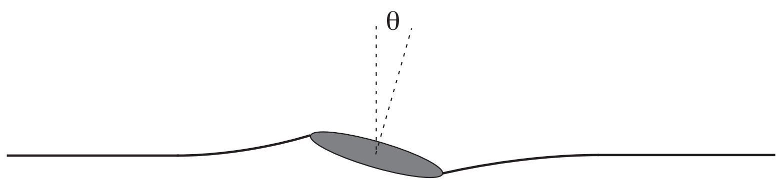
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2003

Practice Exam #3

Problem 1 (35 points) Impurities in a Film



Small self-organized structures of macromolecules, micelles, can exist as impurities in free-standing liquid crystal films. The disk shaped structures prefer to be resting in the plane of the film but can make small departures from this orientation as indicated in the figure. If the film defines the $z = 0$ plane, the Hamiltonian for a single micelle is given by

$$\mathcal{H} = \frac{L_x^2}{2I} + \frac{L_y^2}{2I} + \frac{1}{2}K\theta_x^2 + \frac{1}{2}K\theta_y^2$$

where θ_x (θ_y) indicates the angular displacement of the axis of the micelle away from the z axis in the $x\,(y)$ direction and L_x (L_y) is the canonically conjugate angular momentum. I is a moment of inertia and K is a the equivalent of a spring constant. The film and the micelles are in thermal equilibrium at a temperature T.

a) Find the joint probability density $p(\theta_x, \theta_y, L_x, L_y)$, including the correct normalization.

b) Find the partition function for a collection of N similar but distinguishable micelles.

c) If K depends on the area of the film as $K = K_0(A/A_0)^\gamma$, find the contribution of the micelles to the surface tension $\mathcal{S}$ of the film.

Problem 2 (35 points) Free Expansion of a Gas

A classical, monatomic, non-ideal gas has the equation of state

$$P(T,V) = \frac{NkT}{(V - bN)} - a\left(\frac{N}{V}\right)^2$$

where a and b are positive constants. The term $-bN$ adjusts the volume available to a given atom to compensate for the volume occupied by the other atoms. The term $-a\left(\frac{N}{V}\right)^2$ adjusts the pressure to take into account the long range (van der Waals) attractive interaction between the atoms. Neither of these corrections changes the constant volume heat capacity of the gas.

The gas is held in a container of negligible mass which is isolated from its surroundings. The gas is initially confined to 1/3 of the volume V_0 by a partition and is in thermal equilibrium at $T = T_i$. At $t = 0$ a hole is opened in the partition allowing the gas to expand irreversibly into the rest of the container, thus attaining a final volume V_0.

a) Which thermodynamic quantities are conserved in this process?

b) What is the new temperature of the gas when thermal equilibrium has finally been re-established ?

Problem 3 (30 points) Use of a Carnot Cycle

Two identical bodies with temperature independent heat capacities C_0 are initially at different temperatures T_H and T_C. A Carnot cycle is run between them (with infinitesimal steps) until they have been reduced to a common temperature T_F.

a) Find T_F in terms of T_H and T_C.

b) Find the total work done on the outside world in this process. Is it positive or negative?

PARTIAL DERIVATIVE RELATIONSHIPS

Let x, y, z be quantities satisfying a functional relation $f(x, y, z) = 0$. Let w be a function of any two of x, y, z. Then

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w = \left(\frac{\partial x}{\partial z}\right)_w$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

INTEGRALS

$$\int_0^\infty \frac{x^n}{a}\, e^{-x/a}\, dx = n!\, a^n$$

$$\int_{-\infty}^\infty \frac{x^{2n}}{\sqrt{2\pi\sigma^2}}\, e^{-x^2/2\sigma^2} = 1 \cdot 3 \cdot 5 \cdots (2n-1)\sigma^{2n}$$

$$\int e^{ax}\, dx = \frac{1}{a}\, e^{ax}$$

STERLING'S APPROXIMATION

$$\ln K! \approx K \ln K - K \quad \text{when} \quad K >> 1$$

SOLID ANGLE INCREMENT

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

WORK IN SIMPLE SYSTEMS

System	Intensive quantity	Extensive quantity	Work
Hydrostatic system	P	V	$-PdV$
Wire	$\mathcal{F}$	L	$\mathcal{F}dL$
Surface	$\mathcal{S}$	A	$\mathcal{S}dA$
Reversible cell	E	Z	EdZ
Dielectric material	$\mathcal{E}$	$\mathcal{P}$	$\mathcal{E}d\mathcal{P}$
Magnetic material	H	M	HdM

THERMODYNAMIC POTENTIALS

For a system in which the increment of work done on the system is dW = Xdx

Energy	E	dE=TdS+Xdx
Helmholtz free energy	F=E-TS	dF=-SdT+Xdx
Gibbs free energy	G=E-TS-Xx	dG = -SdT-xdX
Enthalpy	H=E-Xx	dH=TdS-xdX