Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 8 — Issued October 26, 2003
Due November 5, 2003

Problem 1: Variable Reluctance Motor

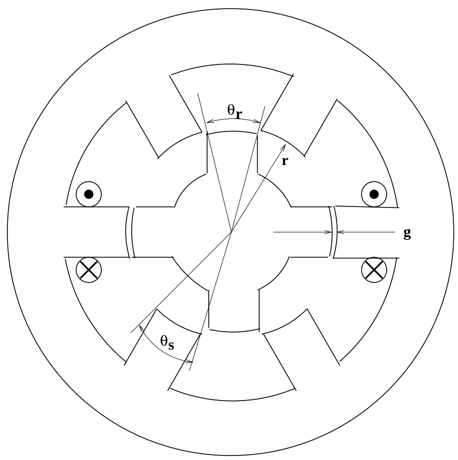

Figure 1: Crude Cartoon of a Variable Reluctance Machine

Figure 1 is a very crude picture of a "6-4" variable reluctance motor. The objective of this problem is to take a first-order look at operation of this machine. To start, assume that the flux vs. current pattern of the machine is as shown in Figure 2. For currents less than some "saturation" level, inductance of one winding of the motor is as shown in Figure 3. For currents greater than the "saturation" level, the incremental inductance is some lower level *Lsat* which is not a function of rotor position, so that flux is:

$$\lambda(\theta, I) = L(\theta)I_s + L_{\min}(I - I_s)$$

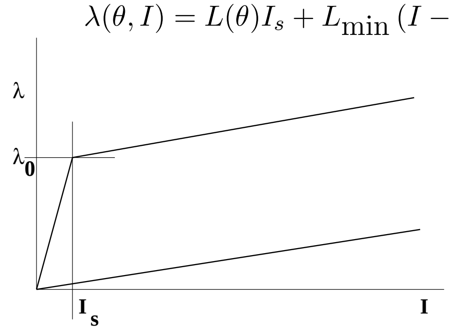

Figure 2: VRM Saturation Characteristic

Assume the following set of parameters:

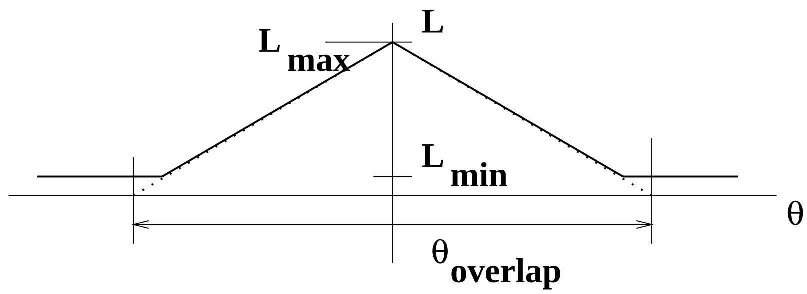

Figure 3: Inductance vs. Alignment Angle

Maximum Inductance	L_{max}	0.5 Hy
Minimum Inductance	L_{min}	0.05 Hy
Saturation Current	I_s	2 A
Overlap Angle	θ_{overlap}	$\pi/3$

The machine is to be operated as shown in Figure 4. For each phase, voltage is applied, driving flux up to the maximum at which point the switches are turned off. Current continues to flow through the diodes of the drive circuit (not shown here), putting negative voltage across the phase until current (and flux) goes to zero. Note that the angle at which the voltage pulse starts, θ_0, could vary over a wide range and is a control parameter. For the purposes of this problem assume that the width of the triangular pulse of flux is the same as the overlap angle ($\pi/3$).

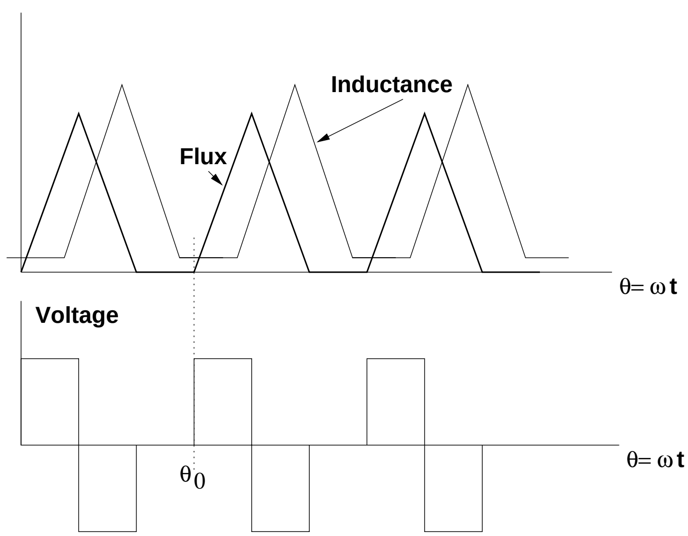

Figure 4: Operating Strategy

- Using Figures 2 and 3, calculate current in the one phase winding as a function of rotor angle and with flux as a parameter. Use values of flux of 0.2, 0.4, ... 2.0 Wb, and a range of angles from $-\pi/2$ to $\pi/2$.
- Using the principal of virtual work (Coenergy), calculate torque produced by that one phase as a function of flux and angle. Plot the results for the same range of angles and

flux values as you used for the first part.

- Now the machine is to be operated at a steady speed of 1000 RPM. Find and calculate the time average torque as a function of the starting angle θ_0. Assume the machine is operated as shown in Figure 4 with flux having a triangular pulse form and consequently voltage having positive and negative square pulses. Remember your voltage pulse may overlap more than one variation in inductance.
- Pick two starting angles, one near peak motoring torque and one near peak generating torque. Plot, vs. angle (corresponding to time), inductance, flux, voltage, current and instantaneous torque for each of these two cases.

Problem 2: Permanent Magnets

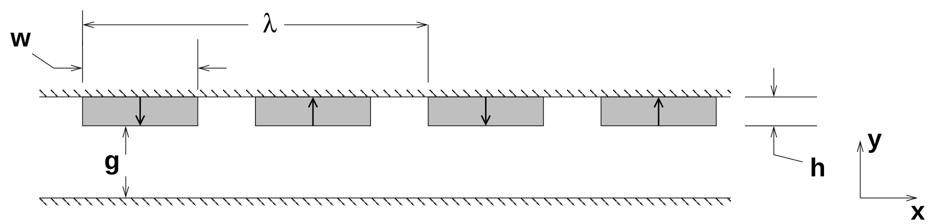

Figure 5: Surface Magnet Arrangement

Figure 5 shows an array of permanent magnets. Assume that this array is periodic (that is, there are enough magnets that you can treat the fields as if they were periodic) and that the depth of the problem in the z-direction is large enough that variations in that direction may be ignored. Dimensions are:

Magnet height	h	1	cm
Magnet width	w	4	cm
Wavelength	λ	12	cm

Assume the remanent flux density of the magnets is $B_r = 1.0$ T. This problem is meant to be worked using a Fourier Series in the x- direction. Be sure to use enough space harmonics to get a good representation of the actual fields.

1. To start, assume that the lower ferromagnetic boundary is far away. Calculate and plot, as a function of x, both x- and y- directed magnetic flux density for planes (that is positions in y) which are 1 wavelength, 1/4 wavelength and 1/10 wavelength below the magnets. Do not plot these on the same scale (if you do you will miss some of the interesting detail).
2. Next, assume that the lower ferromagnetic boundary is located 2 cm away from the magnets ($g = 2$ cm). Plot the y-directed flux density at that lower boundary, halfway between the magnets and the boundary, and at the magnet surface.
3. Repeat the previous part but this time with $g = 0.5$ cm. How does this compare with fields calculated using the 'narrow-gap' assumption?