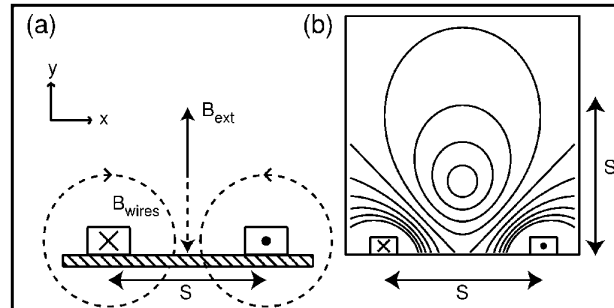


Exam #3

Problem 1 (45 points) Guiding Neutral Atoms on a Chip



“We demonstrate the guiding of neutral atoms by the magnetic fields due to micro-fabricated current-carrying wires on a chip. Atoms are guided along a magnetic field minimum parallel to and above the current carrying wires.” Thus begins the abstract of a Physical Review Letter in 2000 by Mara Prentiss and her collaborators at Harvard. The figure, adapted from the Letter, shows lines of constant potential energy in the $x - y$ plane. $x = 0$ half way between the two wires, $y = 0$ on the substrate, and z is the guiding direction parallel to the wires.

Model the system by a gas of N classical, non-interacting, indistinguishable atoms. We will approximate the confining potential as $V(x, y, z) = (\alpha/2)x^2 + (\beta/2)(y - y_0)^2$ where α and β are constants. In this model the allowed space for the gas extends to infinity in the x and y directions but is confined in the z direction by rigid walls at $z = \pm L/2$. The gas is in thermal equilibrium at a temperature T .

- a) Find the probability density for the velocity of a single atom in the y direction, $p(v_y)$.
- b) Find the probability density for the y position of a single atom, $p(y)$.
- c) Find the partition function for the gas, $Z(N, L, T)$.
- d) Find force exerted by the gas on the wall, f_z , as a function of N , L , and T .
- e) The gas is cooled adiabatically by decreasing the value of β . Find the final temperature in terms of the initial temperature and the initial and final values of β .

Problem 2 (30 points) Modeling the Behavior of a Fluid

Near room temperature the equation of state of a complex fluid can be approximated by the expression

$$V(T, P) = V_1 \exp(T/T_1 - P/P_1)$$

where P_1 , V_1 and T_1 are simply parameters and may not be close to the room temperature values of the pressure, volume and temperature. The heat capacity at constant volume can be approximated by $C_V = DT^3$.

- a) Within this approximation find an expression for the entropy of the fluid as a function of temperature and volume, $S(T, V)$.
- b) Find an expression for the heat capacity at constant pressure as a function of temperature and volume, $C_P(T, V)$.
- c) Find the differential of the Helmholtz free energy, $dF(T, V)$, in terms of its natural variables T and V . You do not need carry through to find $F(T, V)$ itself.

Problem 3 (25 points) Cooling Liquid Helium

In a certain experiment liquid helium at an initial temperature of 1.0 K is to be cooled further by bringing it into contact with a paramagnetic salt which is initially at a lower temperature T_0 . The heat capacity of the liquid may be expressed as $C_L = aT^3$ where $a = 128 \times 10^4$ ergs K^{-4} , and that of the salt may be expressed as $C_S = bT^{-2}$ where $b = 15 \times 10^4$ ergs K . Assume the mixture to be mechanically and thermally isolated from the rest of the universe and no work is done in the process.

- a) What must be the initial temperature of the salt, T_0 , if the final temperature of the system, helium plus salt, is to be 0.5 K?
- b) Find the net change in the entropy of the universe during this process. To facilitate grading, and to minimize the chance of math errors, leave this answer in terms of the parameters a and b .

PARTIAL DERIVATIVE RELATIONSHIPS

Let x, y, z be quantities satisfying a functional relation $f(x, y, z) = 0$. Let w be a function of any two of x, y, z . Then

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w = \left(\frac{\partial x}{\partial z}\right)_w$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

INTEGRALS

$$\int_0^\infty \frac{x^n}{a} e^{-x/a} dx = n! a^n$$

$$\int_{-\infty}^\infty \frac{x^{2n}}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2} dx = 1 \cdot 3 \cdot 5 \cdots (2n-1)\sigma^{2n}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

STERLING'S APPROXIMATION

$$\ln K! \approx K \ln K - K \quad \text{when } K \gg 1$$

SOLID ANGLE INCREMENT

$$d\Omega = \sin \theta d\theta d\phi$$

WORK IN SIMPLE SYSTEMS

System	Intensive quantity	Extensive quantity	Work
Hydrostatic system	P	V	$-PdV$
Wire	\mathcal{F}	L	$\mathcal{F}dL$
Surface	\mathcal{S}	A	$\mathcal{S}dA$
Reversible cell	E	Z	$E dZ$
Dielectric material	\mathcal{E}	\mathcal{P}	$\mathcal{E}d\mathcal{P}$
Magnetic material	H	M	HdM

THERMODYNAMIC POTENTIALS

For a system in which the increment of work done on the system is $dW = Xdx$

Energy	E	$dE = TdS + Xdx$
Helmholtz free energy	$F = E - TS$	$dF = -SdT + Xdx$
Gibbs free energy	$G = E - TS - Xx$	$dG = -SdT - xdx$
Enthalpy	$H = E - Xx$	$dH = TdS - xdx$