The problem 1&2 are about normal modes; 3&4 are about PE; Only problem 1&2 are required; but by finishing 3&4, you can earn extra points.

Problem 1

Based on the finite difference representation of the modal eigenvalue problem derived in the previous homework, complete a normal mode modeling framework in Matlab, including the following components:

a. Use Sturm sequences to determine the number of propagating modes in your 3-layered waveguide.

b. Use bisection in combination with Sturm sequences to define wavenumber intervals containing only a single mode each.

c. Use Newton’s method to determine the modal wavenumbers for all propagating modes.

d. Inverse iteration for computing the modal eigenfunctions.

e. Perturbational treatment of seabed and subbottom attenuation.

f. Computation of transmission loss on a depth-range receiver grid.

Problem 2

As a test example add a 20 m thick sediment layer to the Pekeris waveguide in COA Fig. 2.25, maintaining a water depth of 100 m. Assume the sediment to have sound speed 1650 m/s, density 1.7 g/cm$^3$, and attenuation 0.5 dB/λ. Assume the subbottom to have attenuation 0.2 dB/λ.

a. Compute and plot the modal shapes for the propagating modes at 50 Hz.
b. Perform a convergence analysis for the modal wavenumbers vs the discretization mesh, and show its consistency with the expected convergence rate.

c. For source depth 35 m and receiver depth 45 m, and ranges out to 15 km, compare the modal result with the corresponding wavenumber integration result. Discuss the differences in terms of accuracy and numerical efficiency.

**(optional) Problem 3**

Complete your Matlab modeling toolbox for range-independent media by adding a Split-step Parabolic Equation solver following the recipe in Appendix A6 of Computational Ocean Acoustics, including the following:

a. At least one sediment layer over a fluid halfspace, with proper handling of density contrast at the interfaces.

b. Standard Gaussian starting fields.

**(optional) Problem 4**

As a test example for comparison of all your models, use the test case in Problem 2, i.e. add a 20 m thick sediment layer to the Pekeris waveguide in COA Fig. 2.25, maintaining a water depth of 100 m. Assume the sediment to have sound speed 1650 m/s, density 1.7 g/cm$^3$, and attenuation 0.5 dB/λ. Assume the subbottom to have attenuation 0.2 dB/λ.

a. Perform a convergence analysis vs. discretization for the PE calculation of transmission loss vs range and depth at 50 Hz for a source at 35 m depth, for ranges out to 15 km, using the standard Gaussian starter.

b. Compare the PE solution to the solutions obtained using wavenumber integration and normal modes, and discuss the differences in terms of accuracy and numerical efficiency.