21-cm Radio Astrophysics

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Measurement of the Doppler spectrum of interstellar atomic hydrogen and the dynamics of the galactic rotation. A 2.5-meter computer-controlled alt-azimuth parabolic dish antenna, located on a roof of MIT, is used with a heterodyne measurement chain and digital correlator to observe the Doppler spectrum of the 21-cm hyperfine line of interstellar atomic hydrogen in various directions along the Milky Way. Features of the spiral-arm structure of the Galaxy are deduced from the measured radial velocities of the hydrogen clouds in the galactic disc.

PREPARATORY PROBLEMS

1. Explain the origin of the 21-cm line of atomic hydrogen.

2. Describe the size and shape of the Galaxy, giving our position from the center in light-years and parsecs (pc). What is the maximum Doppler shift you can expect to observe in the 21-cm line?

3. The kinetic temperature of the sun's corona is several million degrees. Why aren't we burned to a crisp? The surface temperature of the sun is ∼6000 K. The temperature of the sun measured by this experiment is somewhere in between these two extremes.

4. The parabolic dish antenna used in this experiment has a diameter of 2.5 m. Suppose it is set so that a source of 21-cm radio emission drifts through its field of view. Plot the signal strength at the output of the diode detector as a function of angle from the center of the field of view.

5. Explain the 21-cm spectrum at 150° and 60° as shown in Figure 3.

6. Explain how the Fourier Transform Spectrometer works.

INTRODUCTION

The following description of the Galaxy is a summary of the material contained in Galactic Astronomy by Mihalas and Binney, a very useful reference for this experiment. It also contains in Chapter 8 a detailed presentation of the theoretical basis for interpreting the data on galactic kinematics obtained from observations of the Doppler profile of the 21-cm line.

The sun is one among the approximately 10^{11} stars that comprise our Galaxy. From a distant vantage point this vast array of stars would appear as a rotating spiral galaxy, rather like our famous neighbor galaxy, M31, in the constellation of Andromeda. Detailed examination would reveal a spheroidal component and a concentric disk component. The spheroidal component consists mostly of small (M$< 0.8M_\odot$), old (10-15 billion years) stars (so-called population II) of which most are concentrated in a central "bulge" with a diameter of 3 kpc and the rest are distributed in an extensive "halo" that extends to a radius of 30 kpc or more. The disk component is a thin, flat system of stars, gas and dust ∼200 pc thick and ∼30 kpc in radius. The disk stars (called population I) are extremely heterogenous, ranging in age from newborn to the age of the halo stars, and with masses from 0.1 to 100 $M_\odot$.

The solar system is imbedded in the disk component at a distance of 8.0±0.5 kpc from the center at which radius the circular motion of the Galaxy has a period of approximately 2x10^8 years. The naked eye can distinguish about 6,000 of the nearby stars (apparent brightness down to 6 mag) over the entire sky; the myriads of more distant stars of the disk blend into what we perceive as the Milky Way, which is our Galaxy seen edge on.

Between the stars of the disk is the interstellar medium (ISM) of gas and dust comprising approximately 10% of the total mass of the disk. The gas consists primarily of hydrogen and helium with a mass density ratio of 3 to 1 and an average total number density of about 1 atom cm^{-3}. The dust is composed mostly of graphite, silicates and other compounds of the light and common elements in microscopic grains containing a small fraction of higher-Z elements, primarily iron. The effect of dust in blocking the light of more distant stars is clearly seen in the "dark lanes" that are conspicuous features of the Milky Way in the region of, e.g., the constellations Cygnus and Aquilla.

The pressure in the ISM is conveniently characterized by the product nT, where n is the density in atoms cm^{-3} and T is the temperature in Kelvins. It is roughly constant in the disk and of the order of 3000 K cm^{-3}. A wide range of observations across the electromagnetic spectrum show that the ISM tends to exist in one of three different states: hot, warm and cool, with temperatures of 10^6, 10^4, and 10^2 K, respectively, and corresponding low, medium and high densities. Within the cool regions there are particularly dense cold regions (T≈ 20K) known as molecular clouds, containing H_2, OH, and other more complex molecules. These clouds are the birthplaces of stars that are formed by gravitational contraction of the cloud material. When a burst of star formation occurs in a molecular cloud, the massive stars heat the surrounding ISM to form a warm region of 10^4 K. After short lives of a few million years the massive stars explode as supernovas that spew into the ISM portions of the heavy elements that have been synthesized by nuclear fusion in their interiors and by neutron capture during their explosions. The explosions blow "bubbles" of 10^6 K gas that merge to form an interconnecting network of hot regions. The hot gas eventually cools and is recycled through molecular clouds in a continuous process that gradually enriches the ISM with the Z>2 elements of which the Earth and we are composed. Dynamical studies show that there also exists "dark matter" of unknown nature with a mass comparable to or even larger than the total mass of the luminous stars.

An essential key to the development of modern astrophysics was the invention of the spectroscope and its application with photography near the turn of the century to the study of stars and hot nebulae in the visible range of the spectrum. Comparison of stellar spectra with the line spectra of elements in the laboratory yielded information on the composition and temperature of celestial objects, and measurement of wavelength shifts due to the Doppler effect provided determinations of radial velocities which revealed the dynamical properties of systems such as double stars and the general expansion of the universe in the motions of distant galaxies. Most of the interstellar medium, however, is too cold to radiate in the visible part of the spectrum and remained undetectable and its properties largely unknown until fifty years ago. Then, in the midst of World War II, a young Dutch astronomer, H. van de Hulst, examined the theoretical possibilities for detecting cosmic radio waves of some distinct frequency, i.e. a spectral line in the radio portion of the electromagnetic spectrum which would permit measurements of physical conditions and radial motions like those available in the visible spectrum. He predicted that the spin-flip transition of atomic hydrogen would produce such a line at a wavelength near 21 centimeters, and that the unique conditions of low density and temperature in interstellar space are such as to allow time for hydrogen atoms, excited by collision to their hyperfine triplet state (F=1), to decay by radiation to the ground state (F=0). The line was observed in 1951 by Ewing and Purcell at Harvard, by Christiansen in Sydney, and by Muller and Oort in the Netherlands. Radio observations at 21 cm soon became a major tool of astronomy for exploring and measuring the structure of our Galaxy and the many distant galaxies accessible to the giant radio telscopes such as the giant single dish antenna at Arecibo in Puerto Rico, the 20-km diameter aperture synthesis array (Very Large Array) in New Mexico, and the transcontinental aperture synthesis array called the VLBA (Very Long Baseline Array).

Atomic hydrogen is the principal constituent of the interstellar medium, and one of the most interesting tracers of galactic structure. Whereas visible light emitted by stars is heavily obscured by interstellar dust, photons with a wavelength of 21 centimeters, emitted in spin-flip transitions of atomic hydrogen in its electronic ground state, reach us from all parts of the Galaxy with little absorption. Measurements of the Doppler shifts of the frequency of the 21-centimeter line determine the radial components (projections onto the line of sight) of motions of the interstellar medium. Analysis of the Doppler shifts in various directions around the galactic plane (i.e. around the Milky Way) reveal the kinematic structure of the galaxy, e.g., the tangential velocity of the matter in the Galaxy as a function of the distance from the center, and, by implication, the geometry of the spiral arms.

With modern electronics the radiation from Galactic atomic hydrogen is easily detected. Several years ago Professor Bernard Burke initiated a project to develop a radio astrophysics experiment for the Junior Lab to measure the spectra of 21-cm atomic hydrogen emission from the Milky Way. The equipment (developed by Burke, Jack Barrett of the radio astronomy group, and Grum Teklemariam of the Junior Lab) consists of: (1) a radio telescope; (2) a low-noise radio receiver; (3) a heterodyne amplifier which mixes the 1420 MHz hydrogen signal down to a readily digitizable 0-3 MHz 'baseband' signal and (4) a PC that controls the telescope, digitizes the signal, calculates the power spectrum and displays the results. Valuable contributions to the software were made by Andrew Kirmse '95. Part (4) was upgraded during the summer of 2000 and now incorporates a software based total power monitor and power-spectrum calculation.

EXPERIMENTAL APPARATUS

The radio telescope is a refurbished 2.5-meter diameter parabolic dish antenna that was used in research by the radio astronomy group in the 1970's and has since been resting on the roof of the Compton Laboratory.
It is mounted on an equatorial mount with roller bearings and rack-and pinion gears driven by stepping motors powered by pulse trains initiated by the computer.

To initiate an observation the observer enters (via the PC keyboard) the galactic latitude ('b') and galactic longitude ('l') of the position to be observed. A summary of celestial geometry is provided in Appendix A. The software performs a check as to whether this portion of the galaxy is currently visible within the mechanical limits of the dish and indicates whether the target is accessible or not. The control program computes the number of pulses required to turn the telescope in Hour Angle (HA) and Declination (DEC) to that position, and the PC then sends the pulses to the stepping motors. When

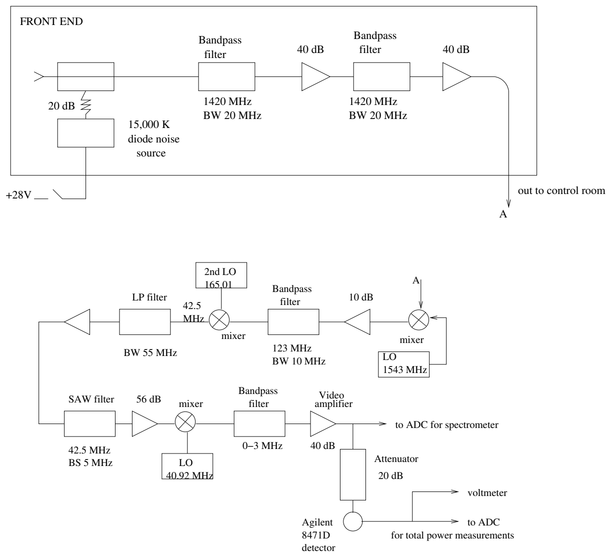

FIG. 1: Schematic of Radio Astrophysics Experiment

the telescope reaches the position, if "tracking mode" is on, the PC continues to generate HA pulses at the rate required to compensate for the earth's rotation so as to keep the telescope pointing to the specified position in the sky.

The diameter of the telescope aperture is about 12.5 times the 21-cm wavelength of the hydrogen line. The corresponding angular resolution is approximately 5°, sufficient to map the distribution of hydrogen as a function of galactic longitude, but not fine enough to resolve the latitude structure.

A focal plane detector and feed was built and a radio receiver of standard design was assembled from parts gleaned from unused equipment in the radio astronomy laboratory and items purchased with the help of a matching funds grant from the National Science Foundation. The system is interfaced to a PC computer that sends control signals to the radio telescope, digitizes the signal following the last stage of heterodyne mixing, calculates the power spectrum and performs the signal averaging.

RECEIVER

Figure 2 shows a block diagram of the radio receiver and subsequent signal processing stages. Radio power arriving from directions close to the axis of the parabolic antenna is focussed by reflection to a simple dipole feed which is connected by a short coaxial cable to the preamplifier located in a container immediately behind the feed. Amplification of 80 db is provided by two off-the-shelf transistor amplifiers (MITEQ AM-4A-0515), with an effective noise temperature of about 300 K. The preamplifier includes two 20 MHz -wide bandpass filters (K & L Microwave 3C45-1420.4/20-NP/OP) to exclude out-of-band interference, and is preceded by a directional coupler through which calibrated noise signals can be injected.

The amplified signal is then transmitted by coaxial cable from the telescope to the control room. There the signal is converted from 1420 MHz to 122.6 Mhz by heterodyning (mixing) with a crystal-controlled 1543 MHz local oscillator. The intermediate-frequency signal is amplified by a standard broad-band amplifier, with a bandpass filter that reduces the signal bandwidth to 15 MHz centered on 122.5 MHz. The next stage mixes the signal with the output of a frequency synthesizer that converts the signal to a second intermediate frequency of 42.5 MHz. The 42.5 MHz signal is then passed through a narrow-band filter, 5 MHz wide. The final frequency conversion, to baseband (0-3 MHz) is accomplished with a crystal oscillator at 40.92 MHz. After video amplification, the signal can be routed for either total power measurements or spectroscopy.

Mixers are circuit elements that form the product of two analog waveforms (in NMR these elements are also called "phase detectors"). It's two inputs and output are related by the trigonometric relationship:

$$\cos\omega_1 t \cos\omega_2 t = \frac{1}{2}\cos(\omega_1+\omega_2) + \frac{1}{2}\cos(\omega_1-\omega_2) \quad (1)$$

Note that both sum and difference frequencies are generated in a mixer. For our application, we band-pass filter the output and throw away the sum frequency contribution. A "balanced" mixer is one in which only the sum and difference frequencies, and not the input signals or their harmonics, are passed, see Reference [7] for more information on mixers.

For total power measurements, the signal is passed through an attenuator to an Agilent 8471D detector. This "square-law" detector produces a voltage proportional to power ($P = \frac{V_{rms}^2}{R_{load}}$ which is read by an analog-to-digital converter (ADC) in the computer (National Instruments Model 5102). For spectroscopy measurements, the signal is sampled directly by the ADC for further (power spectrum) analysis by the computer (see the appendix). For further details of RF power detection, please see Chapter 13 in Reference [7].

A temperature calibration signal can be fed to the system from a "noise tube" whose intensity and spectral distribution is equivalent to the thermal noise of a blackbody at a temperature of approximately 15,000K. The signal from the tube passes through a 20 db precision attenuator which reduces the noise power by a factor of 100 to yield a 150 K calibration signal to the measurement chain. With the noise tube on, the system response is equivalent to that which would be obtained if the entire field of view were filled by a black body at 150 K.

A frequency calibration signal can be fed to the system by using the Marconi frequency synthesizer. The signal is carried by coaxial cable to a small antenna outside the control room. This transmits the signal that is then picked up by the 2.5 meter dish antenna. The synthesizer does not oscillate above 1 GHz so you can set it to exactly half of the hydrogen frequency (1,420.4050 MHz/2=710.2025 MHz) and use the fact that every antenna radiates some energy in the second harmonic.

SPECTROMETER

The line profile, or power spectrum, of the hydrogen signal is calculated using LabVIEW (see the appendix). This software implementation allows the investigator to select the desired spectral resolution of the power spectrum (32-channels gives ±100KHz or ±20km/s resolution). Higher resolutions, up to a practical maximum of about 512 channels are user selectable. The tradeoff for higher resolution is, of course, a lower signal-to-noise ratio.

OBSERVATIONS

You will first test and calibrate the equipment by an observation of the radio emission of the Sun. Point the telescope at the sun and verify that it gives a signal approximately equal to that expected from a black body of diameter 30 arc minutes (0.5°) at a temperature of the order of 100,000 K. Check that the sun is centered in the beam with the settings you prescribed. Then set the telescope at the DEC of the sun and at an HA about 30 minutes (i.e. 7.5°) greater than that of the sun. Make sure the HA drive (i.e. the sidereal tracking pulses) is off and allow the sun to drift through the antenna beam along a diameter as the earth rotates. Calibrate the response with a signal from the standard noise generator. The chart record of the traversal and calibration will contain the essential information required for a quantitative interpretation of the solar signals.

Verify the functioning of the amplifier chain and spectrometer by transmitting the calibration signal from the Marconi frequency synthesizer. The spectrum display should show a line at the expected frequency. You can tune the frequency of the signal to calibrate the spectrometer's channel spacing.

Finally, observe the power spectrum of the 21-cm hydrogen emission from a sequence of positions along the Milky Way. Derive the Galactic rotation curve, and estimate the mass of the Galaxy interior to the circle tangent to the line of sight among your observations that is farthest from the Galactic center. Observe the non-uniformities in the hydrogen distribution revealed by the doppler-induced structures of the line profiles. The multiple-component line profiles are evidence of the spiral-arm structure of the Galaxy.

DETAILED PROCEDURE

Each session of Observation will begin with work outdoors unlocking the antenna and will end with securing the antenna.

At the beginning of your first session, you should meet with one of your TA's or a member of the Junior Lab Staff who will introduce you to the experiment. One of them will explain the procedure for checking out the room key and then escort you to the room where the control and data system of the radio telescope are located.

You will go out on the roof to unlock the telescope. There are two mechanical 'lockouts' on the antenna, one for each axis of rotation. Each lockout depresses a plunger-type rocker-switch which signals the computer that the telescope is unlocked permitting remote controlled operation. In its locked position the telescope is pointed to the zenith, the declination (DEC) of which is our latitude, +42° 21' 39" (42.36°). The hour angle of the zenith is, by definition, zero with positive hour angles corresponding to westward targets. The right ascension (RA) of the zenith is equal to the local sidereal time (LST) which is the hour angle (HA) of the vernal equinox measured west around the polar axis of the earth's rotation (See Appendix). Eastern Standard Time (EST) is used in combination with our local longitude and the current date to give us LST. The software program will automatically calculate LST as long as the computer's clock is correctly set to EST.

Inside the equipment cabinet at the base of the dish, there is a black 'jogger box' for use in returning the dish to zenith when locking it down or for backing it off a limit switch. Two toggle switches permit the investigator to select either the DEC or HA stepper motor and the desired direction of rotation (N/S or E/W). At the beginning of each session, you should ascertain that the interface 'IF' computer cable is connected to the plywood breakout panel prior to returning to the control room. If this is not done, the telescope will not respond to the signals from the computer controller. With the locks open, the telescope pointed at the zenith, and the 'IF' computer cable connected, you can return to the control room and begin taking data.

At the end of the session you should run the software command which returns the antenna to zenith; you must then go outside, and if necessary, use the jogger box to fine adjust the antenna position to the exact zenith so that you can engage the HA and DEC lockouts. To do this, replace the 'IF' cable connector at the plywood breakout panel with the one from the jogger box. Remember to disconnect the jogger box from the AC power strip when you are done using it.

The program does a pretty good job of keeping track of where the antenna is pointed based on the assumption that it was at zenith at the start. However, if it gets confused, you must go outside and return the antenna manually to zenith before restarting. Furthermore, during a very long session, you may want to command the dish to the zenith and then go out and manually make fine adjustment to compensate for accumulated errors. In principle, if you command the antenna to point to a direction outside the normal limits, the antenna will not move and you'll get an error signal. However, in the event that the telescope goes beyond its limits and encounters the limit switches, it is possible to override them and return the telescope to within its allowed range. This should be done very carefully to avoid jamming the motors.

To manually control the dish, disconnect the 'IF' computer cable and connecting instead the plug from the black 'jogger box'. Make sure the jogger box is plugged in and select the proper motor and direction combination to back the dish away from the limit switch. One partner will have to physically push (not too hard please!) the dish away from the limit switch until the jogger box control kicks in. The dish should then move towards the zenith. As soon as the cam is clear of the limit switch you can stop physically pushing the dish. This should require only about a second or two of light pushing.

If you ever need to manually adjust the telescope for any reason using the jogger box, you must move it back to zenith (manually) and then restart the software so that the control system gets initialized correctly!

MEASUREMENTS

The power to all components except the noise tube should be left on at all times. This includes the power strip inside the telescope's equipment cabinet which powers the low-noise amplifiers via the Hewlett Packard power supply. When you first run the LabVIEW based 21-cm program (available from the Windows desktop), it will prompt you to create a log file. Make sure you create a descriptive log file name and save it in your own directory within the shared folder on jls.mit.edu. This way you can retrieve your data set later on for analysis!!!

The program calculates the Local Sidereal Time and indicates the antenna direction on the screen in both Equatorial (RA and DEC) and Galactic ('l' and 'b') coordinate systems.

The program requires the computer clock to be set to Eastern Standard Time (EST). Please do not correct this for Daylight Savings or it's pointing algorithms will fail!

When you first point the telescope after initializing the program, you will see an error message "Error - 10008 occured at DIO Port Write". Click on continue and ignore this error.

Please ignore the tracking indicator. It is not necessary nor desirable to have the telescope track during an observation as the motors will introduce substantial noise into your measurements.

The first object you will observe is the Sun. Point the antenna at the Sun. Make sure the 0-3 MHz signal is feeding the detector. Check the pointing by manually "tweaking" the position of the telescope while you monitor the readings on the voltmeter. The detector produces a *negative* output; *i.e.* a brighter source produces a signal with a larger negative voltage. Point the antenna one-half hour (7.5°) due west of the sun's present position (pointing more to the west means adding a positive number to the hour angle). Turn on the noise tube to establish a temperature scale. Turn off the noise tube and allow the sun to drift through the beam of the stationary telescope (tracking off). In one hour, your drift scan will be complete. From your data you can determine the angular response function of the antenna and the conventional measure of its effective width, the FWHM of the recorded curve.

You may find that the RF detector exhibits a slight non-linear behaviour in its response function. The best (known) way to account for this is to turn it on at 5 different points during the scan: wings, half-height points and peak, and then to rescale the sun's signal by some equation correcting for this 'problem'.

Due to diffraction, the radio telescope receives a signal from a small circular area on the celestial sphere subtending a constant angle (to be calculated by you) at the antenna. This angle is independent of where on the celestial sphere the antenna is pointed. However, on a plot of the sky in celestial coordinates (RA-Declination) the response curve is circular only when the declination is zero (See Fig. 1). Although the hour angle of the Sun increases by one hour for each elapsed hour (hence its name), the time it takes to sweep through the diffraction cone of our antenna will be a function of the sun's declination. You must take this into account when calculating the angular width of the antenna beam.

Once the signal has returned to the background level, turn on the noise tube to get a second reference level. You can compare the amplitudes of the two signals and derive an estimate of the apparent temperature of the solar corona at the wavelength of 21 cm. Note that the fluctuating voltage, i.e., the electrical "noise", generated by the noise tube has a white (flat) spectrum over a wide range of radio frequencies that includes 1420 MHZ, and is equivalent to the Johnson noise of a resistor at a temperature of 15,000K over that range of frequencies. The calibration source is connected to the system

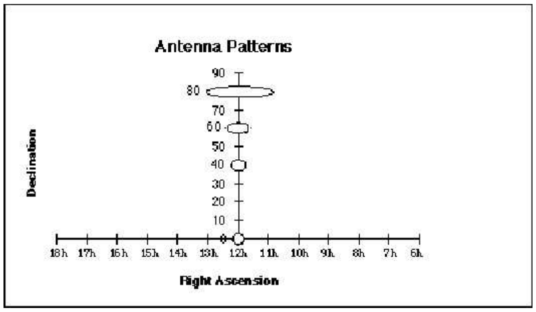

FIG. 2: The response of a hypothetical antenna with a 6° diffraction pattern

through a 20 db directional coupler that leaks 1.0% of the noise power into the transmission line to the preamplifier. Note, further, that the kinetic temperature of the sun's atmosphere ranges from 4800 K at the photosphere to several million K in the corona, and that the solid angle subtended by the sun and its corona is a small fraction of the solid angle of the antenna beam.

The next part of the experiment consists of measuring the profile of the 1420 MHz (21 cm) hydrogen line at various positions along the galactic equator where radial motions cause Doppler shifts that produce distinct features in the profile. First you will generate a dummy signal at the exact unshifted hydrogen frequency (1420.4057 MHz) using the Marconi signal synthesizer which is connected to a small dipole antenna on the roof near the telescope. The synthesizer does not operate as high as 1420 MHz, but sufficient power is generated in the second harmonic when the synthesizer is set to 710.2025 MHz. This radio signal provides a sharp reference signal and helps to verify that the equipment is working properly. Enough power in this calibration signal is received in the "side lobes" of the antenna to produce a sharp spectrum peak regardless of telescope orientation. The only control you will need to touch is the button marked "carrier on/carrier off". With the carrier off (the word "off" should appear in the lower right display) and the antenna pointed towards the zenith, take and store a reference spectrum. Next, take an antenna spectrum with the carrier on. The data collected from these measurements are automatically saved into text files with filenames created using a timestamp calculation.

Record a series of 21-cm line profiles of the galaxy by pointing the antenna at various positions along the galactic equator (b^{II}=0°). For this work you must toggle the tracking mode "on" . The computer only tracks once per minute while in the tracking mode. When you point the antenna using galactic coordinates, the dish will go to the desired location. You can direct the computer to accumulate either a sample spectrum or a background spectrum. The bottom graph automatically displays the most recently acquired sample spectrum background corrected using the most recently acquired background spectrum. These data may be saved to a text file using the control button labeled "save data". The control labeled "Blocks to Avg" tells the computer how much averaging to perform. A reasonable default value is 50 which takes about 30 seconds on a 800MHz PC.

Figure 3, taken from a paper published in Leiden in May 1954 (H. C. van de Hulst et al.), shows various spectra at different galactic longitudes all in the galactic plane. Note that the longitudes used in this paper are old system (l^{I} , b^{I}). In 1958 the International Astronomical Union adopted a new system (l^{II} , b^{II}) with the conversion $l^{II} = l^{I} + 32.4°$ and $b^{II} = b^{I} + 1.4°$.

The software command "View Galactic Map" will help you convert between galactic coordinates and celestial coordinates. The areas outside the blue box correspond to those parts of the sky to which the antenna can not point. The allowed region corresponds to -27° < DEC > +67°, -75° < allowed HA > +75°.

ANALYSIS

Compare your plot of the angular response function of the antenna, obtained in the drift scan of the sun, with the theoretical diffraction pattern of a circular aperture. You may need to consult a text on physical optics and use of a computer computation of a Bessel function.

Derive an estimate of the brightness temperature of the sun at 21 cm from your measurements and calibration, taking account of the fraction of the effective solid angle of the antenna response function that is occupied by the sun. Here you may need to study the portion of Shklovski's book or other reference dealing with brightness temperatures, antenna temperatures and the relation to actual source temperatures.

Reduce the data of your 21-cm line profiles to plots of relative antenna temperature against radial velocity relative to the sun, taking account of the motion of the earth and the antenna around the sun.

With the help of the discussions presented in Mihalas and Binney and by Shu, derive from your data a plot of the velocity curve of the Galaxy as a function of radius. Why are you only able to do so for locations interior to the radial position of our solar system?

SUGGESTED THEORETICAL TOPICS

1. Hyperfine splitting of the hydrogen ground state.
2. Radiative processes in the sun.
3. Antenna theory.
4. The structure and dynamics of our Galaxy.
5. Radiative transfer.

FIG. 3: Survey of hydrogen line profiles at various galactic longitudes. (from H.C. van de Hulst et. al., Bull. of the Astron. Inst. of the Netherlands, XII, 117, May 14, 1954) Note: Pictures are reversed from our images and longitude is given in the old system (l^I,b^I). $l^{II} = l^I + 32.31°$

[1] K. Rohlfs and T. L. Wilson. 1996, Tools of Radio Astronomy (second edition; Springer). A modern handbook of radio astronomy techniques. Section 5.5 describes the relationship between antenna temperature and brightness temperature.[**?**] Chapter 6 provides a useful discussion of antenna theory.

[2] H. C. van de Hulst, et al., Bull. of the Astron. Institutes of the Netherlands XII,117 (1954)

[3] D. Mihalas and J. Binney 1968. Galactic Astronomy. (San Francisco:W. H. Freeman).

[4] I. S. Shklovski 1960. Cosmic Radio Waves. (Cambridge; Harvard University Press).

[5] F. J. Kerr 1969. The Large Scale Distribution of Hydrogen in the Galaxy. Ann. Rev. Astron. Astroph., Vol.7, p 39. (Available in the CSR Reading Room, 5th floor, Bldg. 37.)

[6] Shu, F. H., The Physical Universe - Chapter 12, University Science Books, Mill Valley, CA 1982. This reference gives a clear description of the interpretation of 21cm spectra in terms of the rotation curve of the Galaxy.

[7] Horowitz, H., and Hill, w., The Art of Electronics, 2nd Edition, Cambridge University Press, 1989

[] Note, however, the following errors in Section 5.5: The equation between equation (5.59) and equation (5.60) should read $I_\nu = 2kT/\lambda^2$. Equation (5.62) should read $W = \frac{1}{2} A_e \int\int \frac{2kT_b(\theta,\phi)}{\lambda^2} P_n(\theta,\phi) d\Omega$.

CELESTIAL GEOMETRY

Celestial coordinates (RA and DEC) are the spherical coordinates used to specify the location of a celestial object (Figures 4 and 5). The north celestial pole (NCP) is the direction of the earth's rotation axis, and the celestial equator is the projection onto the sky of the plane of the earth's equator.

Right ascension (RA) is the celestial analog of geographic longitude. RA is measured eastward along the celestial equator from the vernal equinox ("V" in Figures 4 and 5) which is the ascending node of the plane defined by the sun's apparent motion (caused by the orbital motion of the earth around the sun) and the celestial equator. In catalogs of celestial objects RA is generally specified in units of hours, minutes and seconds from 0 to 24 hours, but it is often more conveniently specified in degrees from 00 to 360° with a decimal fraction. Declination (DEC) is the celestial analog of geographic latitude. DEC is measured north from the celestial equator along a celestial meridian which is a great circle of constant RA. In catalogs DEC is generally specified in degrees, arc minutes (') and arc seconds ("), but it is also often more conveniently specified in degrees from -90° to +90° with a decimal fraction. (1 hour of RA at constant DEC corresponds to an angle of 15°*cos(DEC) degrees subtended at the origin).

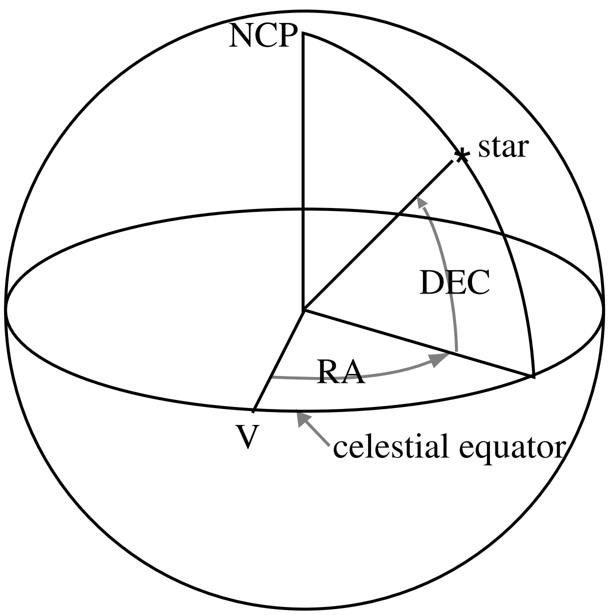

FIG. 4: Diagram of the celestial coordinate system

The **ecliptic** is the intersection of the earth's orbital plane with the celestial sphere. To an observer on earth the sun appears to move relative to the background stars along the ecliptic with an angular velocity of about 1 degree per day. The angular velocity is not exactly constant due to the eccentricity of the earth's orbit (e=0.016722).

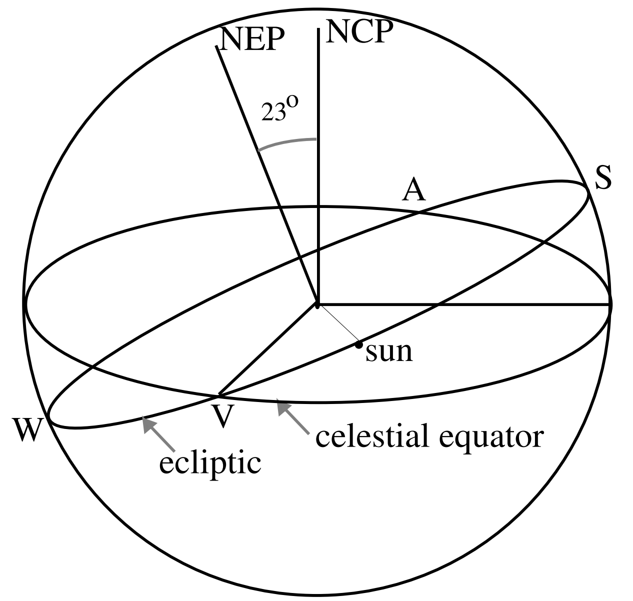

FIG. 5: Diagram of the celestial sphere, showing the celestial north pole (NCP), the celestial equator, the northe ecliptic pole (NEP) and the ecliptic. The points labeled V, S, A, and W are, respectively, the vernal equinox, summer solstice, autumnal equinox, and winter solstice corresponding to the directions of the sun on March 21, June 21, September 21, and December 21. The point labeled "sun" is the direction of the sun on approximately April 21

The period of the earth's orbit is 365.256 days. The **inclination** (ie) of the earth's equator to the ecliptic is 23° 27'. The **ascending node** of the ecliptic with respect to the celestial equator is the intersection of the ecliptic and the celestial equator (the vernal equinox) where the sun in its apparent motion crosses from south to north declinations on March 21. **Precession of the equinoxes** is the motion of the equinoxes along the ecliptic due to precession of the earth's rotational angular momentum about the ecliptic pole. The precession is caused by the torque of the gravitational attractions between the sun and moon and the earth's equatorial bulge. The period of the precession is approximately 25,000 years.

Ecliptic coordinates (Figure 6) are generally used to specify the positions and orientations of objects in the solar system. **Ecliptic longitude** (elon) is measured along the ecliptic eastward from the vernal equinox. **Ecliptic latitude** (elat) is measured along a great circle northward from the ecliptic.

The orientation of the orbit of a planet is specified by 1) the ecliptic longitude of the ascending node of the orbital plane and 2) the inclination of the orbit to the ecliptic. Similarly, the orientation of a planet's rotation or the rotation of the sun itself, as illustrated in Figure 6, is specified by the ecliptic longitude (ELON) of the ascending node of its equator and the inclination (INCL) of the equator to the ecliptic.

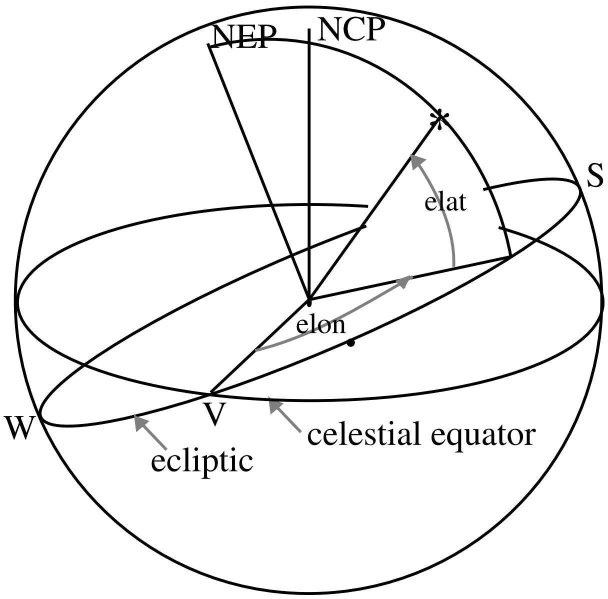

FIG. 6: Diagram showing the relation between the ecliptic and celestial coordinate systems

We turn, finally, to the practical problem of pointing a telescope to observe an object at some given time of the day. Times will be expressed in degrees according to the relation 1 hour = 15°. We first define the following quantities (see Figure 7):

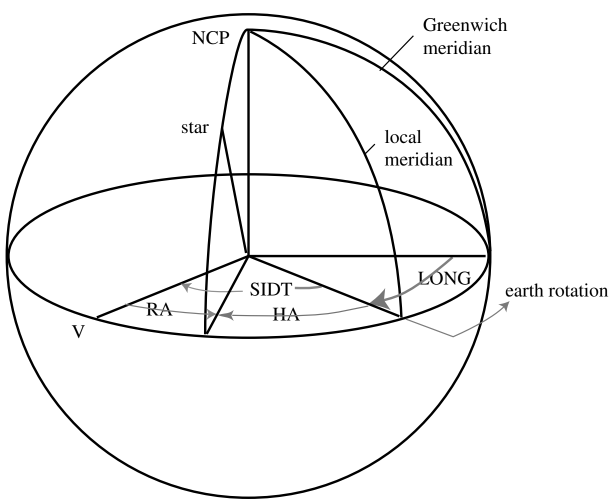

FIG. 7: Diagram of the relations between the quantities involved in setting the position of the telescope about the polar (HA or RA) axis

Hour circle of an object. A great circle on the celestial sphere passing through the celestial poles and the position of the object. Hour angle (HA). The angle, measured westward through 360°, between the celestial meridian of an observer and the hour circle of the celestial object. (The hour angle of the "mean sun" advances with time like the hands of a 24-hour clock, completing one revolution in one solar day. The hour angle of a fixed star completes one revolution in one sidereal day of 23.934 hours.) Universal time (UT). The hour angle of the mean sun at Greenwich plus twelve hours. Greenwich sidereal time (GST). The hour angle at Greenwich of the vernal equinox. Eastern standard time (EST) UT+15°n, where n is the number of time zones west of Greenwich. (n=5 at Boston, so 12 noon EST is 17:00:00 UT.) Local sidereal time (LST) The hour angle of the vernal equinox at the location of the observer. Suppose the telescope is on a standard two-axis polar mounting at geodetic longitude LONG and latitude LAT with the RA axis (polar axis) parallel to the earth's rotation axis and the DEC axis perpendicular to the RA axis. This is the most common arrangement of telescopes and the way the radio telescope antenna is set up.

SIGNAL PROCESSING IN 21-CM RADIO ASTROPHYSICS

The signal presented to the A/D converter is centered on 1.5 MHz and has a 3 MHz bandwidth. According to the Nyquist Theorem, sampling the signal at twice the bandwidth, or at 6 Msamples/sec, completely recovers all the information in the signal. The A/D coverter (National Instruments 5102) samples at 6.67 Msamples/sec; thus, we are slightly oversampling the signal and no information is lost.

For a voltage $v(t)$ measured in the time domain we can compute the Fourier transform $V(\nu)$:

$$V(\nu) = \int v(t)e^{-2\pi i\nu t}dt \tag{2}$$

Note that $V(\nu)$ is *complex*, giving us an amplitude and a phase for each frequency component. In our experiment we are not interested in the phase information. Taking the square modulus of the Fourier transform gives us the voltage power spectrum:

$$S(\nu) = [V(\nu)]^2 \tag{3}$$

$S(\nu)$ is the quantity plotted by the data acquisition software. For the voltage power spectrum, the units are volts^2 (rms), and a sine wave with amplitude $V_o$rms will produce a signal of amplitude $V_o$rms/4, which becomes $V_o$rms/2 in the one-sided spectrum that is displayed (see below). The spectrum displayed on the Junior Lab computer screen has units dBm, which is power expressed in dB referred to one milliwatt. The spectrum values are converted to milliwatt units by taking into account the characteristic impedance of the transmission line, $Z_o = 50\ \Omega$. This is done according to $dBm = 10log\frac{Vrms^2}{.2236^2}$ where the 50 Ω's in the numerator and denominator have canceled. The power you measure $\frac{V_{rms}^2}{Z_0}$ is the power going down the cable to the A/D converter, related to, but not equal to, the power received at the antenna.

The above expression for the Fourier Transform, refer to a continuous signal measured for an infinite length of time. In practice, instead of $v(t)$, we in fact measure $v_i(t)$, discrete samples separated by a time interval δt, over a period of time Δt. Thus the number of points measured is $N = \Delta t/\delta t$. Our Fourier transform is now the sum

$$V_j(\nu) = \sum_{i=1}^{N} v_i(t)e^{-2\pi i\nu_j t}\delta t \tag{4}$$

This differs from the ideal $V(\nu)$ computed above in some important ways. We have to consider the effects of *sampling* and *windowing*. We also note that the power spectrum has the property of *symmetry about zero frequency*.

Sampling: We sample the signal at a rate $1/\delta t$, which means that only signals with frequency $2/\delta t$ or smaller can be reconstructed. In a properly designed spectrometer the sampling range and the *anti-aliasing filter* are chosen with this in mind. This is the reason for the 3 MHz bandpass filter (see Figure 2) and the sampling rate of just over 6 MHz. A strong signal with frequency larger than 6 MHz that "leaks" through the filter will appear in our power spectrum at an *aliased* frequency with the 0-3 MHz band. Thus, strong interference signals can corrupt your spectra, even though they are outside the bandpass.

Because we use the Fast Fourier Transform (FFT) algorithm to compute the Fourier transform, the spectra are also sampled with values known only at certain values of the frequency. Our frequency "channels" are centered at these values and are separated by

$$\delta\nu = \frac{1}{N\delta t} \tag{5}$$

Windowing: Because the time series is truncated (i.e., we only measure the voltage signal for a finite period of time), the features in the frequency domain are broadened. In other words, a pure harmonic signal, which in principle would appear as a delta function in the frequency domain, in fact appears as a feature with nonzero width. We can express this effect of the limited time span by multiplying by a "window function" $w(t)$ in the time domain:

$$v_m(t) = w(t)v(t) \tag{6}$$

For the simplest window function with no weighting, $w(t)$ is unity during the times of the measurements and zero otherwise:

$$w(t) = 1 \text{ for } -\frac{\Delta t}{2} \leq t \leq \frac{\Delta t}{2} \tag{7}$$

By the convolution theorem, the Fourier transform of $v_m(t)$ is the convolution of the Fourier transforms of $v(t)$ and $w(t)$:

$$V_m = W(\nu) * V(\nu) \tag{8}$$

The Fourier transform of the uniform $w(t)$ function above is a sinc function in the frequency domain (see Figure 1):

$$W(\nu) = \frac{\sin \pi\nu\Delta t}{\pi\nu} \tag{9}$$

The width of the sinc function is proportional to Δt. For example, the half width at the first null (HWFN) is $1/\Delta t$. Thus, each frequency "spike" in the frequency domain is broaded into a feature shaped like a sinc function with a width determined by the length of the data

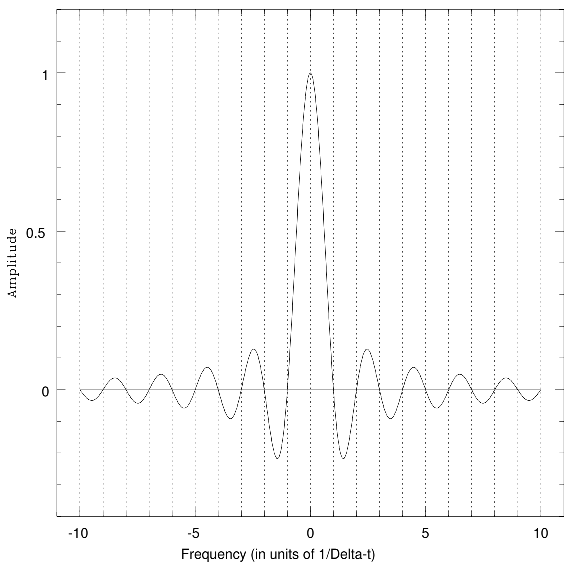

FIG. 8: The Fourier transform of a uniform window of width Δt. The vertical dotted lines show the centers of the frequency channels in the sampled power spectrum, assuming the spectral line is exactly at the center of one channel (which it may not be).

stream. Note that that the spacing between the nulls is equal to $1/\Delta t$ and that this is also the channel spacing. The spectral line is also characterized by "sidelobes," a sort of ringing that can extend far from the main spectral feature. For this choice of window and sampling, the width and sidelobes do not appear in the sampled values of the spectrum if the spectral line is at the center of a channel.

The uniform window is only one possible weighting scheme that can be applied to the data. By changing the weights, we can change the shape of the spectral line in the frequency domain. The choice should be based on considerations of the expected properties of the spectrum. For example, if we were interested in resolving two closely spaced spectral features, we would choose a weighting scheme that would produce a narrow spectral peak. If we were concerned about interfering signals, it would be prudent to choose a weighting function that produces small sidelobes, even at the cost of a broader spectral peak.

Symmetry About Zero Frequency It can be shown that the Fourier transform of a real function is symmetric about zero frequency. For nonzero frequency components, the power is equally divided between $\pm\nu$. When we display the spectrum, we just sum the frequency components at $\pm\nu$ and present a "one-sided" spectrum. Thus, the first frequency bin is at $\nu = 0$, and the last frequency bin is at $\nu = \delta\nu(\frac{N}{2} - 1)$.

Implementation of Fourier Transform Spectroscopy in the Junior Lab Experiment

The radio astrophysics data acquisition program presents you with choices of:

Blocks to average = N_{blocks}
2 Sided PS bins = N_{bins} (corresponds to "n" above)
Window

From these quantities the duty cycle and the frequency channel spacing are computed and displayed.

Blocks to average Each "block" of data consists of $2^{19} = 524288$ samples collected in the time domain (taking approximately 79msec at a sampling rate of 6.67Msamples/sec). This is the minimum amount of data the program can take. After collecting a block of data, the computer is occupied for some time calculating the power spectrum from $n\delta t$ segments of the data, applying any user selected windowing function and converting to dBm. A power spectrum is calculated for each block of data, and these spectra are averaged. The fraction of time that the computer is collecting data (as opposed to processing it) is the "duty cycle" that is displayed on the screen. As computers get faster and faster, this duty cycle should approach unity.

2 Sided PS bins The signal we measure is real, so the power spectrum we compute is symmetric about zero frequency. The spectrum presented on the PC screen has been "folded" so that the channels at plus and minus the same frequency have been summed. The channel spacing, $\delta\nu$, displayed on the screen is

$$\delta\nu = \frac{1}{N_{\text{bins}}\delta t} = \frac{6.67 \text{ MHz}}{N_{\text{bins}}}$$

Window The choice of a window determined the weights that are applied to the data points before the computation of the Fourier transform. As discussed above, the shape of a harmonic spectral feature is the Fourier transform of the function describing the weights.