8.044 SOLUTIONS EXAM #4

1. a) $C_V \sim (\text{CLASSICAL RESULT}) \times \left(\begin{matrix}\text{FRACTION OF}\\ \text{ELECTRONS INFLUENCED}\end{matrix}\right) = \left(\frac{3}{2}Nk\right)\left(\frac{kT}{\epsilon_F}\right)$

$\propto T \Rightarrow \underline{\underline{n=1}}$ IN $C_V = \gamma V T^n$

b) $$\underline{\underline{\frac{dE}{dT} = C_V(T) = \gamma V T}}$$

c) $$\underline{\underline{\frac{dE}{dt} = -4\pi R^2 \sigma T^4}}$$ FROM STEFAN-BOLTZMANN LAW

d) $$\frac{dT}{dt} = \frac{dT}{dE}\frac{dE}{dt} = -4\pi R^2 \sigma T^4 / \gamma \left(\tfrac{4}{3}\pi R^3\right) T$$

$$\underline{\underline{\frac{dT}{dt} = -\frac{3\sigma}{\gamma R} T^3}}$$

SOLUTION (NOT REQUIRED)

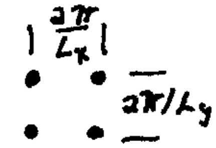
$$-\frac{dT}{T^3} = \frac{3\sigma}{\gamma R}\, dt$$

$$-\int_{T_0}^{T(t)} \frac{dT}{T^3} = \frac{3\sigma}{\gamma R} t = \frac{1}{2}\left(T(t)^{-2} - T_0^{-2}\right)$$

$$T^{-2}(t) = T_0^{-2} + \frac{6\sigma}{\gamma R} t = \frac{1}{T_0^2}\Big(1 + \underbrace{\frac{6\sigma T_0^2}{\gamma R}}_{\equiv \tau^{-1}} t\Big)$$

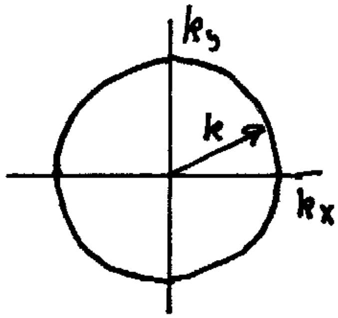
$$\underline{\underline{T(t)/T_0 = (1 + t/\tau)^{-1/2}}}$$

2\. a)



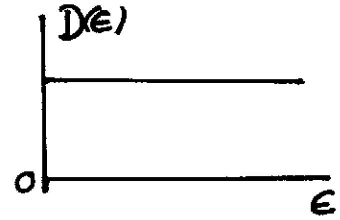
$$D(\vec{k}) = \frac{1}{\frac{2\pi}{L_x}\frac{2\pi}{L_y}} = \underline{\underline{\frac{A}{(2\pi)^2}}} \quad \forall\, \vec{k}$$

b)



$$\# = \overset{\text{SPIN}}{2}\, D(\vec{k})\, \pi k^2 = \frac{2A\pi}{(2\pi)^2}\,\frac{2m\epsilon}{\hbar^2}$$

$$D(\epsilon) = \frac{d\#}{d\epsilon} = \underline{\underline{\frac{Am}{\pi\hbar^2}}}$$

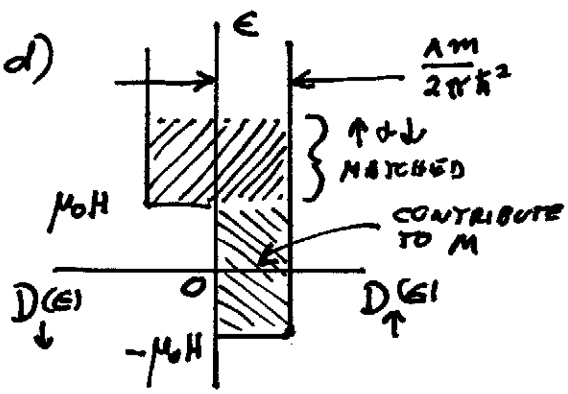


c) $$N = \int_0^{\epsilon_F} D(\epsilon)\,d\epsilon = \left(\frac{Am}{\pi\hbar^2}\right)\epsilon_F \quad \Rightarrow \epsilon_F = \frac{\pi\hbar^2}{m}\,\frac{N}{A}$$

$$E = \int_0^{\epsilon_F} \epsilon\, D(\epsilon)\,d\epsilon = (\ \ )\,\frac{1}{2}\epsilon_F^2 = \frac{1}{2}N\epsilon_F$$

$$E_{\text{Electron Gas}} = \frac{\pi\hbar^2}{2m}\,\frac{N^2}{4\pi R^2} = \underline{\underline{\frac{\hbar^2}{8m}\,\frac{N^2}{R^2}}}$$

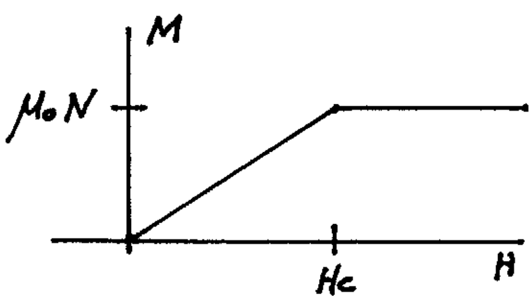
d)



$$M = \mu_0\left(2\mu_0 H\right)\frac{Am}{2\pi\hbar^2}$$

$$= \frac{\mu_0^2 H m}{\pi\hbar^2}\,4\pi R^2$$

$$\underline{\underline{M = \frac{4m\mu_0^2R^2}{\hbar^2}\,H \qquad M < \mu_0 N}}$$



$$\frac{4m\mu_0^2R^2H_c}{\hbar^2} = \mu_0 N$$

$$H_c = \frac{N\hbar^2}{4mR^2\mu_0}$$

3. a) $\langle \mu \rangle = \iint p(\theta,\varphi)\left[\mu_0 \cos\theta\right] d\Omega$

BUT $\dfrac{dZ_1}{d\eta} = \iint \cos\theta\, e^{\eta\cos\theta}\, d\Omega$

$$= \frac{Z_1}{\mu_0}\iint\left[\mu_0\cos\theta\right] p(\theta,\varphi)\, d\Omega = \frac{Z_1}{\mu_0}\langle\mu\rangle$$

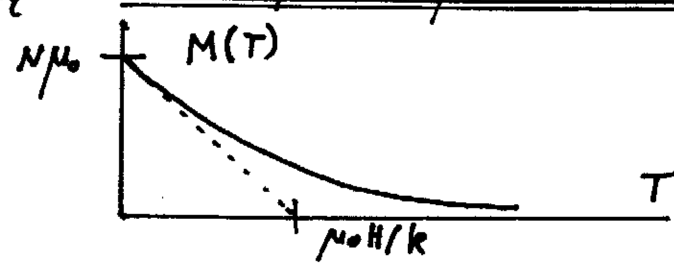
THUS $M = N\langle\mu\rangle = \underline{\underline{N\mu_0 \dfrac{1}{Z_1}\dfrac{dZ_1}{d\eta}}}$

b) $M = N\mu_0 \dfrac{\eta}{\sinh(\eta)}\left[\dfrac{1}{\eta}\cosh(\eta) - \dfrac{1}{\eta^2}\sinh(\eta)\right]$

$\underline{\underline{M = N\mu_0\left[\coth(\eta) - \dfrac{1}{\eta}\right]}}$

c) $\lim_{\eta\to 0} M(\eta) = N\mu_0\left(\frac{1}{3}\eta\right) = \underline{\underline{\dfrac{N\mu_0^2}{3}\dfrac{H}{kT}}}$ FOR HIGH T

$\lim_{\eta\to\infty} \underline{\underline{M(\eta) = N\mu_0\left(1 - \dfrac{kT}{\mu_0 H}\right)}}$ FOR LOW T



d) CURIE LAW $\Rightarrow M \propto \dfrac{H}{T}$ FOR HIGH T $\underline{\underline{\text{YES}}}$

ENERGY GAP $\Rightarrow e^{-\Delta/kT}$ LEADING T DEP, AT LOW T

$\underline{\underline{\text{NO}}}$ BECAUSE THERE IS NO GAP

e) $-kT\ln Z = -NkT\ln Z_1(\eta) = G(T,H)$

$S = -\left.\dfrac{\partial G}{\partial T}\right|_H = Nk\left(\ln Z_1(\eta) + T\dfrac{d\ln Z_1}{d\eta}\dfrac{\partial\eta}{\partial T}\right)\Bigg|_H = \underline{\underline{Nk\left(\ln Z_1(\eta) - \dfrac{\eta}{Z_1}\dfrac{dZ_1}{d\eta}\right)}}$