

# Basic MR image encoding

**MGH-NMR Center**

# Physical Foundations of MRI

**What is NMR?**

**The basic signal we excite and detect.**

**Tricks of NMR**

**The gradient and spin echo**

**How do we encode an image?**

**slice select, frequency and phase encoding.**

**What are some problems (artifacts) relevant to our application.**

# Physical Foundations of MRI

**NMR:** 60 year old phenomena that generates the signal from water that we detect.

**MRI:** using NMR to generate an image

Three magnetic fields (generated by 3 coils)

- 1) static magnetic field  $B_0$
- 2) RF field that excites the spins  $B_1$
- 3) gradient fields that encode spatial info  
 $G_x, G_y, G_z$

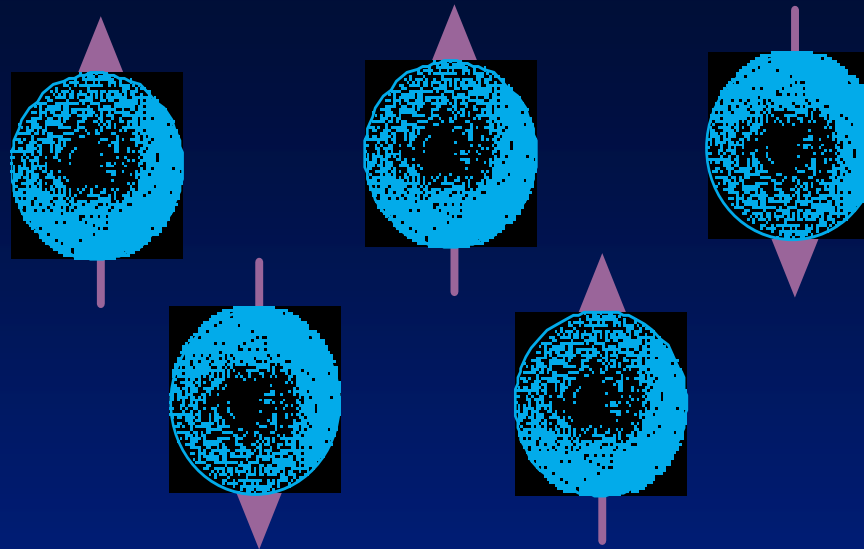
# What is NMR?

**N**UCLEAR

**M**MAGNETIC

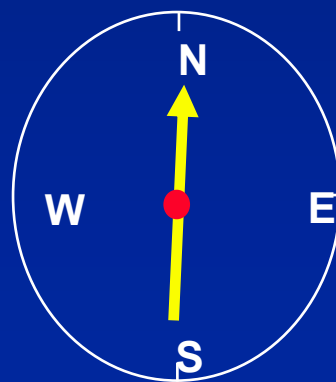
**R**ESONANCE

A magnet, a glass of water,  
and a radio wave source and detector....



**protons**

**Earth's  
Field**



**compass**

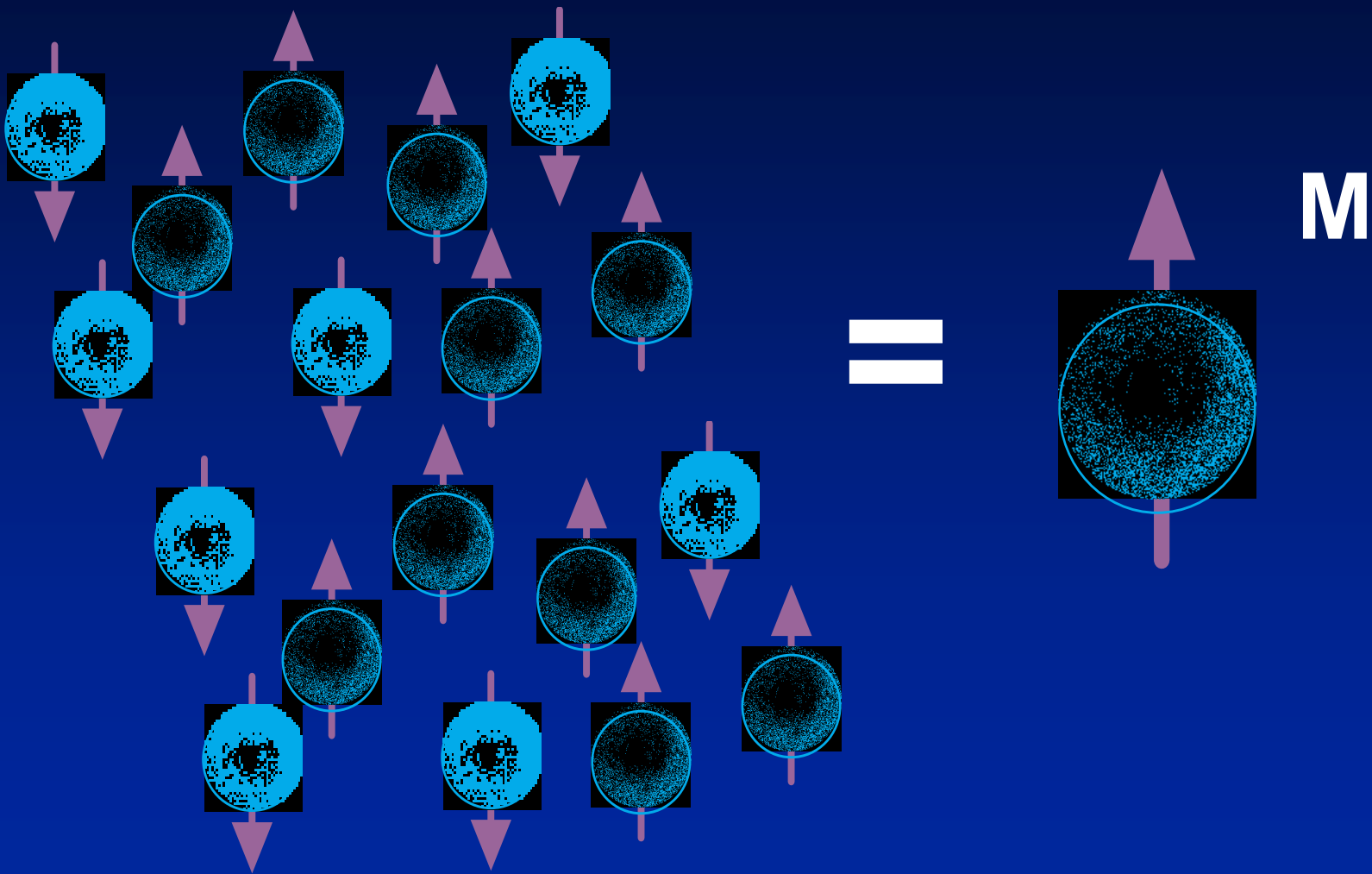
# Nuclei and Magnetic Fields

Not every nucleus lines up with applied magnetic field.

Why?

Direction of spins becomes randomized by thermal motion.

protons at 1.5 Tesla, at room temperature  
net # aligned with field is 1 part in 100,000

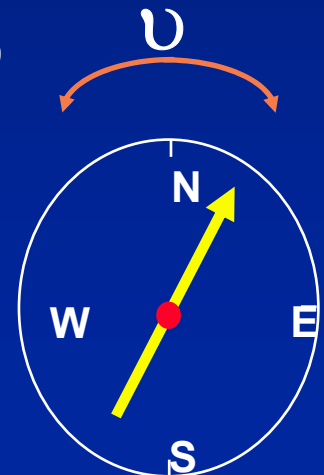


# Compass needles

The vector sum of all the nuclei can be viewed as a compass needle.

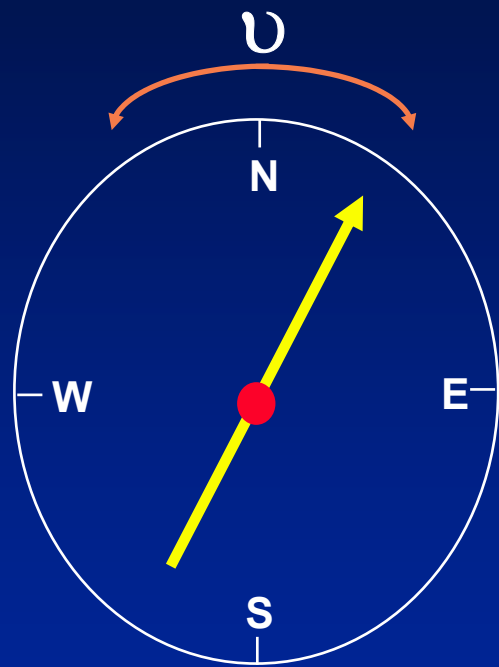
Points North. (aligns along the magnetic field lines of the external field (earth or MR magnet))

If displaced from North, it will wobble about north with a characteristic frequency (called Larmor freq.)





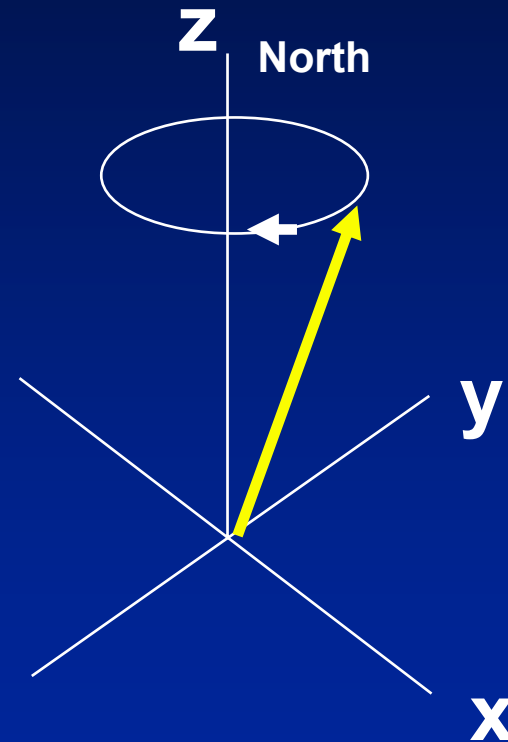
# Compass needles



Earth's  
Field



$$\text{Freq} = \gamma B$$



Main  
Field  
 $B_0$



$$42.58 \text{ MHz/T}$$

# ***EXCITATION* : Displacing the spins from Equilibrium (North)**

**Problem:** It must be moving for us to detect it.

**Solution:** knock out of equilibrium so it  
oscillates

How? 1) Tilt the magnet or compass  
suddenly

2) Drive the magnetization (compass  
needle)

with a periodic magnetic field

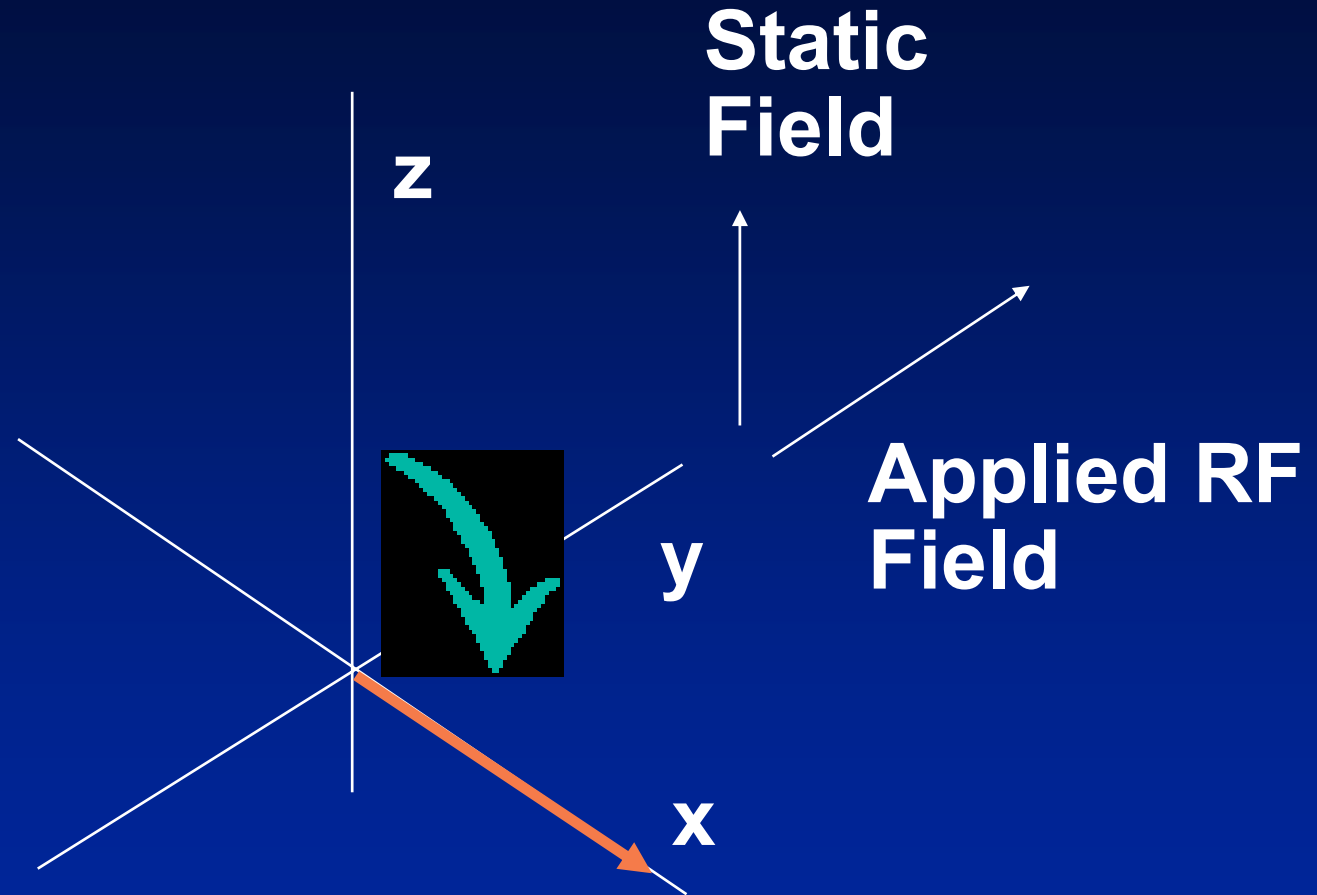
# Excitation: Resonance

Why does only one frequency efficiently tip protons?

**Resonant driving force.**

It's like pushing a child on a swing in time with the natural oscillating frequency.

**z** is "longitudinal" direction  
**x-y** is "transverse" plane



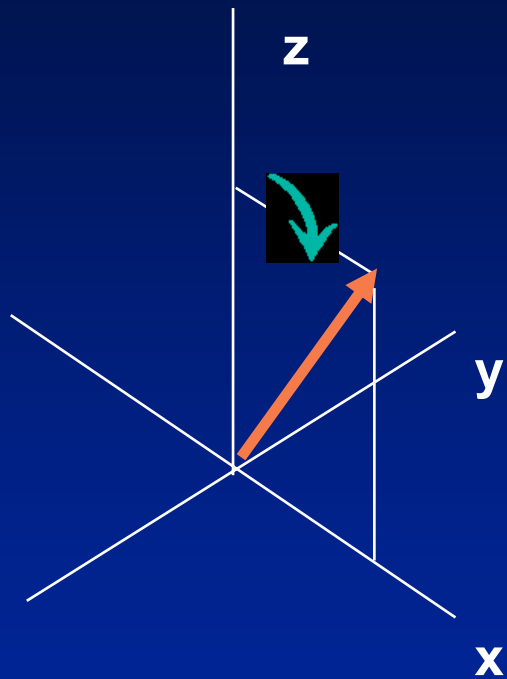
**The RF pulse rotates  $M_0$  about applied field**

# "Exciting" Magnetization

Magnetization processes about new axis (of oscillating RF B field) as long as resonant field is applied.

Total amount vector processes is called the "tip angle" of the excitation.

# "Exciting" Magnetization tip angle



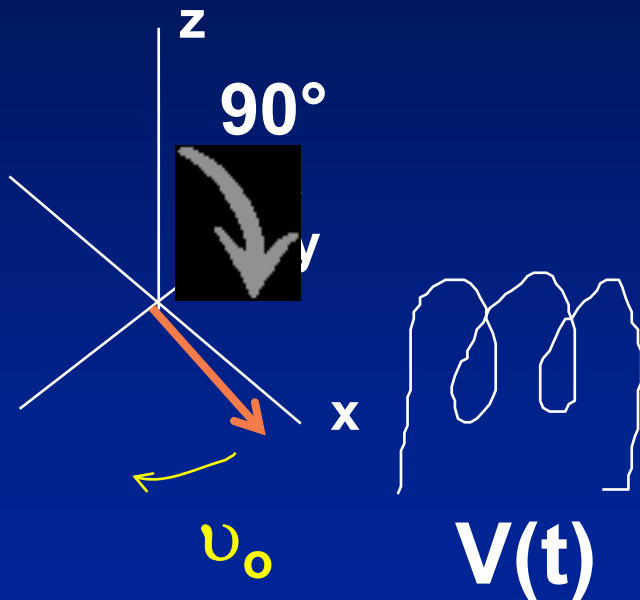
45°



90°

# Detecting the NMR Signal

A moving bar magnet induces a Voltage in a coil of wire.  
(a generator...)



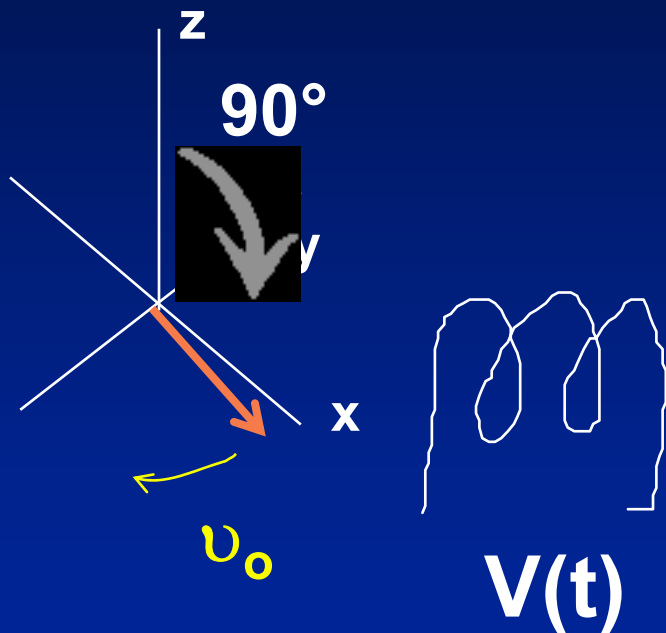
The RF coil design is the #1 determinant of the system SNR

# Detecting the NMR: the noise

Noise comes from electrical losses in the resistance of the coil or electrical losses in the tissue.

For a resistor:  
 $P_{\text{noise}} = 4kTRB$

- Noise is white.  
>> Power  $\propto$  bandwidth
- Noise is spatially uniform.
- R is dominated by the tissue.  
>> big coil is bad.





# Signal to Noise Ratio in MRI

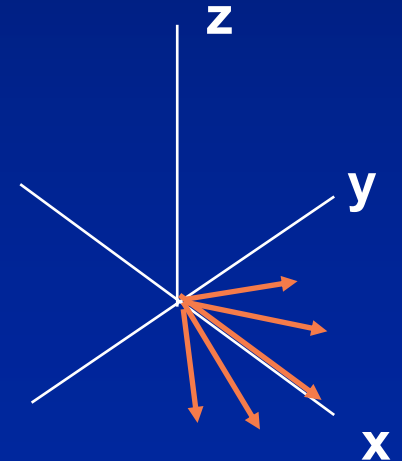
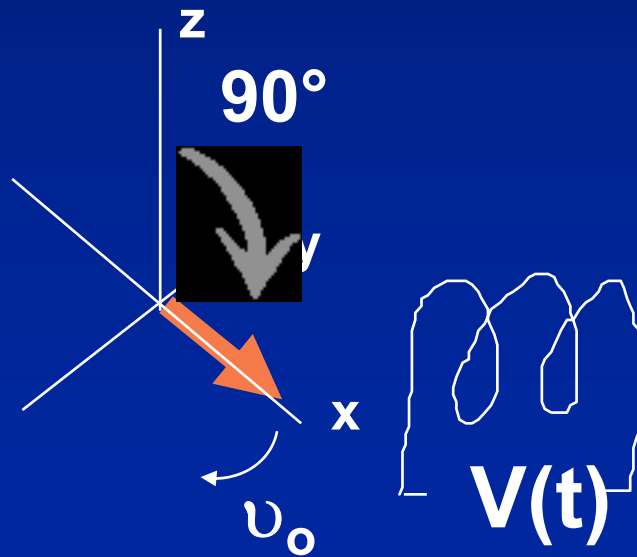
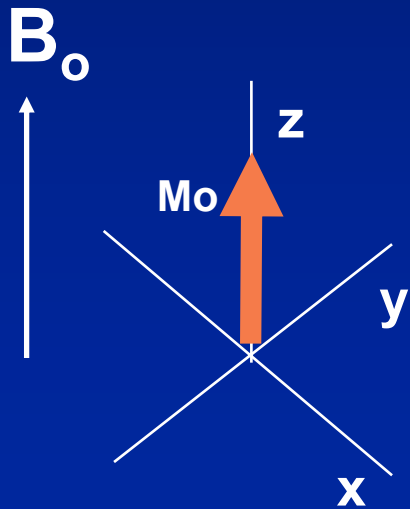
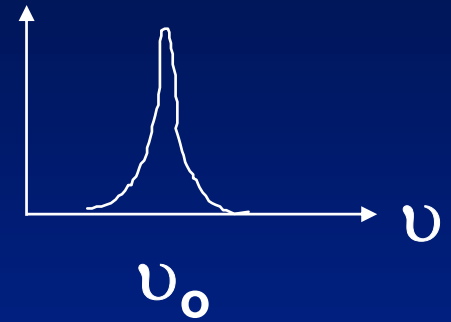
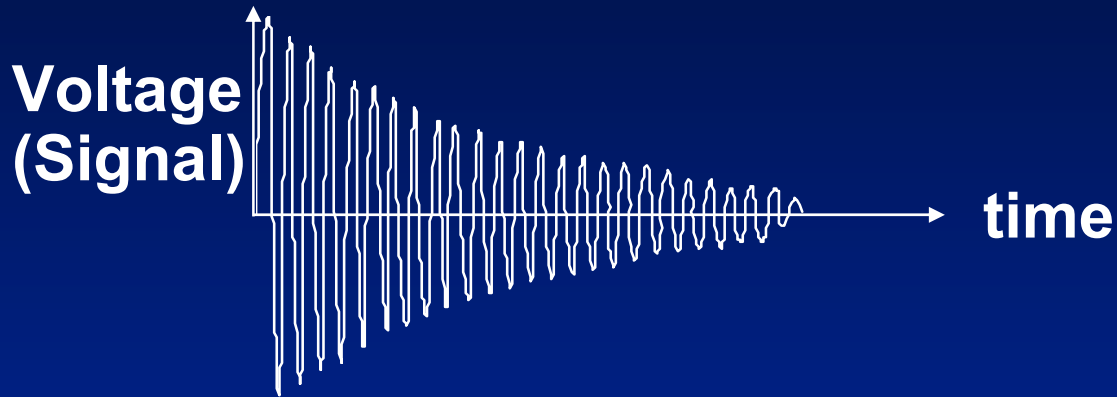
Most important piece of hardware is the RF coil.

$\text{SNR} \propto \text{voxel volume} \quad (\# \text{ of spins})$

$\text{SNR} \propto \text{SQRT}(\text{total time of data collection})$

SNR is also dependent on the amount of signal you throw away to get contrast.

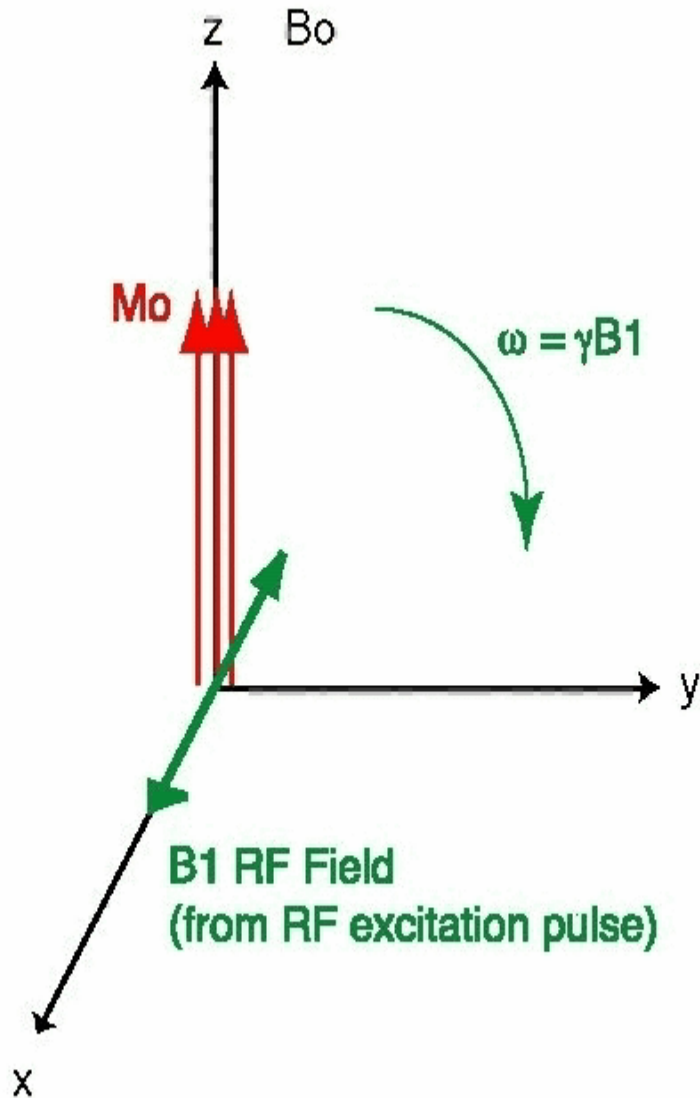
# Review: the NMR Signal



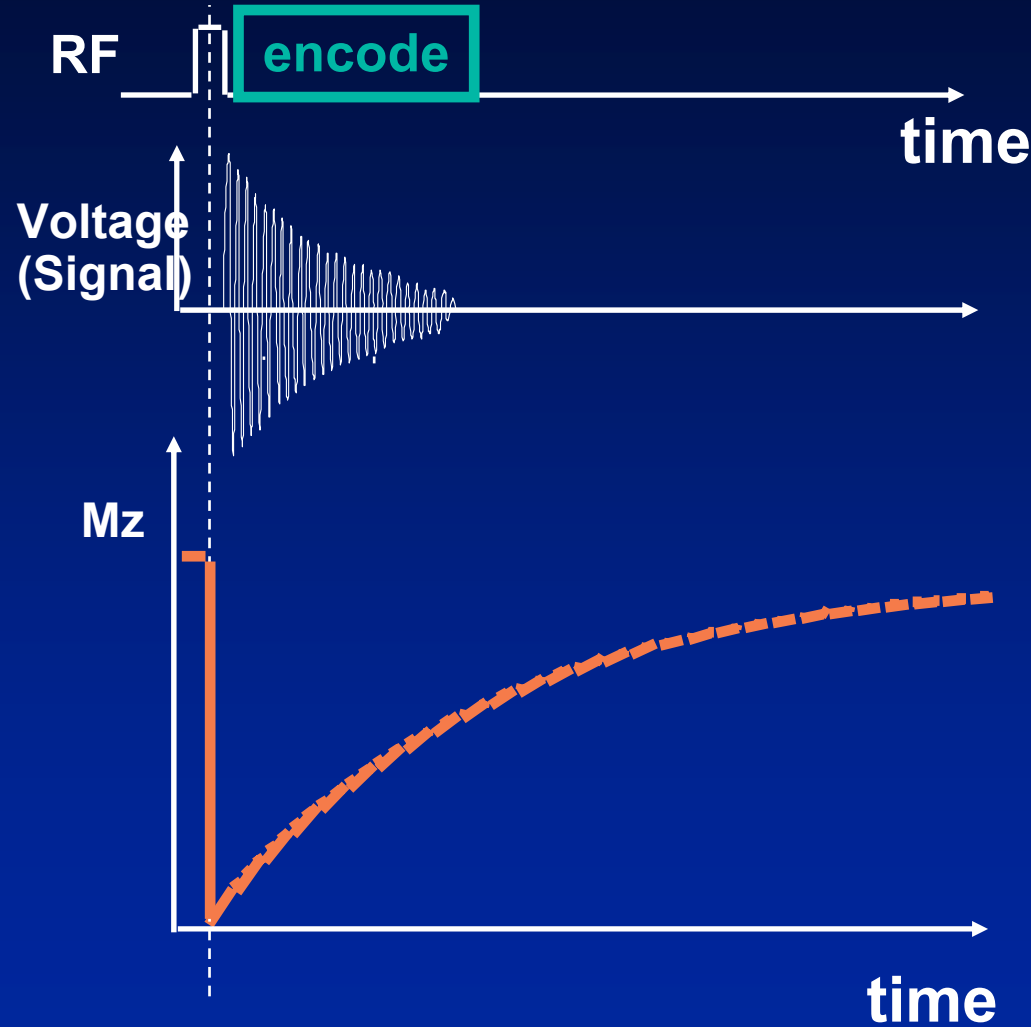
# Three Steps in MR:

- 0) Equilibrium (magnetization points along  $B_0$ )
- 1) RF Excitation (tip magn. away from equil.)
- 2) Precession induces signal, dephasing (timescale =  $T_2$ ,  $T_2^*$ ).
- 3) Return to equilibrium (timescale =  $T_1$ ).

# Magnetization vector durning MR



Wald, MGH-NMR



# Three places in process to make a measurement (image)

0) Equilibrium (magnetization points along  $B_0$ )

1) RF Excitation (tip magn. away from equil.)

proton  
density  
weighting



2) Precession induces signal, allow to dephase  
for time  
 $TE$ .

$T_2$  or  $T_2^*$   
weighting



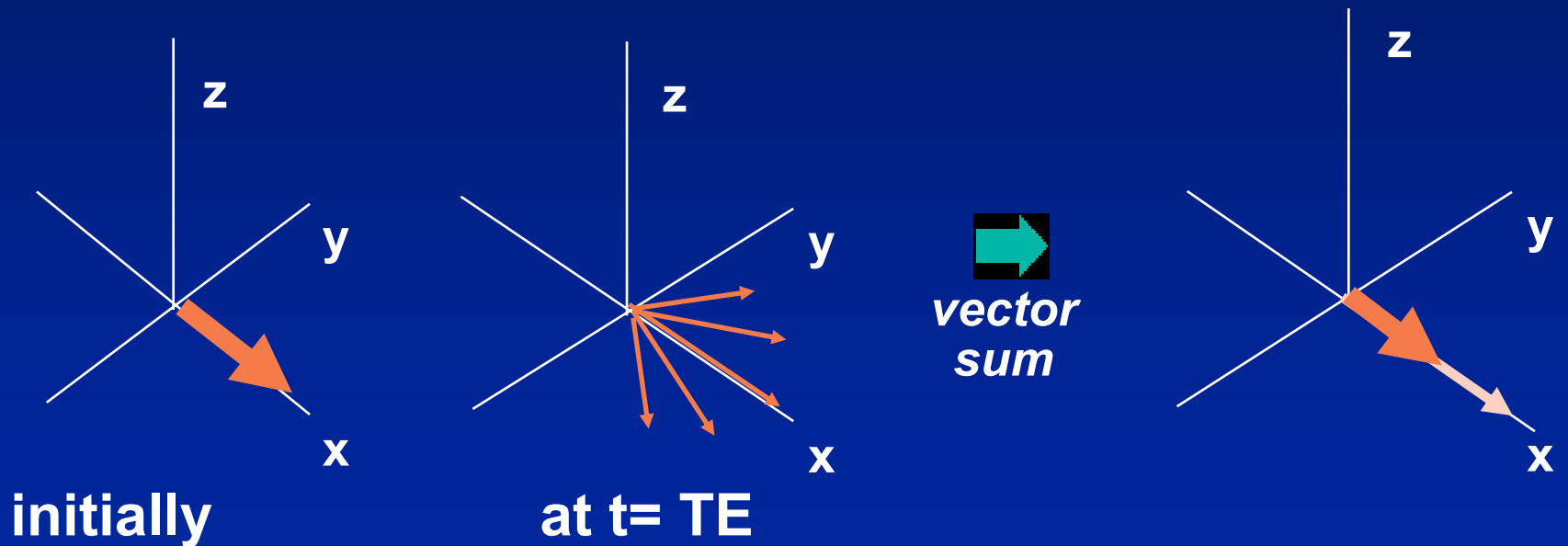
3) Return to equilibrium (timescale =  $T_1$ ).

$T_1$  Weighting



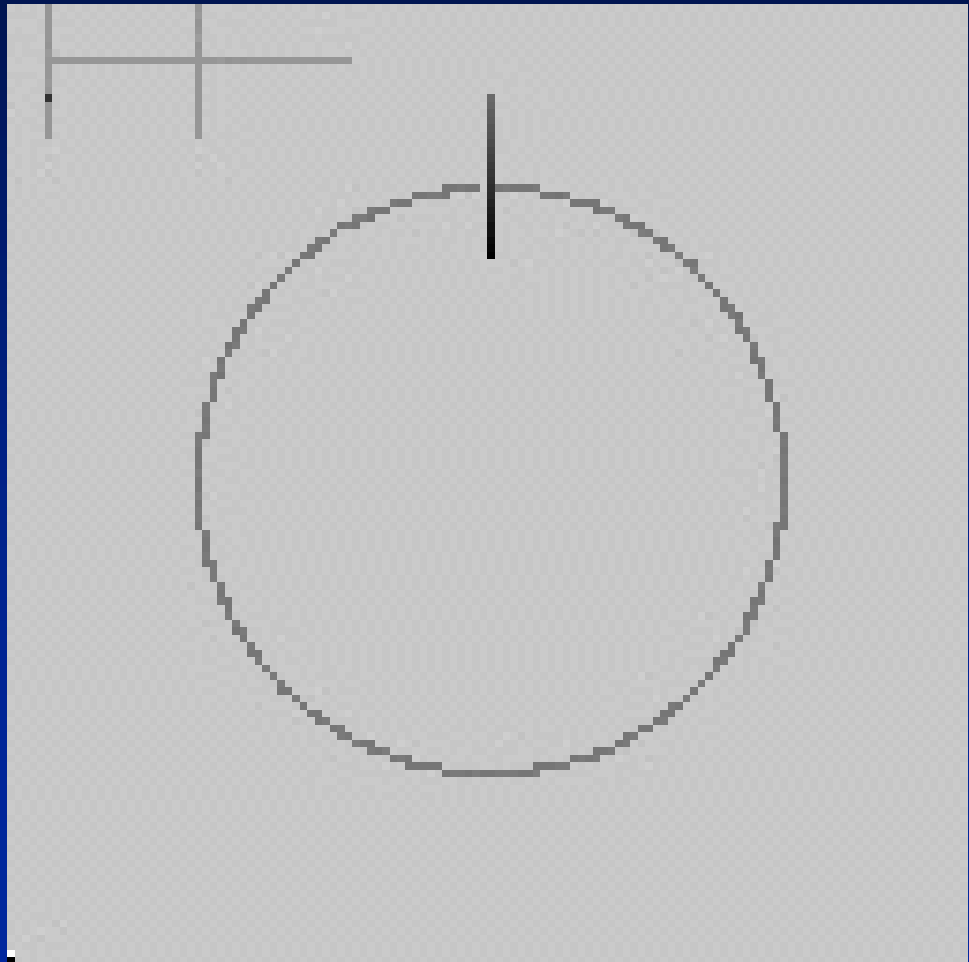
# T2\*-Weighting

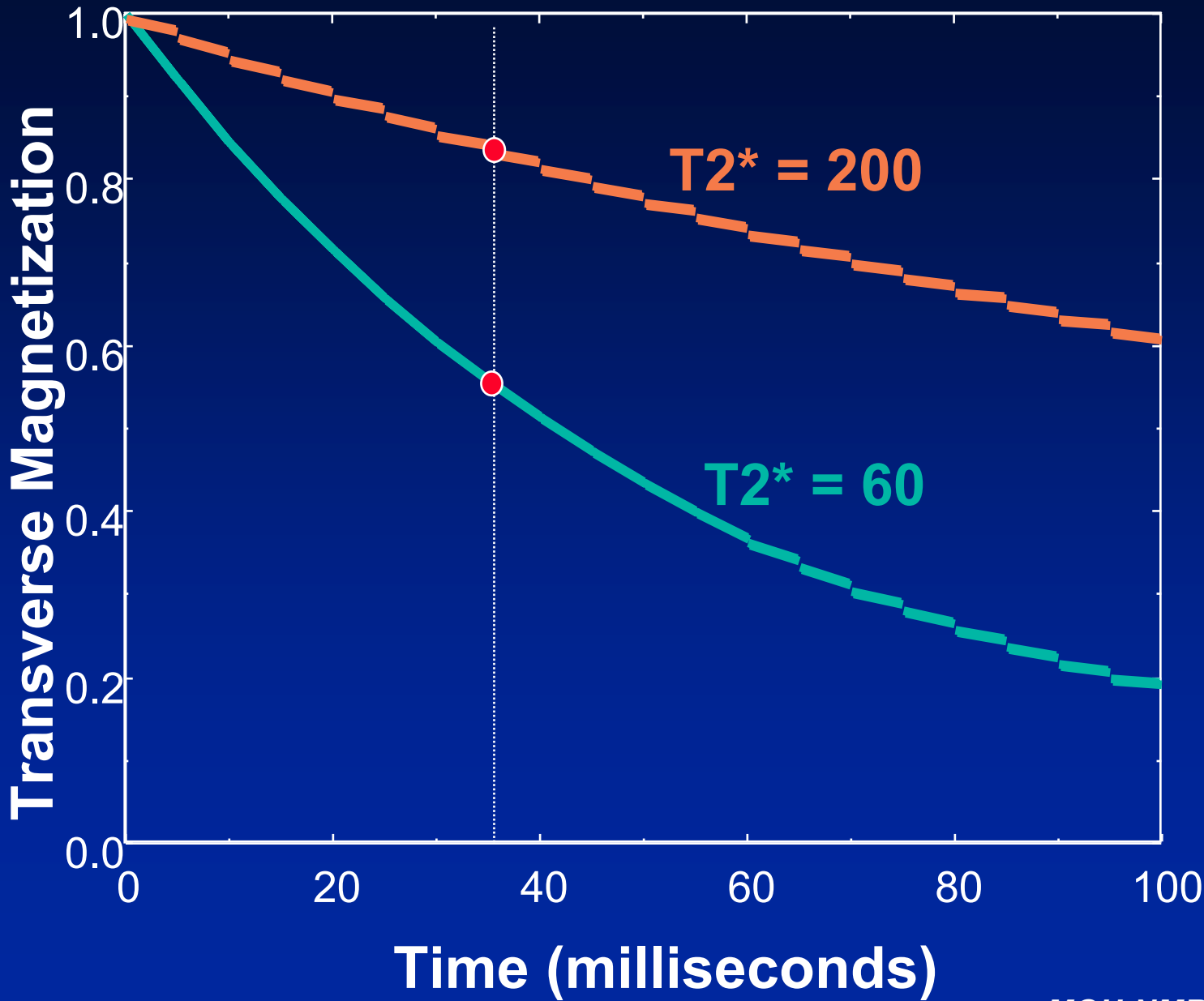
Wait time TE after excitation before measuring M.  
Shorter T2\* spins have dephased



# T2\* Dephasing

Just the tips of the vectors...





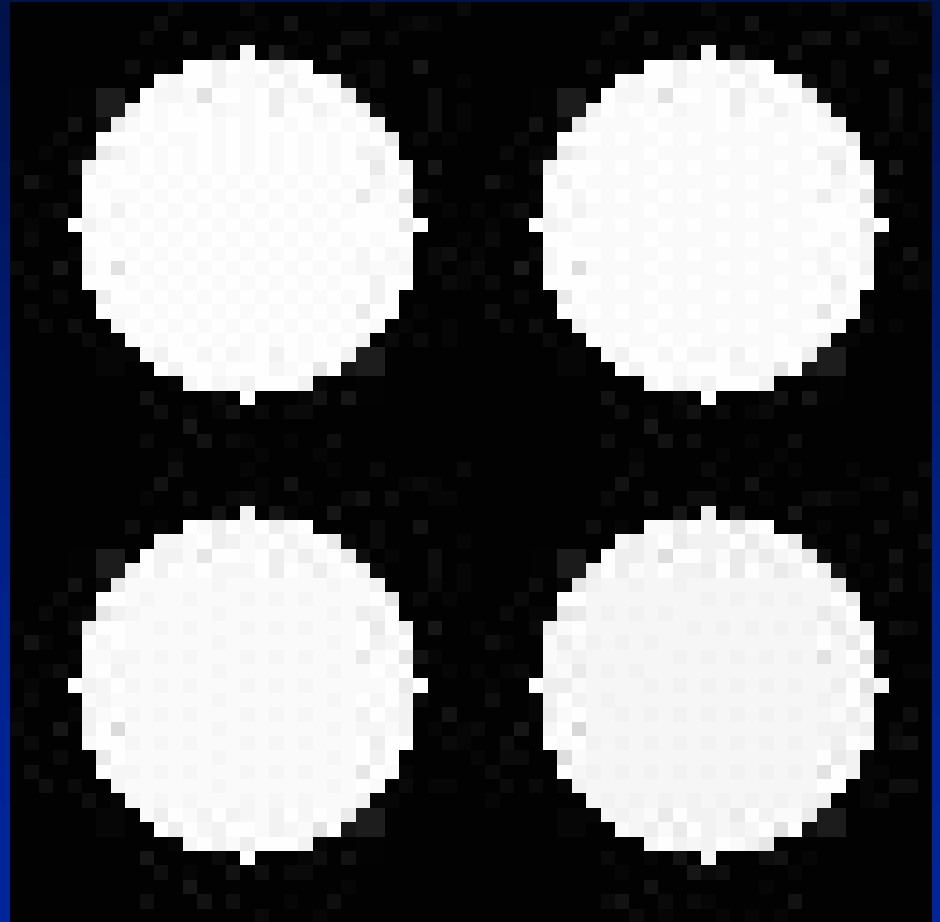


# T2 Weighting

Phantoms with  
four different T2  
decay rates...

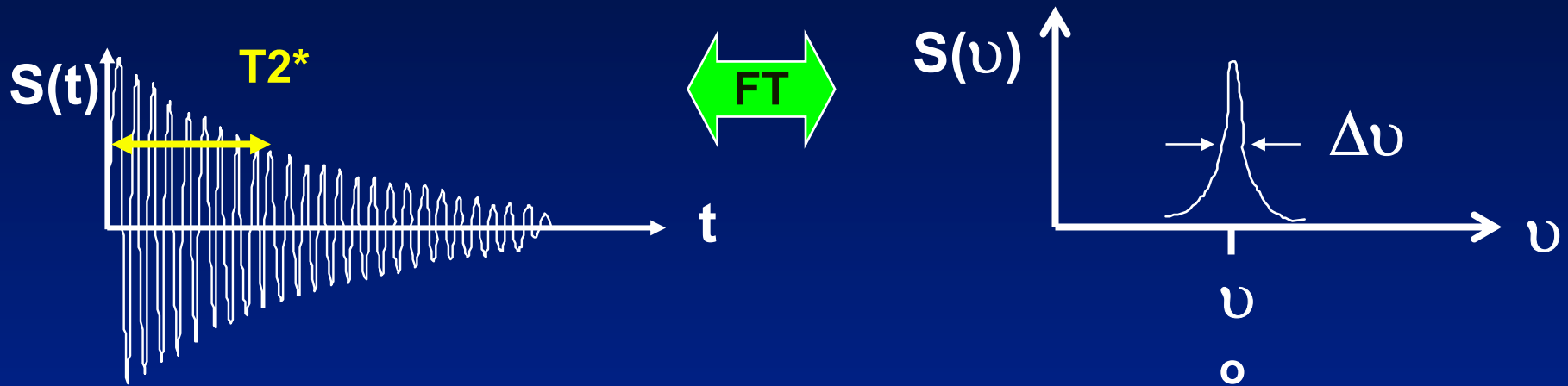
There is no contrast  
difference  
immediately after  
excitation, must wait  
(but not too long!).

Choose TE for max.  
inten. difference.

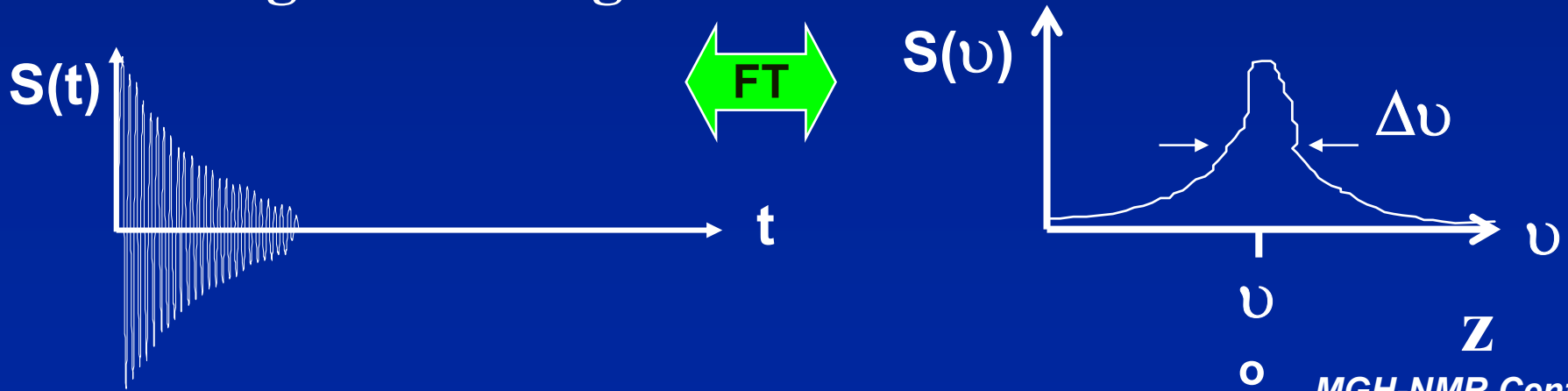


# Dephasing: local field variations

homogeneous magnet.

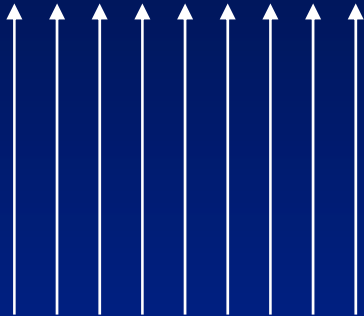


inhomogeneous magnet.



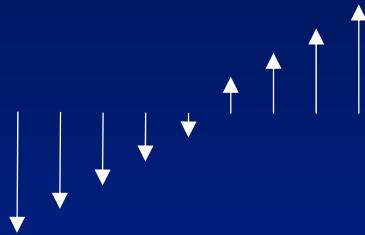
# Aside: Magnetic field gradient

$B_0$



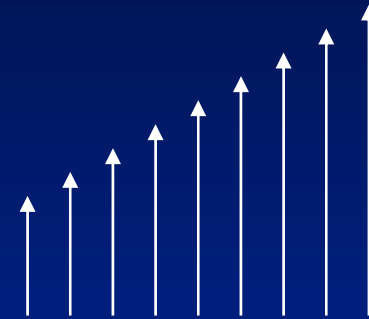
Uniform magnet

$G_x x$

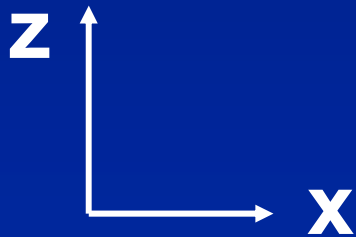


Field from  
gradient  
coils

$B_0 + G_x x$



Total field

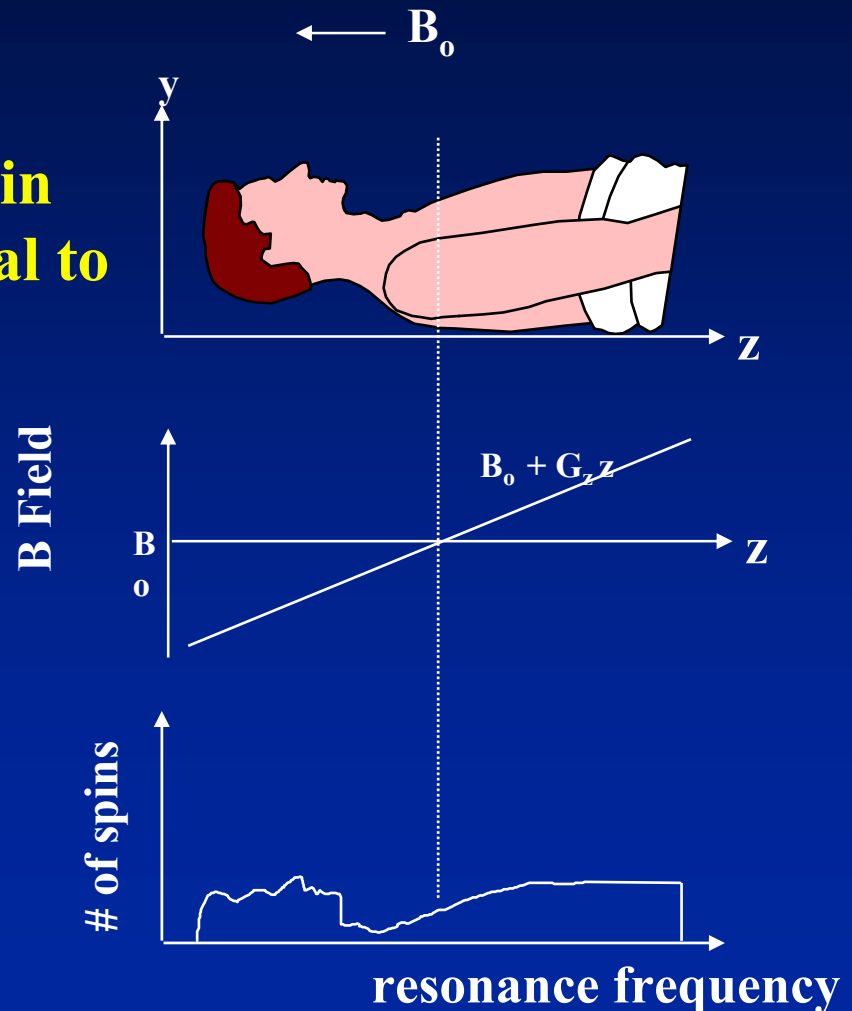
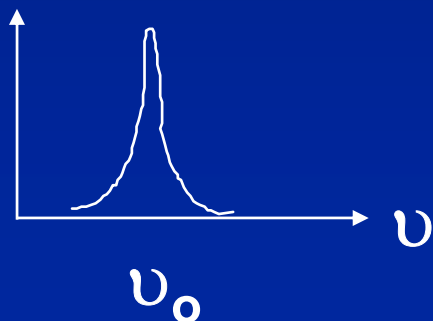


$$G_x = \partial B_z / \partial x$$

# A gradient causes a spread of frequencies

MR frequency of the protons in a given location is proportional to the local applied field.

$$\nu = \gamma \mathbf{B}_{\text{TOT}} = \gamma (\mathbf{B}_0 + \mathbf{G}_z \mathbf{z})$$



# A gradient causes dephasing

I caused it, I can reverse it...

## Gradient echo

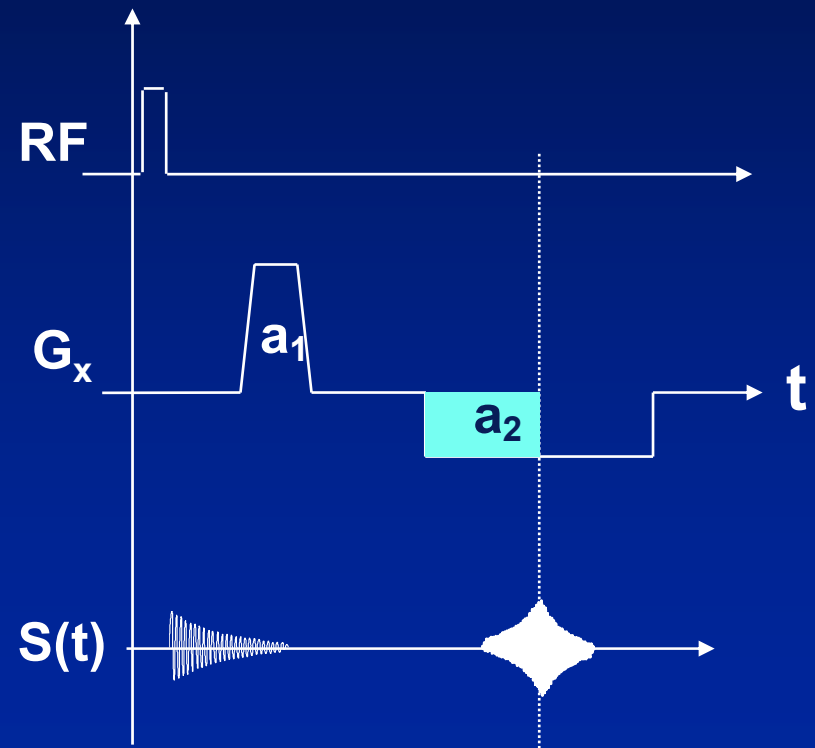
$$\nu = \gamma \mathbf{B}_{\text{TOT}} = \gamma B_0 + G_z z$$

$$\Delta\nu = \gamma \Delta \mathbf{B}_{\text{TOT}} = \gamma G_z z$$

$$\Delta\theta = \Delta\nu \tau = \gamma G_z z \tau$$

Gratuitous manipulation...  
(?)

What happens if the spin moves?

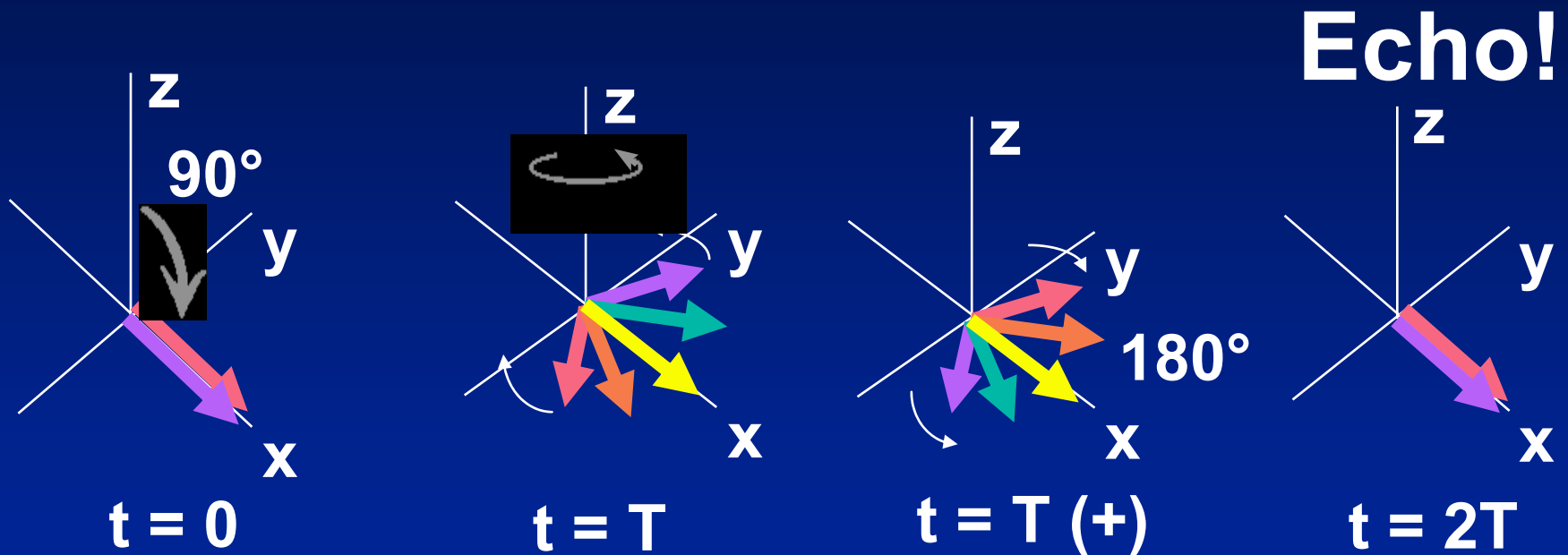


# **Less trivial manipulation... the Spin Echo**

**Refocus the dephased signal  
without resorting to direct  
control of the  $B_0$  field.**

# Spin Echo

Some dephasing can be refocused because its due to static fields.



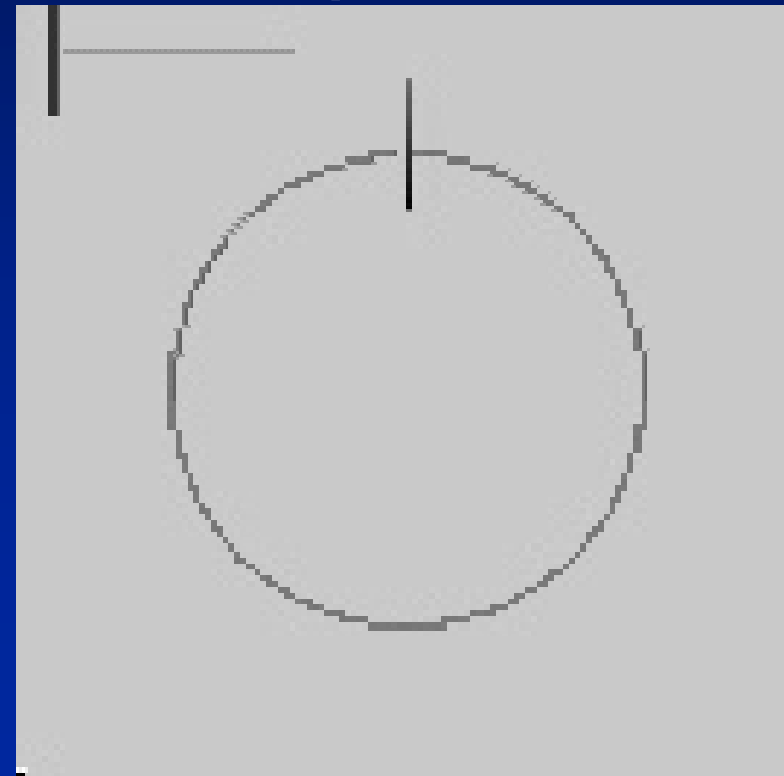
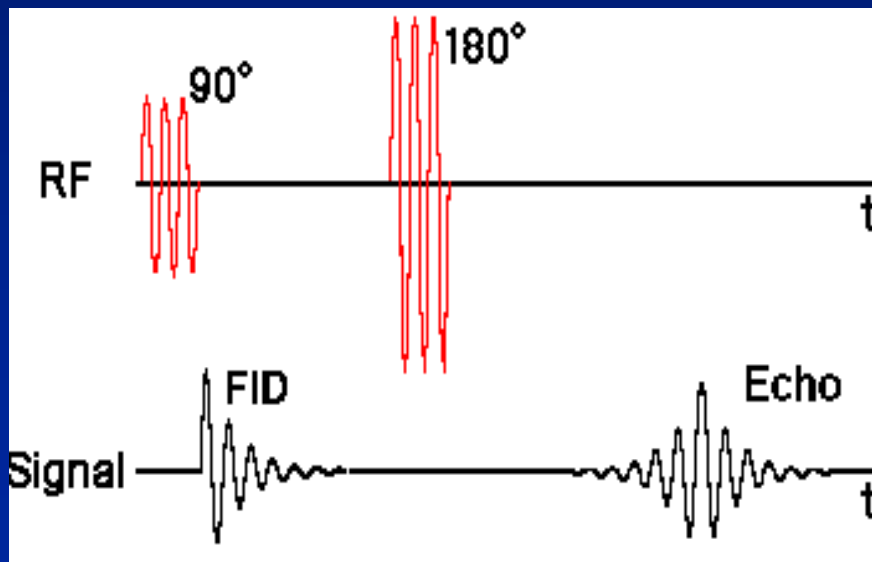
Blue/Green arrows precesses faster due to local field inhomogeneity than red/orange arrow

# Spin Echo

$180^\circ$  pulse only helps cancel static inhomogeneity

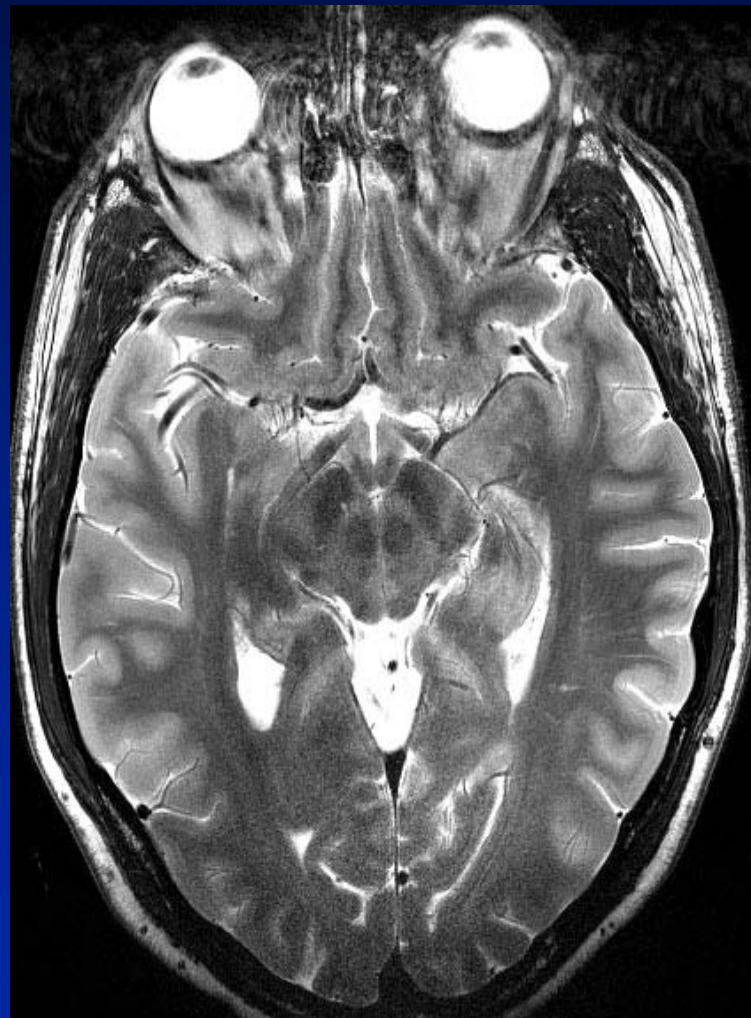
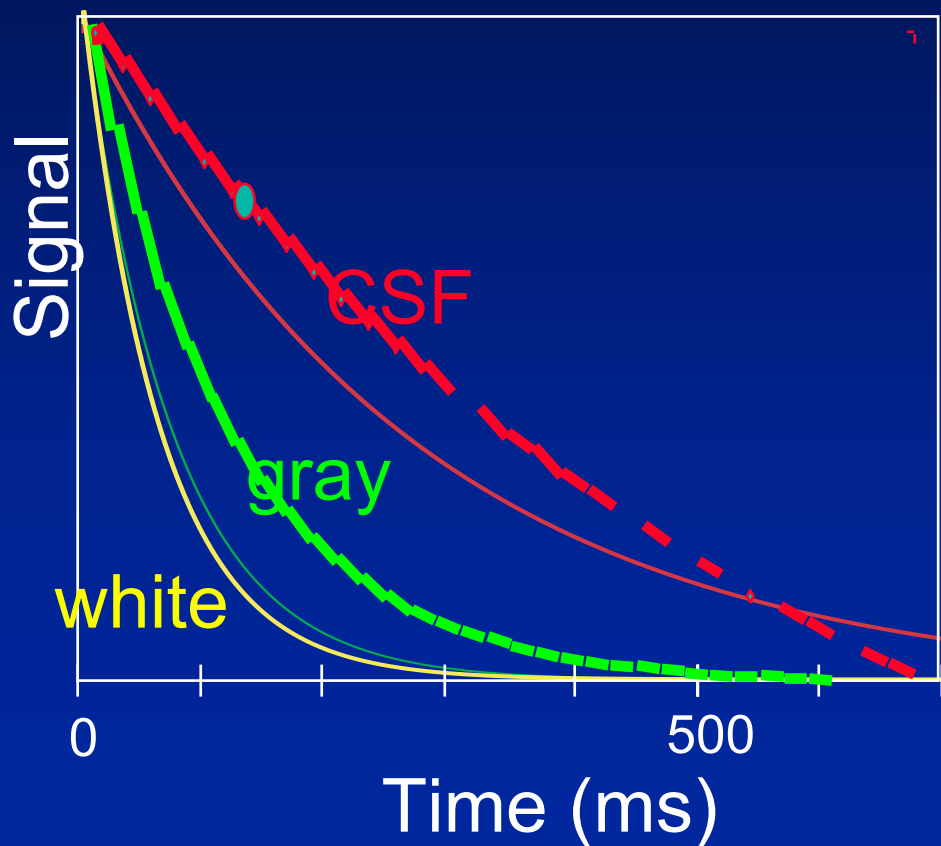
The “runners” can have static speed distribution.

If a runner trips, he will not make it back in phase with the others.





# T2 weighed image



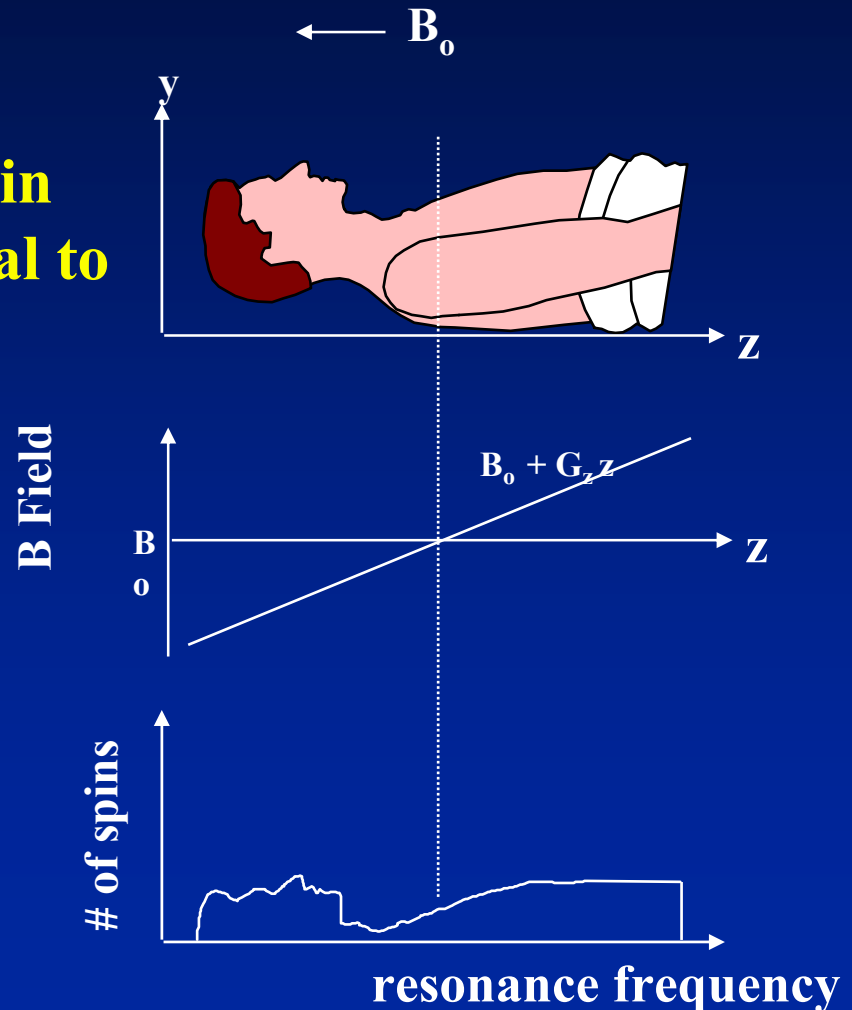
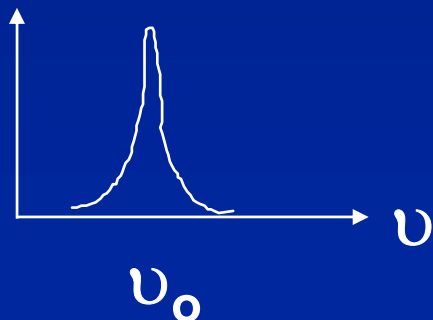
# Part II

## Image encoding

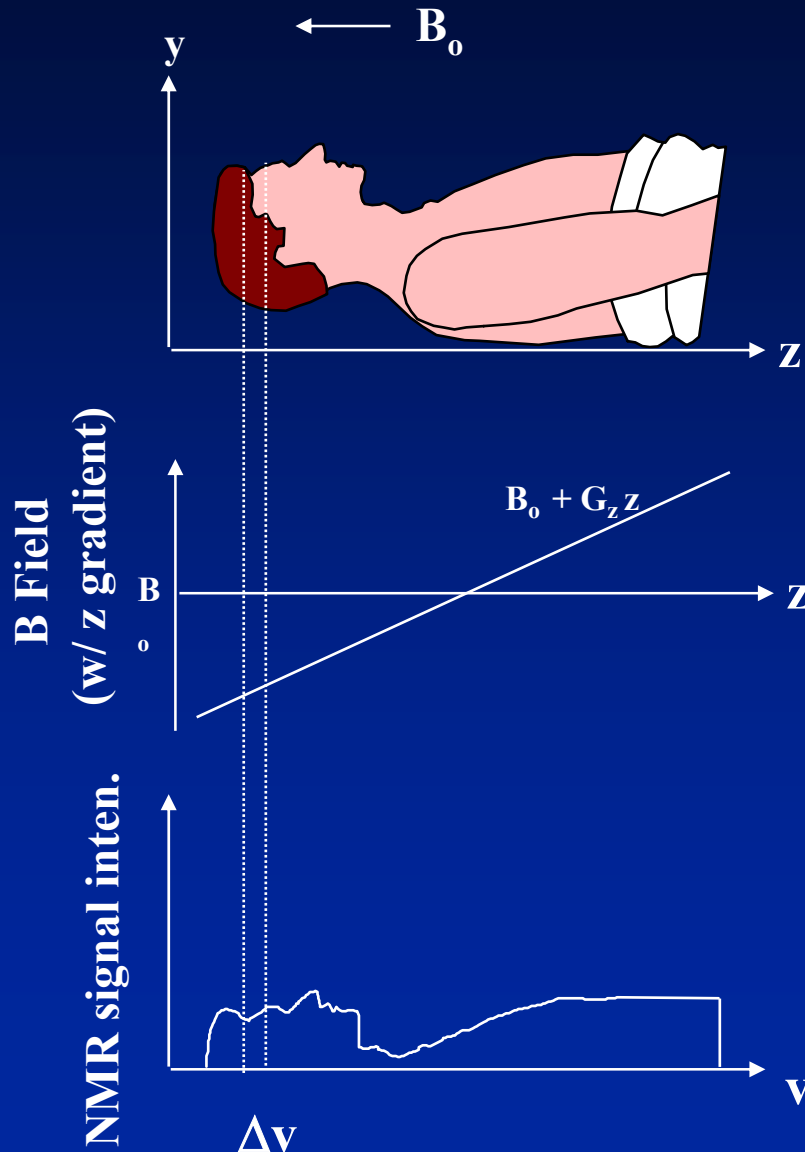
# 1D projection image

MR frequency of the protons in a given location is proportional to the local applied field.

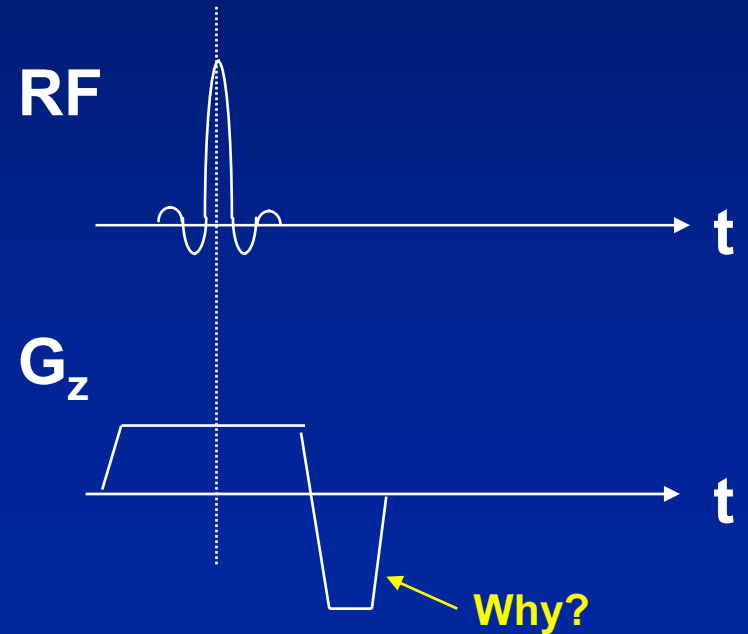
$$\nu = \gamma \mathbf{B}_{\text{TOT}} = \gamma (\mathbf{B}_0 + \mathbf{G}_z \mathbf{z})$$



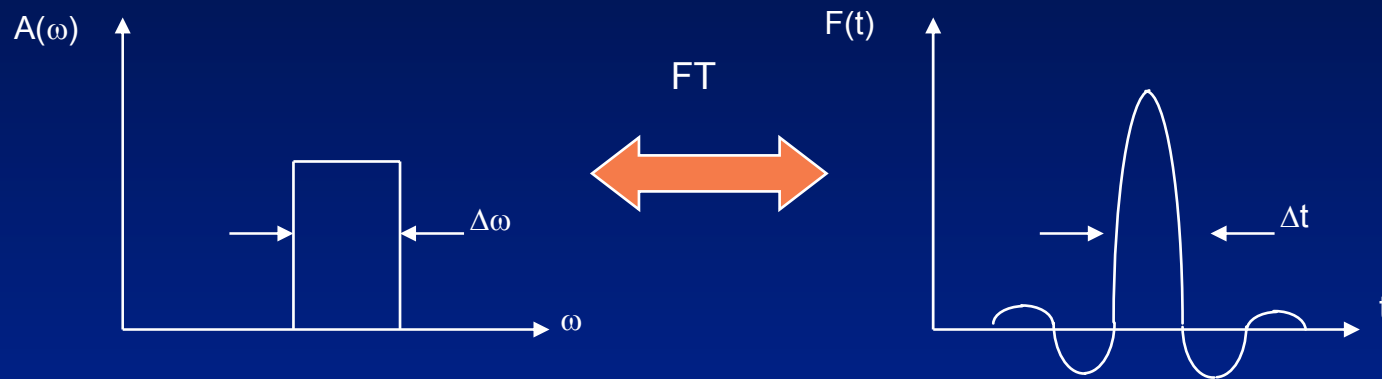
# Step one: excite a slice



While the grad. is on, excite only band of frequencies.

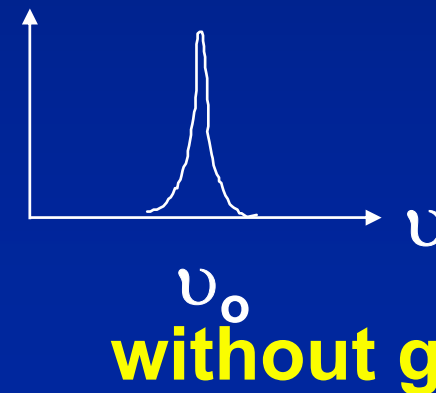
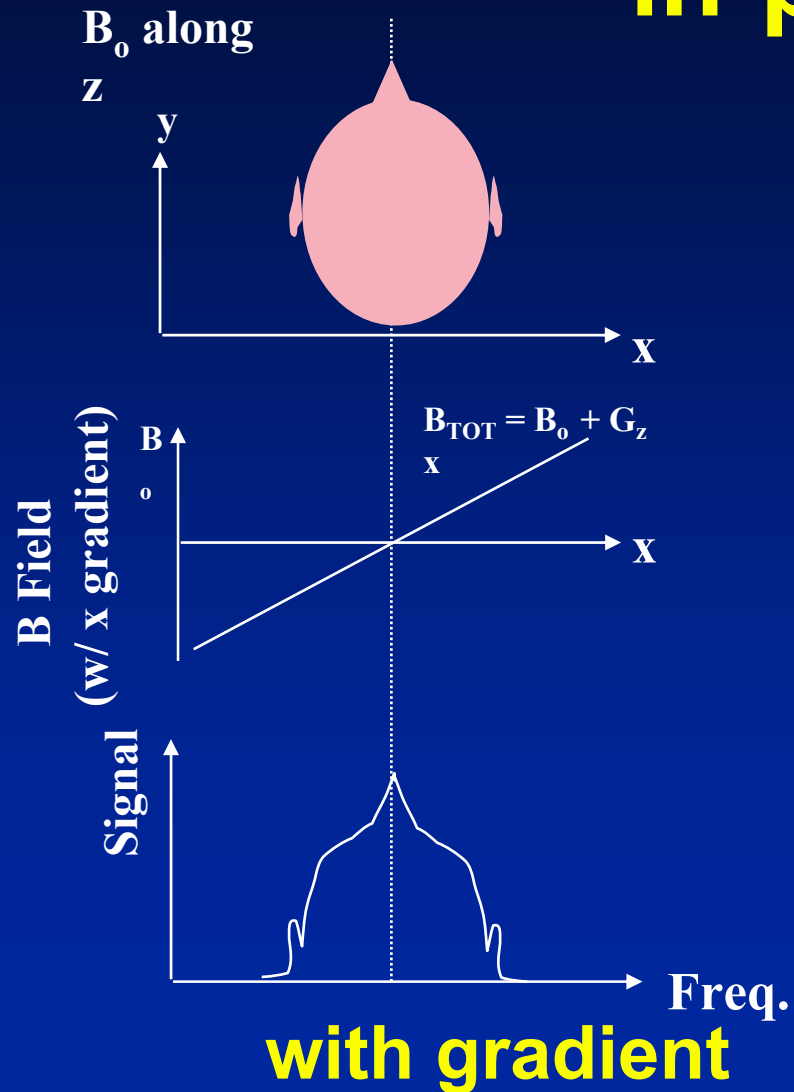


# Slice profile considerations

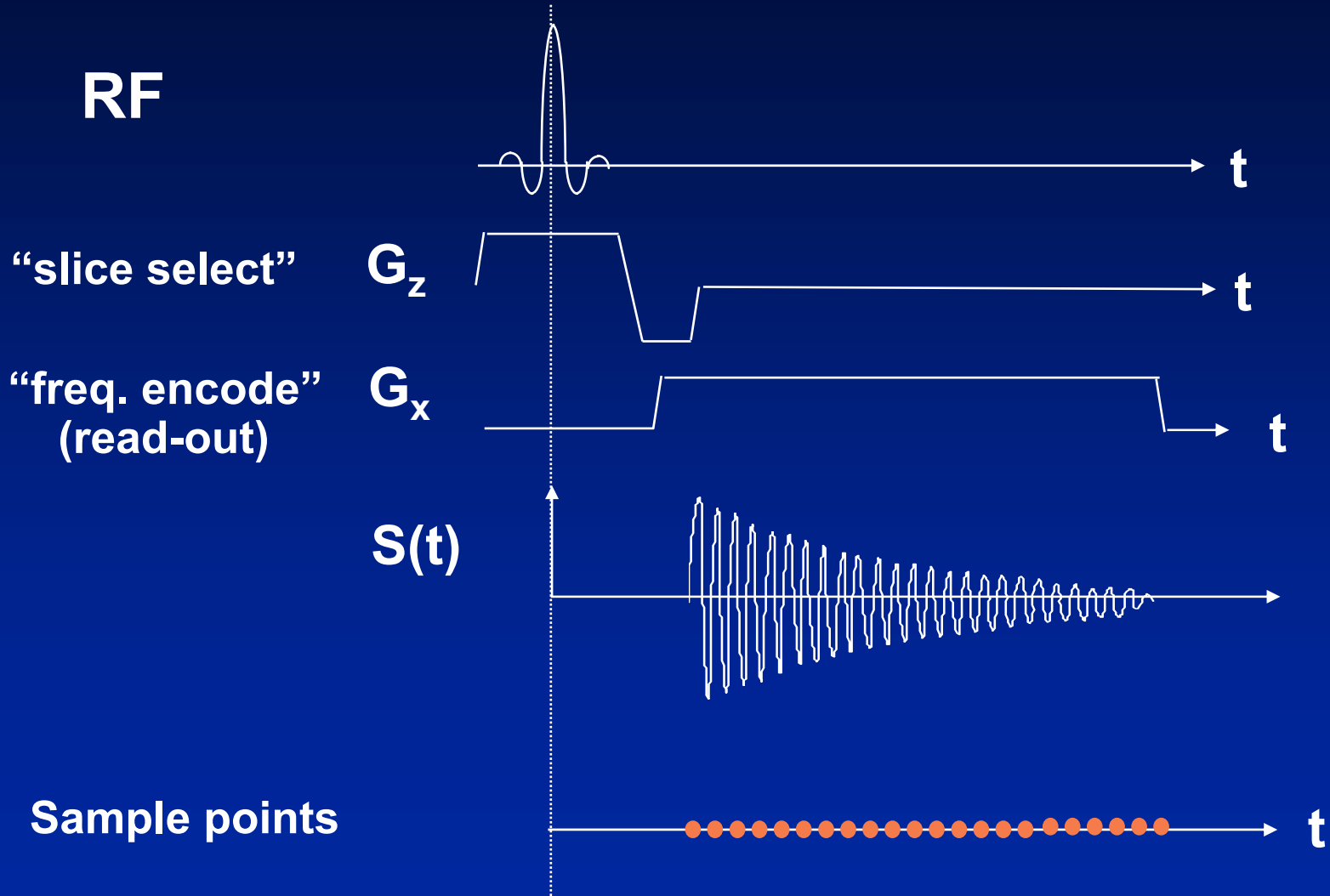


# Step two: encode spatial info. in-plane

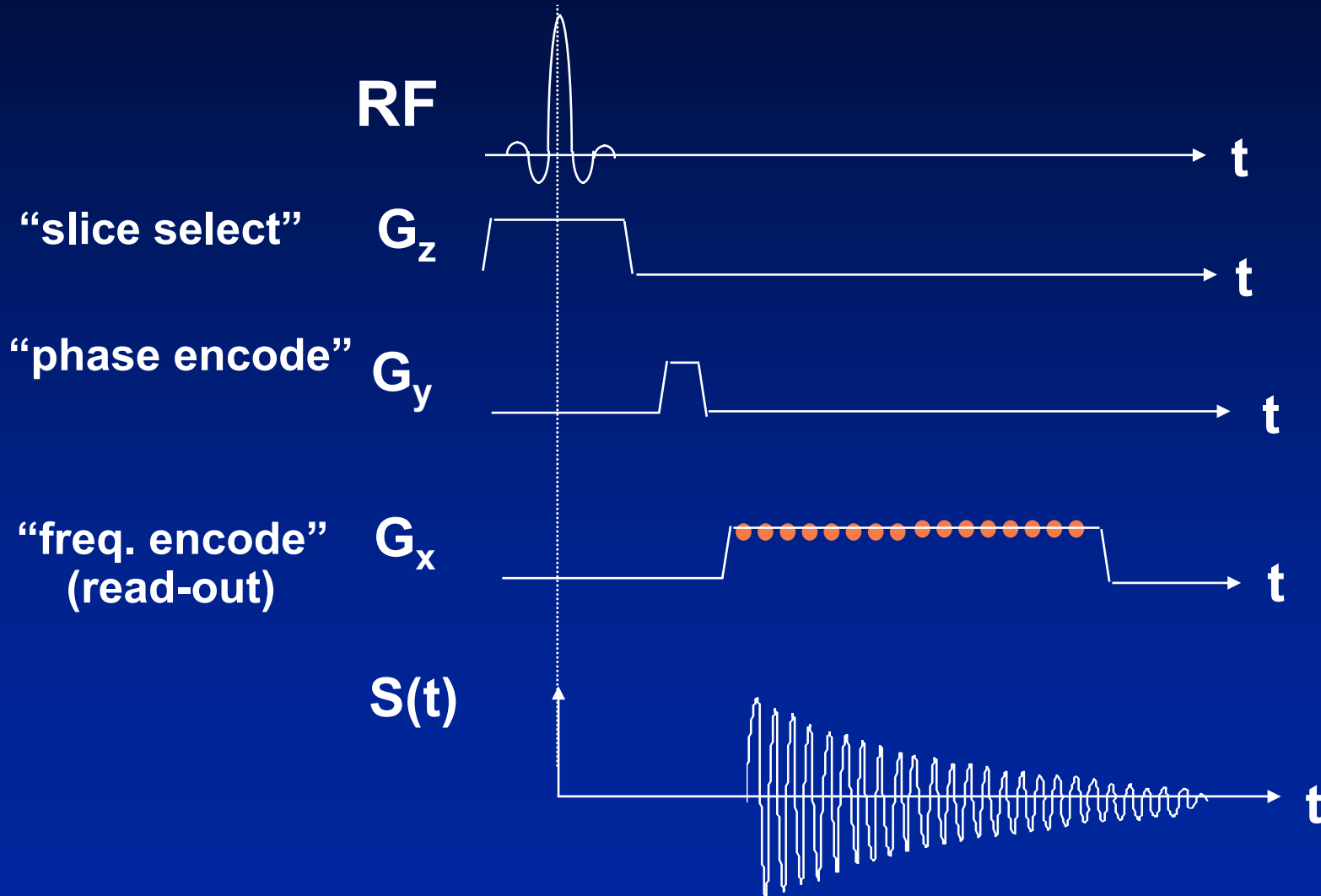
“Frequency encoding”



# 'Pulse sequence' so far

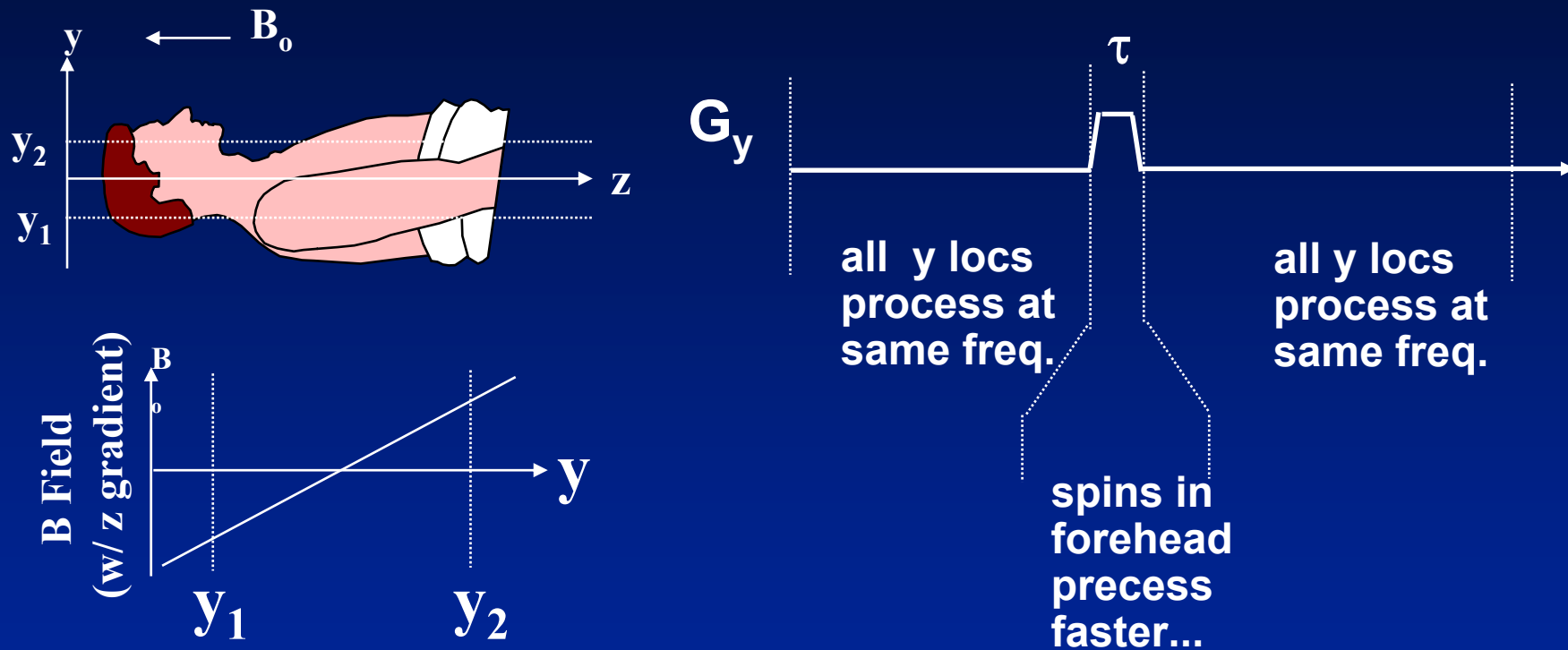


# “Phase encoding”





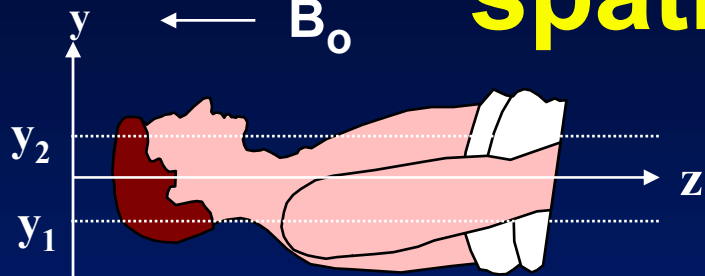
# How does blipping on a grad. encode spatial info?



$$\nu(\mathbf{y}) = \gamma \mathbf{B}_{\text{TOT}} = \gamma \mathbf{B}_0 \Delta \mathbf{y} \mathbf{G}_y$$

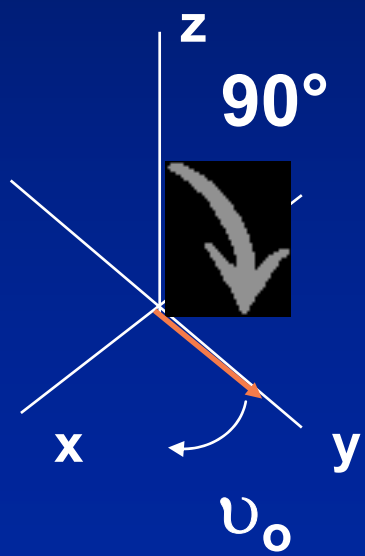
$$\theta(\mathbf{y}) = \nu(\mathbf{y}) \tau = \gamma \mathbf{B}_0 \Delta \mathbf{y} (\mathbf{G}_y \tau)$$

# How does blipping on a grad. encode spatial info?

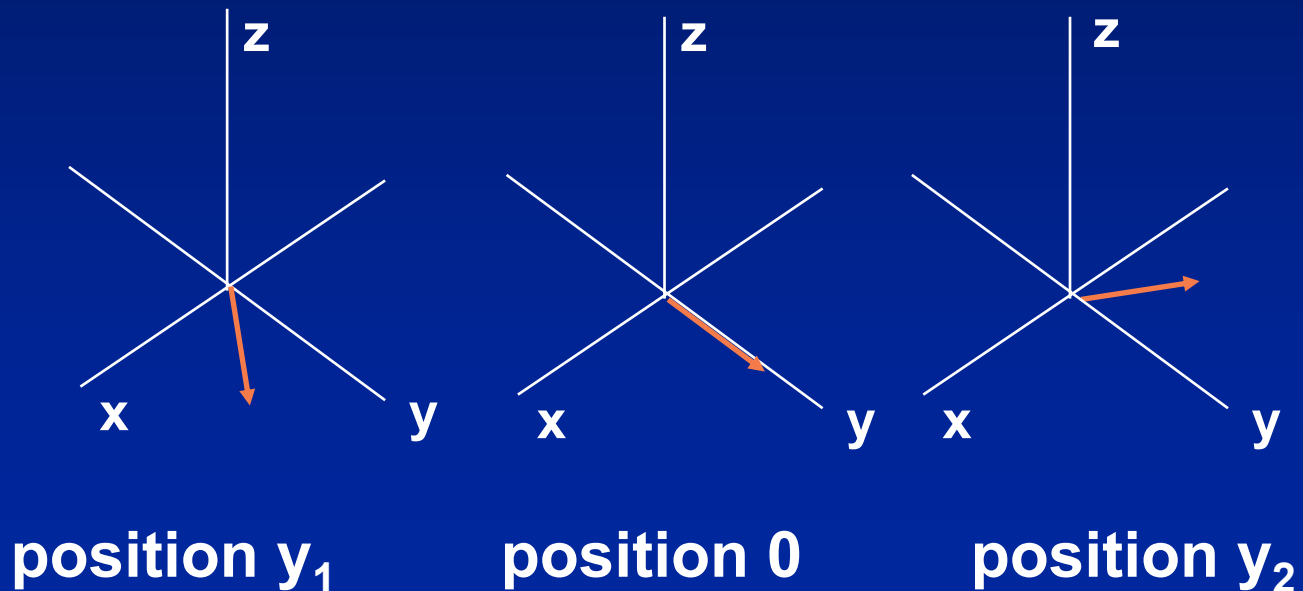


$$\theta(\mathbf{y}) = \nu(\mathbf{y}) \tau = \gamma \mathbf{B}_0 \Delta \mathbf{y} (\mathbf{G}_y \tau)$$

after RF



After the blipped y gradient...



# How does blipping on a grad. encode spatial info?

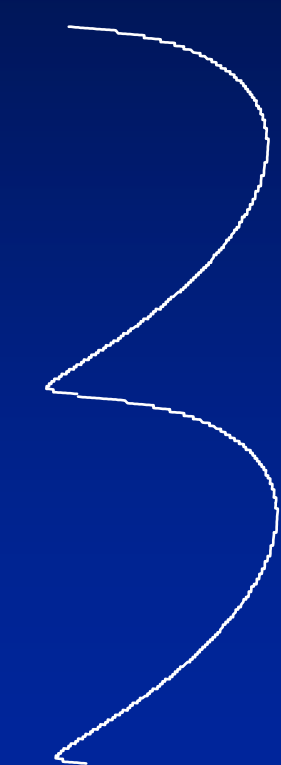
The magnetization vector in the  $xy$  plane is wound into a helix directed along  $y$  axis.

Phases are 'locked in' once the blip is over.

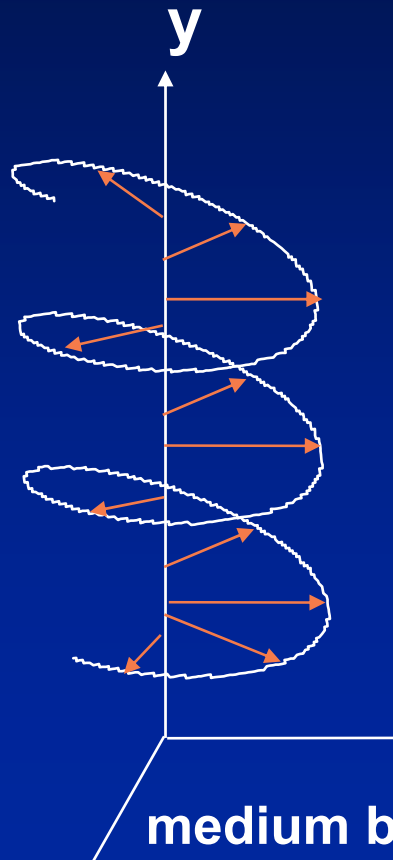


# The bigger the gradient blip area, the tighter the helix

$$\theta(y) = v(y) \tau = \gamma B_0 \Delta y (G_y \tau)$$



small blip



medium blip



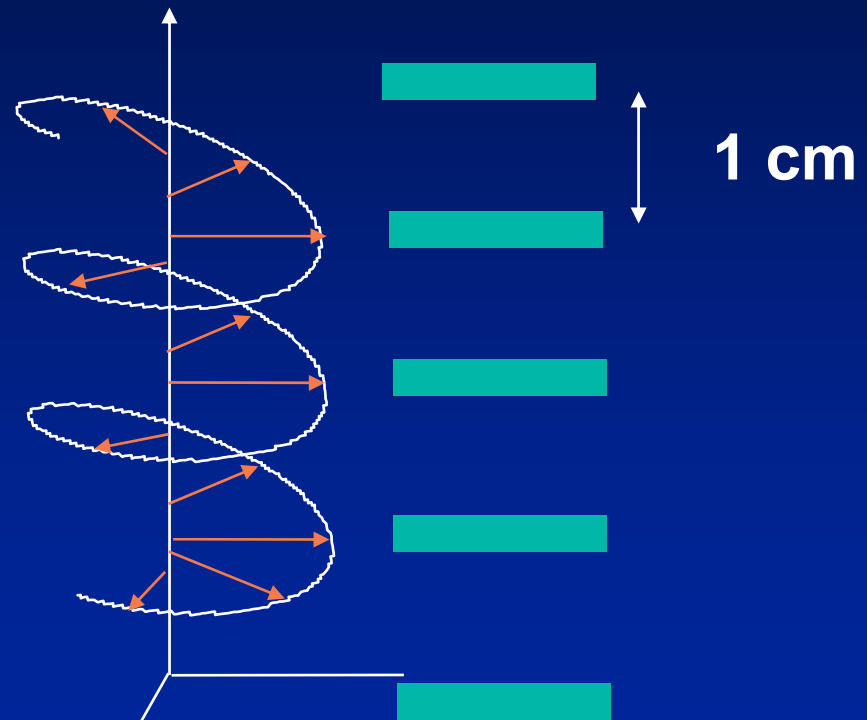
large blip

# What have you measured?

Consider 2 samples:

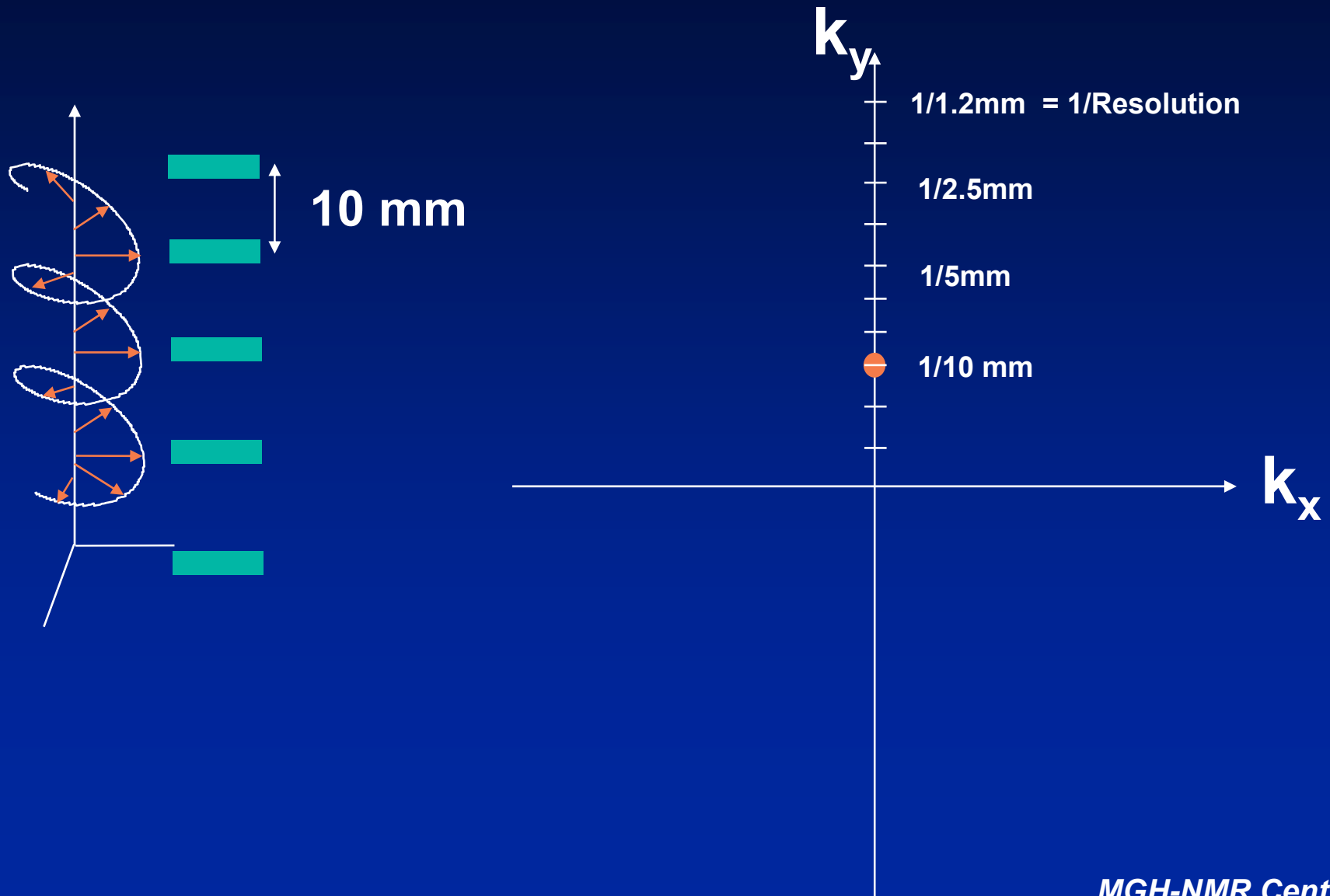


no signal observed

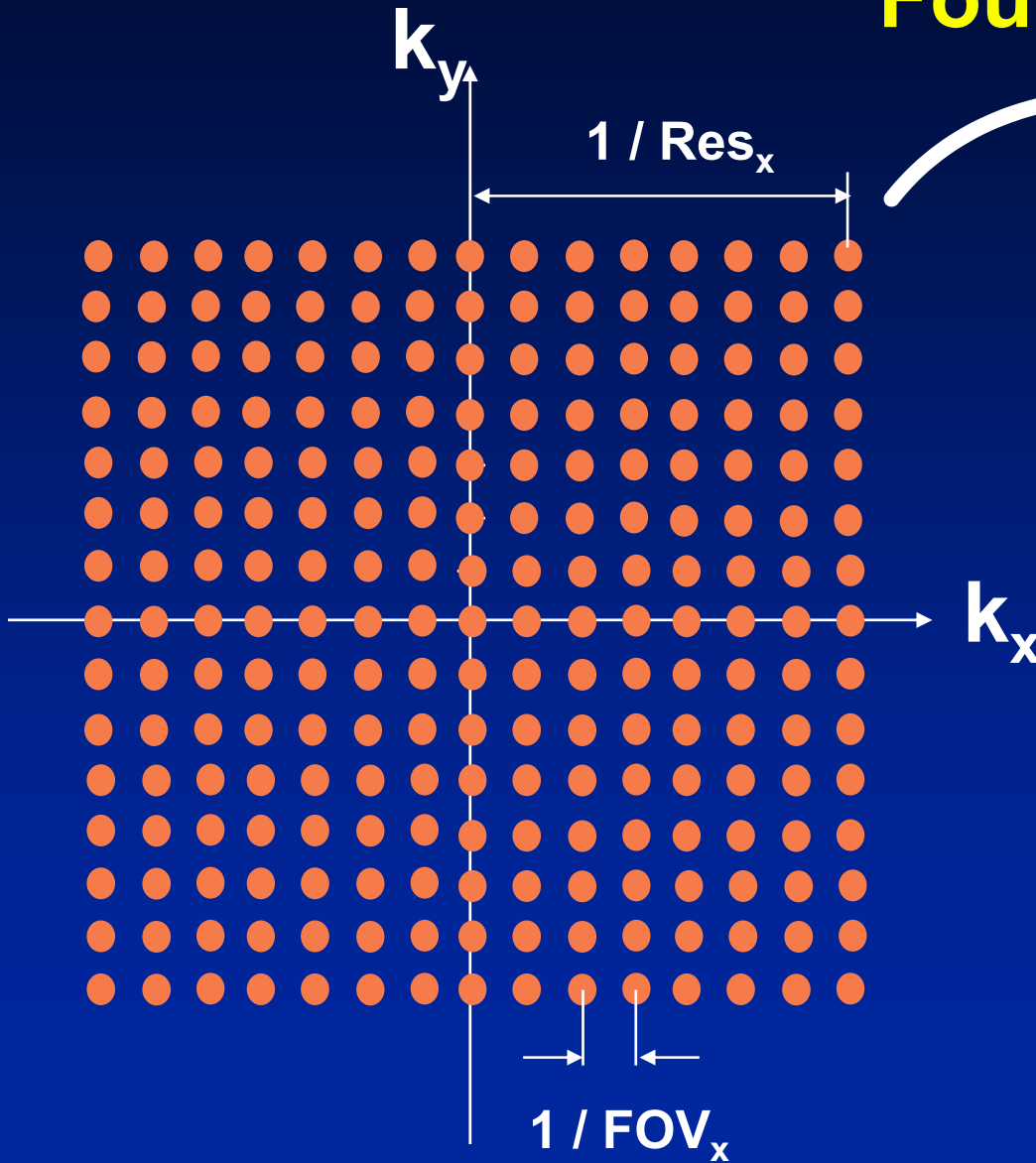


signal is as big as if no gradient

# Measurement intensity at a spatial frequency...

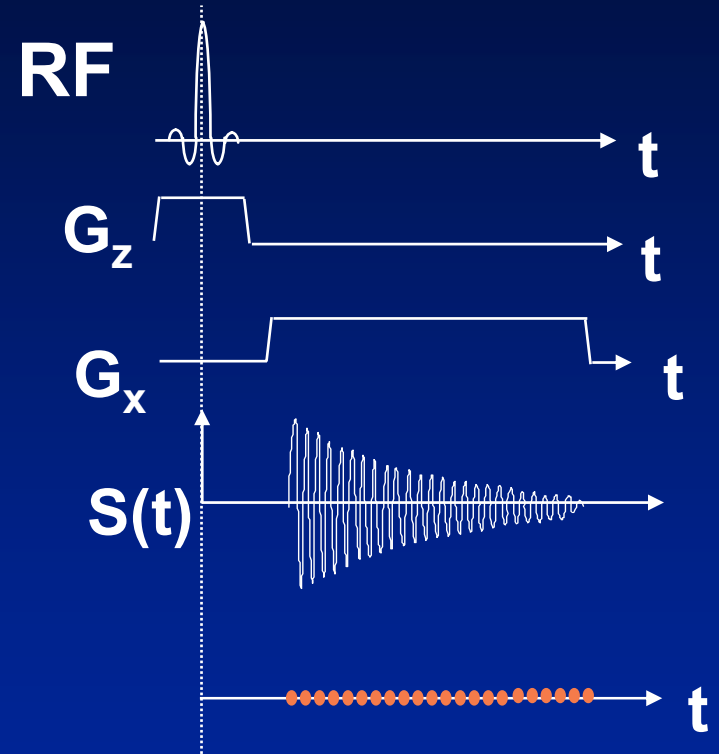
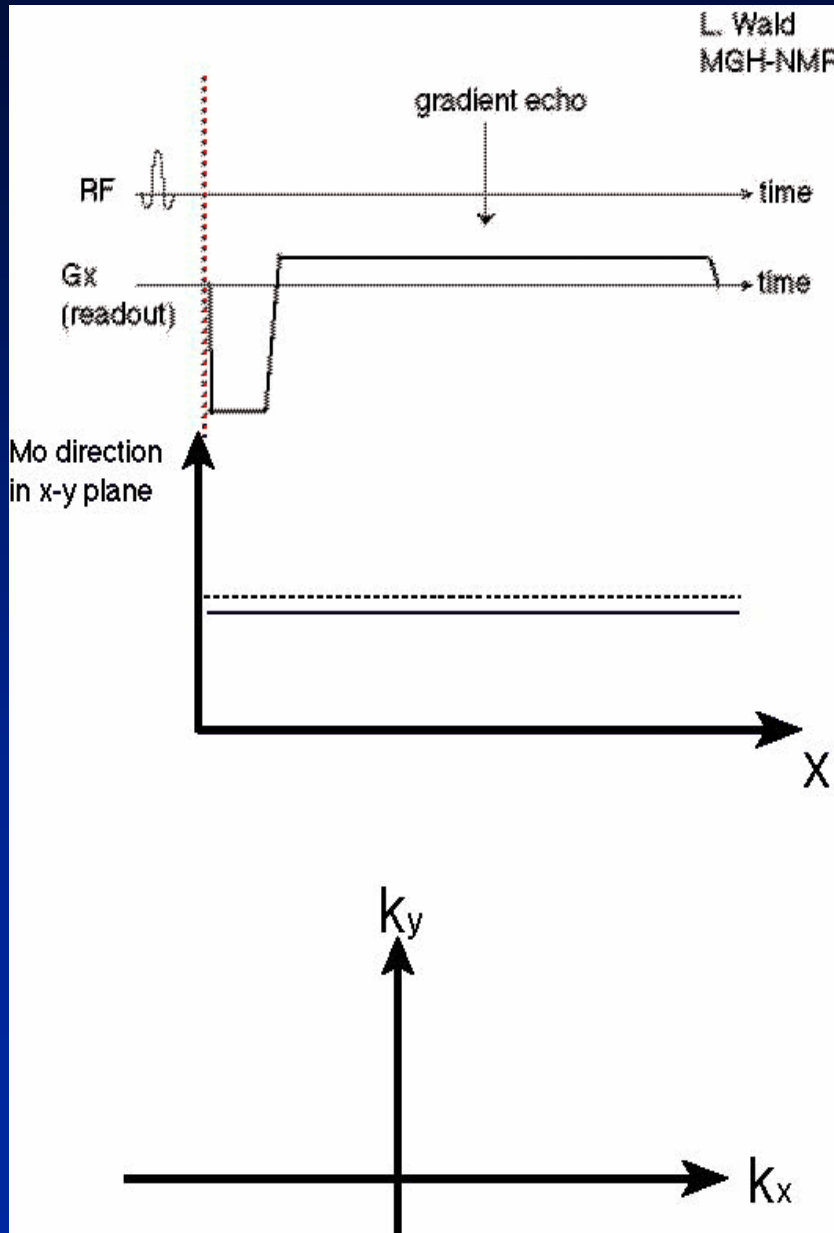


# Fourier transform



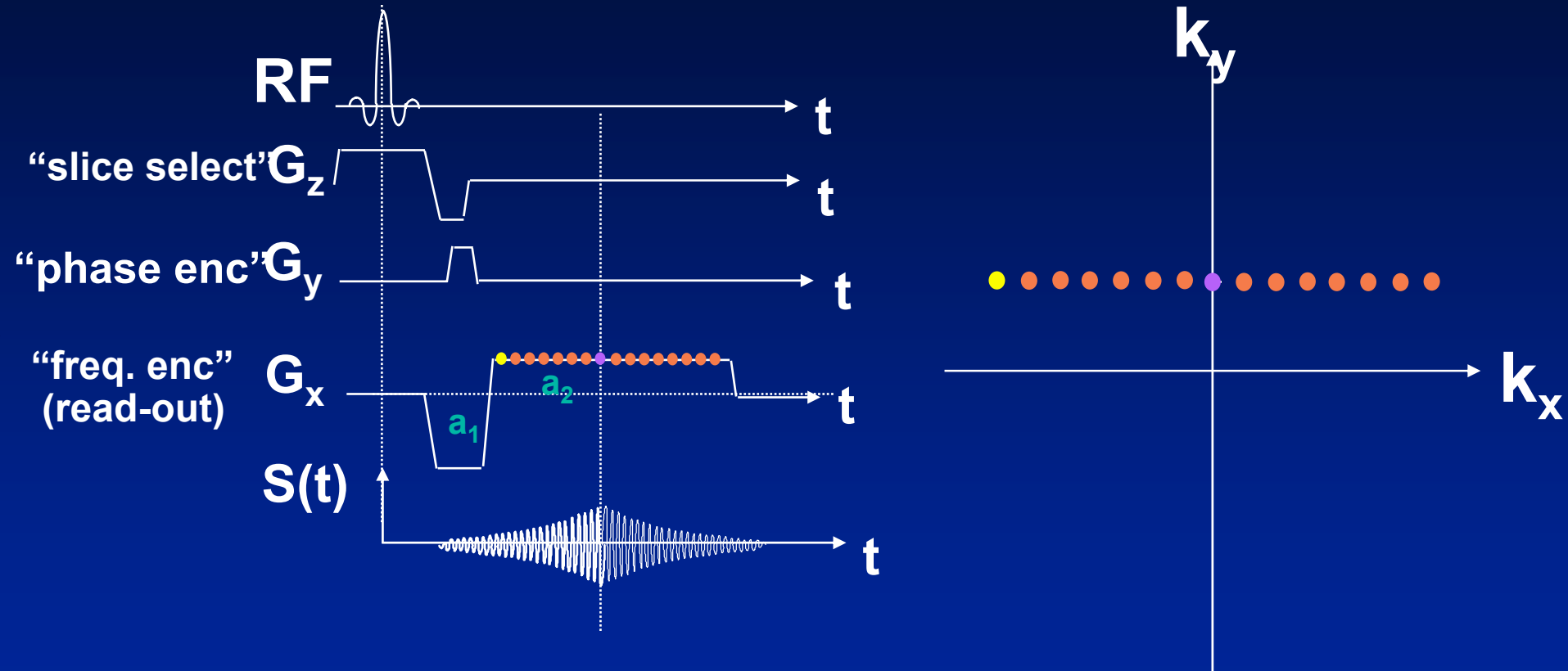
Horizontal double-headed arrow below the MRI image labeled  $\text{FOV}_x = \text{matrix} * \text{Res}_x$

# Frequency encoding revisited





# “Spin-warp” encoding



one excitation, one line of kspace...

# “Spin-warp” encoding mathematics

The “image” is the spin density function:  $\rho(x)$

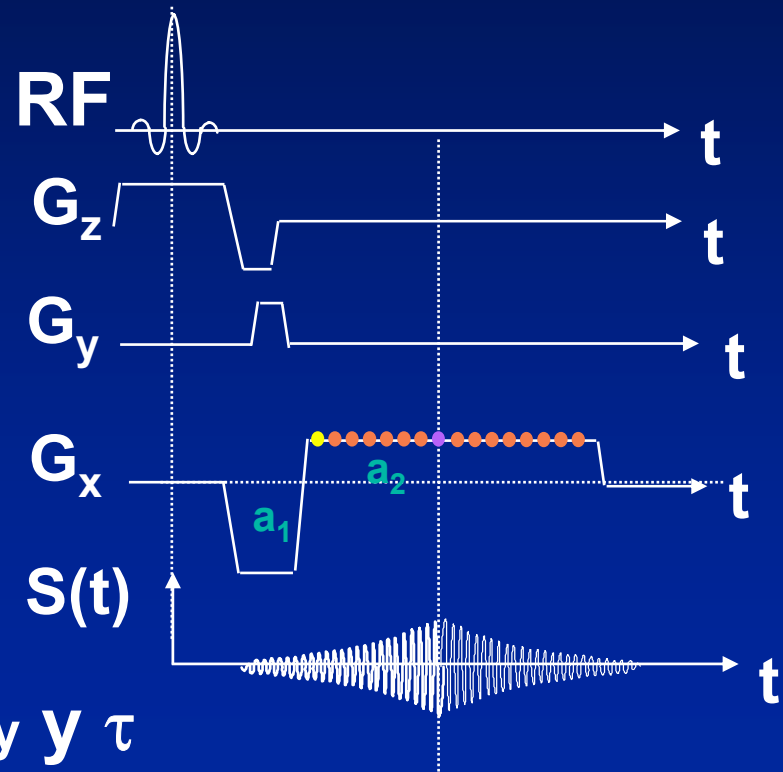
Phase due to readout:

$$\theta(t) = \omega_0 t + \gamma G_x x t$$

Phase due to P.E.

$$\theta(t) = \omega_0 t + \gamma G_y y \tau$$

$$\Delta\theta(t) = \omega_0 t + \gamma G_x x t + \gamma G_y y \tau$$



# “Spin-warp” encoding mathematics

Signal at time  $t$  from location  $(x,y)$

$$S(t) = \rho(x,y)e^{i\gamma G_x x t + i\gamma G_y y t}$$

The coil integrates over object:

$$S(t) = \iint_{\text{object}} \rho(x,y)e^{i\gamma G_x x t + i\gamma G_y y t} dx dy$$

Substituting  $k_x = -\gamma G_x t$  and  $k_y = -\gamma G_y t$ :

$$S(k_x, k_y) = \iint_{\text{object}} \rho(x,y)e^{-ik_x x - ik_y y} dx dy$$

# “Spin-warp” encoding mathematics

View signal as a matrix in  $k_x, k_y \dots$

$$S(k_x, k_y) = \iint_{\text{object}} \rho(x, y) e^{-ik_x x - ik_y y} dx dy$$

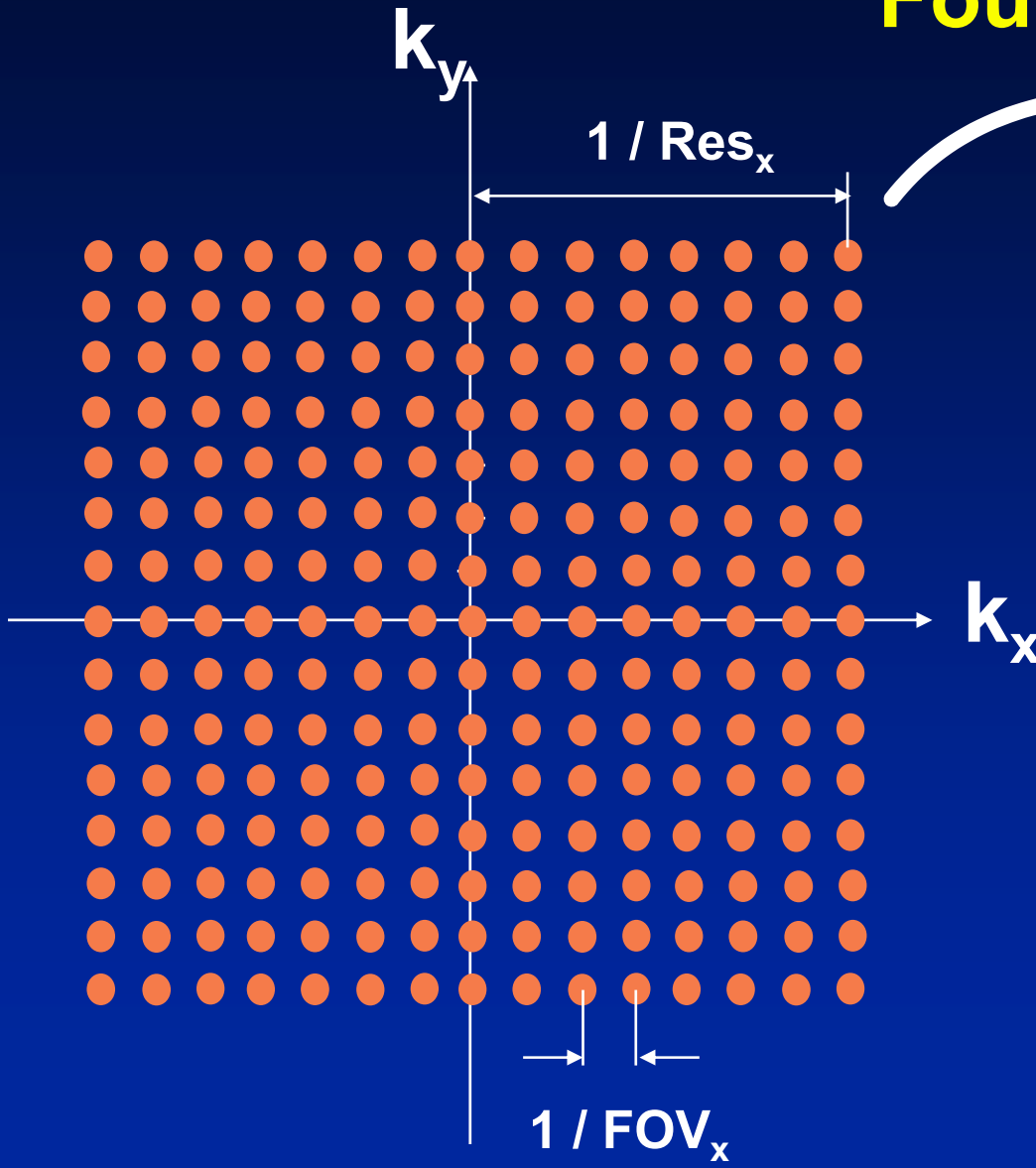
:

Solve for  $\rho(x, y)$

$$\rho(x, y) = FT^{-1} [S(k_x, k_y)]$$

$$\rho(x, y) = \iint_{\text{kspace}} S(k_x, k_y) e^{ik_x x + ik_y y} dk_x dk_y$$

# Fourier transform



$\text{FOV}_x = \text{matrix} * \text{Res}_x$

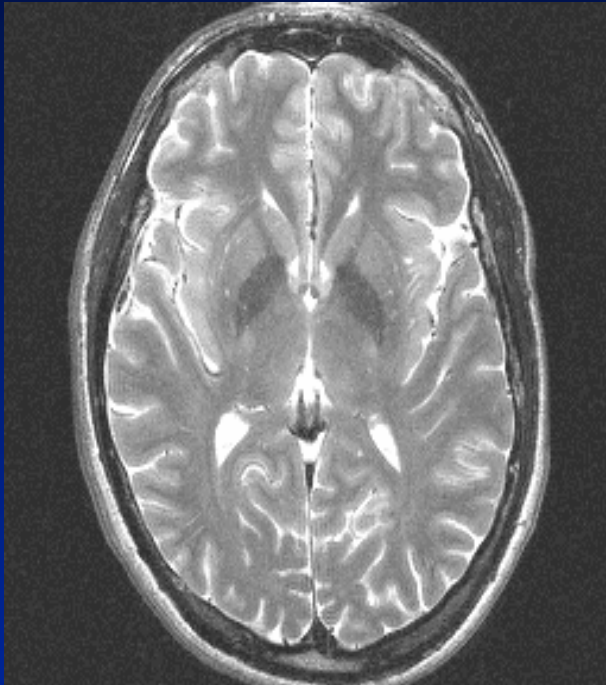
# Kspace facts

Resolution is determined by the largest spatial freq sampled.

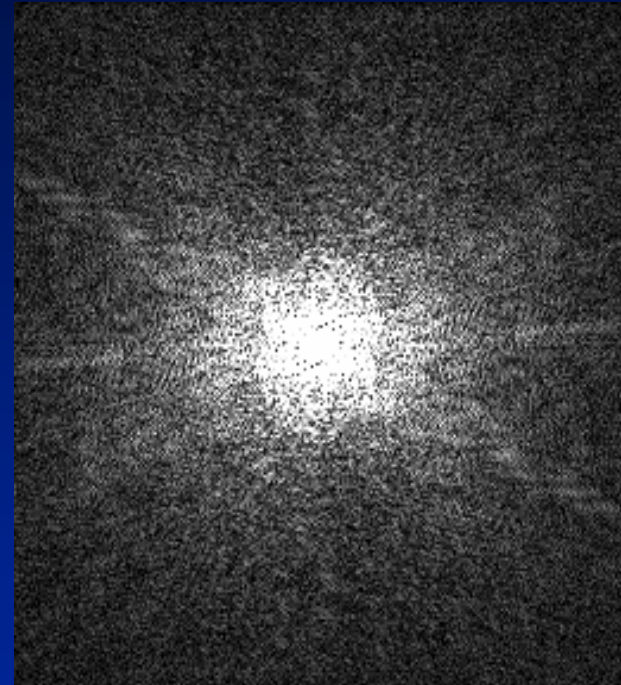
FOV = matrix \* resolution

If the object is real, half the information in kspace matrix is redundant. We only need to record half of it.

# kspace

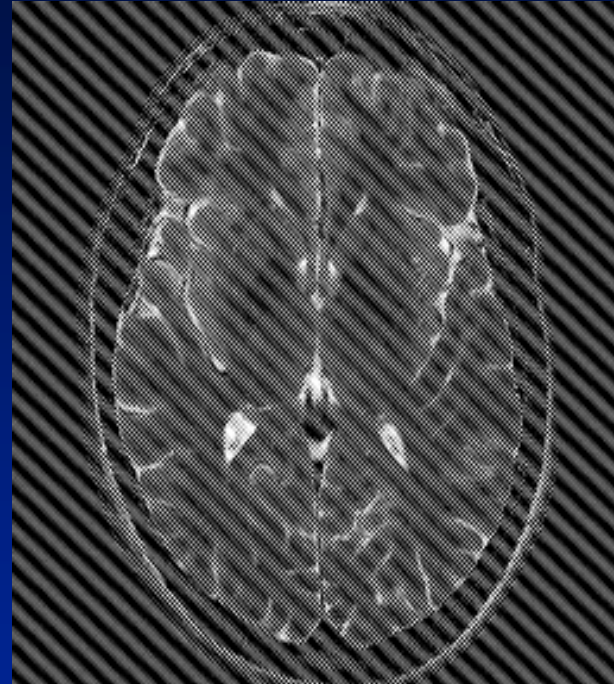
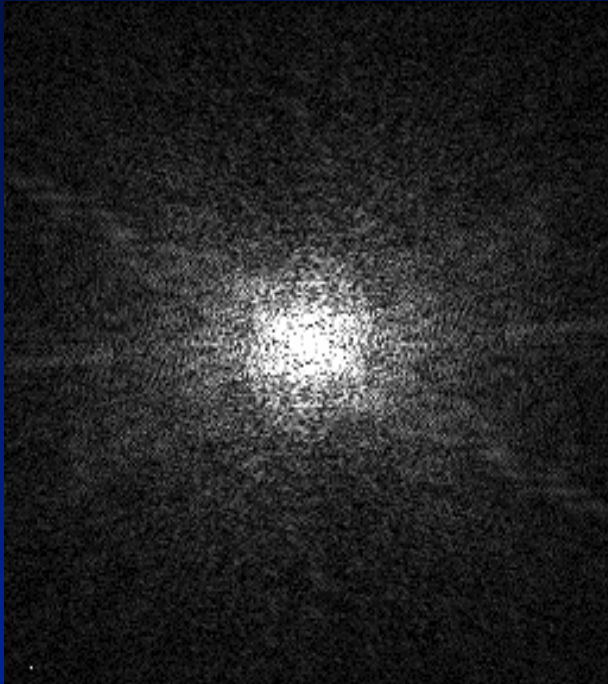


**Image space (magnitude)**



**k-space (magnitude)**

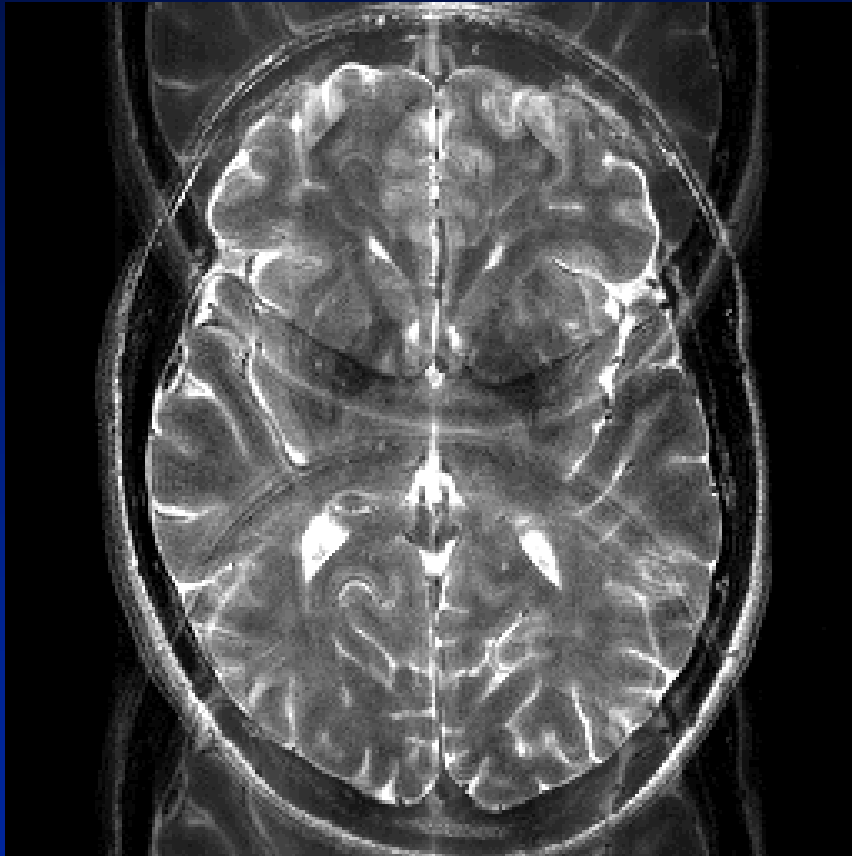
# kspace artifacts: spike



One “white pixel” in kspace from a electric spark



# Kspace artifacts: Symmetric N/2 ghost



Even numbered lines got  
 $\exp(i\phi)$

Odd numbered lines got  
 $\exp(-i\phi)$

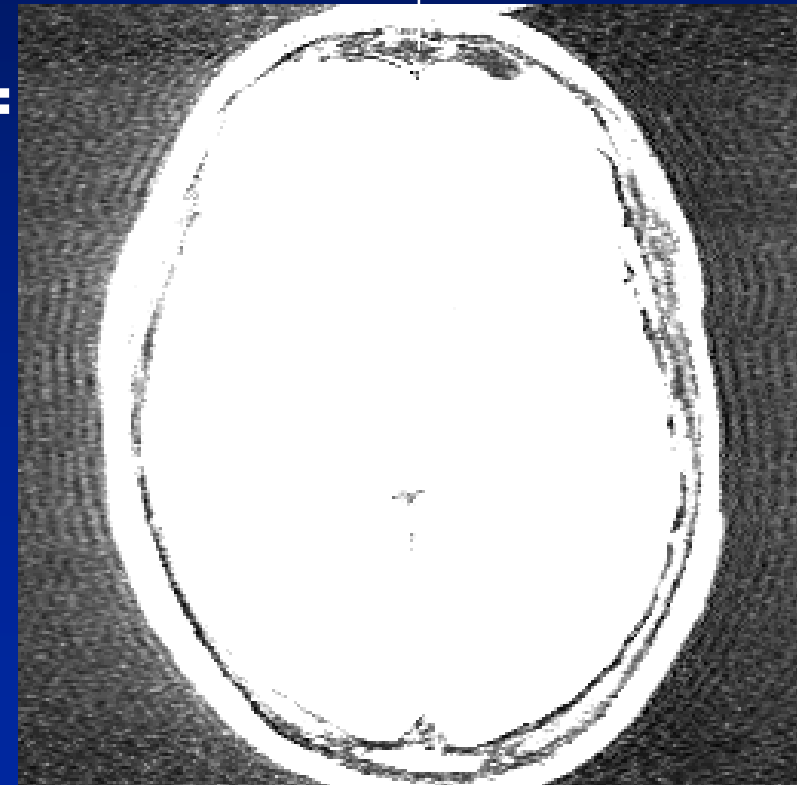
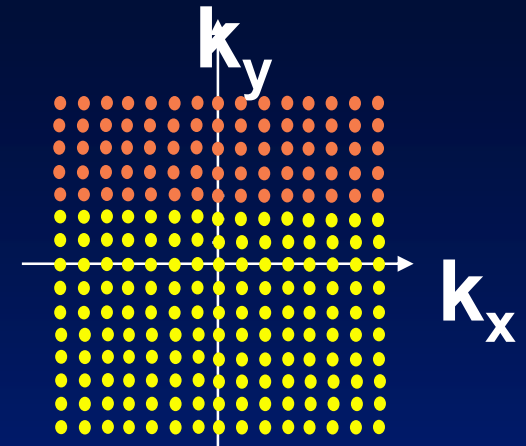
$\phi = 12$  degrees

# k-space artifacts: subject motion

Yellow = position1  
Orange = moved 2 pixels

Movement in real space =  
linear phase shift across  
k-space.

=> Orange points have  
linear phase  $\theta = a k_y$



# Fast Imaging

*“Dost thou love life?  
Then do not squander time,  
for that’s the stuff  
life is made of.”*

- Benjamin Franklin

# Requirements for brain mapping

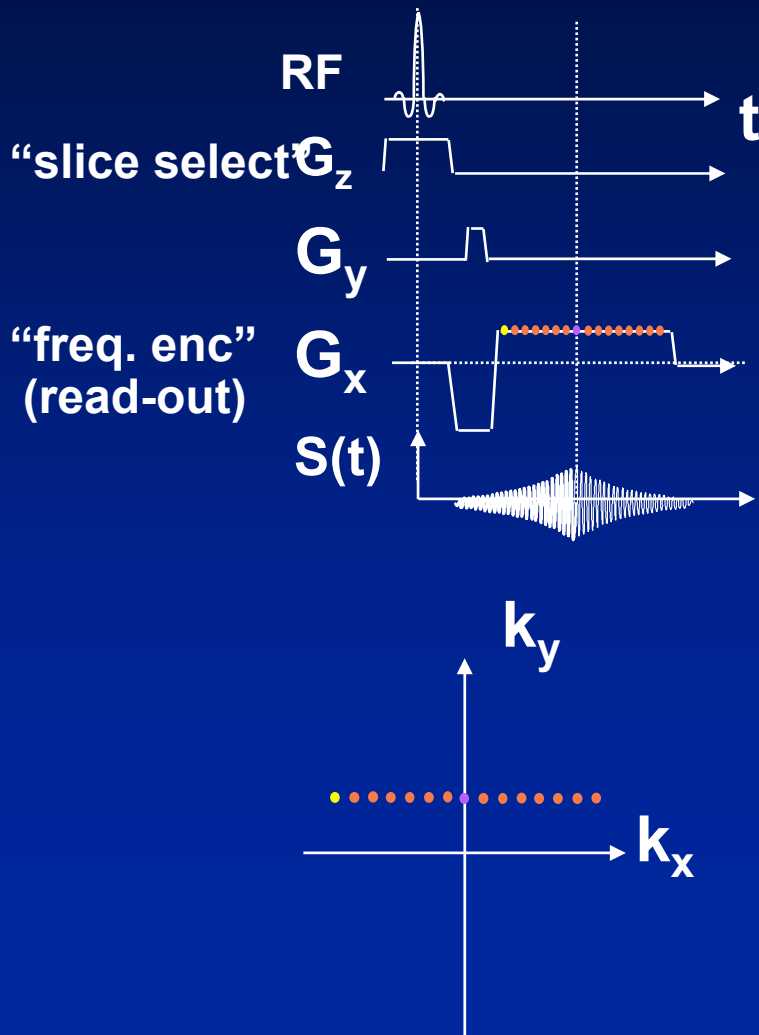
## Considerations:

- Signal increase = 0 to 5% (small)
- Motion artifact on conventional image is 0.5% - 3%
- Need to see changes on timescale of hemodynamic changes (seconds)

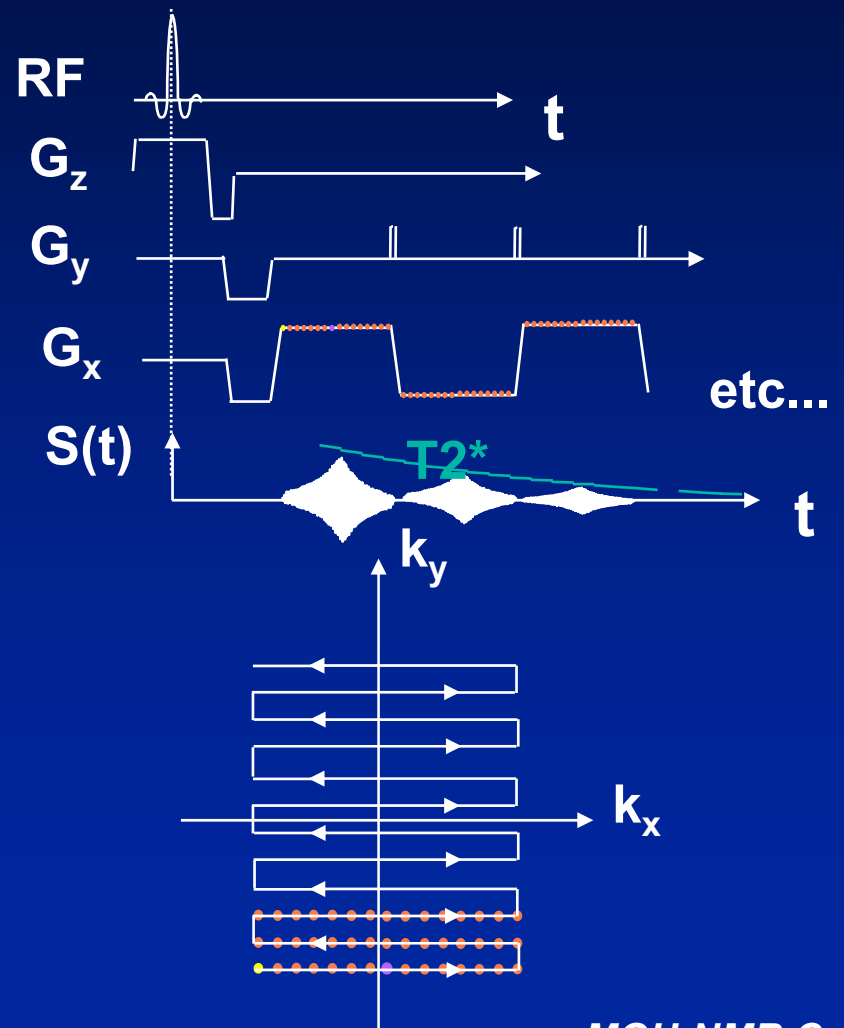
**Requirement:** Fast, “single shot” imaging, image in 80ms, set of slices every 1-3 seconds.

# What's the difference?

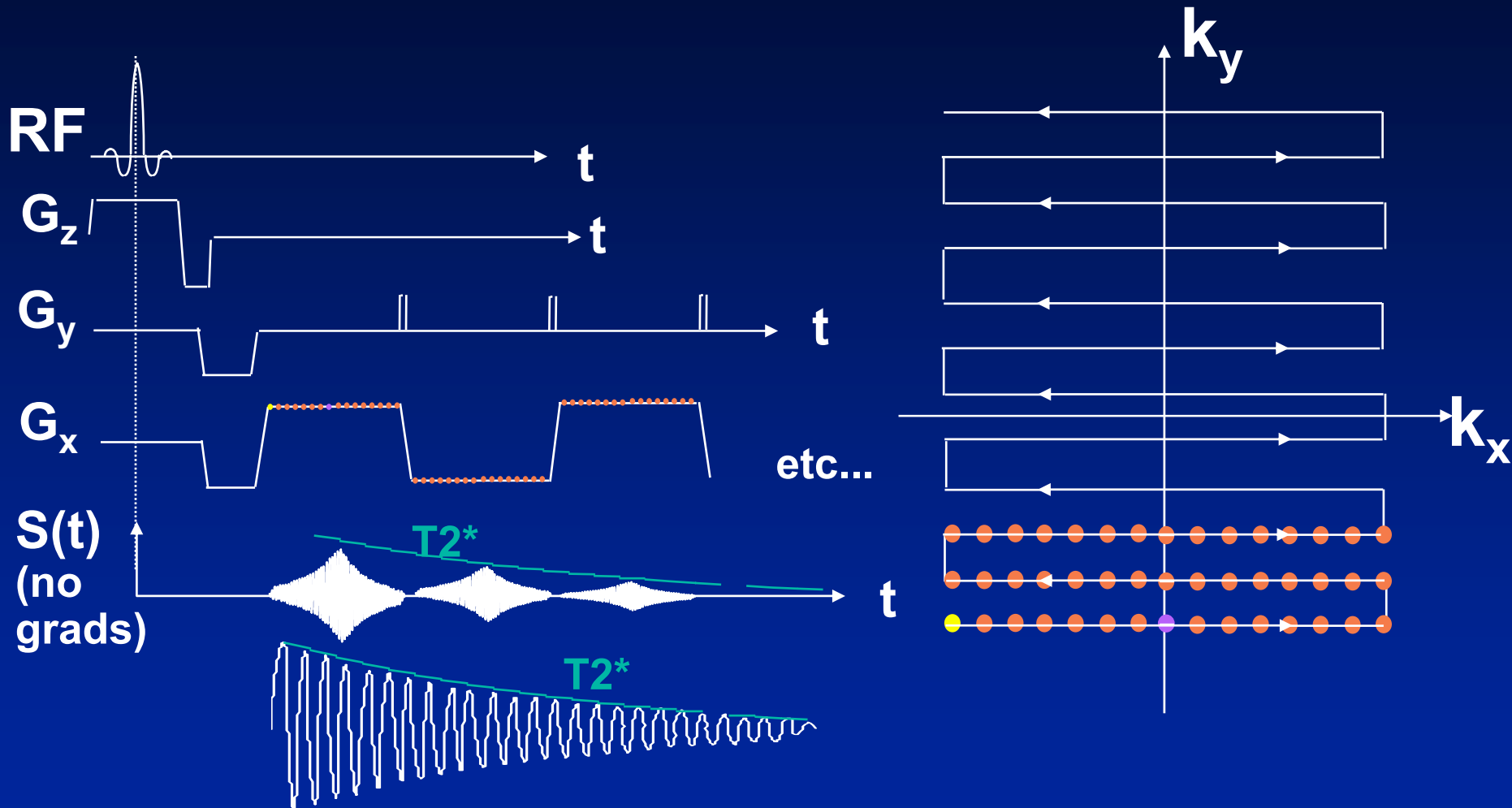
## conventional MRI



## echoplanar imaging



# “Echo-planar” encoding

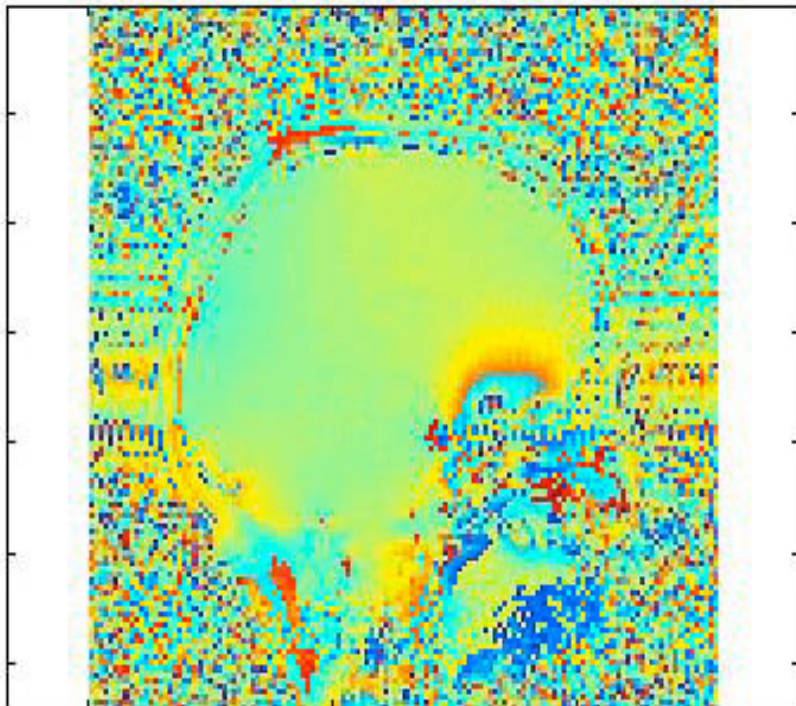


one excitation, many lines of kspace...

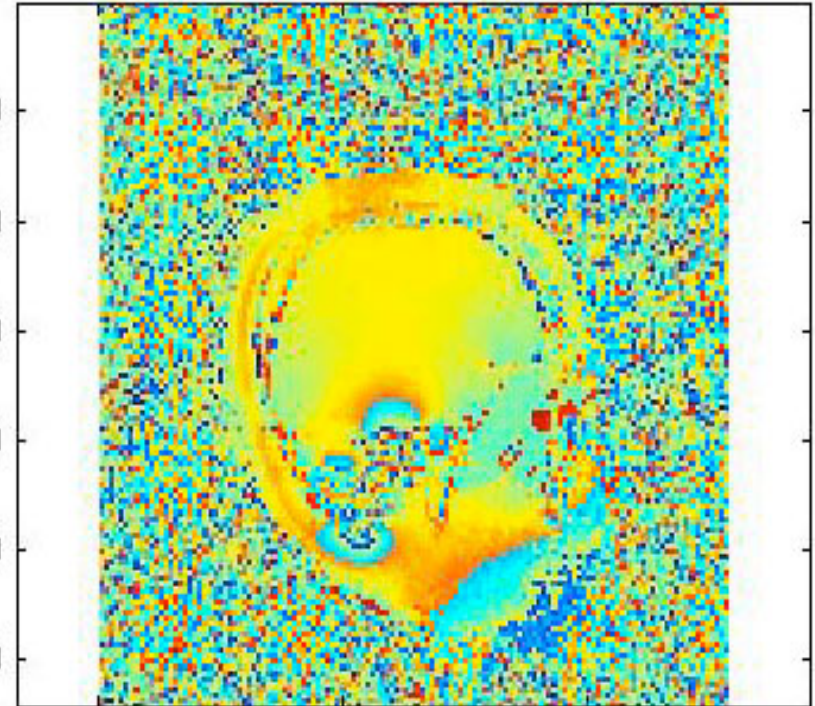


# Enemy #1 of EPI: local susceptibility gradients

Orbitofrontal susceptibility region



Lateral temporal susceptibility region



**$B_0$  field maps in the head**

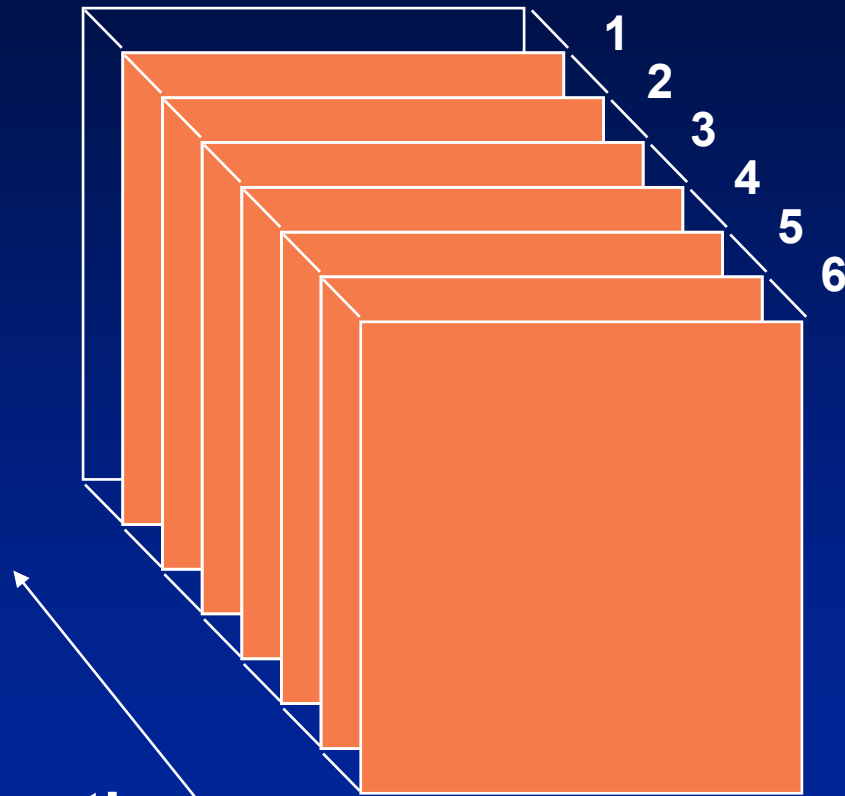


# **EPI: Local susceptibility gradients**

**Local susceptibility gradients have 2 effects:**

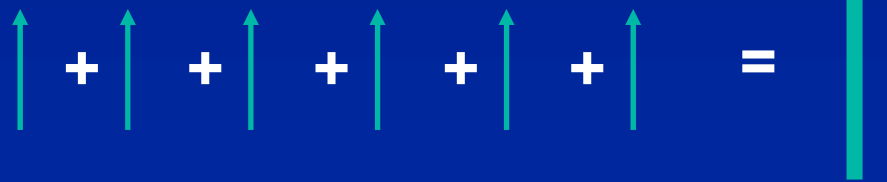
- 1) Local dephasing of the signal (signal loss) mainly from thru plane gradients**
- 2) Local geometric distortions, mainly from local in-plane gradients.**

# Susceptibility: thru plane dephasing

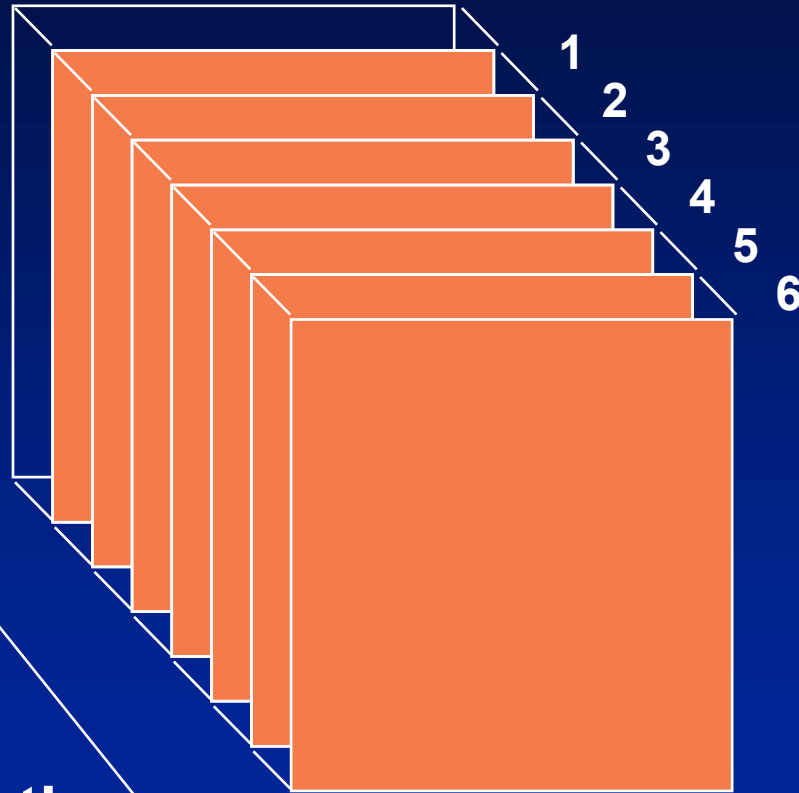


Magnetic  
Field Uniform

Signal from whole slice comes from adding together the MR vectors. When in phase, add constructively, SNR increases like slice thickness.

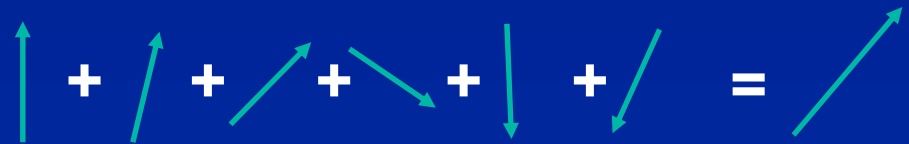


# Susceptibility Artifact and Slice Thickness



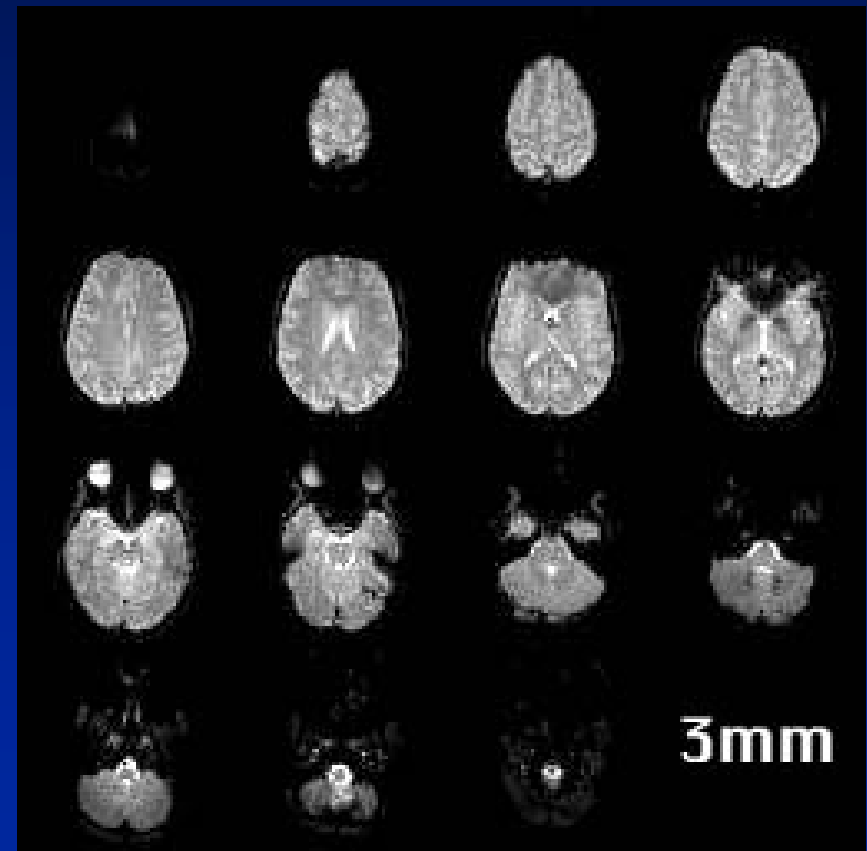
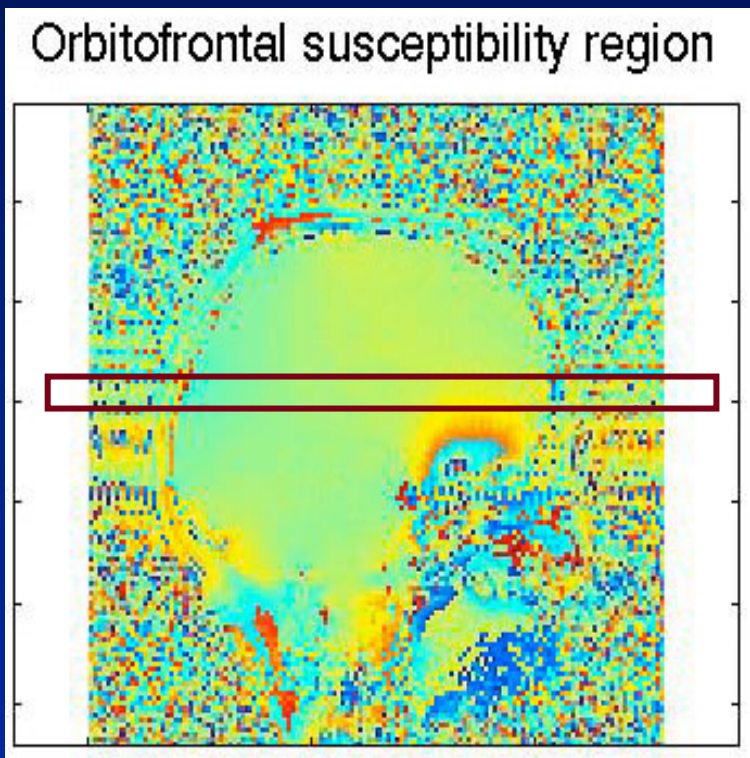
Magnetic  
Field NONUniform

Signal from whole slice comes from adding together the MR vectors, which get out of phase when the magnetic field is not uniform



# Local susceptibility gradients: thru-plane dephasing

Bad for thick slice above frontal sinus...



# Local gradients: geometric distortion

Local gradient alters the helix of phase we have so carefully wound.

Phase error accumulates over entire kspace.  
(conventional imaging phase is reset every line)

>> faster encoding is better.

Readout points are taken close together (~5us)

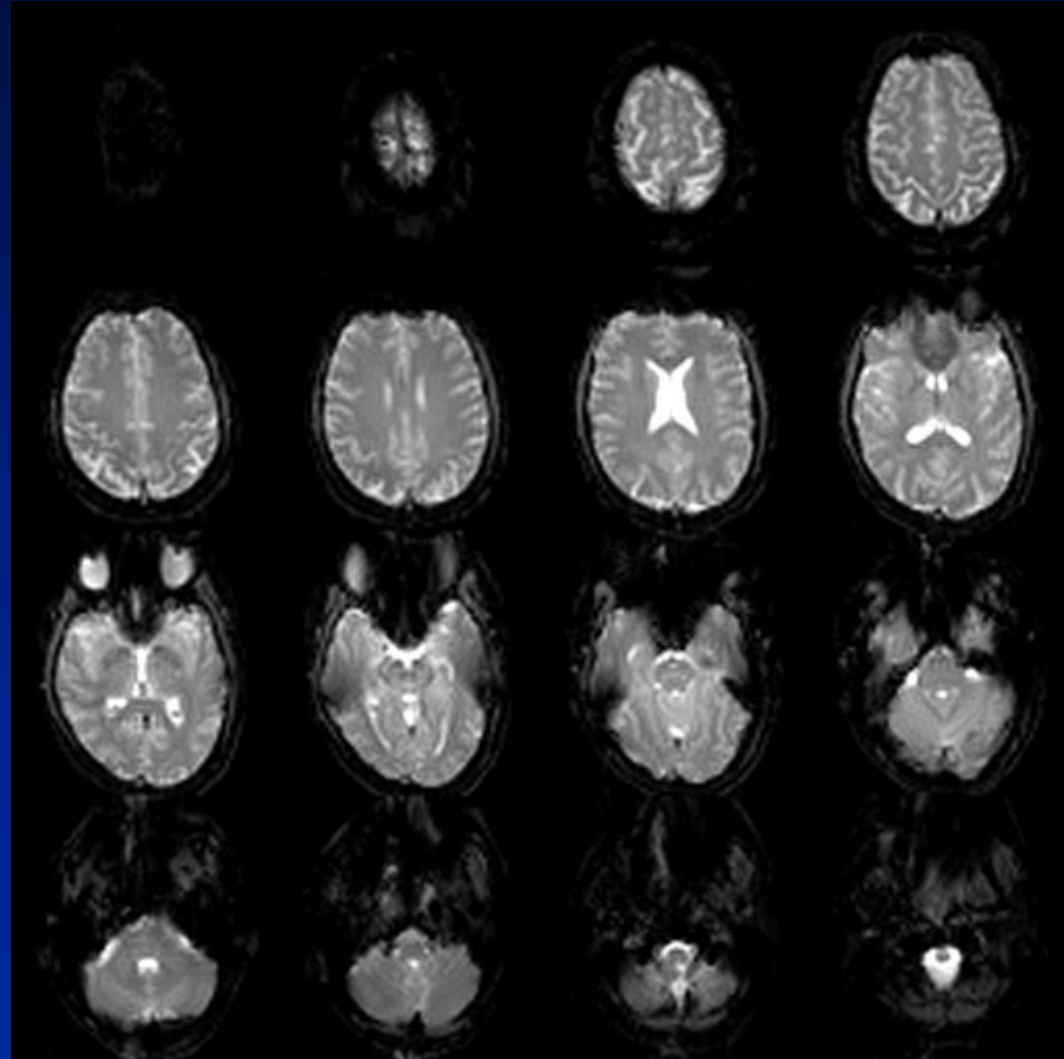
Phase encode points are taken farther apart (~500us)

>> distortion occurs in P.E. direction.

# Local gradients: geometric distortion

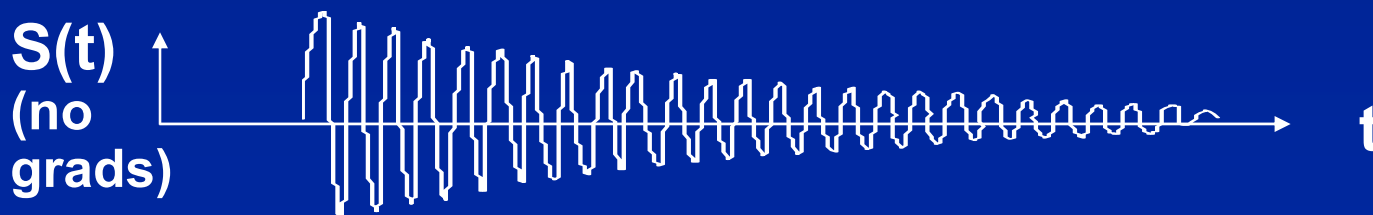
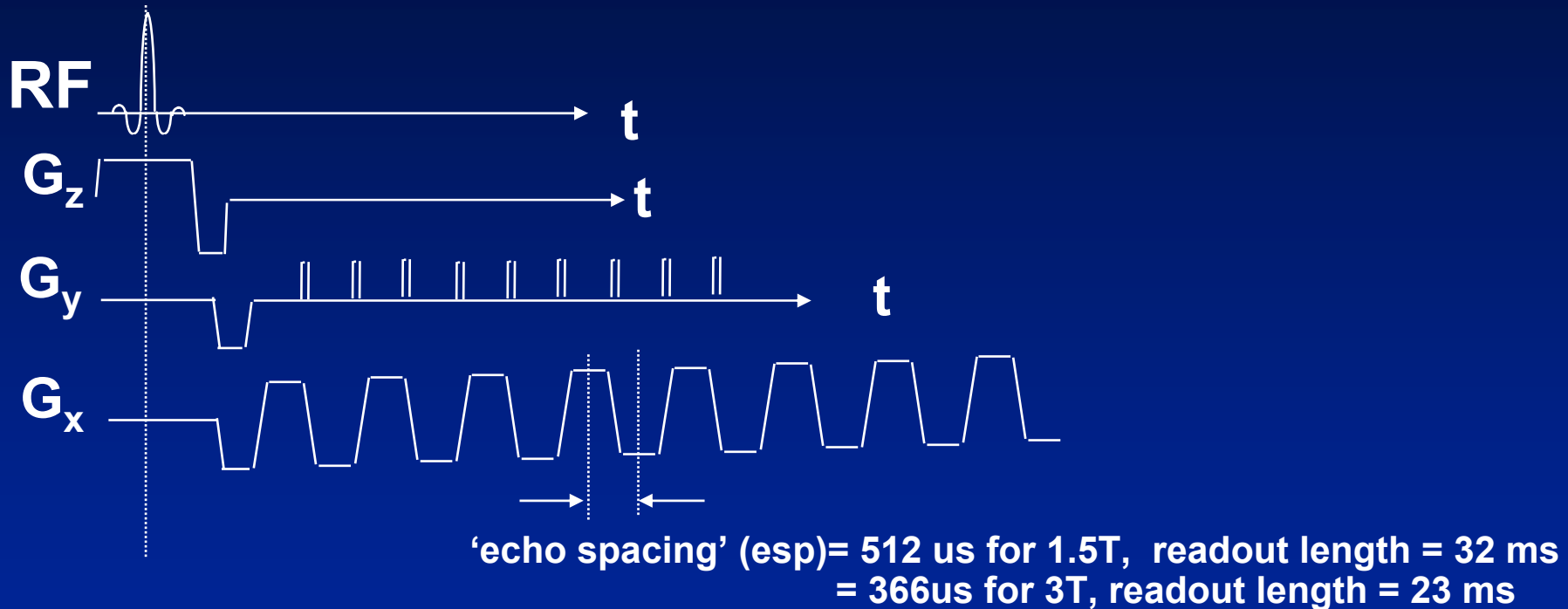
Two sets of EPI:

- 1) encode in 32ms
- 2) encode in 23ms



# Characterization of grad. performance

- length of readout train for given resolution (requires fast slew and high grad amplitude)

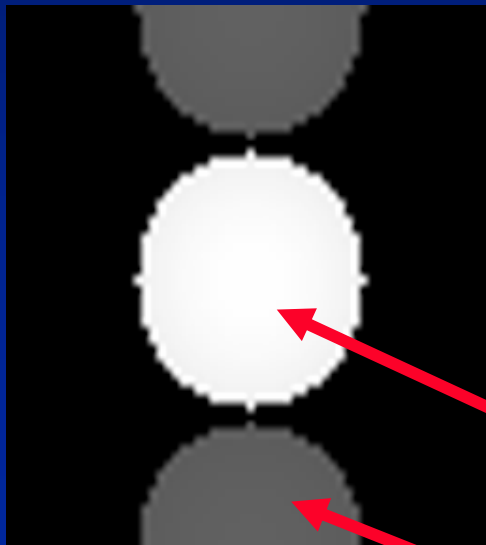


# EPI problems: N/2 ghost

Asymmetry in alternate lines gives N/2 image ghost.

Asymmetry from:

Eddy currents  
receiver filter  
receiver timing  
head coil tuning.



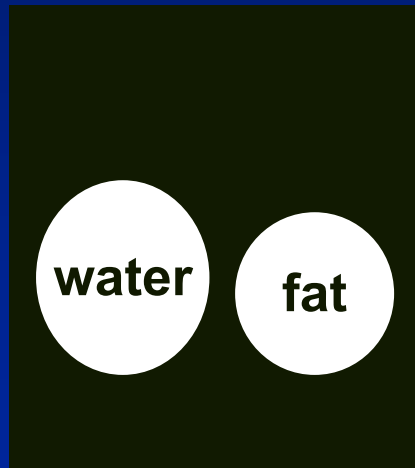
object

N/2 ghost

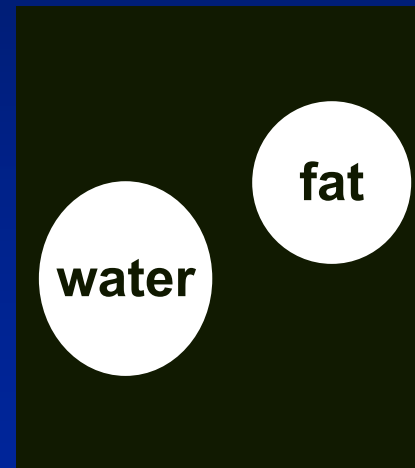


# EPI problems: frequency offset

If one object has a different NMR frequency (e.g. fat and water) it gets shifted in PE direction. (why?)

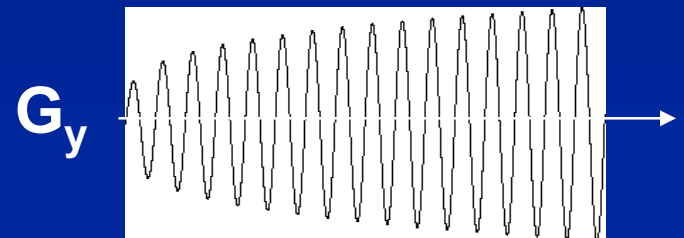
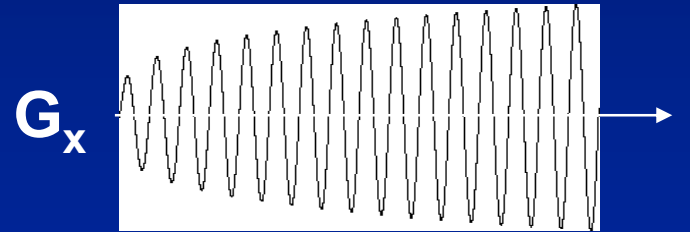
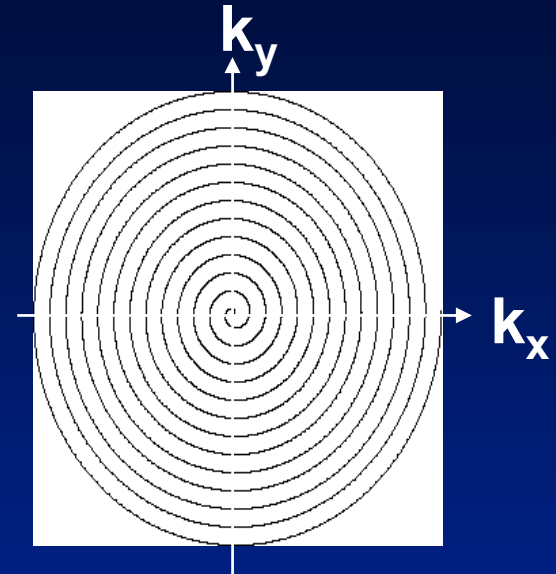
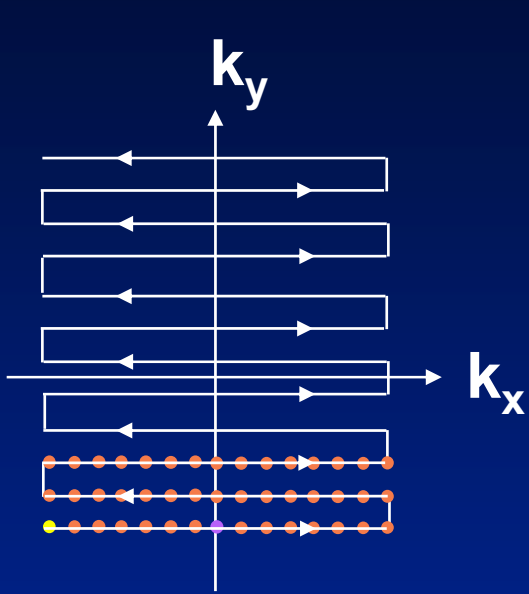


True location



Echoplanar image

# EPI and Spirals



## EPI

## Spirals

**Eddy currents:**

**ghosts**

**blurring**

**Susceptibility:**

**distortion,  
dephasing**

**blurring  
dephasing**

**$k = 0$  is sampled:**

**1/2 through**

**1st**

**Corners of kspace:**

**yes**

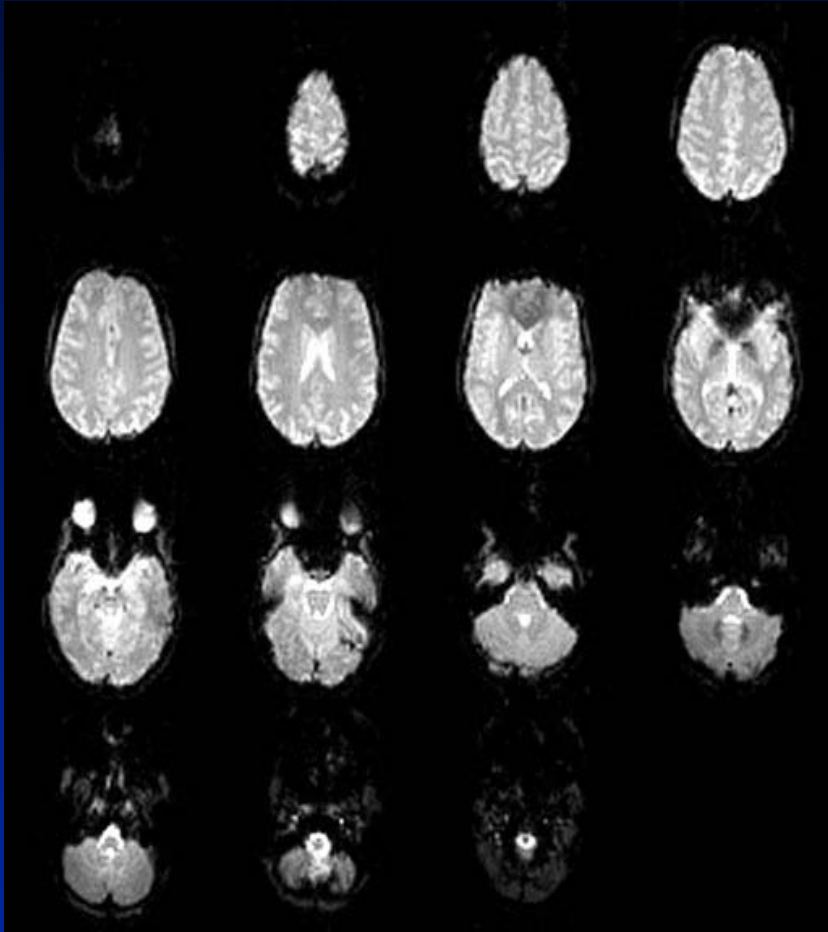
**no**

**Gradient demands:**

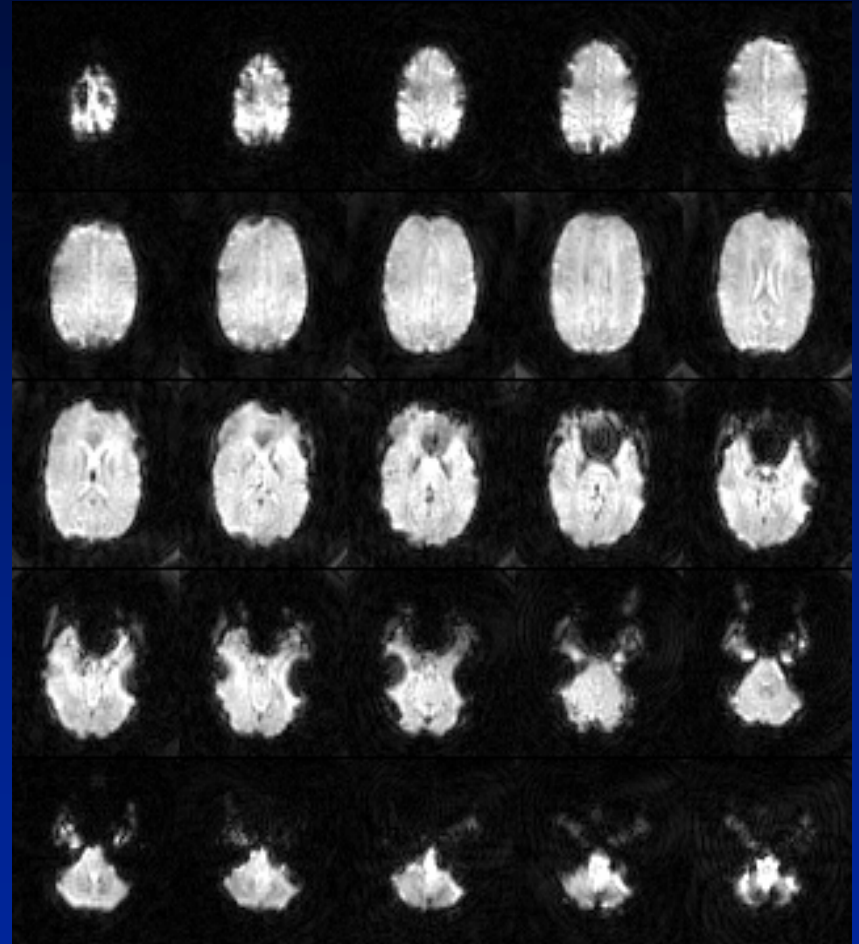
**very high**

**pretty high**

# EPI and Spirals



**EPI at 3T**



**Spirals at 3T  
(from G. Glover)**