MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2003

Exam #2

Problem 1 (25 points) Bose Gas

In a weakly interacting gas of Bose particles at low temperature the expansion coefficient α and the isothermal compressibility $\mathcal{K}_T$ are given by

$$\alpha \equiv \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P = \frac{5}{4}\frac{a}{c}T^{3/2}V^2 + \frac{3}{2}\frac{b}{c}T^2V^2$$

$$\mathcal{K}_T \equiv -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T = \frac{1}{2c}V^2$$

where a, b and c are constants. It is known that the pressure goes to zero in the limit of large volume and low temperature. Find the equation of state $P(T, V)$.

Problem 2 (35 points) Hydrostatic System

The internal energy U of a certain hydrostatic system is given by

$$U = AP^2V$$

where the constant A has the units of (pressure)$^{-1}$.

a) Find the slope, dP/dV, of an adiabatic path ($đQ = 0$) in the P-V plane in terms of A, P and V.

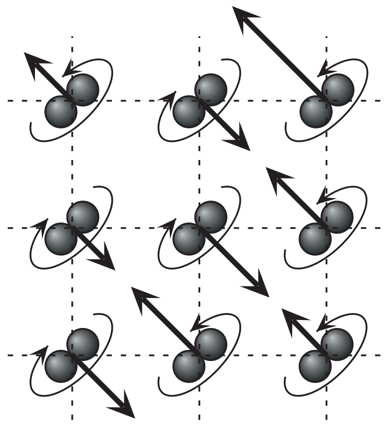
Assume that one also knows the thermal expansion coefficient α and the isothermal compressibility $\mathcal{K}_T$.

$$\alpha \equiv \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P \qquad \text{and} \qquad \mathcal{K}_T \equiv -\frac{1}{V}\left.\frac{\partial V}{\partial P}\right|_T$$

b) Find the slope, dP/dV, of an isothermal path in the P-V plane.

c) Find the constant volume heat capacity, C_V, in terms of the known quantities.

Problem 3 (40 points) Molecular Solid



In a particular molecular solid the individual molecules are localized at specific lattice sites and possess no center of mass motion. However, each of the N molecules is free to rotate about a fixed direction in space which we will designate as the z direction. As far as the rotational motion is concerned the molecules can be considered to be non-interacting. The classical microscopic state of each molecule is specified by a rotation angle $0 \leq \theta < 2\pi$ and a canonically conjugate angular momentum $-\infty < l < \infty$ about the z axis. The energy of a single molecule is independent of θ and depends quadratically on l. Thus the Hamiltonian for the system is given by

$$\mathcal{H} = \sum_{i=1}^{N} \frac{l_i^2}{2I}$$

where I is the moment of inertia of a molecule about the z axis.

a) Represent the system by a microcanonical ensemble where the energy lies between E and $E + \Delta$. Find an expression for the phase space volume Ω. Use Sterling's approximation to simplify your result. [It may be helpful to consult the attached information sheet.]

b) Based on your calculations in a) find the probability density $p(\theta)$ for the orientation angle of a single molecule and explain your method.

c) The probability density $p(l)$ for the angular momentum of a single molecule can be written in the form $p(l) = \Omega'/\Omega$ where $\Omega = \Omega(E, N)$ is the quantity you found in a). Find Ω'. Do not try to simplify your answer. Do explain how to eliminate E from your expression for $p(l)$.

d) Find the energy of the system as a function of temperature, $E(T, N)$.

PARTIAL DERIVATIVE RELATIONSHIPS

Let x, y, z be quantities satisfying a functional relation $f(x, y, z) = 0$. Let w be a function of any two of x, y, z. Then

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w = \left(\frac{\partial x}{\partial z}\right)_w$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

COMBINATORIAL FACTS

There are $K!$ different orderings of K objects. The number of ways of choosing L objects from a set of K objects is

$$\frac{K!}{(K-L)!}$$

if the order in which they are chosen matters, and

$$\frac{K!}{L!(K-L)!}$$

if order does not matter.

STERLING'S APPROXIMATION

When $K \gg 1$

$$\ln K! \approx K \ln K - K \quad \text{or} \quad K! \approx (K/e)^K$$

DERIVATIVE OF A LOG

$$\frac{d}{dx} \ln u(x) = \frac{1}{u(x)} \frac{du(x)}{dx}$$

VOLUME OF AN α DIMENSIONAL SPHERE OF RADIUS R

$$\frac{\pi^{\alpha/2}}{(\alpha/2)!} R^\alpha$$

LIMITS

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0$$

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$$

$$\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad (\text{any } x)$$

$$\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad (\text{any } x)$$

WORK IN SIMPLE SYSTEMS

System	Intensive quantity	Extensive quantity	Work
Hydrostatic system	P	V	$-PdV$
Wire	$\mathcal{F}$	L	$\mathcal{F}dL$
Surface	$\mathcal{S}$	A	$\mathcal{S}dA$
Reversible cell	E	Z	EdZ
Dielectric material	$\mathcal{E}$	$\mathcal{P}$	$\mathcal{E}d\mathcal{P}$
Magnetic material	H	M	HdM