

8.044 SOLUTIONS EXAM #1

1. a)



$$b) \langle f \rangle = 2 f_0(\frac{1}{2}) + f_0(\frac{1}{3}) + 4 f_0(\frac{1}{6}) = f_0 \left(\frac{3}{3} + \frac{1}{3} + \frac{2}{3} \right) = \underline{\underline{2 f_0}}$$

$$c) \langle f^2 \rangle = 4 f_0^2(\frac{1}{2}) + f_0^2(\frac{1}{3}) + 16 f_0^2(\frac{1}{6}) = f_0^2 \left(\frac{6}{3} + \frac{1}{3} + \frac{8}{3} \right) = 5 f_0^2$$

$$\text{Var}(f) = \langle f^2 \rangle - \langle f \rangle^2 = (5 - 4) f_0^2 = \underline{\underline{f_0^2}}$$

d) ALL BUT ONE ISOTOPE MUST HAVE $A = n$.
 THE OTHER (64 CHOICES) MUST HAVE $A = n + 1$

$$\Rightarrow p(M = (64n + 1)m_0) = \underline{\underline{64 \cdot \frac{1}{3} \cdot \left(\frac{1}{2}\right)^{63}}}$$

e) USE THE CENTRAL LIMIT THEOREM

$$p(M) \approx \left(\frac{m_0}{\sqrt{2\pi \text{Var}(M)}} e^{-\frac{(M - \langle M \rangle)^2}{2 \text{Var}(M)}} \right) \cdot \sum_{i=1}^{\infty} \delta(M - i m_0)$$

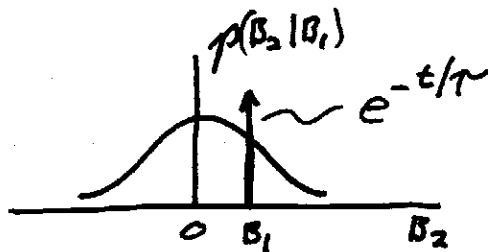
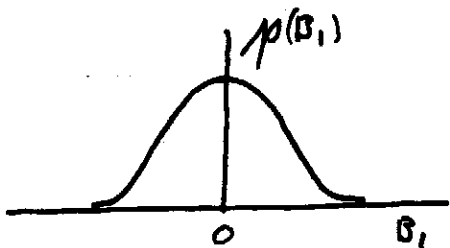
WHERE $\langle M \rangle = 64 m_0 \langle A \rangle$

$$\text{Var}(M) = 64 m_0^2 \text{Var}(A)$$

$$\begin{aligned}
 2. \quad a) \quad p(B_1) &= \int_{-\infty}^{\infty} p(B_1, B_2) dB_2 = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-t/\tau} e^{-B_1^2/2\sigma^2} \\
 &\quad + \frac{1}{\sqrt{2\pi\sigma^2}} (1 - e^{-t/\tau}) e^{-B_1^2/2\sigma^2} \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-B_2^2/2\sigma^2} dB_2}_1 \\
 &= \underline{\underline{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-B_1^2/2\sigma^2}}}
 \end{aligned}$$

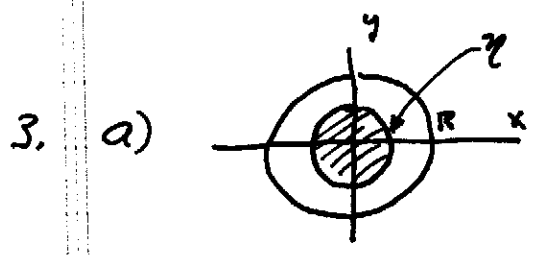
NOTE THAT THIS DOES NOT DEPEND ON τ

$$b) \quad p(B_2 | B_1) = \frac{p(B_1, B_2)}{p(B_1)} = \underline{\underline{e^{-t/\tau} \delta(B_2 - B_1) + (1 - e^{-t/\tau}) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-B_2^2/2\sigma^2}}}$$



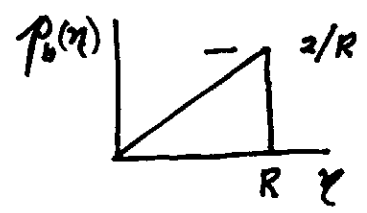
c) B_1 & B_2 ARE NOT S.I. BECAUSE $p(B_1, B_2) \neq p(B_1)p(B_2)$

NOTE THAT $p(B_1, B_2)$ DEPENDS ON τ BUT $p(B_1)$ AND $p(B_2)$ DO NOT.

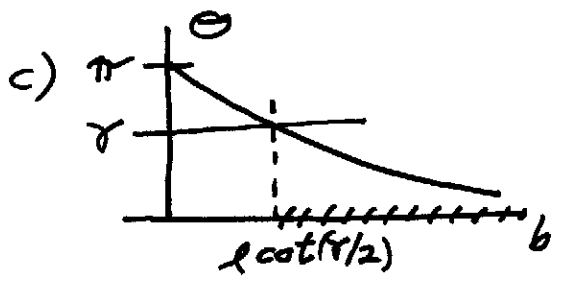


$b < r$ IN SHADED REGION
 $P_b(r) = (\pi r^2) \left(\frac{1}{\pi R^2} \right) = r^2/R^2$

$p_b(r) = \frac{dP_b(r)}{dr} = 2r/R^2 \quad 0 < r < R$
 $= 0 \quad \text{ELSEWHERE}$



b) LARGE $b \rightarrow$ SMALL θ
 LARGEST $b = R \Rightarrow \theta_{MIN} = 2 \text{ ARCCOT}(R/l)$



$P_\theta(r) = \int_{l \cot(r/2)}^{\infty} p(b) db$

$p_\theta(r) = \frac{d}{dr} P_\theta(r) = (-1) \left(-\frac{l}{2} \csc^2(r/2) \right) \left(2 l \cot(r/2) / R^2 \right)$
 $= \frac{l^2}{R^2} \frac{1}{\sin^2(r/2)} \frac{\cos(r/2)}{\sin(r/2)}$
 $= \frac{l^2}{R^2} \frac{\cos(r/2)}{\sin^3(r/2)} \quad \theta_{MIN} \leq r \leq \pi$
 $= 0 \quad \text{ELSEWHERE}$

