8.044 SOLUTIONS EXAM #2

1. $dP = \frac{\partial P}{\partial T}\Big|_V dT + \frac{\partial P}{\partial V}\Big|_T dV$

$$\frac{\partial P}{\partial V}\Big|_T = \frac{1}{\frac{\partial V}{\partial P}\Big|_T} = \frac{1}{-V K_T} = -\frac{2c}{V^3}$$

$$\frac{\partial P}{\partial T}\Big|_V = \frac{-1}{\frac{\partial T}{\partial V}\Big|_P \frac{\partial V}{\partial P}\Big|_T} = \frac{\frac{1}{V}\frac{\partial V}{\partial T}\Big|_P}{-\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T} = \frac{\alpha}{K_T} = \frac{5}{2} a T^{3/2} + 3bT^2$$

$$P(T,V) = \int \frac{\partial P}{\partial T}\Big|_V dT + f(V) = aT^{5/2} + bT^3 + f(V)$$

$$\frac{\partial P}{\partial V}\Big|_T = f'(V) = -\frac{2c}{V^3} \Rightarrow f(V) = +\frac{c}{V^2} + \cancel{\text{CONSTANT}} \quad 0 \text{ SINCE } P \to 0 \text{ AS } V \to \infty$$

$$\underline{\underline{P(T,V) = aT^{5/2} + bT^3 + \frac{c}{V^2}}}$$

2. USE P+V AS INDEPENDENT VARIABLES

a/ $dU = AP^2 dV + 2APV dP \qquad đW = -P dV$

$$đQ = dU - đW = 2APV dP + P(1+AP) dV$$
$$= 0 \text{ FOR ADIABATIC CHANGES}$$

$$\Rightarrow \underline{\underline{\frac{dP}{dV}\Big|_{đQ=0} = -\frac{(1+AP)}{2AV}}}$$

b/ $\frac{dP}{dV}\Big|_{\text{ISOTHERMAL}} = \frac{\partial P}{\partial V}\Big|_T = \frac{1}{\frac{\partial V}{\partial P}\Big|_T} = \frac{-1/V}{-\frac{1}{V}\frac{\partial V}{\partial P}\Big|_T} = \underline{\underline{-\frac{1}{K_T V}}}$

c/ $\frac{đQ}{dT}\Big|_V = 2APV \frac{\partial P}{\partial T}\Big|_V = 2APV \left(\frac{-1}{\frac{\partial T}{\partial V}\Big|_P \frac{\partial V}{\partial P}\Big|_T} \right)$

$$= 2APV \frac{\frac{\partial V}{\partial T}\Big|_P}{\left(-\frac{\partial V}{\partial P}\Big|_T\right)} = \underline{\underline{\frac{2A\alpha PV}{K_T}}}$$

3 a/ COMPUTE THE CUMULATIVE VOLUME IN PHASE SPACE FIRST.

$$\phi = \left[\int_0^{2\pi} d\theta\right]^N \int_{\sum_{i=1}^N \frac{l_i^2}{2I} < E} \{dl_i\} = (2\pi)^N \times \text{VOLUME OF N DIMENSIONAL SPHERE OF RADIUS } \sqrt{2IE}$$

$$= (2\pi)^N \pi^{N/2} (2IE)^{N/2} \frac{1}{(N/2)!}$$

$$\approx (2\pi)^N (2\pi IE)^{N/2} \frac{1}{\left(\frac{N}{2e}\right)^{N/2}} = (2\pi)^N \left(\frac{4\pi e I E}{N}\right)^{N/2}$$

$$\Omega = \Delta \frac{d\phi}{dE} = \underline{\underline{\left(\frac{N\Delta}{2E}\right)(2\pi)^N \left(\frac{4\pi e I E}{N}\right)^{N/2}}}$$

b/ $p(\theta) = \Omega'(\text{ONE ANGLE FIXED})/\Omega = \frac{(2\pi)^{N-1}}{(2\pi)^N}$

$$= \underline{\underline{\frac{1}{2\pi} \quad 0 \le \theta < 2\pi, \quad = 0 \text{ ELSEW.}}}$$

c/ Ω': SET $N \to N-1$, $E \to E - \frac{l^2}{2I}$ IN ANGULAR MOMENTUM CONTRIBUTION

$$\Omega' = \underline{\underline{\left(\frac{(N-1)\Delta}{2(E - l^2/2I)}\right)(2\pi)^N \left(\frac{4\pi e I (E - l^2/2I)}{N-1}\right)^{(N-1)/2}}}$$

E WILL BE REPLACED BY $\underline{\underline{N\langle l^2\rangle/2I}}$

d/ $S = k \ln \phi$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial E}\right|_N = k \frac{1}{\phi} \left.\frac{\partial \phi}{\partial E}\right|_N = k \frac{1}{\phi}\left(\frac{N}{2} \frac{1}{E} \phi\right) = \frac{Nk}{2E}$$

$$\underline{\underline{E = \tfrac{1}{2} NkT}}$$