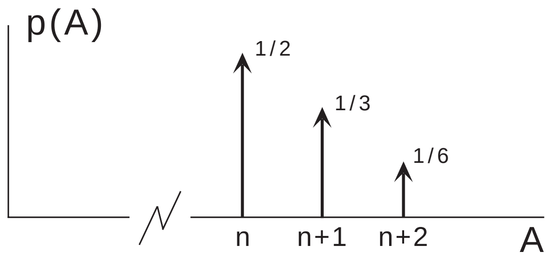
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2003

Exam #1

Problem 1 (35 points) Isotopic Abundance



A certain element has three stable isotopes with atomic weights $A = n, n+1$, and $n+2$. n is a known integer. The probability of occurrence of each, p(A), is shown in the figure. The scattering of neutrons from the isotopes is governed by an atomic-weight-dependent scattering amplitude $f(A)$. It is known that

$$\begin{aligned} f(n) &= 2f_0 \\ f(n+1) &= f_0 \\ f(n+2) &= 4f_0 \end{aligned}$$

where f_0 is a constant.

a) Make a carefully labeled sketch of the cumulative function $P(A)$ which displays all of its important features.

b) Find $< f >$. Coherent neutron scattering from a crystal is proportional to $< f >^2$.

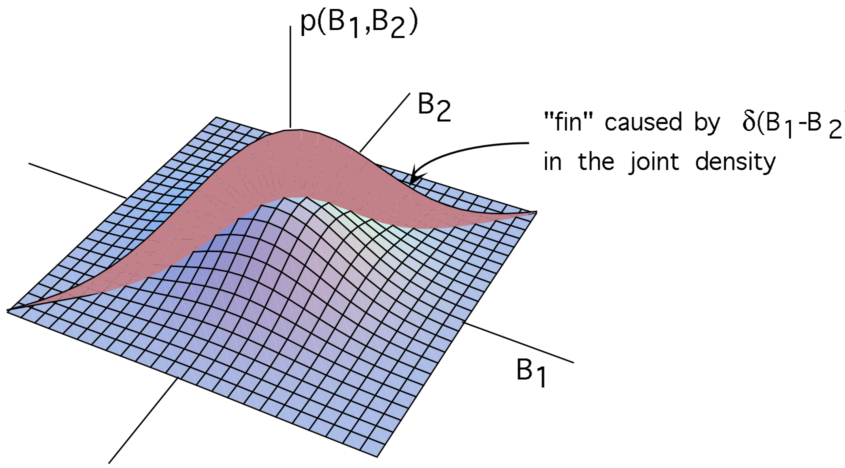
c) Find the variance of f, $\mathrm{Var}(f) \equiv < (f - < f >)^2 >$. Incoherent neutron scattering from a crystal is proportional to $\mathrm{Var}(f)$.

Chemists are able to grow nanocrystals of this element, each containing exactly 64 atoms. Let M be the total mass of a nanocrystal.

d) The minimum possible value of M under these circumstances is $64n\, m_0$ where m_0 is the proton mass. What is the exact probability that a nanocrystal will have a mass equal to $(64n+1)m_0$?

e) What is the approximate probability density $p(M)$ for the mass M of a nanocrystal in terms of $< A >$, $\mathrm{Var}(A)$ [do <u>not</u> calculate either of these], and m_0?

Problem 2 (30 points) Field Reversals



The earth's magnetic field changes suddenly at random times as the earth evolves. A possible model for this behavior gives the following joint probability density for the magnetic fields B_1 and B_2 measured at two different times separated by t years.

$$\begin{aligned} p(B_1, B_2) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-t/\tau]\, \delta(B_1 - B_2) \exp[-B_1^2/2\sigma^2] \\ &+ \frac{1}{2\pi\sigma^2} (1 - \exp[-t/\tau]) \exp[-(B_1^2 + B_2^2)/2\sigma^2] \end{aligned}$$

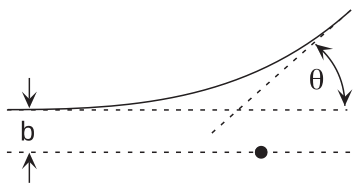
τ is a parameter of the order of 5×10^5 years and σ is a parameter of the order of 1/2 gauss.

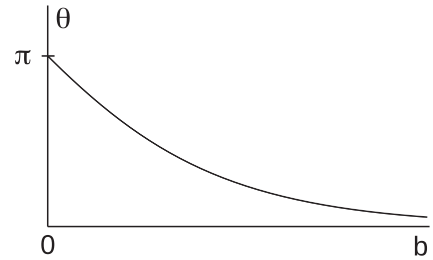
a) Find $p(B_1)$. Sketch the result.

b) Find the conditional probability density $p(B_2\,|B_1)$. Sketch the result.

c) Are B_1 and B_2 statistically independent? Explain your reasoning.

Problem 3 (35 points) Rutherford Scattering



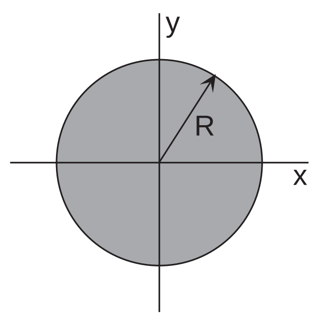


In Rutherford scattering of mono-energetic α particles from nuclei, the dependence of the scattering angle θ on the impact parameter b is given by

$$\theta = 2\ \mathrm{arccot}(b/l)$$

(as shown in the figure above) where l is a characteristic length. The impact parameter b is the closest distance the α particle would come to the nucleus if there were no Coulomb interaction.

In the following assume that the α particle flux is uniform over a disk of radius R centered on the nucleus. Thus b is in the range from 0 to R.



a) Find $p(b)$ and sketch the result.

b) What is the smallest possible scattering angle?

c) Find $p(\theta)$ and sketch the result.

Derivatives of Trigonometric Functions

$$\frac{d\sin x}{dx} = \cos x.$$

$$\frac{d\cos x}{dx} = -\sin x.$$

$$\frac{d\tan x}{dx} = \sec^2 x.$$

$$\frac{d\cot x}{dx} = -\csc^2 x.$$

$$\frac{d\sec x}{dx} = \sec x \tan x.$$

$$\frac{d\csc x}{dx} = -\csc x \cot x.$$

Definite Integrals

For integer n and m

$$\int_0^\infty x^n e^{-x}\, dx = n!$$

$$\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\, dx = \sqrt{\pi}$$

$$(2\pi\sigma^2)^{-1/2} \int_{-\infty}^\infty x^{2n} e^{-x^2/2\sigma^2}\, dx = 1 \cdot 3 \cdot 5 \cdots (2n-1)\, \sigma^n$$

$$\int_0^\infty x\, e^{-x^2}\, dx = \frac{1}{2}$$

$$\int_0^1 x^m (1-x)^n\, dx = \frac{n!m!}{(m+n+1)!}$$