### 14.03 Problem Set 3

Due in Class \#9

## Theory:

1) Suppose that a consumer's indirect utility is given by $V\left(P_{x}, P_{y}, I\right)=\ln I-a \ln P_{x}-b \ln P_{y}$ A) What are the uncompensated ("Marshallian") demand functions $d_{x}\left(P_{x}, P_{y}, I\right)$ and $d_{y}\left(P_{x}, P_{y}, I\right)$ ? (Hint: You'll need to use Roy's Identity to solve this problem. See p. 161 of Nicholson)
B) What is the expenditure function $E\left(P_{x}, P_{y}, U_{0}\right)$ that corresponds to the indirect utility function in (A)?
C) What are the compensated ("Hicksian") demand functions $h_{x}\left(P_{x}, P_{y}, U_{0}\right)$ and $h_{y}\left(P_{x}, P_{y}, U_{0}\right)$ ?
2) Continue with the utility function from problem set $\# 2$ where $U(x, y)=-1 / x-1 / y$. Let $I=18, P_{x}=1$ and $P_{y}=4$.
A) What are the Marshallian demands for $x$ and $y$ (okay to fill in answers from previous problem set)?
B) Suppose that the government gives the consumer an income subsidy of 12. What are the Marshallian demands now?
C) Suppose that the government instead decides to give an in-kind transfer of 12 units of $x$ that cannot be resold. What are the new consumption levels?
D) What utility does the consumer attain at these consumption levels (from part C)?
E) What is the minimum expenditure level required to attain the utility level calculated in part D if the consumer was buying the goods on the market at $P_{x}=1$ and $P_{y}=4$ ? F) What is the cash equivalent of the in-kind transfer, that is, how much would the government need to subsidize a consumer's income beyond the 18 she started with in order to make her as well off as with the in-kind transfer of 12 units of $x$ ?
G) Assume now that $x$ is food and $y$ is all other goods. You are a Senator about to plead before Congress for reauthorization of the Food Stamps program, an in-kind transfer program for low-income families that provides non-resaleable coupons redeemable for food at grocery stores. One of your assistants presents you with the analysis above and says, "Clearly, Senator, we should convert Food Stamps to a cash transfer program. We could make people
better off and save money." Although your assistant's math is sound (we assume!), you feel that this policy would be harmful to recipients. State clearly in three sentences or fewer why you believe that converting food stamps to a cash transfer program would be a bad policy. (Even if you do not believe these arguments, you should be able to articulate them. If you like, you may also write three sentences rebutting your arguments.)
3) Now suppose that the consumer's utility is $U(x, y)=(x y)^{0.5}$. Fix $P_{x}=P_{x 0}$ and $P_{y}=P_{y 0}$ and income equals $I_{o}$.
A) Suppose the government taxes good $x$ with a tax $t$ (hence $P_{x(t)}=P_{x 0}+t$ ), but compensates each consumer with a rebate so that the consumer can still remain on the original indifference curve $U(x, y)=V\left(P_{x}, P_{y}, I_{0}\right)$. This is called a "compensated" tax. Given the prices and the tax, what "rebate" is required to keep the consumer on the original indifference curve? (Equivalently, what difference in expenditures is required for a consumer to attain the old utility with the new tax-adjusted prices?)
B. How much tax revenue is raised with the compensated tax as a function of prices, income, and $t$ ? You can calculate this directly using the information on tax revenues and rebates, or as the difference between consumer expenditures at the post-tax consumption bundle and the income received by stores at that bundle.
C) What is the deadweight loss from taxation? Must it always have the same sign? Can it ever be zero? If so, when?
4) Continuing with the same utility function, suppose initially that $P_{x}^{0}=1, P_{y}^{0}=2$ and that they increase to $P_{x}^{1}=2, P_{y}^{1}=3$. We use different price indices to estimate the change in the cost of living. For each, give the formula for arbitrary prices and incomes and then plug in to calculate the answer for this example.
A) What is the change in the ideal cost of living index?
B) What is the change in the Laspeyres and Paasche indices?
C) Draw two graphs, each of which illustrates the bias in the Laspeyres and Paasche indices. Do you use the same "reference utility" in each exercise? Explain.

## Application: CPI articles \& the Deadweight Loss of Christmas

5) The Boskin commission argues that the presence of new discount retail outlets (like Costco or Sam's Club) leads to a bias in the CPI. What is the assumption embodied in the CPI calculation that leads to this bias? If you were presented evidence that these retail outlets were growing rapidly in market share, would you count this as evidence supporting the assumption in the CPI calculation or against it? Why or why not?
6) The editorial distributed in class from the St. Louis Post-Dispatch argues that, "If the average family can afford only chicken instead of beef, does this mean inflation is being overstated? Hardly." Does this argument make sense? Explain briefly and precisely.
7) Katherine Abraham, Commissioner of the Bureau of Labor Statistics (the agency that puts out the CPI), in public comments disputing the relevance of the substitution effect said, "If the price of dryers went down, I don't think people would buy more washing machines." Is this a valid argument against the Boskin Commission's findings? Explain briefly and precisely why or why not?
8) Cellular telephone service was introduced in the U.S. in 1983, but not included in the CPI until 1998. What do you think happened to the price of cell phone service between 1983 and 1998, and what does this imply for the CPI? Would there still be a bias in the CPI if the Bureau of Labor Statistics had started including cell phones in 1983? Explain.
9) Joel Waldfogel writes in his article The Deadweight Loss of Christmas that, "Estimates in this paper indicate that between a tenth and a third of the value of holiday gifts is destroyed by gift giving." (A) Explain in one or two sentences and/or with a diagram what Waldfogel means. (B) Offer a brief Revealed Preference argument to suggest that Waldfogel's conclusion may be misleading. You may want to take into account the utility of the gift giver in addition to the utility of the gift receiver. (Bear in mind that giving a cash gift is always an option.) (C) Explain how your Revealed Preference argument is consistent with the results in Table 3: Gift Yield and Tendency to Give Cash, by Identity of Giver.
