deviations, for it is the standard deviations which are used to determine the magnitude of meandering which is in turn used to recreate the original velocity profiles. Their are three regions of interest to examine:

1. The effect of meandering at peak values
2. The effect of meandering at inflection points
3. The effect of meandering on near constant profiles

Contrary to intuition, the standard deviation at a peak will not, in turn, be a peak, since at a peak a slight bit of wandering results only in slight changes in the velocity. While the standard deviation can still be relatively high, it will not be a peak value. Inflection points tend to have large values of standard deviations since a little bit of meandering on either side of the inflection point results in large changes in velocity. The third case intuition holds true: small meanderings on a gradual slopes result in low standard deviations.

These three regions of interest are important in reconstructing velocity profiles. When data are collected, the mean velocities and standard deviations are given. The standard deviations contain terms due to laser noise, turbulence, and meandering and it is sometimes difficult to isolate the effect of meandering. Often two components of velocity are needed such as the axial and vertical component. Since the magnitude of the wake defect is much smaller than the range of vertical velocities, the component of standard deviations due to meandering will be much higher in the vertical velocities while the component of standard deviations due to turbulence should be the same for both components of velocity. The effect of laser noise is merely an additive shift. This concept is crucial in determining the magnitude of RAM. Velocities can be reconstructed by meandering a certain profile until the standard deviations and meandered velocities match the data.
Chapter 4

A Numerical Method for Quantifying RAM in Two Dimensions

One of the main goals of the experiments being conducted in the Marine Hydrodynamics Laboratory was to validate Navier-Stokes codes simulating the experiment. Since the codes do not take into account the effect of RAM on the velocity profiles, there is no exact way to determine, by examining the data, whether the codes are overpredicting the maximum values of the vertical velocities and wake defect without having some sort of method for removing the effect of RAM on the velocity profiles. A model of the vortex meandering was created in two dimensions which simulates the effect of RAM on velocity profiles. This information can be used to reconstruct Lagrangian velocity profiles. The term Lagrangian refers to a reference frame moving with the vortex.

4.1 Assumptions

Any model in engineering must make certain simplifying assumptions in order to solve the problem at hand. With a bit of luck, these assumptions are valid so that the model can be reliable to a certain degree of accuracy. The two main assumptions for the Meandering Simulation are that the tangential and axial velocity profiles are idealized profiles and that the vortex meanders in a perfectly random pattern governed by a Gaussian distribution.
function. A third assumption that is made is that the level of turbulence is constant for both the axial and tangential velocities. The third assumption is justified because turbulence has no preferred orientation.

The tangential and axial velocity profiles for an idealized viscous vortex take on the form of a Lamb's equation and Gaussian distribution function respectively [2]. The equations and their forms are shown below.

\[
V_{Tan} = \frac{\Gamma}{2\pi R} \left(1 - e^{-\frac{R^2}{\pi^2}}\right) \quad (4.1)
\]

\[
V_{Axial} = 1 - C_1 \left(e^{-\frac{R^2}{\pi^2}}\right) \quad (4.2)
\]

The use of these profiles can be justified most of the time. With the exception of rollup close to the foil, the concentrated vortex induces a tangential velocity similar to the idealized form. The slight differences outside of the core that occur do not cause significant errors when used in quantifying the magnitude of meandering. The axial velocity for small angles takes on a Gaussian form which fits the data fairly well. Unfortunately for large angles a jetting behavior begins and this assumption breaks down.

The assumption that the vortex meanders in a perfectly random pattern is the issue that causes the most contention among researchers. When observing the vortex during operation
the vortex occasionally appears to move to a location and stick for an arbitrary length of time. However, the vortex does not seem to favor any particular positions during the course of its meandering. Since the outputs from the LDV are time averaged there is no way to break them down to give a time history of the vortex. Also, due to the apparent symmetry of the data there is no easy way to determine if the vortex favors a certain motion. When all of these considerations are taken into account the only logical choice for a meandering pattern is to assume the vortex meanders randomly.

4.2 Randomized Coordinates of the Vortex Center

Using the assumption that the vortex meanders in a random pattern about a mean position, the coordinates of the vortex can be determined such that the vortex has a certain probability of being at a certain radius from the mean location at any point in time. In addition, the vortex must have an equal probability of being at any angular position at a given point in time. Let $W$ denote the relative magnitude of meandering in terms of core radii. Thus if $W = 1$ the vortex will never be more than one radius from the mean position and will move in a perfectly random pattern within one radius from the mean position. If a parameter $R_M$ denotes the percentage of maximum meandering and is given by a Gaussian distribution function the coordinates of the vortex $(\xi, \eta)$ can be determined by:

\begin{align*}
\xi &= WR_M \cos(\beta) \\
\eta &= WR_M \sin(\beta)
\end{align*} \tag{4.3}

where $\beta$ is the angular position of the vortex and is chosen such that the vortex always has an equal probability of being at any angular position. The number of times $N$ the vortex is at any radius $R_v = WR_M$ and angular position $\beta$ are given by

\begin{align*}
N &= 100e^{-\frac{3R_v^2}{2}} \\
\beta &= i \frac{2\pi}{N}
\end{align*} \tag{4.5}

\tag{4.6}

where $i$ ranges from 1 to $N$. It is now possible to create an array of 4119 coordinates reflecting a randomized pattern of wandering such that there are $N (\xi, \eta)$ at any radius $R_v$. 

23
A diagram of the coordinates and how the velocities measured by the LDV are affected is shown in Figure 4.2.

### 4.3 Computing Velocities and Standard Deviations

The velocities are calculated along a cut which passes through the mean center of the vortex as shown in Figure 4.2. The one dimensional coordinate along the cut at which the velocities are calculated is \( y \). Given the location of the vortex core at \((\xi, \eta)\) the radius from the vortex to the point is just

\[
R = \sqrt{(Y + \xi)^2 + \xi^2}
\]  

where \( R \) can then be used in Equation 4.1 and Equation 4.2 and to determine the tangential and axial velocities at that point. To obtain the vertical velocity as the LDV would measure it, the tangential velocity must be broken into the vertical and spanwise components. The parameter \( \alpha \) is used to denote the angle between the Y axis and the vortex location as shown in Figure 4.2. The vertical velocity \( v_z \) is then obtained as \( V_{\text{Tan}} \cos(\alpha) \). The axial velocity
$\nu z$ remains unaltered from this shift. For each of the 4119 $(\xi, \eta)$ a value of the vertical and axial velocities is calculated. These values are then averaged and the standard deviation is computed. The output of the program gives $y, \nu z, u z, s dz, s dz$, and the Lagrangian $\nu z$ and $u z$. The parameters $W$ and $\Gamma$ can be varied to make a fit with the data. This will be explained in the next chapter.
Chapter 5

Applying the Method to Data

5.1 Previous Work

While the phenomenon of vortex meandering is quite well known, not much experimental work had been conducted in the field until recently. D.H. Fruman of the National Hydrodynamics Laboratory in France developed a technique in 1994 to determine the magnitude of wandering by using the velocity standard deviations of both components [2]. His technique utilized the presentation of data whereby the standard deviation of the axial velocity was plotted against the standard deviation of the vertical velocity. What he noticed was that there were three main regions in the data. The first region was obvious, for it was merely the offset due to random turbulence and laser noise and appears as an additive shift on both axes. The second region was a linear region of unit slope corresponding to isotropic turbulence which affects both components of standard deviation with equal magnitude. The third region marked a rapid change from the linear portion marks the effect of wandering. Since the standard deviation of the vertical velocity changed much more rapidly then the standard deviation of axial velocity when subjected to wandering, it makes sense that there should be a dramatic increase in slope between the two components. See Figure 5.1.

Fruman then took his data including the axial and vertical velocities and “wandered” then in a manner which was probably similar to the method presented in the previous chapter, although he may have only wandered the profiles in one dimension. He added a term proportional to the axial velocity as an assumed value for the isotropic turbulence

\[1\] Data from the MIT Marine Hydrodynamics Laboratory
common to both standard deviations and terms corresponding to the offsets. He was then able to obtain reasonable estimates of the magnitude of wandering.

5.2 Method of RAMplots

One of the problems with Fruman’s technique is that it is not immediately obvious how the plots of standard deviation will change when the profiles are subjected to increased wandering. Whether the bifurcation point changes or the slope of the third region increases are not questions that can easily be answered. With this in mind, I set about determining a method that relied mainly on one component of velocity which makes it intuitively simple to see the correlation between the magnitude of RAM and outputs from the method. The result was a form of presenting the data known as RAMplots. RAMplots consist only of the standard deviations plotted against the vertical velocity. A sample RAMplot is presented in Figure 5.2 which shows only the effect of RAM. See Appendix B Data Set 6 for an example isotropic turbulence’s effect on the RAMplot.

There are many descriptive features of these plots. The first and most important is the increasing magnitude of the top arc with increasing RAM. This is convenient because it makes it relatively simple to determine whether the appropriate magnitude of RAM has been determined. This curved upper region is indicative of fluctuations within the core.
Figure 5-2: Example of a RAMplot

which should yield high standard deviations according to the theory already presented. The second, and almost equally important, is the decrease of $V_{\text{max}}$ when the magnitude of RAM increases. This is due to the effect of meandering on peaks in sine-like waves previously discussed. The linear regions near the base of the plots which are little affected by increasing RAM correspond to the variations outside of the core which are not important in determining core pressures.

The importance of RAMplots was realized when data from the Tip Vortex experiment was analyzed revealing a complicated combination of wake defect and jetting behavior in the core at angles of attack over 10°. These profiles would have been nearly impossible to model in a reliable fashion. Fortunately with the method of RAMplots this problem is easily surmountable since the tangential velocity profile still takes on its assumed form. The level of isotropic turbulence can still be determined by plotting the standard deviation of the axial velocity against the vertical velocity. This method will become clearer when the data is incorporated.
5.3 Iteratively Determining Vortex Strength and Magnitude of RAM

The iterative steps used to determine $\Gamma$ and $W$ are:

1. Choose a level of isotropic turbulence based on the standard deviation of the axial velocity
2. Choose a value of $\Gamma$ which corresponds to a value of $V_{max}$
3. Choose values of $W$ until the approximate level of RAM is obtained
4. Choose a new value of $\Gamma$ to match $V_{max}$
5. Repeat the process until $W$ and $\Gamma$ converge

Following this outline as a rough procedural guide will help converge on values of $W$ and $\Gamma$ which seem to fit the data. The process used to determine these two parameters could very easily be called guesswork, but in the end, there is only one combination of $W$ and $\Gamma$ which best fits the data and how these values is obtained does not really matter unless significant amounts of time are wasted in the process.

A sample data set with computed values of $W$ and $\Gamma$ is shown in Figure 5.3. This data set corresponds to a $5^\circ$ angle of attack at a distance downstream of 263mm. Only one set of data is included in the body of the thesis to avoid redundancy. For a complete listing of the data see Appendix A for $5^\circ$ angle of attack data sets and Appendix B for $10^\circ$ angle of attack data sets.

5.4 Nondimensionalizing the Results

All of the data have been nondimensionalized with respect to certain parameters. The circulation $\Gamma$ is nondimensionalized with respect to the average circulation $\Gamma_o$ on the foil given by:

$$\Gamma_o = \frac{Lift}{U_\infty \rho S} = \frac{1}{2} C_L U_\infty C$$

(5.1)
where $C_L$ is the lift coefficient of the foil. For the case of an elliptical foil of aspect ratio $A$, the $C_L$ is given by Prandtl’s approximation

$$C_L = \frac{2\pi\alpha}{(1 + \frac{A}{4})} \quad (5.2)$$

For $\alpha = 5^\circ$ and $10^\circ$ the lift coefficients are 0.308 and 0.616. Experimentally the lift coefficients were 0.244 and 0.496, showing a decrease in lift due to the effect of a non-elliptic foil. For the purposes of nondimensionalizing the circulation, the experimentally determined lift coefficients were used. The values of $\Gamma_0$ are 0.14 and 0.28.

The core size is nondimensionalized with respect to the boundary layer thickness of a flat plate at the trailing edge given by

$$\delta_{turbulent} \approx \frac{1}{Re^{-\frac{1}{6}}} = 2.6mm \quad (5.3)$$

The velocities and standard deviations are nondimensionalized by the mean freestream velocity $V_{dp} = 7.25m/sec$.  

Figure 5-3: Sample Data Set with Simulation Results
5.5 Summary of Results

The results of the RAM simulation are presented in Tables 5.2 and 5.3. First, it should be noted that there were many inconsistencies in the data corresponding to $\alpha = 10^\circ$ and thus the results in Table 5.3 may not be correct. However, the data corresponding to $\alpha = 5^\circ$ forms a consistent data set from which accurate conclusions can be inferred.

The main points to be gained from the simulation are:

1. Vorticity is fed into the core as it is convected downstream. Immediately behind the trailing edge the vorticity was 75% of its final measured strength.

2. The radius of the core, in general, increases due to viscous diffusion as the distance
downstream increases. The decrease in $R_{\text{core}}$ immediately behind the foil is due to the formation process.

3. The magnitude of RAM increases as the core moves downstream as well. Again, this makes intuitive sense. If the vortex line is constrained to leaving the foil at a certain location but is free to randomly meander downstream the magnitude of wandering should be almost zero close to the foil and larger downstream of the foil. The results affirm this hypothesis, showing an increase of RAM as the distance downstream increases. However, there must be a downstream position where this hypothesis breaks down because the vortex line infinitely far downstream cannot have an infinite range of motion. In other words there must be a final limit on RAM as the vortex moves downstream. The simulation results show this to some degree, as the change in RAM versus downstream position decreases with downstream position.

4. The spanwise position of the vortex moves inward, as can be predicted by numerical simulations. It is interesting to note that the final vortex position for both angles of attack is the same. This can be easily demonstrated with numerical simulations as well.

5. The isotropic turbulence decreases with downstream position. This implies that the formation of a viscous core serves to make the fluid entrained in the vortex behave in a more uniform fashion. It is interesting to note that the isotropic turbulence in the $10^\circ$ case was double the turbulence level of the $5^\circ$ case.

Based on the RAM results, the maximum measured values differ from the Lagrangian maximum velocities by 20%.
Chapter 6

Tip Vortex Formation

6.1 Background

Tip vortices are created in accordance with Helmholtz’s theorem which states that vortices cannot end in a fluid. For any force that is applied to a fluid a vortex with strength proportional to the force must be generated. Anyone who has stirred a cup of coffee, taken a bath, or paddled a boat has observed them. In the process of observing vortices one might have noticed that they are convected with the velocities around them, for the fundamental reason that vortices cannot sustain forces. The explanation for this is simple. From Newton’s law we know that force is equal to the mass multiplied by its acceleration. For inviscid flow a vortex has no mass. Thus if it were to sustain a force it also must have infinite acceleration, which it cannot. Hence vortices must always be convected with the velocity of the fluid around them. While vortex theory can be rather elegant, implementing it in accurate numerical methods which are simple and computationally efficient has plagued researchers for a hundred years.

Discrete vortices are useful for modeling flow. While in reality the trailing vortex system will actually be a sheet of vorticity shed off of a foil, it is easier to discretize the sheet into a system of discrete vortices such that the total circulation is maintained. Once that has been done, calculating the motion of the vortices should be straightforward, but has traditionally been one of the hardest problems to model. Typically cores are placed around the discrete vortices to remove singularities. The core radius is a parameter which can be varied by the user and presumably requires a great deal of trial and error to determine it’s “best” value, which can either be a function of distance or by specifying the maximum velocity.
outside the Rankine core. The vortices can be moved in various integration schemes such as Range-Kutta or forward Euler. Even with much practice it is very difficult to obtain consistent and accurate results using discrete vortices to model the vortex sheet [3] [8] [11].

The problem with all of the traditional methods for calculating tip vortex formation is that they fail to include the fundamental rotational motion induced by the vortices on one another. Typically when the distance between two vortices goes to zero the vortices induce infinite velocities on each other which must be convected. If the traditional scheme of position is equal to velocity multiplied by time is used then the motion becomes unstable as the vortices are convected away from each other. What the rest of this chapter will try to explain is that the vortices take on a rotational motion about the center of vorticity such that the motion is in fact very stable.

6.2 Motion of Two Discrete Vortices

If two vortices are placed in an inviscid fluid of constant velocity, preferably zero but not necessarily so, they will induce velocities of each other proportional to their strengths. Defining the strength of the vortices as \( \Gamma_1 \) and \( \Gamma_2 \) and the distance between them as \( L \) the velocities induced by the vortices on each other are given by:

\[
V_1 = \frac{\Gamma_2}{2\pi L} \hat{e}_\theta
\]

\[
V_2 = \frac{\Gamma_1}{2\pi L} \hat{e}_\theta
\]

It should be noted that \( \hat{e}_\theta \) is the unit normal perpendicular to the radial vector drawn from each vortex and is defined such that \( \hat{e}_r \times \hat{e}_\theta \) is oriented out of the page. While each of the vortices must move with the velocities induced upon them the direction of motion becomes more complicated. Since the radial and tangential vectors are changing orientation with time, the velocities must also change their orientation with time. For this reason it is easy to see why the straightforward assumption that position can be easily calculated by taking velocity times time breaks down, for this assumption relies heavily on the assumption that the tangential vector does not change orientation over a finite time scale. What happens in
the two vortex system is that the vortices rotate about a centroid given by:

\[ X_{\text{centroid}} = \frac{\Gamma_1 X_1 + \Gamma_2 X_2}{\Gamma_1 + \Gamma_2} \quad (6.3) \]

\[ Y_{\text{centroid}} = \frac{\Gamma_1 Y_1 + \Gamma_2 Y_2}{\Gamma_1 + \Gamma_2} \quad (6.4) \]

If the vortex strengths are equal then the centroid is located midway between the vortices and thus the vortices will rotate about each other in a circular manner with a constant distance between them. On the other extreme, if the vortex strengths are of equal and opposite signs than the denominator is zero. While mathematically this does not make sense for it implies that the vortex pair will not have a rotation. This is in fact the case, since two vortices of opposite and equal strength will be convected along a straight path, mathematically equivalent to a rotation about a center located infinitely far away. Obtaining the motion of the centroid is a simple task, for it is merely the time derivative of the position of the centroid given by:

\[ U_{\text{centroid}} = \frac{\Gamma_1 U_1 + \Gamma_2 U_2}{\Gamma_1 + \Gamma_2} \quad (6.5) \]

\[ V_{\text{centroid}} = \frac{\Gamma_1 V_1 + \Gamma_2 V_2}{\Gamma_1 + \Gamma_2} \quad (6.6) \]

This will be proved in the next section to be equal to zero in the case of a system of vortices in an otherwise stationary fluid. The main results for two vortices are that there is never a radial velocity term so that the distance between the vortices is constant and the vortices are convected in a circular orbit about the centroid.

### 6.3 Motion of an Infinite Number of Vortices

The results from the previous section are easily incorporated into the motion of an infinite number of vortices once the fundamental concepts are understood. Simplifying the equations by combining X and Y into complex notation, the position of a vortex will be denoted as \( Z_j \) and the velocity will be denoted as \( V_j \). The complex potential is given by:

\[ W = i \sum_{i=1}^{\infty} \Gamma_i \log(Z - Z_i) \quad (6.7) \]
\[ V_j = \left. \frac{dW}{dZ} \right|_{Z=Z_j} = i \sum_{\substack{i=1 \atop i \neq j}}^{\infty} \frac{\Gamma_i}{Z_j - Z_i} \] (6.8)

If the centroid is the only nonmoving point relative to the flow field its relative velocity must be equal to zero. This point can be proved by observing that

\[ \sum_{j=1}^{\infty} \Gamma_j V_j = 0 \] (6.9)

because

\[ \frac{\Gamma_i \Gamma_j}{Z_i - Z_j} = -\frac{\Gamma_i \Gamma_j}{Z_j - Z_i} \] (6.10)

Since the position of the centroid is the integral of the velocity with respect to time it must be equal to

\[ Z_{\text{centroid}} = \sum_{i=1}^{\infty} \frac{\Gamma_i Z_i}{\sum_{i=1}^{\infty} \Gamma_i} \] (6.11)

The equations presented so far can be found in Milne-Thomson’s “Theoretical Hydrodynamics” [7]. However, the applications and extensions are, to the best of the author's knowledge, unique to this document.

If \( N \) vortices are rotating about a common centroid then their rotational speed is given by

\[ \omega_{\text{centroid}} = \frac{(N - 1) \sum_{i=1}^{N} \Gamma_i}{\sum_{i=1 \atop i \neq j}^{N} (Z_i - Z_j)^2} \] (6.12)

This formulation is extremely powerful when trying to model the rollup of a system of discrete vortices. Unfortunately, unlike the two vortex scenario, the conclusions are not as straightforward. Lengths between vortices are not necessarily constant. Likewise while the centroid remains fixed with respect to the surrounding fluid the motions of the vortices do not have to be circular about the centroid. In fact the motion is very complicated, being analogous to planetary motion with almost chaotic motion. However, for the case of trailing vortex rollup examining only one side of the total free vortex system, we can hypothesize that for a system of vortices of constant sign the initial undeformed positions
the vortices are at their maximum distance from the centroid of the system, thereby showing that stability must result during the rollup process.

6.4 Discretization of Free Vorticity from Bound Circulation Distribution

The first problem in predicting vortex rollup is discretizing the trailing vortex sheet into a finite number of vortices of proper strength. Fortunately this is a very simple task. With a circulation distribution given by:

$$\Gamma(x) = \sum_{n=1}^{5} A_n \sin(n\pi x)$$  \hspace{1cm} (6.13)

which is then discretized into cosine spaced bound vortex segments of strength

$$\Gamma_i = \sum_{n=1}^{5} A_n \sin\left(\frac{(2i - 1)n\pi}{2N}\right)$$  \hspace{1cm} (6.14)

where \(N\) is the number of bound vortex segments. Since the undeformed wake structure will essentially look like a system of horseshoe vortices, the value of free vorticity at each station is equal to the difference of the bound vorticies on either side of it. Thus the free vorticity strength \(\gamma_i\) in the wake, with the exception of \(\gamma_1\) which is just the same strength as \(\Gamma_1\), is given as:

$$\gamma_i = \sum_{n=1}^{5} A_n \left\{ \sin\left(\frac{(2i + 1)n\pi}{2N}\right) - \sin\left(\frac{(2i - 1)n\pi}{2N}\right) \right\}$$  \hspace{1cm} (6.15)

Once the wake has been discretized, a first order approximation can be made as to the final spanwise position of the concentrated vortex by calculating the center of vorticity of one side of the wing. The constraints for this are that the free vorticity is of constant sign. Since the other side of the wing induces a downwash which tends to pull to vortices near the center of the wing, the final position of the tip vortex will be slightly inboard of the centroid. \(^1\) Fortunately the vortices close to the center of the wing have almost no strength.

\(^1\)It is generally believed that the spanwise center of vorticity should remain constant during the rollup process according to Betz's first rule for the conservation of momentum [8] and shown in Equation 6.9 for
<table>
<thead>
<tr>
<th>$A_1$</th>
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<th>$A_5$</th>
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Table 6.1: Glauert Coefficients and Centroidal Positions

and thus the discrepancy will be very small. Table 6.1 shows various Glauert coefficients and the resulting centers of vorticity.

### 6.5 Formulation of Tip Vortex Rollup Programs

After the wake has been discretized the challenge arise in moving the vortices in an efficient yet reliable manner. The method that will be presented uses an combination of “dumb-timestepping” and centroid theory to move the vortices. The dumb-timestepping refers to moving the vortices based on their velocity and not taking into account any possible rotational motion that might occur during a timestep. The steps used in the program are as follows.

1. Calculate the velocities at each vortex
2. Create a spine of centroids
3. Calculate the rotational speed of each vortex about it’s centroid
4. Determine a timestep
5. Move the vortices
6. Repeat the process with new vortex positions

The first step is straightforward and can be done by knowing the vortex strengths and positions relative to one another. The velocity is calculated by summing the X and Y velocities of all of the vortices on each other.

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a system of vortices in a uniform flow field. However, intuition argues that this might not be true due to the presence of a velocity gradient caused by the downwash from the other half of the wing. Quantitatively, simulation results show a 1% change in the spanwise center of vorticity for the elliptically loaded lifting line. (See Appendix C)
A spine of vortices is arranged by finding the centroid of the first two tipmost vortices \( C_1 \), then the first three vortices \( C_2 \), etc until the vortex corresponding to mid span has been included. Figure 6-1 illustrates this concept with a system of seven vortices in early stages of rollup. It is important to note that this thesis only deals with symmetrical loading such that the motion of the vortices can be mirrored about the mid span location. Just as the position of the centroids can be determined by summing the positions and strengths of the vortices included, the velocities of the centroids can be found by summing the velocities and strengths of the velocities included. In this manner a spine of positions and velocities of centroids can be created.

The spine was created such that there is a centroid corresponding to each vortex except the outmost vortex. For example the centroid corresponding to a vortex \( V_j \), \( C_{j-1} \) would be the centroid that only includes that vortex and all \( j-1 \) others closer to the tip. When \( V_j \) moves, centroid \( C_{j-2} \) must move such that centroid \( C_{j-1} \) moves with its calculated speed such that the center of vorticity does not change relative to the system. The rotational speed of each vortex relative to its centroid can be determined by knowing the tangential
velocity of each relative to the segment connecting them such that

\[ w = \frac{V_{\tan 1} - V_{\tan 2}}{\text{Length}} \]  

(6.16)

where \( w \) is the rotational speed of the system. Unfortunately to complicate matters there is also a contraction term since the vortices are not constrained to move in a circle of constant radius. Fortunately this can be evaluated by using the normal components of velocity in the same fashion. This is shown in Figure 6-2. It should be noted that the normal components of velocity on the two vortices result from influences external to the two vortex system.

A timestep is created such that it is large enough for the vortices to move quickly yet small enough so that large errors do not accumulate. The concession being implied that this method will not give the exact solution. To do this, the rotational rates are used such that the \( w \) corresponding to the outmost vortex not yet rolled into the vortex determines the time which is \( \frac{\pi}{20 \cdot \max \cdot w} \). In other words this says that the fastest rotating vortex not yet incorporated into the concentrated vortex core will not traverse more than one fortieth of a circle.

As has been mentioned the vortices are moved using dumb-timestepping and centroid theory. The criterion for determining which should be used depends on the rotation rate. If the angle given by \( w \cdot \text{time} \) is small such that \( \frac{1}{\cos(w \cdot \text{time})} \) is almost 1 then dumb timestepping can be used and both \( V_j \), its centroid \( C_j \) and centroid \( C_{j-1} \) are moved by velocity times the timestep. The centroid \( C_{j-1} \) is moved in the dumb-timestepping fashion in order to preserve the motion of \( C_{j-1} \). Otherwise if \( \frac{1}{\cos(w \cdot \text{time})} \) is greater than 1, \( V_j \) is moved by rotation about the centroid \( C_{j-1} \) including the problematic contraction term. This rotation
must also move $C_{j-2}$ in order to maintain the proper motion of $C_{j-1}$. In this manner both a spine of centroids and the spokes containing the vortices can be moved in an efficient way.

### 6.6 Results of Rollup Simulation

The simulation program works efficiently and the wake shape converges very well for varying numbers of vortices in terms of the position and strength of the wake sheet and concentrated vortex core. Elliptical loading corresponding to Fourier coefficients of $(1,0,0)$ were used to show convergence of the wake for a system of ten, twenty, and forty spanwise vortices after a certain time period had elapsed. As can be seen in Figure 6-3 the positions of the discrete vortices and the rolled-up vortex core overlap extremely well.

The program was tested with up to 500 vortices with no noticeable difference of the position of the vortex core for a given time elapsed. In less than two seconds with 60 vortices the program can accurately give the final spanwise position of the vortex core. This time increases with the number of vortices because of two effects. For each timestep the number of calculations depends on the square of the number of vortices used. The
timestep will also change with the number of vortices such that it takes more timesteps
for a wake with 500 vortices to evolve over the same amount of absolute time as a wake
with only 60 vortices. The rollup with 500 vortices is shown in Figure 6-4. It is interesting
to observe that numerical instabilities are propagated inboard such that groups of vortices
begin to cluster together in a stable manner before they are fed into the concentrated tip
vortex. The reason for this is that the program assumes that each vortex revolves around
its centroid. While this is true, the program does not try to make any assumptions about
vortices rotating about a common centroid which would stabilize the system.

A modified rollup program was run for a series of other coefficients. These are presented
in Appendix C to show the concept works for a variety of loadings.

6.7 Centroidal Theory in Three Dimensions

Once the concepts previously outlined are properly understood, the extension of the cen-
troidal theory to three dimensions should be straightforward. In the two dimensional case
the vortices rotate about a centroid point. When the transition to three dimensions is made,
the vortex filaments will now rotate about a centroidal line, rotating normal to that line
at a distance commensurate to the radius coupled with its rate of change. While this may
sound complicated, the helical scenario presents an easy case to examine.

Suppose that there are two concentrated tip vortices being shed off of a two bladed propeller in a helical wake. The two vortices will take on a helical shape and will rotate around each other. The centroidal line of these vortex lines will just be a line oriented downstream connecting to the center of the hub of the propeller, and the vortices will rotate about this line. This is fairly obvious using general intuition. However, since the goal is to obtain the configuration of two concentrated helical vortices from a helical vortex sheet this quickly becomes more complicated. The centroidal lines must be calculated at every timestep and might require some complicated geometry. Fortunately the calculations in three dimensions should only include terms proportional to the number of vortices squared and will not accumulate significantly longer times to run the rollup program. While this explanation may seem vague, no more should be said in this thesis due to the fact that for now these extensions are merely hypothetical.

An easier method for extending two dimensional centroidal theory to three dimensions is to use the Munk-Jones slender wing theory [11]. The three dimensional wake structure can be initially constructed by computing the perturbation velocities in a crossflow plane and moving the vortices in the crossflow plane. The plane can then be moved with the local unperturbed velocity. The first iteration would essentially create a three dimensional grid upstream of the crossflow plane with infinite line vortices downstream of the plane. Further adaptations of the rollup simulation compute the rollup until 99% of the vorticity is in the vortex core in under 80 timesteps, making it practical to include three dimensional effects in this manner.
Chapter 7

Conclusions

The method presented for quantifying RAM and reconstructing the Lagrangian velocity profiles of the moving vortex core appears to work well for reconstructing the vertical velocity profiles inside the core. A concession will first be made that outside of the core the assumed vertical velocity profile does not conform to the data. However, for predicting RAM and the maximal Langrangian velocity the model works very well. Unfortunately, the axial velocity profiles can deviate dramatically from the assumed profiles, especially at large angles of attack when a jetting behavior occurs, and the method breaks down. To circumvent the problem of complex velocity profiles it would be possible to "wander" the data using interpolation schemes to achieve similar results. Indeed if there is a future demand for reconstructed grid profiles this will have to be the approach taken since the grid profiles are too complex to model with idealized profiles. The method shows experimentally the growth and motion of the vortex core very well. This same growth and motion can be shown numerically. In Appendix C, the results from the simulation program for the Glauert coefficients of (1,0.1,0.03), which approximately correspond to the loading of the foil, are presented. The numerical and experimental cases overlap well, especially considering the Glauert loading is not the actual loading on the foil. This brings up the second and more important concept presented in this work, the centroidal theory.

The implications of the centroidal theory are very broad. While amalgamation algorithms and methods using finite cores yield the same answers as centroidal theory, even "better" answers in terms of obtaining the spiral shape, centroidal theory utilizes the physics of the problem to obtain answers orders of magnitudes faster than either. Centroidal theory
does not rely on arbitrary user inputs and will accurately predict to vortex core formation without user trial and error. In addition, it can be shown that centroidal theory will always remain stable for the basic reason that in the limit two vortices will merely rotate about each other when the distance between them goes to zero. Centroidal theory also removes the restriction that the timesteps must be very small in order to maintain stability and allows the wake to deform over relatively large timescales. Centroidal theory will make an elegant addition to two dimensional and three dimensional vortex lattice codes as it can predict longterm vortex behavior with minimum computations.
Appendix A

Data Sets at 5° Angle of Attack

A summary of the results presented in Chapter 5 for the 5° angle of attack data sets is shown to summarize the results of the RAM simulation. The experiment was conducted with the foil/halfbody configuration in the “side” orientation. The angle of attack was set by placing the foil at zero lift using the dynamometer to measure the lift. The results of the data overlap well with previous data sets, giving a certain measure of reliability to the accuracy of setting the angle of attack. The axial velocity plots from the simulation are not included since the axial velocity profiles deviate greatly from the assumed profiles in the 10° case. In addition, the standard deviation of the axial velocity is plotted against the vertical velocity to show consistency in the RAMplot format and to be consistent with the presentation of the 10° data sets.

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Figure A-1: Data Set 1: 7mm Downstream with RAM Simulation Results
Figure A-2: Data Set 2: 70mm Downstream with RAM Simulation Results
Figure A-3: Data Set 3: 163mm Downstream with RAM Simulation Results
Figure A-4: Data Set 4: 263mm Downstream with RAM Simulation Results
Figure A-5: Data Set 5: 343mm Downstream with RAM Simulation Results
Appendix B

Data Sets at $10^\circ$ Angle of Attack

There were several flaws in the $10^\circ$ data sets. The angle of attack may be off by a degree due to uncertainties in the dynamometer. The laser window was scratched and slightly warped causing, in addition to extremely low data rates, poor quality of the data obtained. These flaws can be seen easily in Data Sets 9 and 10. In both, there is a sharp decreases in the magnitude of vertical velocity standard deviation in the very center of the vortex. This, based on a great deal of similar data, is not indicitive of what is really happening inside the core. There should be high standard deviations due to isotropic turbulence and even more so due to the RAM effect.

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<th>$W$</th>
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Figure B-1: Data Set 6: 7mm Downstream with RAM Simulation Results
Figure B-2: Data Set 7: 70mm Downstream with RAM Simulation Results
Figure B-3: Data Set 8: 163mm Downstream with RAM Simulation Results
Figure B-4: Data Set 9: 263mm Downstream with RAM Simulation Results
Figure B-5: Data Set 10: 343mm Downstream with RAM Simulation Results
Appendix C

Vortex Rollup Simulation Results

The outputs from the simulation program presented in Figures C-1, C-2, C-3, and C-4 corresponding to the Glauert Coefficients (1,0,0), (1,0.2,0.08), (1,0.2,0.6), and (1,0.1,0.03) show the formation of the vortex core as it is convected downstream. The final spanwise locations of the vortex cores for three of the test cases agree well with the predicted locations given in Table 6.1. The simulation made the constraint on the timestep chosen that the outermost vortex not yet included in the core could not rotate more that the angle between the outermost vortex, its centroid, and the next outermost vortex.

In this version of the simulation program the rotation of the core was constructed by using the “centroidal spine” and attempted to model the rotation of the core as a solid-body rotation about the centroid. In future versions of the program this effect will be better modeled and even larger timesteps will be used. The running time for the simulation program was about 6 seconds.

While this program works well for the cases in which there is only one concentrated vortex being formed, it will have to be modified to account for multiple rollups due due flaps or other loading conditions. In addition, unstable cases such as a sheet of constant vorticity in which several cores will be formed must be taken into account as well. In essence, the program needs to be “smarter”.

As was best said by Churchill:

This is not the end, nor is it the beginning of the end. It is merely the end of the beginning.
WAKE SHAPE FOR ELLIPTICAL LOADING

100 SPANWISE VORTICES
35% VORTICITY FORMS "CORE"
80 TIME STEPS

97.93% OF VORTICITY IN CORE
TIP VORTEX POSITION 76.58
CENTER OF VORTICITY 77.45
THEORETICAL CENTROID 78.54

Figure C-1: Vortex Rollup with Glauert Coefficients (1,0,0)
Figure C-2: Vortex Rollup with Glauert Coefficients (1,-0.2,-0.08)
Figure C-3: Vortex Rollup with Glauert Coefficients (1,0.2,0.06)
WAKE SHAPE FOR (1.1,0.03) LOADING

100 SPANWISE VORICES
35% VORTICITY FORMS "CORE"
80 TIME STEPS

97.63% OF VORTICITY IN CORE
TIP VORTEX POSITION 82.46
CENTER OF VORTICITY 83.43
THEORETICAL CENTROID 84.45

Figure C-4: Vortex Rollup with Glauert Coefficients (1.0,1.0,0.03)
Bibliography


