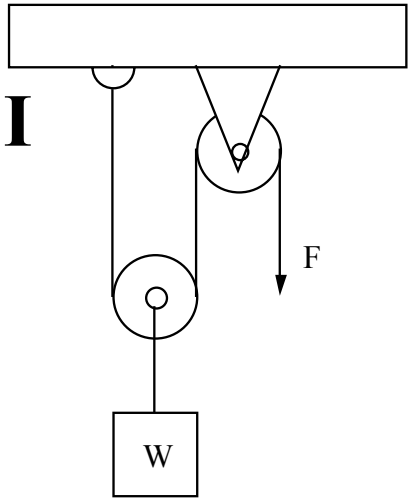
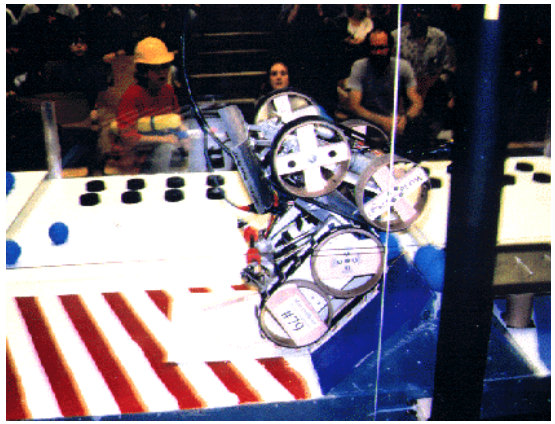
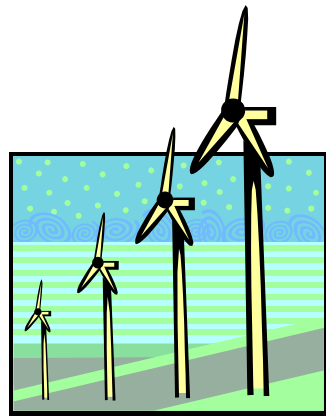
Topic 5
Power Transmission Elements I

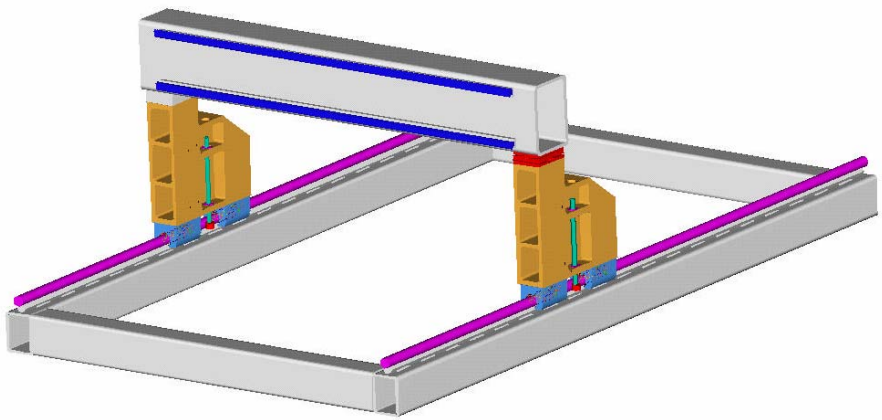




Topics:

- Transmissions
- Pulleys
- Winches
- Belts & Cables
- Wheels
- Steering & Suspensions
- Clutches & Differentials
- Cams
- Shafts
- Couplings





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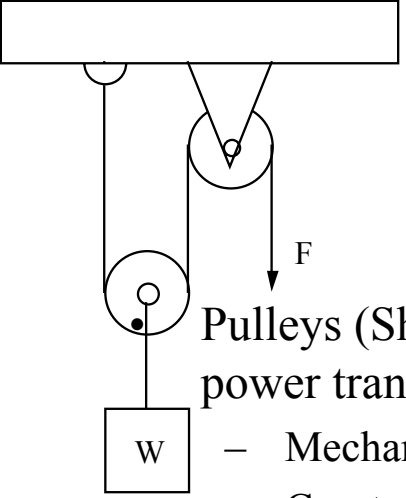
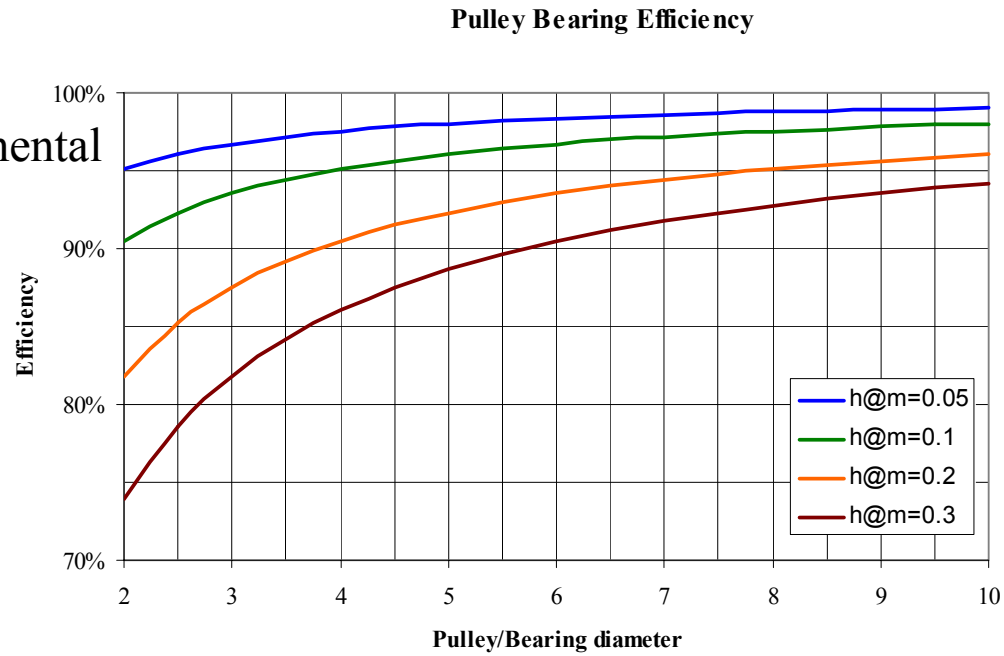
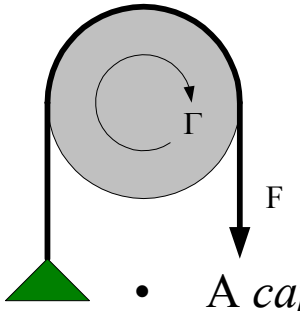
Pulleys



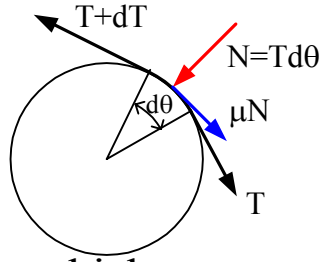
Pulleys (Sheaves) are a most fundamental power transmission element

- Mechanical advantage
- Capstans
- Efficiency
- Tracking
- Mounting





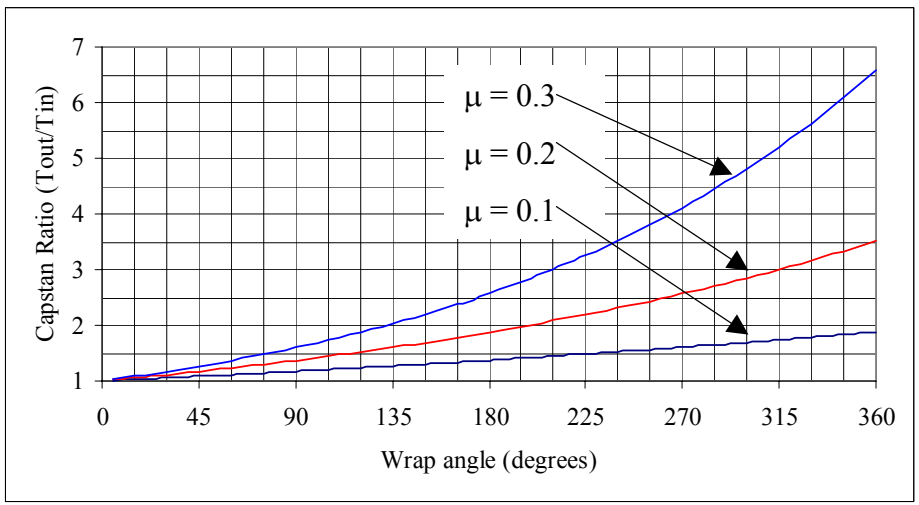
Pulleys: *Capstans*



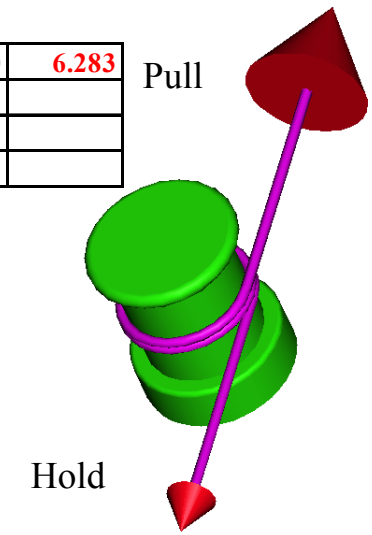
- A *capstan* is typically a fixed, or controlled rotation, body-of-revolution which a cable wraps around
  - A capstan can also form the basis for a band brake, where a band is anchored to a structure, and then wraps around a shaft…
- A cable wrapped around a capstan by θ radians with coefficient of friction m and being held with a force F_{hold}, can resist the pull of a cable with many times higher force F_{pull}

$$F_{hold} = F_{pull} e^{-\mu\theta}$$

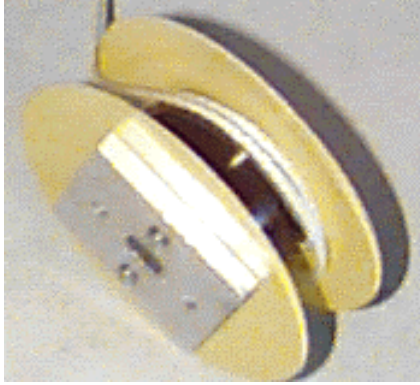
- If a belt or cable runs around a fixed shaft, then there is a lot of friction between the belt and the shaft, and the efficiency is low:



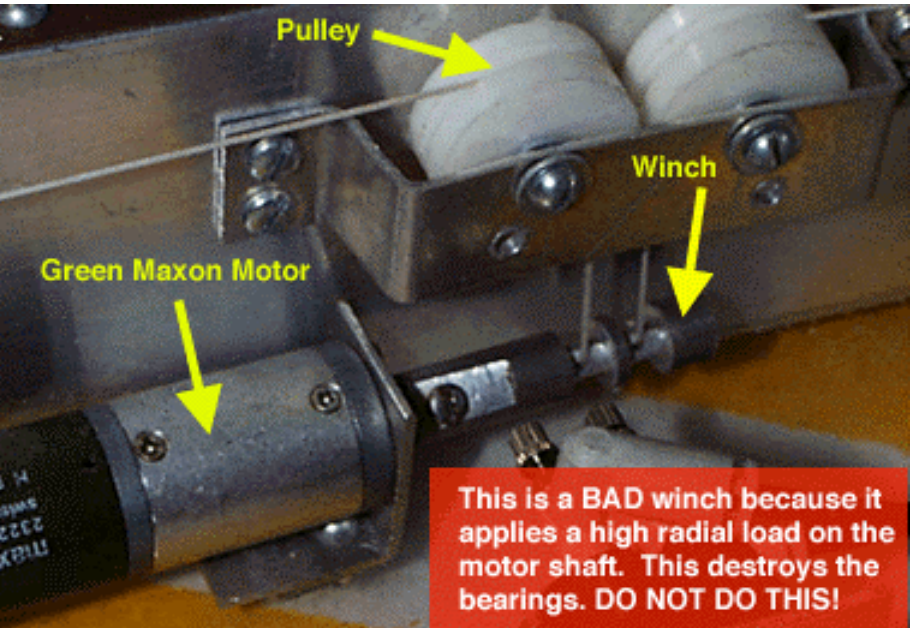
Angle of wrap (degrees, radians) θ	**720**	**6.283**
Coefficient of friction μ	**0.2**	
Holding force Fhold	**1**	
Pulling force that can be resisted Fpull	**3.52**	

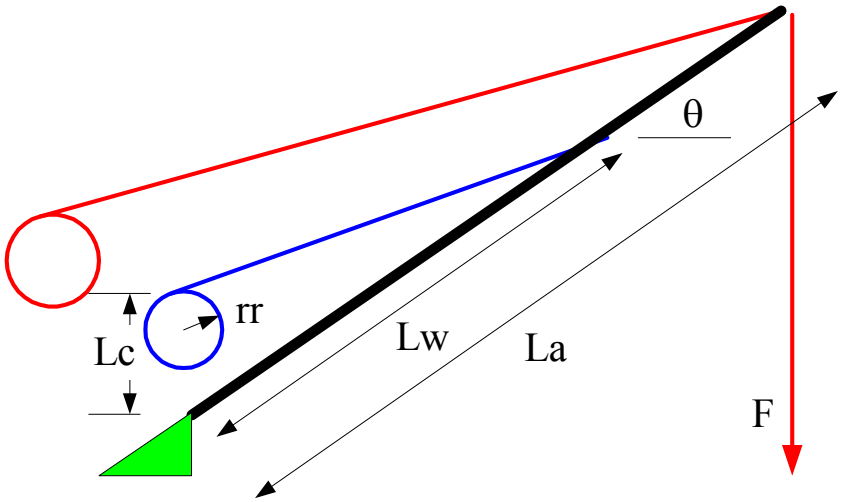


Winches

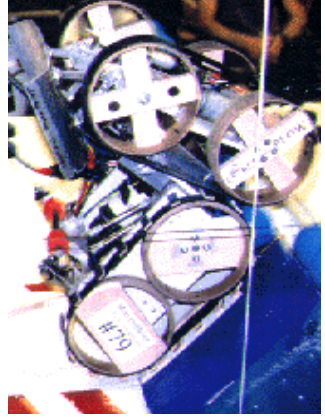


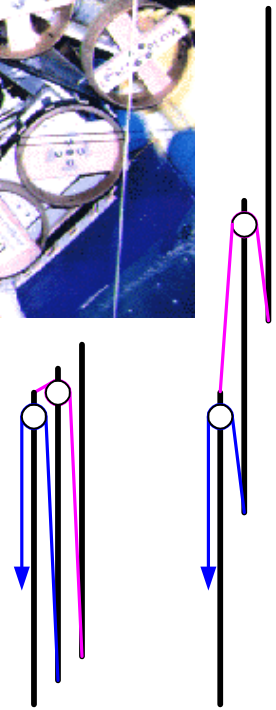
- A winch is a motorized drum that controls cable tension
  - A single wrap of cable on the drum requires a long or large diameter drum
    - Effective drum diameter and winch force capability remain constant
  - Multiple wraps of cable on the drum allow for more cable in a smaller place
    - Drum diameter and winch force capability vary
  - A fairleader is a device to control the input/output of the cable so it winds on the drum in an orderly fashion
    - The simplest design just uses smooth rounded static features
    - Vertical and horizontal roller designs reduce cable wear



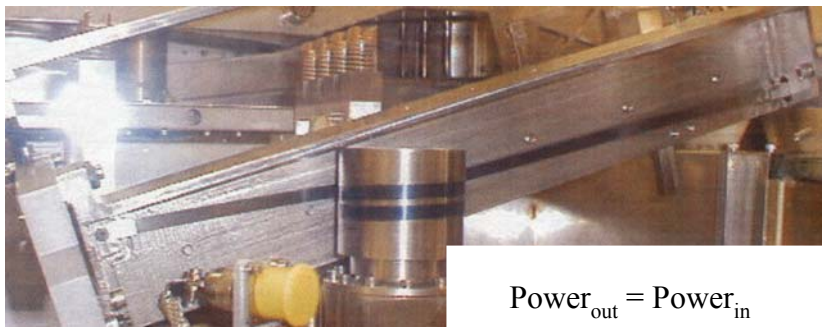


Belts & Cables





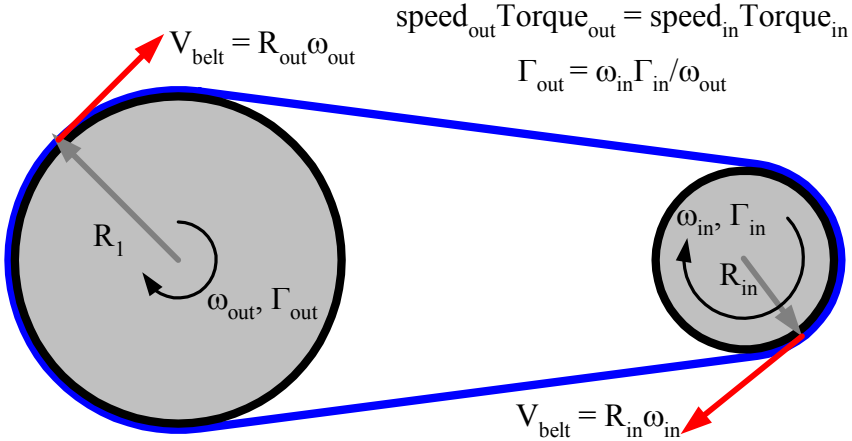
- Applications and Engineering of Belts & Cables
  - Linear motion
  - Rotary motion
  - Crawler tracks
  - Manufacturing & Assembly

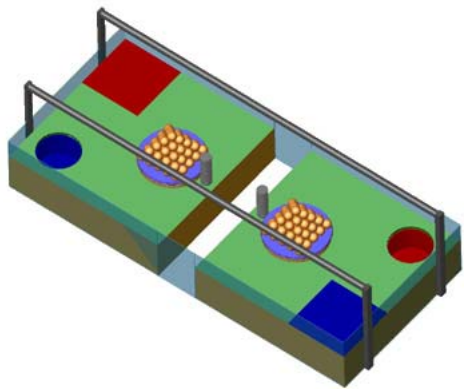


$$\text{Power}_{out} = \text{Power}_{in}$$

$$\text{speed}_{out}\text{Torque}_{out} = \text{speed}_{in}\text{Torque}_{in}$$

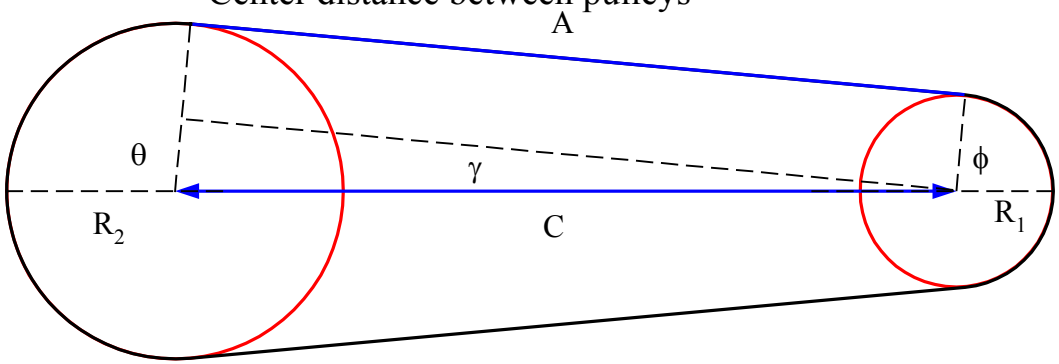
$$\Gamma_{out} = \omega_{in}\Gamma_{in}/\omega_{out}$$



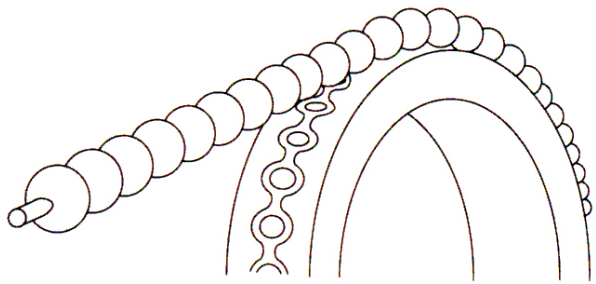


Belts & Cables: *Stress, Tension, & Center Distance*

- Belts and cables are very robust elements, but they require engineering of 3 basic details:
  - Stress from wrapping a belt or cable around a pulley
  - Tension in the belt or cable
  - Center distance between pulleys



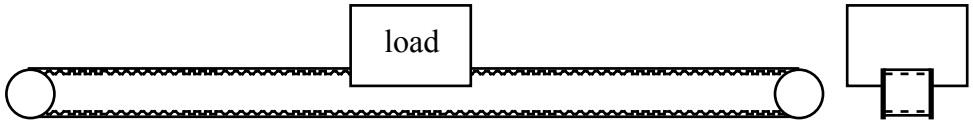
Bandstress.xls	
By Alex Slocum	
Last modified 4/22/02by Alex Slocum	
Stress in a flat belt wrapped around a pulley	
Enters numbers in BOLD, Results in RED	
Belt parameters	
Thickness, t (mm)	**0.10**
Width, w (mm)	**5**
Modulus, E (N/mm^2)	**2.00E+05**
Poisson ratio, n	**0.29**
Forces	
Load to be carried, F (N)	**10**
Belt stress, sigF (N/mm^2)	**20.0**
Pulley wrap stresses	
Pulley diameter, D (mm)	**50**
stress	**437**
Motor torque required (N-mm)	**250**
Capstan effect	
coefficient of friction, mu	**0.2**
Wrap angle, q (degrees)	**180**
required pre-tenion, pT (N)	**5.3**
Belt stress, sigT (N/mm^2)	**10.6**
Total stress	**467**
Total strain	**0.23%**
Check: Tension in the belt (pluck it like a guitar string)	
Measured frequency of lateral vibration (hz)	**150**
Free-length (mm)	**300**
density (g/mm^3)	**7**
mass per unit length (g/mm)	**0.004**
Tension (N)	**7.1**



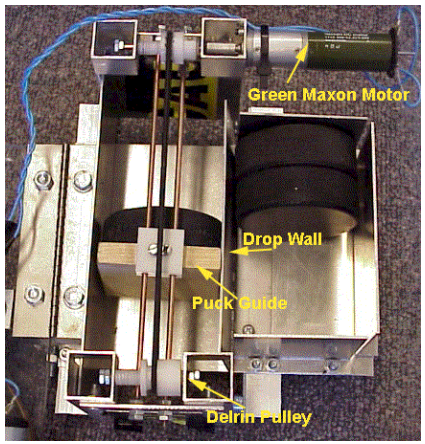
Bead belt drive

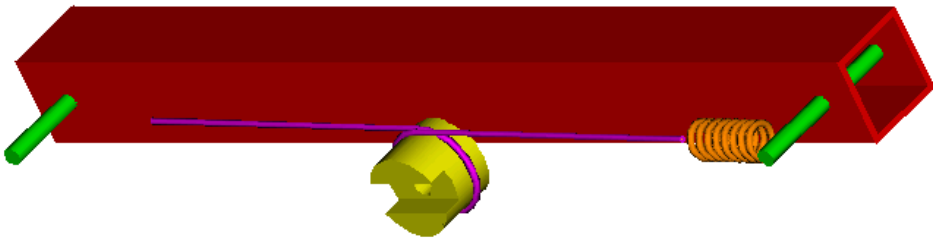
Belts & Cables: *Linear Motion*

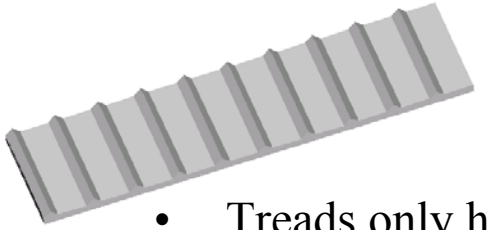
- Belts & Cables are a very effective way to convert rotary to linear motion



- The force F in a belt with tension T on a pulley of diameter D that can be generated by the torque Γ can be conservatively estimated by:
  - $F = 2\Gamma/D$ *for toothed belts,* and $F = T\mu D/2$ *for flat belts*
    - A more exact model would consider the capstan effect
    - Play with *bandstress.xls*
  - The speed is simply
    - $V_{linear}=2\omega_{motor}*D$
- Belts run on pulleys
  - For flat belts, the pulleys must be crowned to prevent the belt from coming off the pulley due to pulley misalignment
  - Timing belt pulleys must also be carefully aligned to prevent premature failure

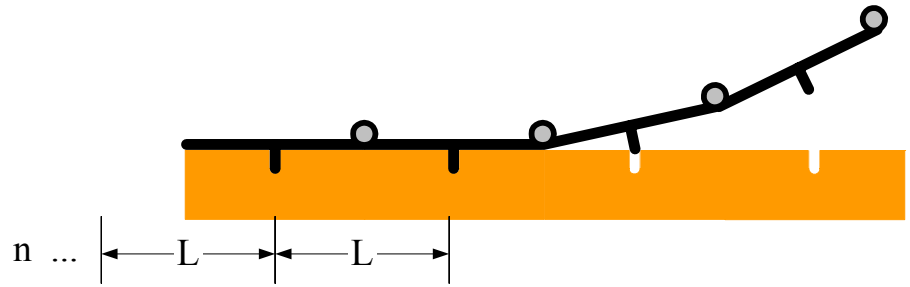


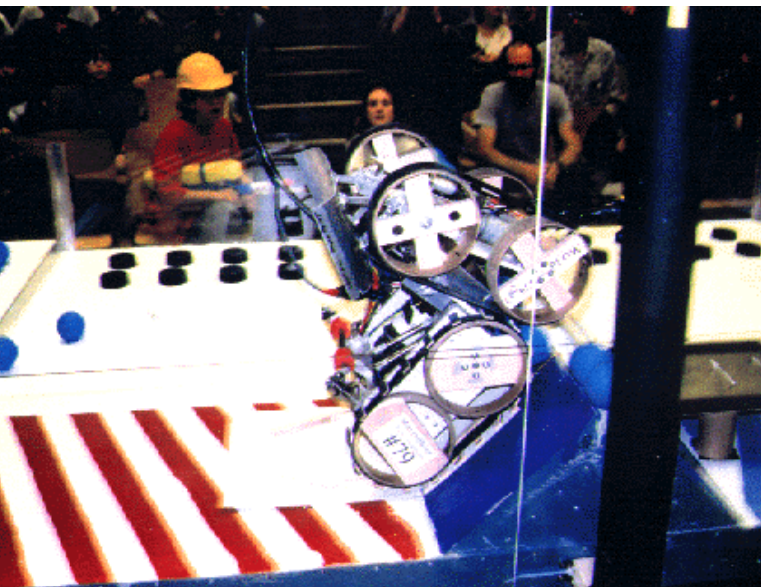


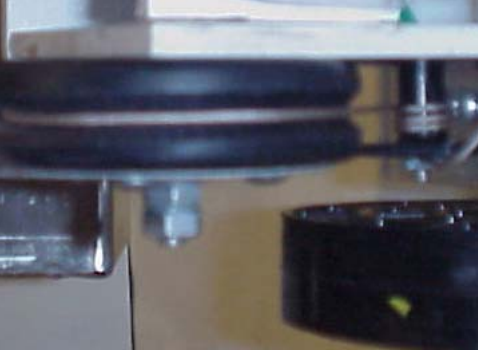


Belts: *Crawler Tracks*

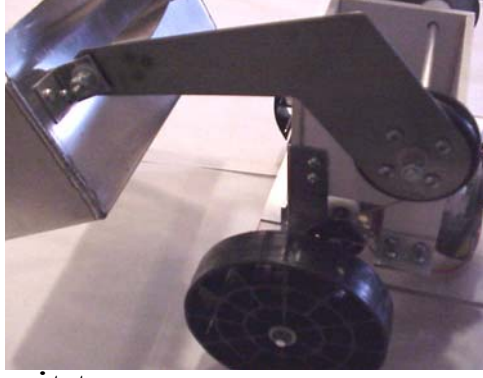
- Treads only help when there is loose media or a surface into which they can dig
- Treads DO NOT help on smooth surfaces
  - Smooth surfaces often are covered with a dust layer, and sharp-groove treads can help
- Treads can be created by cutting angled slices from a rubber strip, and gluing them onto the belt surface



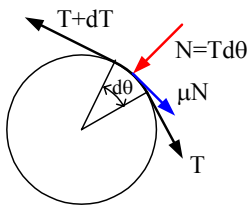


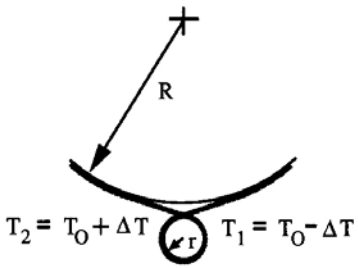


Belts & Cables: *Rotary Motion*



- Flat belts and cables (string drives) require higher tension to transmit torque
  - Conservatively, the belt needs to be held in tension equal to the desired torque divided by the coefficient of friction and the small pinion radius
- Vee-belts use the the *principle of self help*:
  - Increased tension caused by power being transmitted, wedges the belt in a Vee-shaped pulley groove, so it can transmit more torque…
- *Synchronous Drives (timing or gear belts)* can transmit torque between shafts and also achieve a transmission ratio
  - They combine the positive timing action of gears with the flexibility, speed and low noise level of belts
    - For an in-depth discussion on synchronous drive design, see Stock Drive Products on-line tech library: http://www.sdp-si.com/Sdptech_lib.htm





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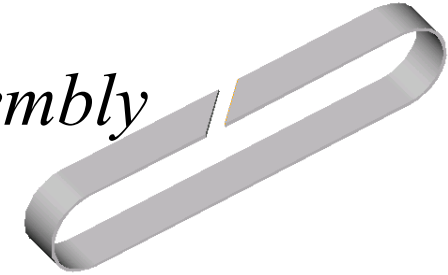
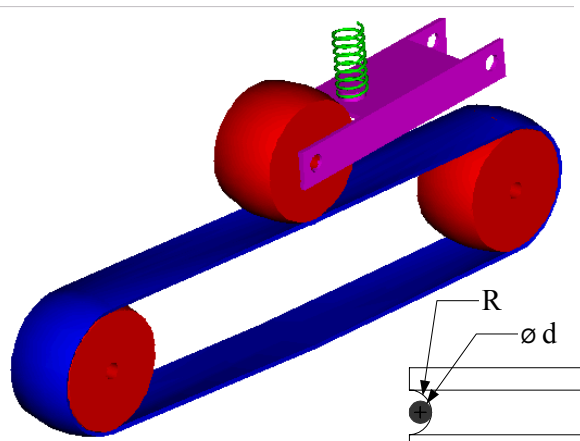
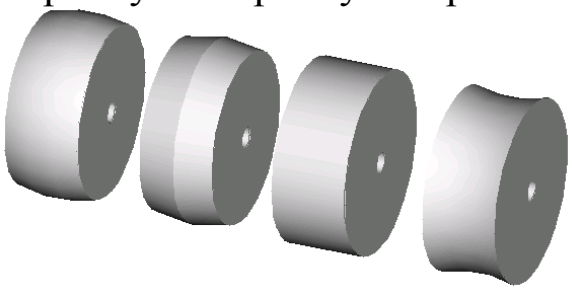
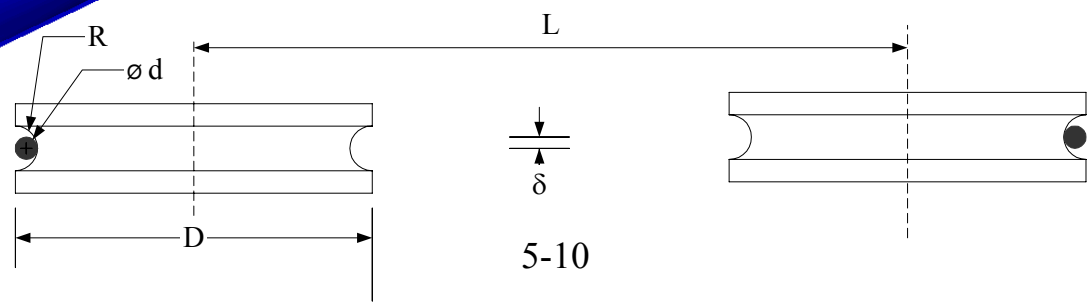
Belts & Cables: *Manufacturing & Assembly*

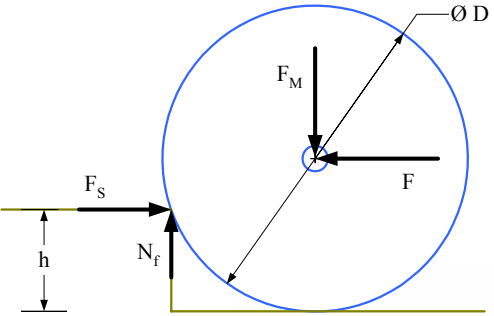


- Pulleys can be grooved to provide a guide for a cable
- Two pulleys' axes of rotation can never be perfectly parallel, so a flat belt will want to drift off (tracking)
  - Pulleys must be crowned (round profile) to keep a belt from walking off
    - On a concave surface, the side with more belt in contact will cause the belt to drift further to that side until it falls off
    - A flat pulley is at best neutrally stable
    - Great, OK, bad, & horrid pulleys:
  - Tension must be maintained either by proper pulley center distance and belt elasticity, or a mechanism to tension the pulleys or a pulley that pushes sideways on the belt

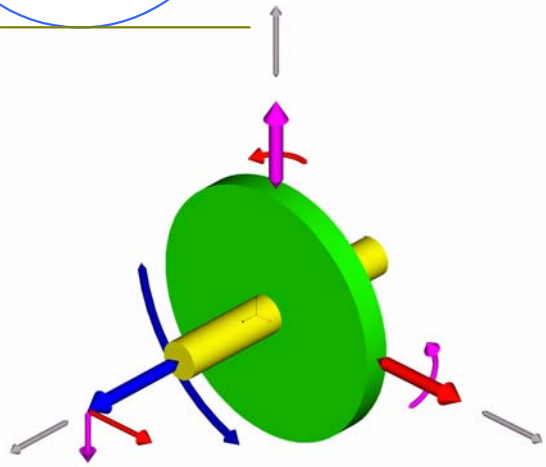




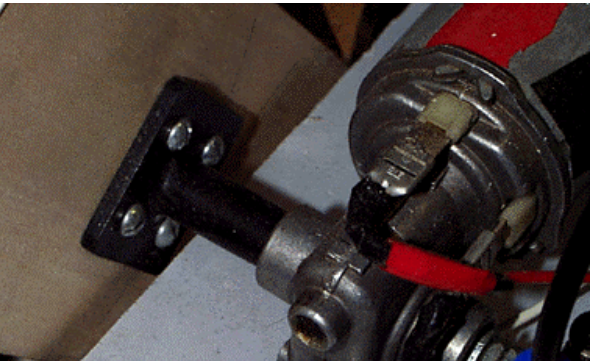


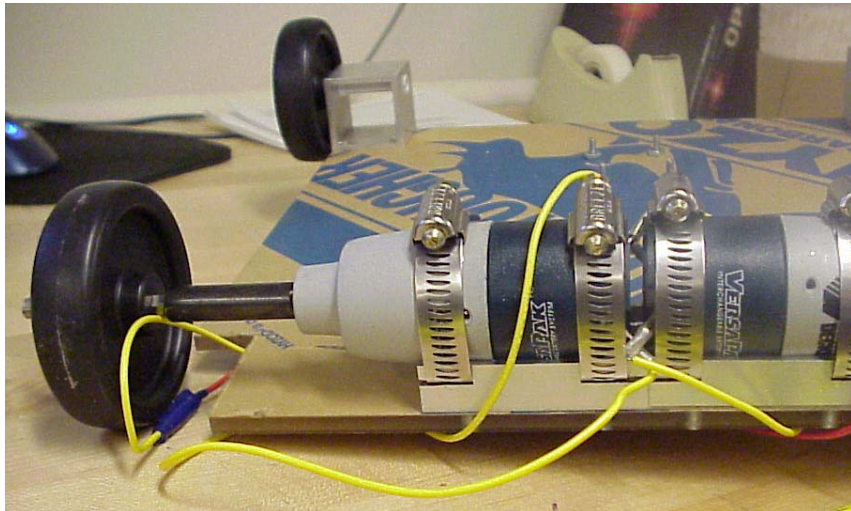


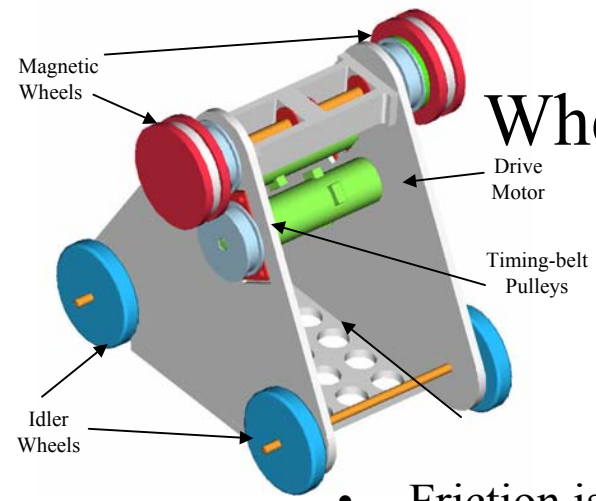
Wheels



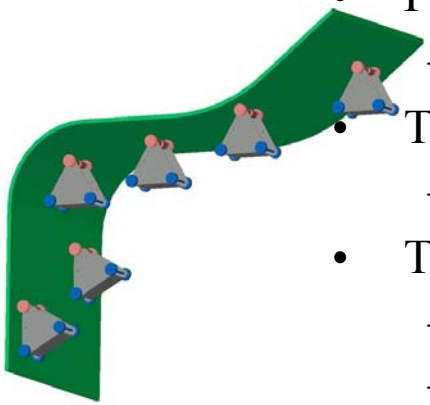
- Traction & Controllability
- Size, Torque & Contact Pressure
- Manufacturing & Mounting



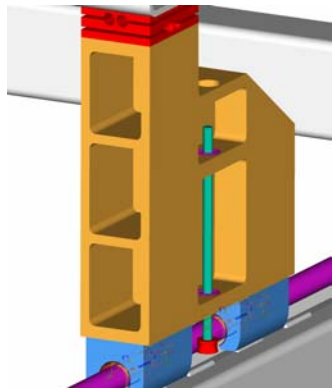


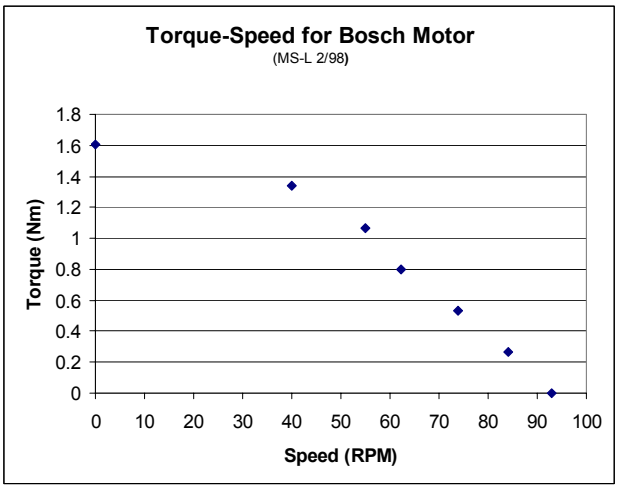


Wheels: *Traction & Controllability*



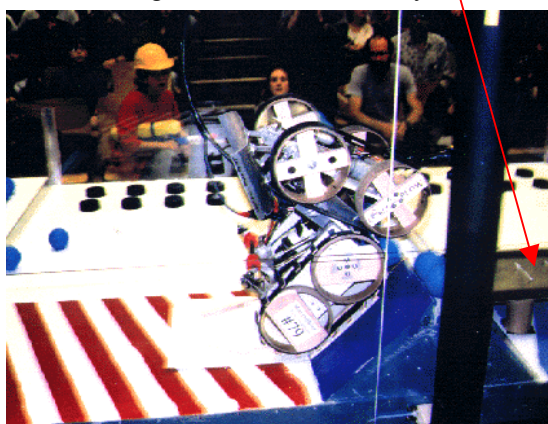
- Friction is independent of surface finish, unless there is physical engagement
  - Like gear teeth on a rack, sandpaper grips carpet, plastic…
- Two-Wheel Drive vehicles are simple to build
  - Against a 4-Wheel Drive or a Tracked vehicle, they lose
- The smaller the wheel:
  - The greater the traction force the motors can cause
  - The slower the vehicle per unit input to the controller!





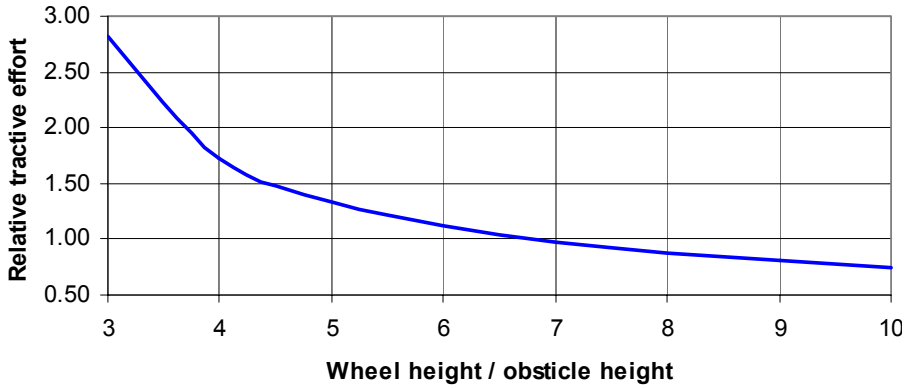
This was a very successful 2WD vehicle with sandpaper covered rear driven wheels

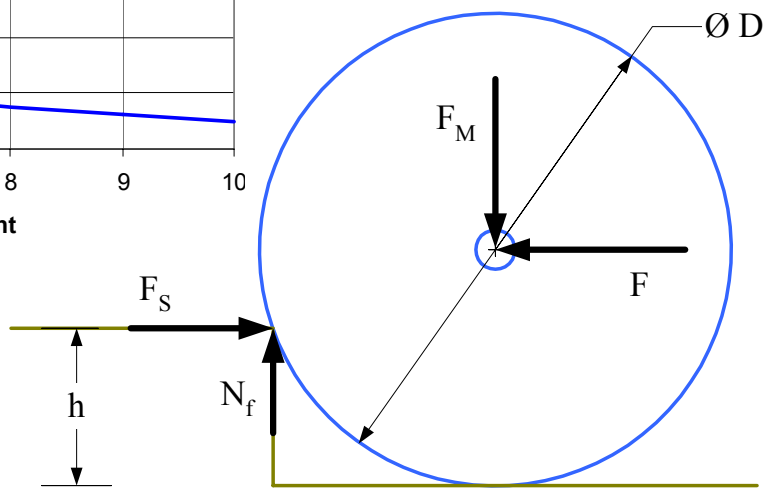
A sandpaper covered platform enabled this machine to get to the other side every time!

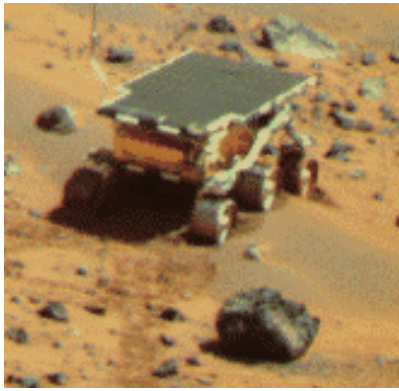


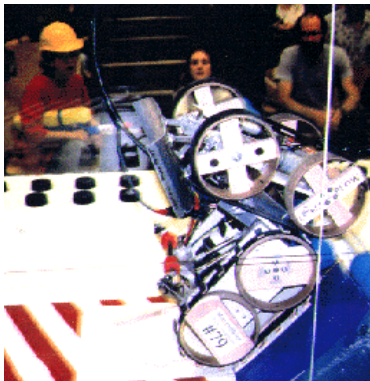
Wheels: *Size, Torque and Contact Pressure*

- How big must a wheel be to make sure it can overcome obstacles?
  - Yet the smaller the wheel diameter, the less the torque to turn the shaft
    - Ideally, the wheel just slips when maximum torque is applied
      - This keeps you from stalling the motor and potentially burning it out
  - And the smaller the diameter, the higher the contact pressure, and the greater the wear
    - Hertz Contact stress theory can determine if wheels are too heavily loaded!









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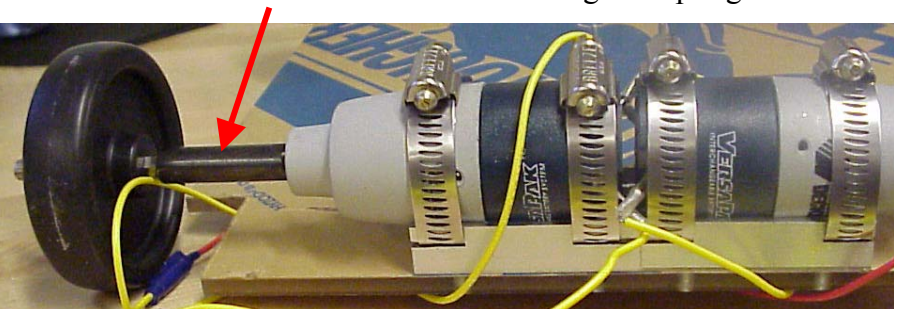
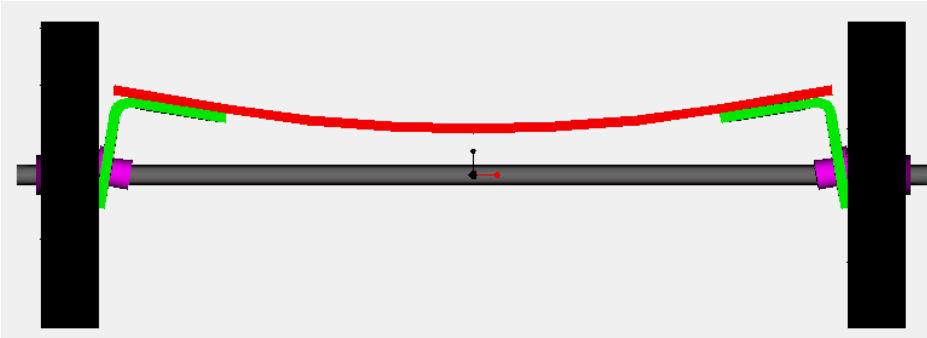
Wheels: *Manufacturing & Mounting*

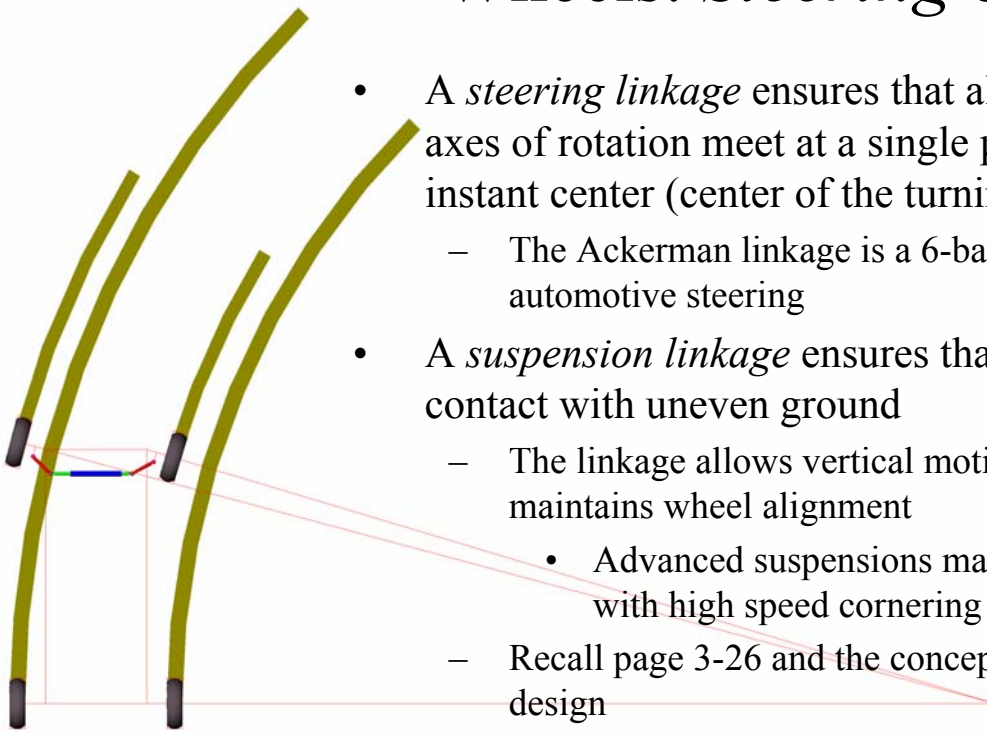
- A positive engagement is required to effectively transmit torque from a shaft to a wheel
  - A hexagon hole can be broached into pulleys and wheels
  - Square and hexagon shafts can be threaded!
  - A round shaft's ends can be machined square to fit into a square-broached hole
- The wheel-on-shaft subassembly should overhang the gearbox as little as possible!
  - If the overhung shaft has a lot of compliance, an "outrigger" bearing can be added without over constraining the shaft, if it confines the shaft to its nominal central position
- Beware of system deformations and the fact that they can take up all the clearance in a system and cause it to bind

Too much overhang (remember Saint-Venant!). Consider the use of an outrigger bearing, or mount the wheel on a properly supported shaft and drive the shaft with the motor using a coupling

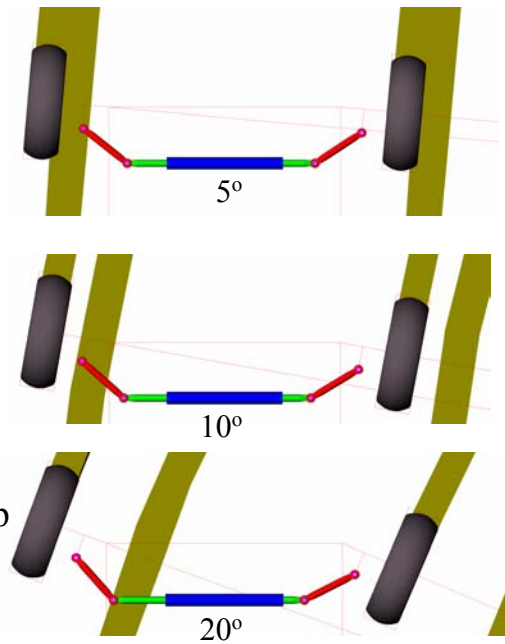




Wheels: *Steering & Suspensions*



- A *steering linkage* ensures that all the vehicle's wheels axes of rotation meet at a single point, which is the instant center (center of the turning radius)
  - The Ackerman linkage is a 6-bar linkage developed for automotive steering
- A *suspension linkage* ensures that all wheels keep contact with uneven ground
  - The linkage allows vertical motion of the wheel, but maintains wheel alignment
    - Advanced suspensions may cant (tilt) wheels to help with high speed cornering
  - Recall page 3-26 and the concept of elastically averaged design
- Vehicles with crawler tracks (e.g., bulldozers) typically allow at least one of the Crawler tracks to pivot to help ensure the treads to maintain ground contact



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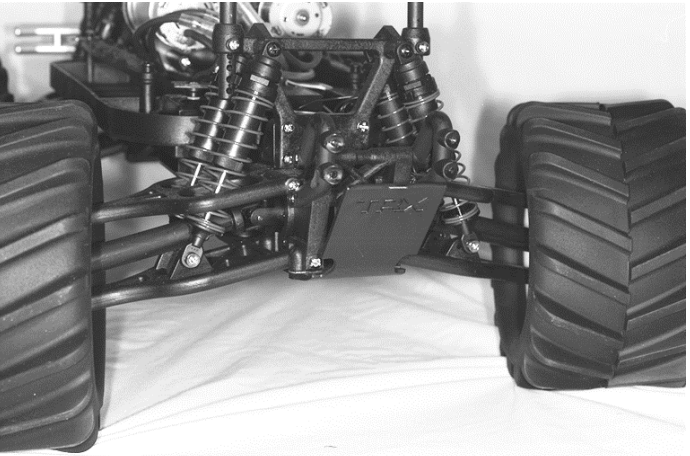
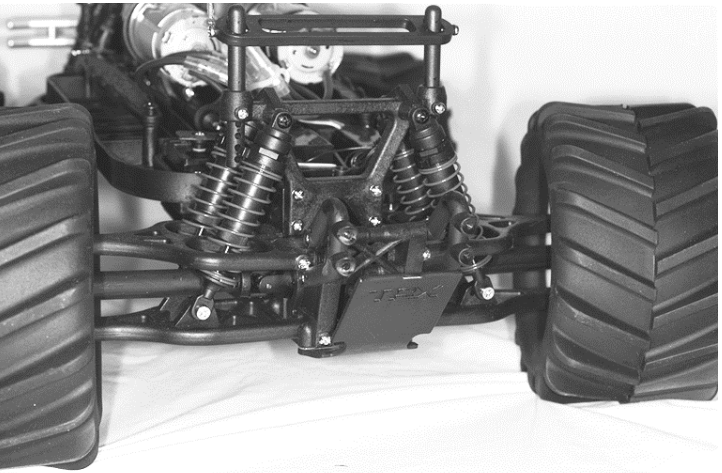
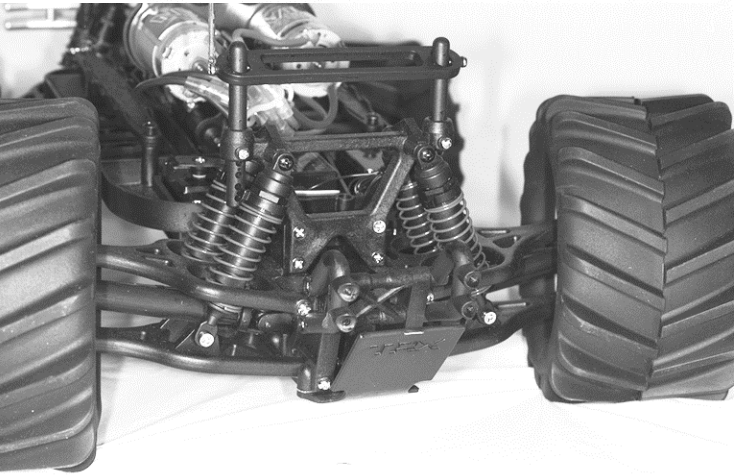
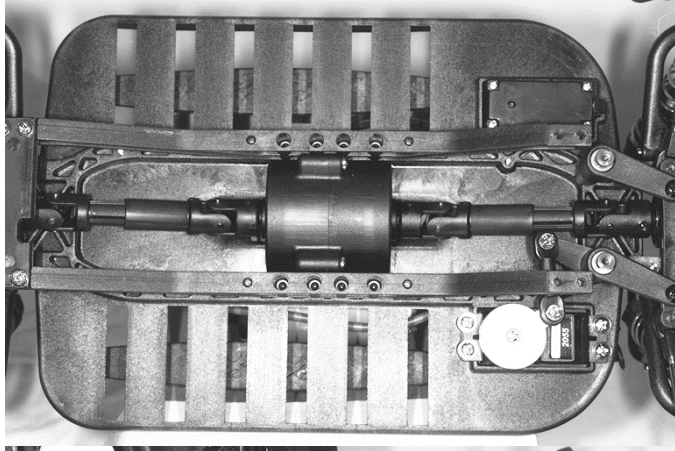


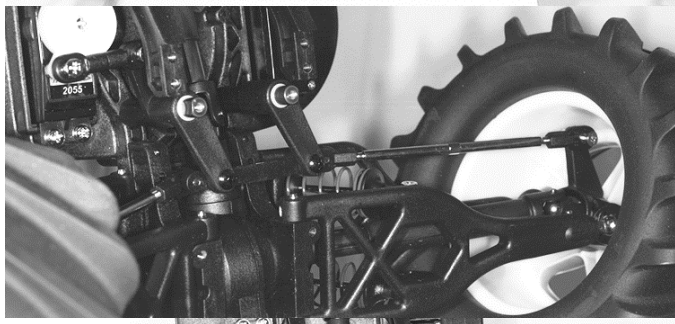


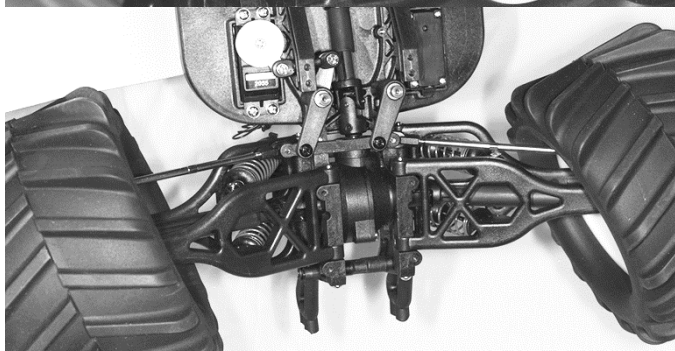


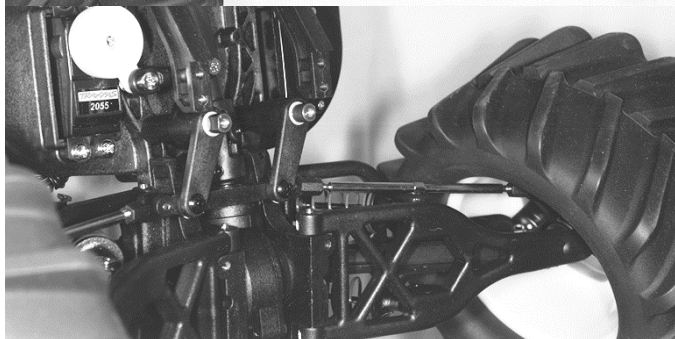
Wheels: *Steering & Suspensions*

- How to transmit power to the wheels and enable them to move with respect to the vehicle to absorb shocks and move across uneven surfaces?
- How to transmit power to the wheels and enable them to steer?





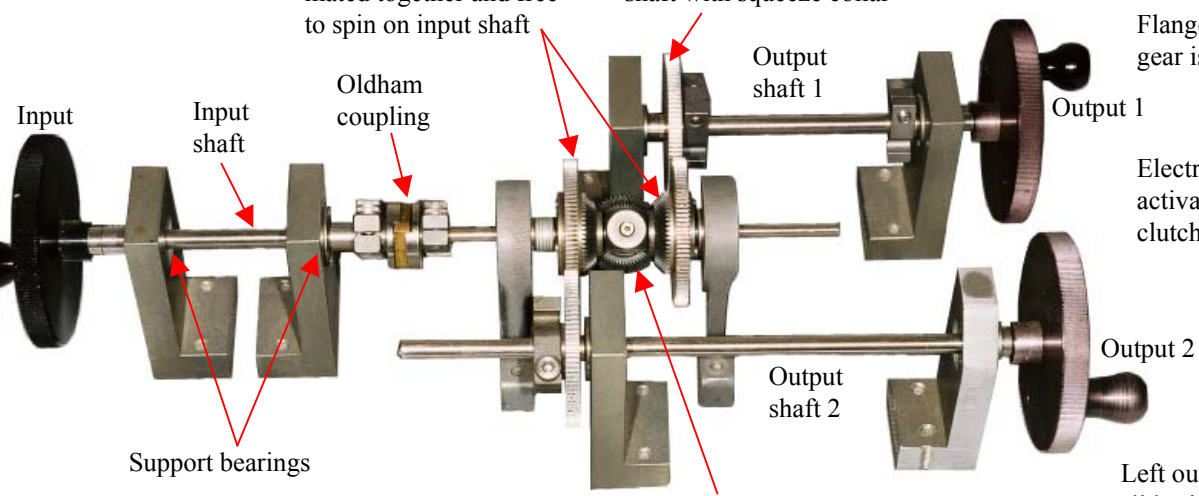


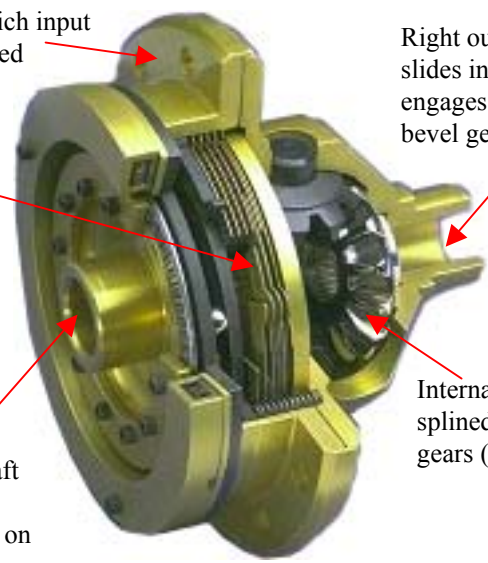


Wheels: *Clutches & Differentials*

- Clutches and differentials allow for differential velocities between rotating shafts
  - A classic example is when a car makes a turn, all the wheels have the same instant center, but the radius from each wheel to the instant center is different
    - Thus each wheel travels a different distance along an arc in the same time, and a *differential* is needed to enable the speed difference to occur
      - However, if one wheel is on ice, and another wheel is on dry pavement, the wheel on ice will spin
    - Limited slip differentials typically use spring-loaded clutch plates to ensure some torque is always transmitted to both wheels
    - Other designs use a centrifugal force to engage locking features
    - New designs use electromagnetic clutches to optimize torque transfer to the wheels
  - Clutches can be activated mechanically or electrically to control the torque transmitted between shafts, while allowing differential velocity to occur (see page 6-17)

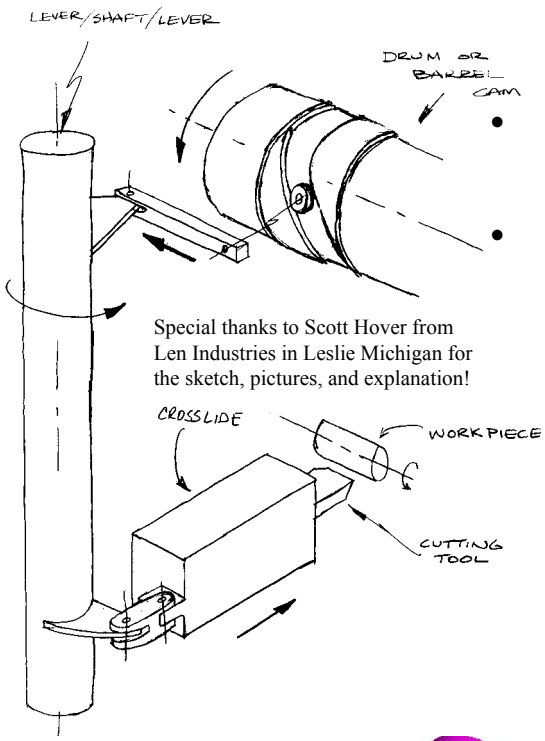
Electric clutch allows limited slip across the differential, or the differential to lock forcing both shafts to be driven. See: http://automotive.eaton.com/product/traction_stability/intellitraxx.html



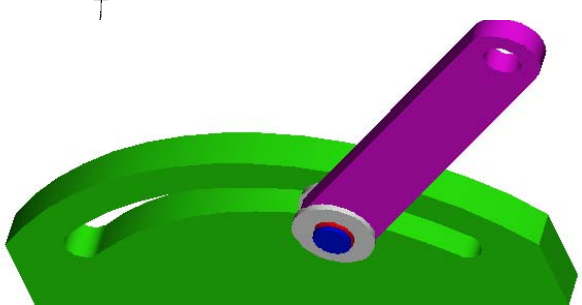


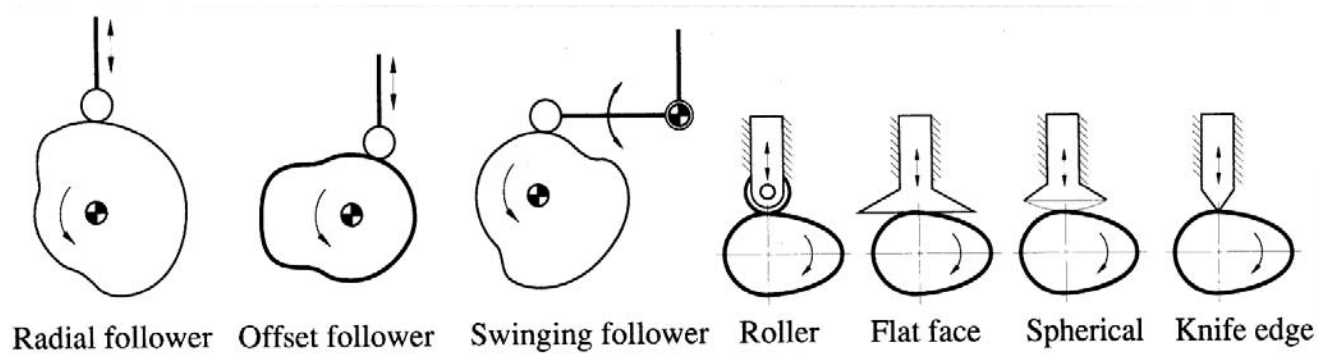
Cams

- A *cam* is a rotating shape whose angular motion is converted into output motion by a *cam follower* which rides on the cam surface
  - Rolling elements provide the highest degree of efficiency, but they take up more space
  - Sliding elements are very compact, and can be efficient of the speed is high enough to maintain oil film lubrication
- A *cam follower* is a rolling or sliding contact element that follows the contour of a surface and transmits the motion to a mechanism
- A *cam profile* can be designed to create corresponding *acceleration*, *velocity*, *position*, and *dwell* profiles in a mechanism (e.g., an engine valve)
  - Specialized cam design software can create virtually any type of cam and provide all required engineering data
  - See url for example http://www.camnetics.com/



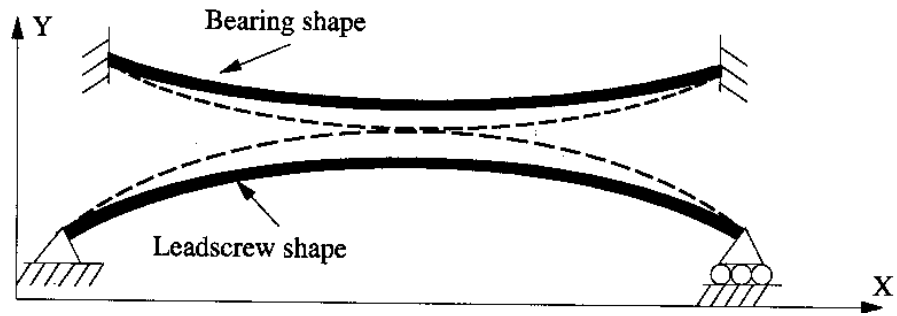
Special thanks to Scott Hover from Len Industries in Leslie Michigan for the sketch, pictures, and explanation!



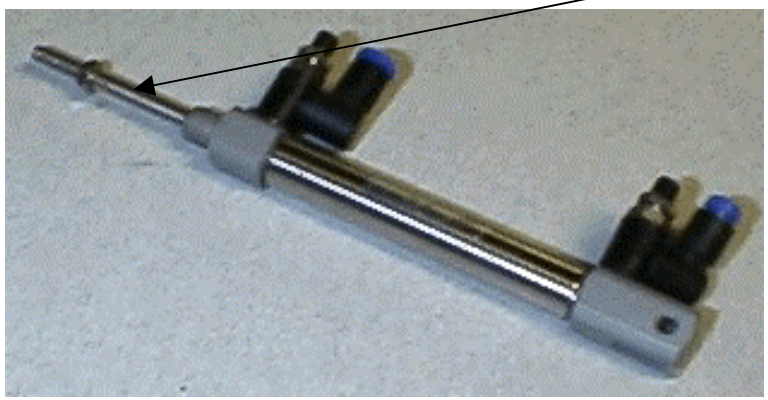


Shafts

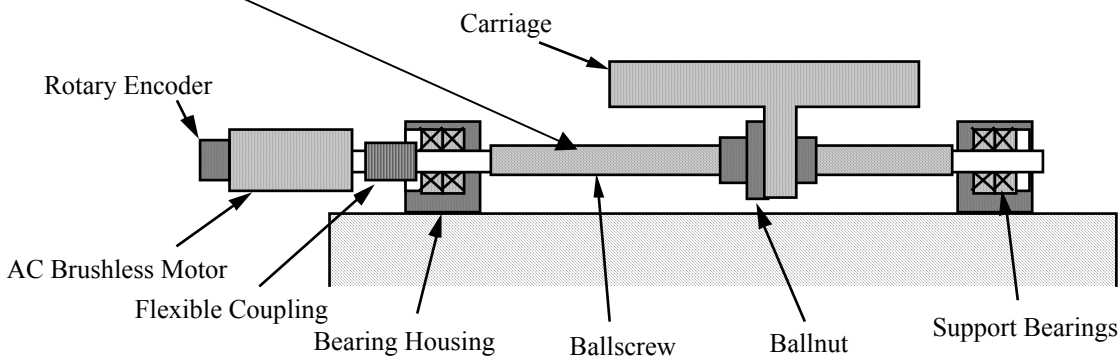
- Shafts transmit rotary and linear power via motors, leadscrews, pistons…
  - Shafts are one of the most common machine elements
- The primary design issues are:
  - Loading conditions and stress concentrations
  - Mounting
  - Component Attachment



Shafts



pneumatic or hydraulic cylinder



Leadscrew driven carriage

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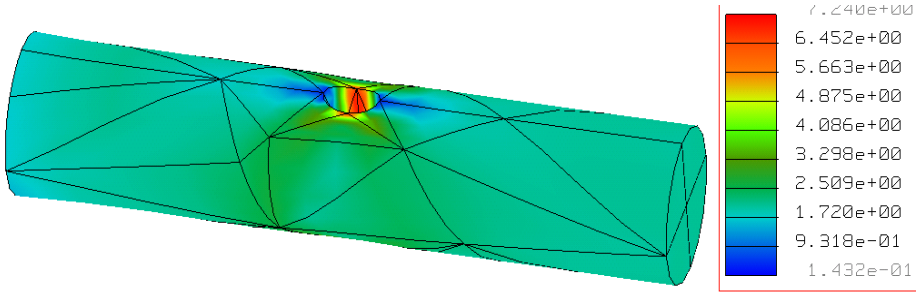
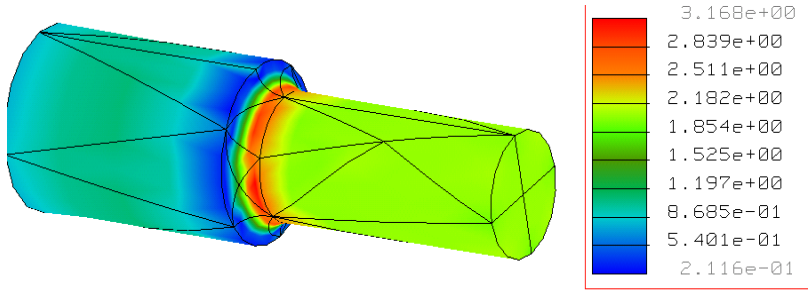
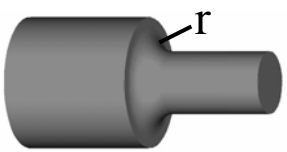
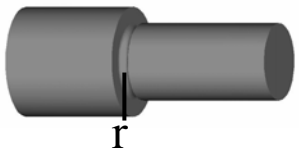
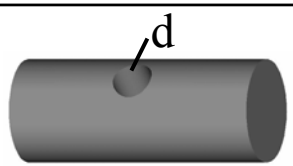
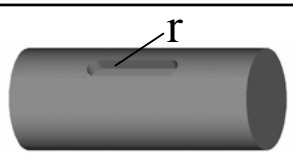
Shafts: *Axial Loads*





	Shaft loaded by axial force F	Stress	Stress concentration factor
Case 1	D  d (r)	$\sigma = \frac{K_t 4F}{\pi d^2}$	$K_t = 1 + \frac{(r/d)^{-0.36-0.2(D/d)}}{5 + 0.12/(D/d-1)}$
Case 2	D  d (r)	$\sigma = \frac{K_t 4F}{\pi d^2}$	$K_t = 1 + \frac{(r/d)^{-0.511-0.34(d/(d-2r)-1)}}{3 + 0.507/(d/d-2r-1)^{0.42}}$
Case 3	D  (d)	$\sigma \approx \frac{K_t F}{0.25\pi D^2 - dD}$	$K_t \approx 1 + 0.65/(d/D)^{0.275}$
Case 4	D  (r) Corner r Width b Depth t	$\sigma \approx \frac{K_t F}{0.25\pi D^2 - bt}$	$K_t \approx 1.5$ B=D/4, t=D/8

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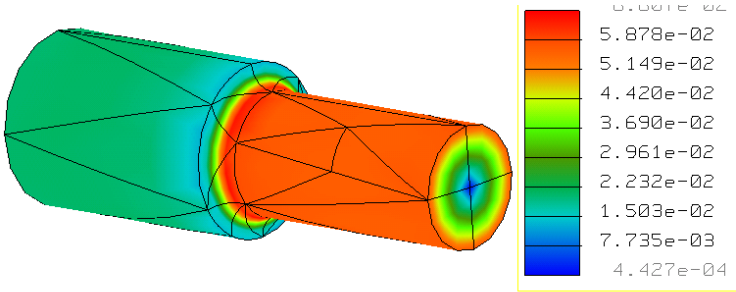
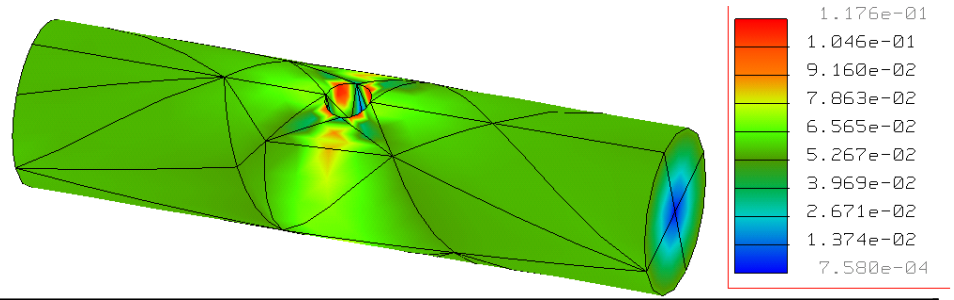
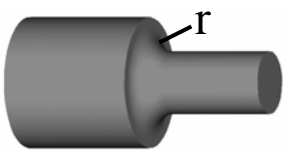
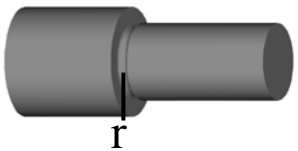
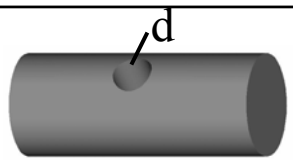
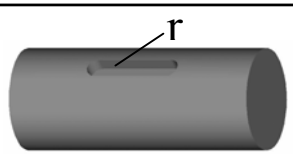
Shafts: *Torsional loads*





	Shaft loaded by Torque Γ	Stress	Stress concentration factor
Case 1	D  d (r)	$\tau = \frac{K_t 16\Gamma}{\pi d^3}$	$K_t = 1 + \frac{(r/d)^{-0.3-0.2(D/d)}}{13 + 0.3/(D/d-1)}$
Case 2	D  d (r)	$\tau = \frac{K_t 16\Gamma}{\pi d^3}$	$K_t = 1 + \frac{(r/d)^{-0.609-0.146(d/(d-2r)-1)}}{5 + 3.73/(d/(d-2r)-1)^{0.252}}$
Case 3	D  (d)	$\tau = \frac{K_t \Gamma}{\pi D^3/16 - d D^2/6}$	$K_t = 1 + 1.47(d/D)^{-0.197}$
Case 4	D  (r) Corner r Width b Depth t	$\tau = \frac{K_t 16\Gamma}{\pi D^3}$	$K_t \approx 1.5$ B=D/4, t=D/8

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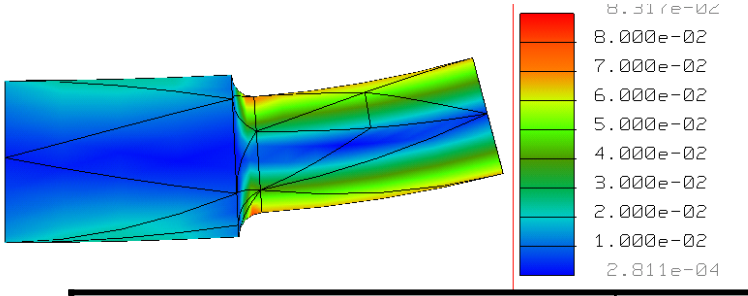
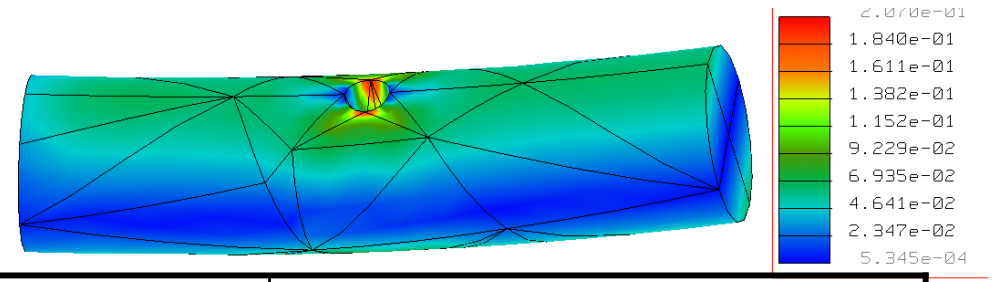
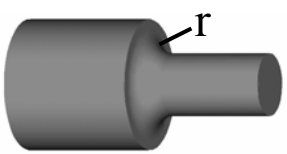
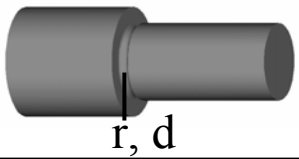
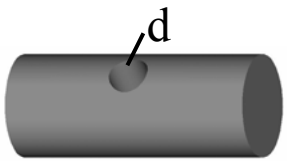
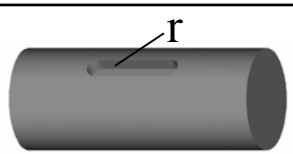
Shafts: *Bending Loads*





	Shaft loaded by Bending moment M	Stress	Stress concentration factor
Case 1	D  d	$\sigma = \dfrac{K_t 32M}{\pi d^3}$	$K_t = 1 + \dfrac{(r/d)^{-0.73-0.42(D/d-1)}}{5 + 4.38/(D/d-1)^{0.16}}$
Case 2	D 	$\sigma = \dfrac{K_t 32M}{\pi d^3}$	$K_t = 1 + \dfrac{(r/d)^{-0.59-0.184(d/(d-2r)-1)}}{5 + 0.081/(d/(d-2r)-1)}$
Case 3	D 	$\sigma = \dfrac{K_t \Gamma}{\pi D^3/32 - d D^2/6}$	$K_t \approx 1 + 0.65/(d/D)^{0.275}$
Case 4	D  Corner r Width b Depth t	$\sigma = \dfrac{K_t 32M}{\pi D^3}$	$K_t = 1 + \dfrac{(r/d)^{-0.66}}{11.14}$ B=D/4, t=D/8

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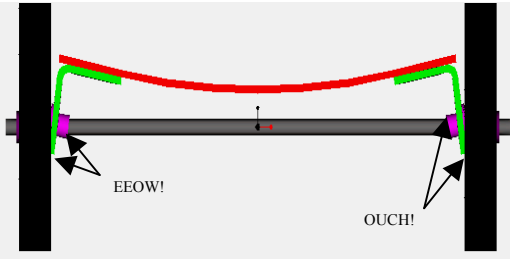
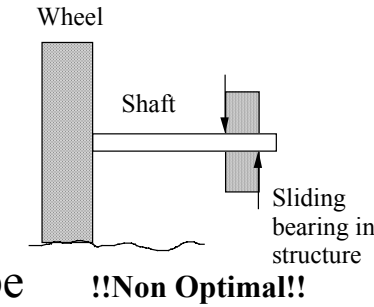
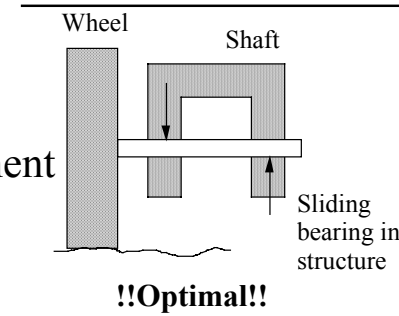
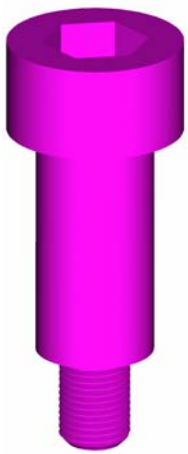


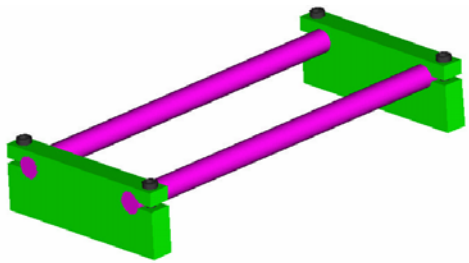
Shafts: *Mounting & Stability*

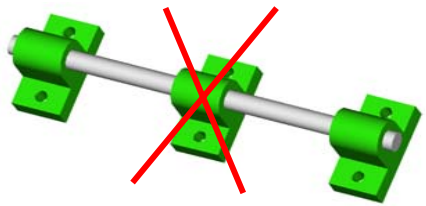




- *Abbe* and *Saint-Venant's* Principles and *Exact Constraint Design* must be carefully considered when designing the mounting of shafts
- Design the system to accommodate misalignment
  - Allow for clearance between bearing and shaft to accommodate misalignment
  - Use flexible couplings or self-aligning (spherical) bearings
  - Use designs that automatically accommodate misalignment
  - Line-bore holes at the same time through mounting plates







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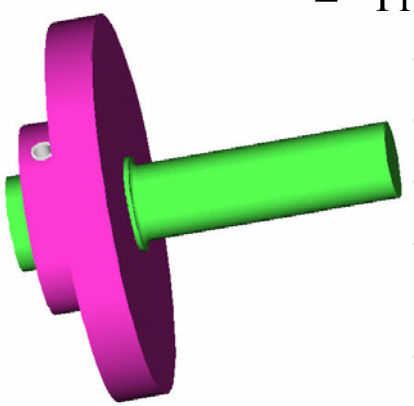
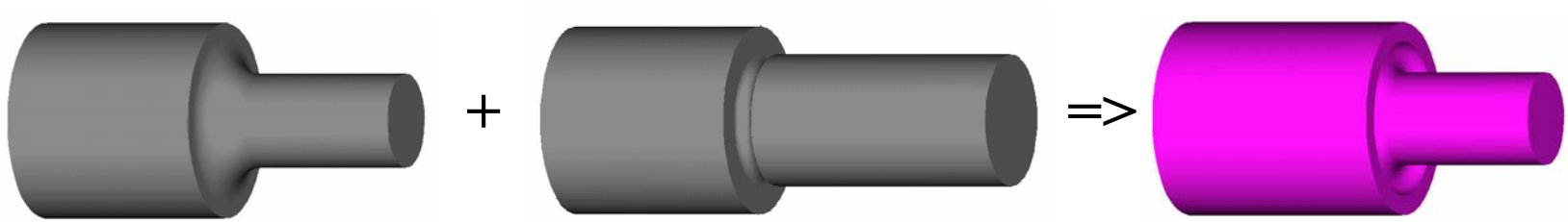
Shafts: *Component Attachment*

- The primary functional requirements for attaching a component to a shaft (in order of goodness) are:
  - Prevent the component from slipping
    - Press-fit: Best torque transmission, but can be difficult to properly manufacture
    - A variation is when the shaft and hub are ground to have a mating lobed fit
    - Spline: Excellent torque transmission, but can be expensive
    - Circumferential clamp: Very good torque transmission, low stress concentration, modest price
    - Keyway: Very good torque transmission, modest stress concentration, low cost
    - Pinned shaft: Good torque transmission, modest stress concentration, low cost
    - Setscrew: Your worst nightmare!



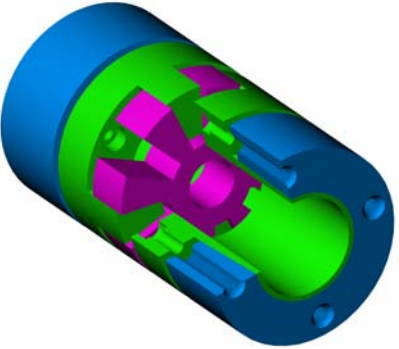
- Minimize stress concentration
  - Raised diameter with radiused corner
    - Requires a lot of material to be removed from a long shaft
    - Rarely required in simple machines

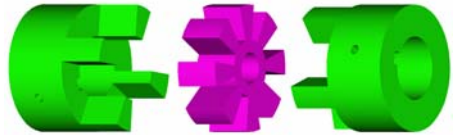


Shafts: *Press-Fits*

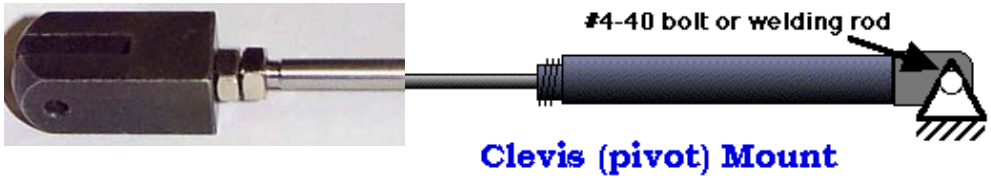
- Press-fits are potentially the most reliable and strongest form of connection between components:
  - The spreadsheet *press_fit.xls* includes the effects of tolerance, differential thermal expansion, and expansion due to high speed rotation (of solid members)

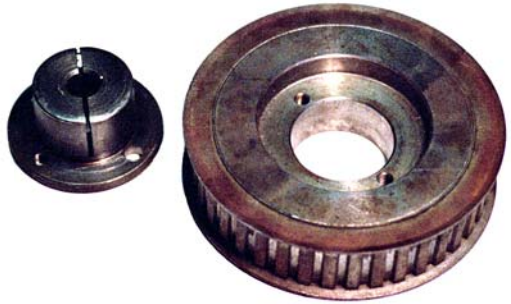
Enter numbers in **BOLD**, Results in **RED**			
Loads		**Required interference parameters**	
Torque to be transmitted, Torque (N-mm)	**100**	Minimum required interface pressure, rPI (N/mm^2)	**0.8017**
Axial force to be transmitted, Force (N)	**100**	Add Poisson diametral interference, ddp (mm)	**6.92E-06**
Coefficient of friction, mu	**0.1**	Add differential thermal expansion diameter, ted (mm)	**7.50E-04**
Operating temperature, ot (^{o}C)	**25**	Add outer body rotating radial displacement, robd (mm)	**1.80E-06**
Stress concentration factor at interface edge, scf	**2**	Subtract inner body rotating radial displacement, ribd (mm)	**4.17E-07**
Rotation speed, omega (rpm)	**100**	Total additional interference amount to be added to ibod, addi (mm)	**0.0008**
Outer body input parameteres		**Interference fit calculations (assumes addi has been added to ibod)**	
Outside diameter, obod (mm)	**20**	maximum interference, maxdelta (mm)	**0.0078**
Interface diameter, obid (mm)	**15**	Maximum resulting interface pressure, maxIP (N/mm^2)	**26.36**
Plus tolerance, obptol (mm)	**0**	minimum interference, mindelta (mm)	**0.0038**
Minus tolerance, obmtol (mm)	**0.002**	Minimum resulting interface pressure, minIP (N/mm^2)	**12.77**
Modulus of elasticity, obe (N/mm^2)	**200000**	Minimum safety margin (min obtained pressure/required pressure)	**15.9**
Yield strength, obsy (N/mm^2)	**300**	**Outer body material stresses at maximum interface pressure**	
Poisson's ratio, obn	**0.29**	Radial displacement of inner surface (mm)	**0.0038**
Coefficient of thermal expansion, obcte (1/^{o}C)	**2.00E-05**	Max radial press-fit stress is at ID, obsr (N/mm^2)	**-26.36**
Density, obrho (g/mm^3)	**7.9**	Max cirumferential press-fit stress is at ID, obsc(N/mm^2)	**94.13**
Inner body input parameters		Axial stress from applied axial Force, obsz (N/mm^2)	**2.29**
Engagement length, L (mm)	**30**	Max shear stress (at ID) from applied Torque, obtau (N/mm^2)	**43.65**
Outside diameter, ibod (mm)	**15.003**	Max cirumferential centrifugal stress is at ID, obcc(N/mm^2)	**0.08**
Plus tolerance, ibptol (mm)	**0.002**	Von Mises stress at ID, obvm (N/mm^2)	**132.74**
Minus tolerance, ibmtol (mm)	**0**	Resulting safety factor (Yield stress)/(scf*Von Mises stress)	**1.13**
Inside diameter, ibid (mm)	**5**	**Inner body material stresses at maximum interface pressure**	
Modulus of elasticity, ibe (N/mm^2)	**200000**	Radial displacement of outer surface (mm)	**-0.0009**
Yield strength, ibsy (N/mm^2)	**300**	Max radial press-fit stress is at OD, ibsr (N/mm^2)	**-26.36**
Poisson's ratio, ibn	**0.29**	Max cirumferential press-fit stress is at OD, ibsc(N/mm^2)	**-32.94**
Coefficient of thermal expansion, ibcte (1/^{o}C)	**1.00E-05**	Axial stress from applied axial Force, ibsz (N/mm^2)	**2.00**
Density, ibrho (g/mm^3)	**7.9**	Max shear stress (at OD) from applied Torque, ibtau (N/mm^2)	**38.19**
Shrink-fit design		Max cirumferential centrifugal stress at OD, ibcc(N/mm^2)	**0.01**
Desired assembly clearance at deltaT, ddt (mm)	**0.005**	Von Mises stress at OD, ibvm (N/mm^2)	**73.54**
Required differential temperature if heating outer body, robdt (^{o}C)	**43**	Resulting safety factor (Yield stress)/(scf*Von Mises stress)	**2.04**
Required differential temperature if cooling inner body, ribdt (^{o}C)	**85**		



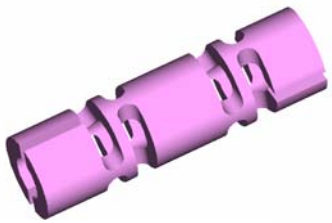


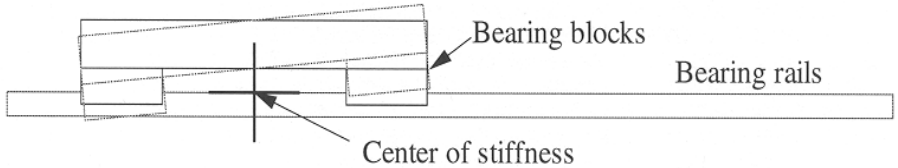
Couplings

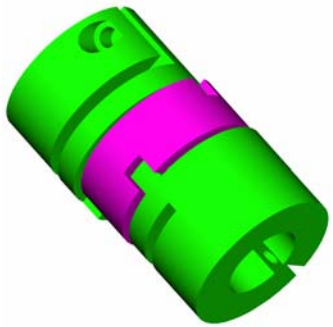


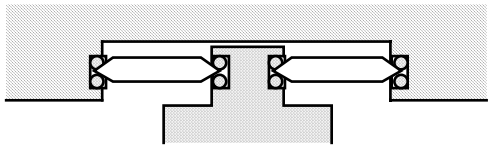


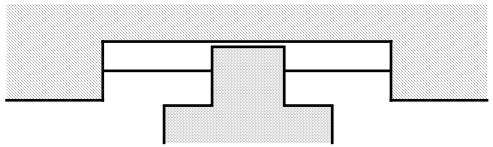
- No two moving components can be perfectly aligned
  - To prevent over constraint from destroying components and robbing your system of power, use couplings
- Identify the degrees of freedom desired, the accuracy (repeatability) needed, and the load capacity and stiffness required in the direction of force or torque transmission
  - Select either a sliding, flexural, or rolling element coupling

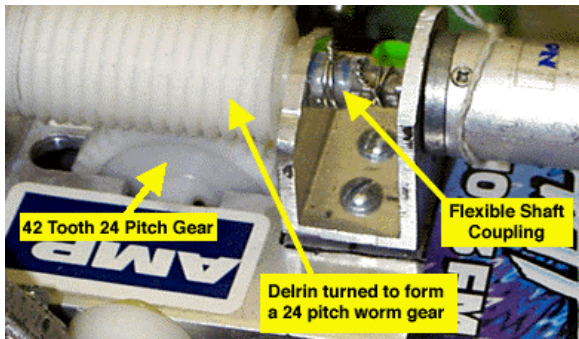












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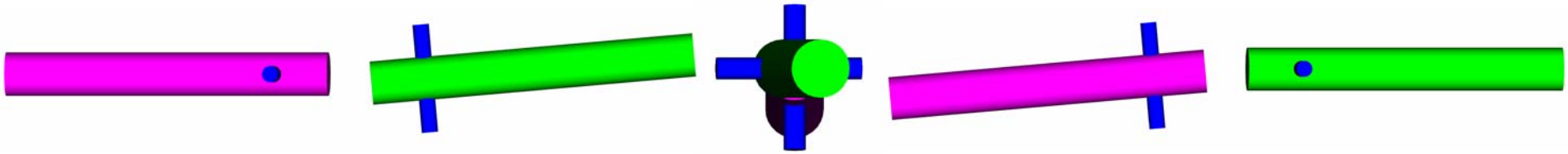
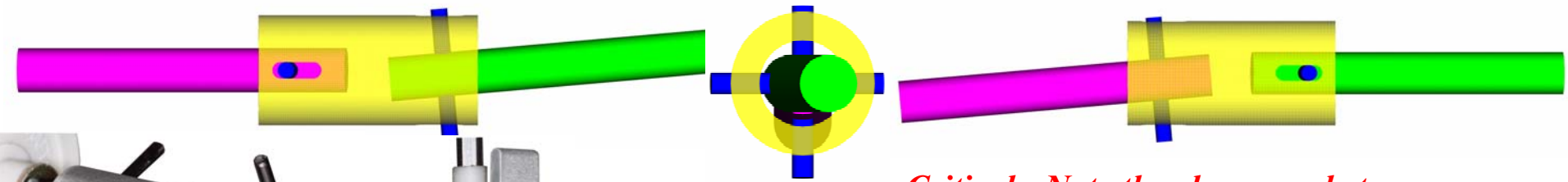
Couplings: *Cheap & Easy Example*

- What about in a robot design contest where two shafts may be linearly misaligned axially, vertically and horizontally, and angularly misaligned?



  - How can you design a simple one-piece coupling to enable one shaft to transmit torque to the other shaft?
    - Can the coupling be made from plastic tube to reduce shock loads?
    - Would O-rings be useful to nominally center it?



Critical: Note the clearance between the coupling bore and both shafts!