1. $E(\vec{v}, \vec{r}) = \frac{1}{2} m \vec{v} \cdot \vec{v} + \frac{1}{2} \alpha x^2 + \frac{1}{2} \beta (y - y_0)^2$

$p(\vec{v}, \vec{r}) \propto e^{E(\vec{v}, r)/kT}$ FACTORS INTO A PRODUCT OF TERMS, EACH OF WHICH CAN BE NORMALIZED BY INSPECTION

$$p(\vec{v}, \vec{r}) = \left[\sqrt{\frac{1}{2\pi kT/m}} \, e^{-\frac{v_x^2}{2kT/m}} \right] \left[\sqrt{\frac{1}{2\pi kT/m}} \, e^{-\frac{v_y^2}{2kT/m}} \right] \left[\sqrt{\frac{1}{2\pi kT/m}} \, e^{-\frac{v_z^2}{2kT/m}} \right]$$

$$\times \left[\sqrt{\frac{1}{2\pi kT/\alpha}} \, e^{-\frac{x^2}{2kT/\alpha}} \right] \left[\sqrt{\frac{1}{2\pi kT/\beta}} \, e^{-\frac{(y-y_0)^2}{2kT/\beta}} \right] \left[\frac{1}{L} \right]$$

a) $p(v_y) = \sqrt{\frac{1}{2\pi kT/m}} \, e^{-\frac{v_y^2}{2 kT/m}}$

b) $p(y) = \sqrt{\frac{1}{2\pi kT/\beta}} \, e^{-\frac{(y-y_0)^2}{2kT/\beta}}$ $\begin{cases} \text{BECAUSE THE } z \text{ INTEGRAL} \\ \text{IS OVER } p \text{ NOT } v. \end{cases}$

c) $Z_1 = (2\pi kT)^{5/2} (\alpha \beta m^3)^{-1/2} L m^3 / h^3$, $Z(N, L, T) = Z_1^N / N!$

d) $F = -kT \ln Z$ $\mathcal{G} = \frac{\partial F}{\partial L}\Big|_{N,T} = -\frac{kT}{Z} N \frac{Z}{L} = -\frac{NkT}{L}$

$\mathcal{G}$ IS DEFINED AS A TENSION $\Rightarrow$ $f_z = -\mathcal{G} = \frac{NkT}{L}$

e) $S = -\frac{\partial F}{\partial T}\Big|_{N,L} = k \ln Z + kT \frac{1}{Z} \frac{5}{2} N \frac{Z}{T} = k \ln Z + \frac{5}{2} Nk$

THUS CONSTANT S (ADIABATIC) $\Rightarrow$ CONSTANT Z

$\Rightarrow T_i^{5/2} \beta_i^{-1/2} = T_f^{5/2} \beta_f^{-1/2}$

$T_f = T_i \left(\beta_f / \beta_i \right)^{1/5}$

2. $\quad V = V_1 \, e^{(T/T_1 - P/P_1)} \quad \rightarrow \quad \ln V/V_1 = T/T_1 - P/P_1$

$\quad P(T, V) = P_1 \left(T/T_1 - \ln(V/V_1) \right)$

a) $\quad dS = \frac{\partial S}{\partial T}\Big|_V dT + \frac{\partial S}{\partial V}\Big|_T dV$

$dQ = T dS \Rightarrow c_V = T \frac{\partial S}{\partial T}\Big|_V \rightarrow \frac{\partial S}{\partial T}\Big|_V = c_V/T = DT^2$

$\frac{\partial S}{\partial V}\Big|_T = \frac{\partial P}{\partial T}\Big|_V \left(\begin{array}{c} \text{USING A} \\ \text{MAXWELL RELATION} \end{array} \right) = P_1/T_1$

$dS = DT^2 dT + \left(P_1/T_1 \right) dV$

$S = \frac{1}{3} DT^3 + f(V), \quad f'(V) = \frac{P_1}{T_1} \Rightarrow f(V) = \frac{P_1 V}{T_1} + \text{CONSTANT}$

$\underline{S = \frac{1}{3} DT^3 + P_1 V / T_1 + S_0}$

b) $\quad c_P = \frac{dQ}{dT}\Big|_P = T \frac{\partial S}{\partial T}\Big|_P \qquad \text{BUT WE HAVE } dS \text{ ABOVE}$

$\frac{\partial S}{\partial T}\Big|_P = \underbrace{\frac{\partial S}{\partial T}\Big|_V}_{DT^2} + \underbrace{\frac{\partial S}{\partial V}\Big|_T}_{P_1/,} \underbrace{\frac{\partial V}{\partial T}\Big|_P}_{V/T_1}$

$\underline{c_P = \underbrace{DT^3}_{c_V} + \frac{P_1 V T}{T_1^2}}$

c) $\quad dF = -S dT - P dV$

$\underline{dF = \left(-\frac{1}{3} DT^3 - \frac{P_1 V}{T_1} + S_0 \right) dT + P_1 \left(\ln V/V_1 - \frac{T}{T_1} \right) dV}$

3. a) $\Delta Q_S = -\Delta Q_L$

$$\int_{T_0}^{1/2} bT^{-2}\,dT = -\int_{1}^{1/2} aT^3\,dT$$

$$-b\left[T^{-1}\right]_{T_0}^{1/2} = -\frac{a}{4}\left[T^4\right]_{1}^{1/2}$$

$$\left(2 - 1/T_0\right) = \frac{1}{4}\frac{a}{b}\left(\frac{1}{16} - 1\right) = -\frac{1}{4}\frac{a}{b}\frac{15}{16} = -\frac{1}{4}\frac{128}{15}\frac{15}{16} = -2$$

$$1/T_0 = 4, \quad \underline{T_0 = 1/4}$$

b) $\Delta S = \Delta S_S + \Delta S_L$ $\qquad\qquad dS = \frac{dQ}{T} = \frac{c\,dT}{T}$

$$= \int_{T_0}^{1/2} bT^{-3}\,dT + \int_{1}^{1/2} aT^2\,dT$$

$$= -\frac{b}{2}\left[T^{-2}\right]_{1/4}^{1/2} + \frac{a}{3}\left[T^3\right]_{1}^{1/2}$$

$$= -\frac{b}{2}\underbrace{\left(4 - 16\right)}_{-12} + \frac{a}{3}\underbrace{\left(\frac{1}{8} - 1\right)}_{-7/8} = \underline{6b - \frac{7}{24}a}$$