Dielectric Resonator Antennas: Theory and Design

by

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Submitted to the Department of Electrical Engineering in partial fulfillment of the requirements for the degree of Master of Engineering in Electrical Engineering and Computer Science at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 1999

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Abstract

Theoretical models for the analysis of Dielectric Resonator Antenna (DRA) are developed. There are no exact solutions to many of the problems in analytical form, therefore a strong focus on the physical interpretation of the numerical results is presented alongside theoretical models. I have used the physical interpretation of the numerical results to lay down some important design rules. A few new inventions associated with the DRA are also included. These are the elliptical DRA, the DRA with a rectangular slot, the adjustable reactance feed, the triangular DRA and the dual band DRA-patch antenna.

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Acknowledgments

It is my pleasure to acknowledge the assistance of the wonderful staff at Qualcomm library for helping me to find many of the research materials. Usually, I only need to send them an email for a paper, and within a few days, the paper would be ready for my perusal. I do try to look for many of the materials myself at the University of California at San Diego (UCSD) library. This is possible because the UCSD library extended to me via Qualcomm Library level 9 privilege to use their materials. Also important is the funding from Qualcomm via the VI-A internship program as well as discussions regarding the thesis subject with Ali Tassoudji who is also my Qualcomm thesis supervisor. Yi-Cheng Lin and Randy Standke assisted me on using the HFSS which is a finite element simulation tool. The email correspondences with Professor Jin Jiang-Ming from University of Illinois on the subject of Method of Moments and Finite Elements, and Professor Allen Glisson from University of Mississippi on the subject of the Body of Revolution, were also very helpful.

It is also my pleasure to acknowledge the many good professors who have taught me important concepts in electromagnetism. These professors are Professor Jin Au Kong who taught me 2 semesters of graduate Electromagnetism, Professor Bose who taught me 1 semester of Acoustics, and Professor John Belcher who often responded so quickly to my questions on electromagnetism despite that he was never my professor for any subject. The Junior Lab training - the experiments, the oral presentations, the Q and A sessions after those presentations were all very helpful in shaping this thesis.
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Chapter 1

Introduction

1.1 Background

This thesis is the documentation of the research that was performed on Dielectric Resonator Antennas when I was at Qualcomm. The significance of this thesis is that it resulted in some important theoretical models for resonant frequency, radiation, and input impedance. It also gives the design rules for tuning the frequency and the input impedance. In the course of gaining this understanding, several novel radiating elements were invented.

Dielectric Resonator Antennas (DRAs) are ceramic resonators that radiate energy into space when excited appropriately. Besides applications in antennas, these ceramic resonators are used in microwave circuits in areas such as filter and combiner applications. Usually they would be encased in a casing to prevent energy lost through radiation when used as such devices. The figures of merit for these resonators are their Q factor and temperature stability. The Q factor can range from 20 to 50000. The relative dielectric constant ($\epsilon_r$) for commercially available dielectric resonators (DRs) can range up to about an $\epsilon_r = 90$. The temperature coefficients of resonant frequency ($\tau_f$) can range from about -6 to 4 ppm/$^\circ$C. The general characteristics of dielectric resonators were described very comprehensively by D. Kajfez and P. Guillon [24].

In recent years, DRAs have enjoyed a strong following among antenna engineers. At QUALCOMM, the interests in such antennas are due to its prospect of shrinking down the sizes of antennas for satellite phone operations and its potential in reducing the cost and ease of manufacturing. DRAs are one of the most recent progresses in antenna technology, taking its place on par among the more common antennas, such as wire, microstrip, horn,
and reflector antennas. Perhaps, for the reason that it is such a recent development that there is still much room for development but with very little published material for reference.

The review paper by Mongia and Bhartia [26] is an excellent paper that outlines the major characteristics of the DRA with respect to modes and their respective resonant frequencies. The cylindrical DRA is emphasized in their paper. The rectangular and spherical DRAs were also touched on briefly. It is a good place to start. Equations for the resonant frequency and bandwidth of the cylindrical DRA were provided based on empirical formulas. The drawback is that it was not intended to contain information such as impedance models or radiation on finite ground plane. There was not enough space for them to write everything.

A few weeks after I decided to embark on this subject as my thesis topic, a publication appeared in the June issue IEEE Magazine entitled, *Recent Advances in Dielectric Resonator Antenna Technology* [27]. This is a paper that describes the current DRA technology being pursued at Communications Research Center in Canada that is done in collaboration with the Royal Military College of Canada. The technologies being pursued are novel DRA elements for wide-band, compactness, circular polarization, high gain and active applications. I suspect that they are pursuing DRA array technology. Their paper has an unusually strong emphasis on couple slot feed, circular polarization (CP) DRA, and arrays.

DRA has the inherent merit of having no metallic loss. This merit is especially good for high frequency applications where conductor loss is proportional to the frequency. The exploitation of this characteristic is in millimeter wave satellites where they can be used for satellite to satellite communication. The earth atmosphere is opaque to the electromagnetic spectrum at millimeter wave due to absorption by $O_2$ and $N_2$ molecules in the atmosphere. This means that information transmitted from one satellite to another satellite via millimeter wave will have increased security due to the impossibility to tap such signals from earth.

Another advantage is that because it is usually operated at high permittivity, it allows the antenna to be shrunk in physical dimensions because the wavelength is reduced by approximately a factor of $\sqrt{\varepsilon_r}$. DRAs also offer easy coupling schemes to transmission lines by simply varying the position of the feed points. And there is much room for flexibility in design to optimize for bandwidth. On the manufacturing side, the DRA can be easily tuned to its resonant frequency by using a tuning screw attached above the DRA. The large
degree of freedom means that design is difficult because a lot of variables interplay with each other. However, if the physics is well understood, it is possible to design the DRA to operate at nearly any rational input impedance, thus eliminating the need of a matching circuit. The complexity of the DRA can be exploited.

I will now give the outline of what is to come. In the later part of this chapter, I would show some of the salient features of the DRA. In the next chapter, I derived, using the magnetic model, the expressions for the resonance frequency for the cylindrical DRA with emphasis on the $HE_{11d}$ mode. The topic of radiation is discussed next. Its relation to the multipole expansion of the field distribution is important because it is this relation that determines the radiation pattern of the DRA. Also important is the discussion on finite ground plane effects [35], [36], [37], [38], [51]. The equivalence principle formulation [14], [1] will be used to explain the radiating mechanism. In chapter 4, an impedance model for the DRA is introduced based on data collected at QUALCOMM and also experimental results published in [39]. Chapter 5 presents the analysis for the resonant frequency and bandwidth of the ring resonator. The ring resonator is important because it allows the bandwidth to be tuned. Some DRA related inventions are also discussed.

1.2 Basic Characteristics

1.2.1 Physical Dimensions and Electrical Properties

Like most good things in life, DRAs can come in many forms of shapes and sizes. The most common shape of the DRA is the cylindrical DRA. This is shaped like a cylinder with its radius, $a$, and height, $h$. It usually has an aspect ratio - the ratio of radius over height ($a/h$) - of about 0.5 to 4. The DRA usually sits on top of a ground plane and is excited using a probe or aperture. The radiation pattern and the feeding method depend on the mode of interest. The DRA can resonate at many different modes. For the cylindrical DRA, the modes are analyzed and indexed in a similar manner as that of the dielectric waveguide. As with the dielectric waveguide, the three spatial coordinates are the radius, $\rho$, the azimuthal angle, $\phi$, and the axial length, $z$. The coordinate system is shown in figure 1-1.
1.2.2 Modes

Transverse modes are defined as modes with transverse E or H fields to the z axis. In waveguides, it is advantageous to solve Maxwell’s equations by considering the transverse E and transverse H separately. The axial vector gives all the information needed to calculate the rest of the vector components. As with the dielectric waveguide, to satisfy the boundary condition of continuity of tangential fields at the boundary of the dielectric and air, the only transverse fields that can exist are the modes with no azimuthal variation such as $TE_{01\delta}$, $TE_{011+\delta}$, $TM_{01\delta}$, and $TM_{011+\delta}$. All the other modes are hybrid modes just like the dielectric waveguide [14]. Table 1.1 summarizes some of the major modes that can exist in a cylindrical DRA. Note that for the $\cos \phi$ variation, we could replace it with $\sin \phi$ or a linear sum of both. This is a degeneracy as both the orthogonal modes with $\cos \phi$ and $\sin \phi$ variations can both simultaneously exist in a DRA with the same frequency. General
The general feeding mechanism for waveguides and cavities is described in the classic book by Collin [4], [5] and also by Jackson [11]. These feeding structures are usually small electric or magnetic dipoles. The reason for why this is so is because the field distribution for the low order modes when expressed as an expansion in multipole terms would contain strong low order terms such as electric dipole or magnetic dipoles terms. Therefore to couple the energy from the probe into the cavity, electric or magnetic dipole probes are used. To obtain magnetic dipoles, small circular current loops are usually employed. However, in practice, variants of these methods are used. For example, it is more typical to see the DRA being excited by a short monopole antenna rather than a dipole. Also, a half-circular current loop is more common in practice.

The apprehension of the appropriate feed mechanism can be obtained in the following way. Firstly, the natural fields in the cavity can be expanded in multipoles. Van Bladel [21], [22] showed that for modes of any shape of the DR are the non confined type. The dominant term in the expansion of the fields in multipole is the magnetic dipole term. The next term

Table 1.1: Salient Features of Modes of a Cylindrical DR

<table>
<thead>
<tr>
<th>Mode</th>
<th>Plane of Symmetry at z=0</th>
<th>Fields inside Resonator</th>
<th>Far Fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TE_{01\delta}$</td>
<td>Magnetic Wall</td>
<td>$H_z = J_0(hr) \cos (\beta z)$ $E_z = 0$</td>
<td>Mag. dipole</td>
</tr>
<tr>
<td>$TE_{011+\delta}$</td>
<td>Electric Wall</td>
<td>$H_z = J_0(hr) \cos (\beta z)$ $E_z = 0$</td>
<td>Mag. quadrupole</td>
</tr>
<tr>
<td>$TM_{01\delta}$</td>
<td>Electric Wall</td>
<td>$E_z = J_0(hr) \cos (\beta z)$ $H_z = 0$</td>
<td>Elec. dipole</td>
</tr>
<tr>
<td>$TM_{011+\delta}$</td>
<td>Magnetic Wall</td>
<td>$E_z = J_0(hr) \cos (\beta z)$ $H_z = 0$</td>
<td>Elec. quadrupole</td>
</tr>
<tr>
<td>$HE_{11\delta}$</td>
<td>Electric Wall</td>
<td>$E_z = J_1(hr) \cos (\beta z) \cos \phi$ $H_z = 0$</td>
<td>Mag. dipole</td>
</tr>
<tr>
<td>$HE_{21\delta}$</td>
<td>Electric Wall</td>
<td>$E_z = J_2(hr) \cos (\beta z) \cos 2\phi$ $H_z = 0$</td>
<td>Mag. quadrupole</td>
</tr>
<tr>
<td>$EH_{11\delta}$</td>
<td>Magnetic Wall</td>
<td>$H_z = J_1(hr) \cos (\beta z) \cos \phi$ $E_z = 0$</td>
<td>Elec. dipole</td>
</tr>
<tr>
<td>$EH_{21\delta}$</td>
<td>Magnetic Wall</td>
<td>$H_z = J_2(hr) \cos (\beta z) \cos 2\phi$ $E_z = 0$</td>
<td>Elec. quadrupole</td>
</tr>
</tbody>
</table>
is the electric dipole term followed by higher order electric and magnetic multipoles. For resonator shape that is axisymmetric, it can also support confined modes. For confined modes, the electric dipole term contributes dominantly.

Therefore, we can picture that embedded in the DRA are these tiny electric or magnetic dipoles. The exact choice is dependent on which mode we choose to excite. For example, to excite the $HE_{11\delta}$ mode which radiates like a horizontal magnetic dipole, we can create an aperture underneath that radiates a magnetic dipole through the surface of the aperture as shown in Figure 1-2. This is called slot-coupled. We can also use a vertical monopole as shown in Figure 1-3

![Figure 1-2: A DRA fed using a slot aperture.](image-url)
1.2.4 Radiation Patterns

From the last section, we know that the fundamental modes of the DRAs radiate like magnetic or electric dipoles. This is because the field distributions in the cavity for the low order modes support these terms. For radiation study, a natural approach to study is to expand the radiated fields using the multipole expansion technique. In the multipole expansion technique, any arbitrary radiation pattern is decomposed into a sum of dipole, quadrupole and higher order multipole terms. Jackson [11] has a good treatment of this subject in Chapter 16 of his book.

For low profile antennas operated at low order modes, the contribution from the higher order poles are usually weak. Generally, the smaller the radiating element is compared to the free space wavelength, the better this approximation is. H. A. Bethe [33] has applied this method to the study of radiation from a small aperture in a waveguide.

DRAs can also be used to produce circular polarization [29]. There are actually two $HE_{11\ell}$ modes which are orthogonal. Some people prefer to call one of it as odd and the other as even to distinguish between the two. To obtain CP from $HE_{11\ell}$, each of the degenerate mode must be excited at 90° phase difference. Figure 1-4 shows the radiation from a circular DRA with dimensions of $a=0.57$ inch and $h=0.425$ inch with an $\epsilon_r = 45$ for the elevation cut when excited in this fashion. Throughout this thesis, in the polar radiation plots, $D_{\text{max}}$ refers to the maximum directivity, $D_{\text{avg}}$ refers to the average directivity averaged from $-80^\circ \leq \theta \leq 80^\circ$ for the major polarization direction (either Right Hand circular polarized (RHCP) or Left Hand Circular Polarized (LHCP) ) for all $\phi$ and $D_{\text{min}}$ refers to the minimum directivity for the major polarization at $\theta = 80^\circ$.

As a low profile antenna, the radiation from a DRA is sensitive to the ground plane size. This is discussed in Chapter 3 of this thesis. For finite ground plane sizes with large...
Figure 1-4: Radiation Pattern as a function of elevation angle for a typical circular DRA with the two modes excited in quadrature phase

diameter (greater than 1 \( \lambda \)), geometrical theory of diffraction (GTD) can be used [38], [51]. For consumer applications, a smaller antenna size is desirable and therefore the ground plane is usually designed to be as small as possible. A small sized ground plane of less than a quarter wavelength can be used with the DRA without significantly distorting the radiation pattern.

1.3 Method of Analysis

There is no exact analytical solution to the Maxwell’s equations for many of the complicated structures of the dielectric resonators that are used as antennas. In view of this, to attempt to gain some logical understanding, approximate models must be used to obtain answers in closed form. This is in accordance with the early developments of quantum theory. In the beginning, when the radiation from the hydrogen atom was not understood, ad hoc methods such as the Bohr planetary theory of the atom was used to predict the radiation spectra. Of course, this cannot be correct for reasons of stability, because an electron moving in orbit would be accelerating and would thus radiate energy away and in the end collapse.
into the nucleus. The rest is of course history. De Broglie proposed that those atoms were de Broglie waves, and given that they are waves, it was natural for an uncertainty principle to be formulated and the need for a wave equation to describe the space time variation of the wave. Thus, Bohr's theory led to new ideas that suggested the existence of the particle wave duality. Perhaps using a model as ad hoc as it may be might lead to some insights that were not captured from experience and intuition.

According to my research, the most popular models to analyze these DRAs are the magnetic wall model, the transmission line model and numerical methods (Method of Moments [18] or Finite Elements [19], [20]). The magnetic wall model and the transmission line model are good for building physical insight. However, they are not as accurate as numerical methods. Numerical methods are the most accurate but they offer very little physical insight and a good reliable finite element program may take as long as 6 hours to solve a problem at a particular frequency. Thus, for anyone wishing to increase the rate of her design cycle, it is usually good to start off with an analytical model and based on the analytic model refine her design using numerical methods. For the remaining of the chapters, the magnetic wall model will be used to analyze these DRAs. The predicted results using the model will then be corrected by analytical means if possible using perturbation theory or correction factors. Design curves from numerical methods and reported experimental results will also be included.

1.4 Summary

In this chapter, I outlined some salient features of the DRA. Some reasons for using the DRA are its inherent merit of having no metallic loss, small physical size, easily tunable, and the large degree of design freedom in choosing the parameters. The characteristics of the major low order modes can be adequately approximated by keeping the low order terms of the multipole expansion for the field distribution. The radiation pattern of the DRA would ultimately depend on the field distribution inside the DRA. For example, for the $H_{115}$ mode, the fields inside the DRA can be approximated closely by a horizontal magnetic dipole. Therefore the radiation pattern for that DRA at that particular mode will be that of a horizontal magnetic dipole. Methods of feeding the DRA to excite the mode of interest using monopole probe and slot fed method are also discussed.
Chapter 2

Resonance and Bandwidth

2.1 Introduction

The formulation in terms of TE and TM modes in guided wave theory was motivated by the observation that the complicated wave problem can be separated into two main problems of finding the electric and magnetic fields transverse to the direction of propagation. When these two quantities are found, one could obtain the other components by using Faraday’s Law and Ampere’s Law as formulated in Maxwell’s equations. The fields in a resonator are standing wave fields and thus the problem can be solved by assuming that the waves are guided waves propagating in +z and -z directions. To find the transverse components the following equations called homogeneous Helmholtz equations are solved where \( z \) is the direction of propagation:

\[
[\nabla^2 + \omega^2 \mu \varepsilon] E_z = 0
\]

for transverse magnetic (TM) case and,

\[
[\nabla^2 + \omega^2 \mu \varepsilon] H_z = 0
\]

for transverse electric (TE) case.

The other field components are given by Faraday’s and Ampere’s laws after doing some algebraic manipulation to solve for each vector components in terms of \( E_z \) and \( H_z \). The final field expressions are given as:
where $k = \omega^2 \mu \varepsilon$ is the wave number and $s$ is the transverse components. I am using the same notation that was used by Kong [14]. In the case of cylindrical dielectric resonators, the equations are expanded in cylindrical coordinates and substitution $H_z = 0$ for the TM case into the above equations give:

\[
\begin{align*}
E_\rho &= \frac{1}{k^2 - k_z^2} \left[ \frac{\partial^2}{\partial \rho \partial z} E_z \right] \\
E_\phi &= \frac{1}{k^2 - k_z^2} \left[ \frac{1}{\rho} \frac{\partial}{\partial \phi} E_z \right] \\
H_\rho &= \frac{-1}{k^2 - k_z^2} \left[ i \omega \varepsilon \frac{1}{\rho} \frac{\partial}{\partial \phi} E_z \right] \\
H_\phi &= \frac{1}{k^2 - k_z^2} \left[ i \omega \varepsilon \frac{\partial}{\partial \rho} E_z \right]
\end{align*}
\]

### 2.2 Formulation of the Cylindrical Dielectric Resonator using the Magnetic Wall Model

The physical dimensions and coordinate origin is as shown in figure 1-1 and 2-1. The height of the resonator is $L$ and the boundary condition at $z=-L1$ and $Z=L2$ is that of a perfect conductor. For the $HE_{l1\delta}$ mode, $H_z \approx 0$ so we will solve the problem for the TM case. The field within the region $0 < z < L$ is,

\[
E_z = J_1(k_\rho \rho) \cos \phi (A e^{i\beta z} + B e^{-i\beta z})
\]

where $\beta$ is the propagation constant in $z$ direction. Knowing $E_z$ we can find the other field components by using equations (2.1) to (2.4). We get,

\[
E_\rho = \frac{i\beta}{k^2 - \beta^2} \left[ k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho) \right] \cos \phi (A e^{i\beta z} - B e^{-i\beta z})
\] (2.5)
Figure 2-1: The DRA is placed at z=0 and the boundary condition at z=-L1 and L2 is that of a perfect conductor

\[ E = \frac{-i\beta}{\rho(k^2 - \beta^2)} [J_1(k_\rho \rho) \sin(Ae^{i\beta z} - Be^{-i\beta z})] \]  
\[ H_\rho = \frac{i\omega \epsilon}{\rho(k^2 - \beta^2)} [J_1(k_\rho \rho) \sin(Ae^{i\beta z} + Be^{-i\beta z})] \]  
\[ H_\phi = \frac{i\omega \epsilon}{k^2 - \beta^2} [(k_\rho J_\rho(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)) \cos(\phi(Ae^{i\beta z} + Be^{-i\beta z})] \]  

where

\[ k^2 = \omega^2 \mu \epsilon = k_\rho^2 + \beta^2 \]

\[ k_\rho \] and \( \beta \) are the propagation constants in \( \rho \) and \( z \) respectively. The radial propagation constant \( k_\rho \) can be obtained by using the condition that \( H_\phi \) is zero at \( \rho = a \). This is due to the perfect magnetic conductor assumption at the curved surface. This condition yields

\[ k_\rho a J_0(k_\rho a) = J_1(k_\rho a) \]

This gives \( k_\rho a = 0, 1.85, 5.3, \ldots \)

Figure 2-1 shows the segmentation of the region around the DRA. Let region 1 be the region where \(-L1 < z < 0\). The \( E_z \) field here is,

\[ E_{z1} = J_1(k_\rho \rho) \cos(\phi(Ce^{\alpha_1 z} + De^{-\alpha_1 z})) \]
Similarly, the fields in region \(-L_1 < z < 0\) are:

\[
E_{\rho 1} = \frac{\alpha_1}{k_1^2 - \alpha_1^2} [k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)] \cos \phi (C e^{\alpha_1 z} - D e^{-\alpha_1 z}) \tag{2.10}
\]

\[
E_{\phi 1} = -\frac{\alpha_1}{\rho(k_1^2 - \alpha_1^2)} J_1(k_\rho \rho) \sin \phi (C e^{\alpha_1 z} - D e^{-\alpha_1 z}) \tag{2.11}
\]

\[
H_{\rho 1} = \frac{i \omega e_0}{\rho(k_1^2 - \alpha_1^2)} J_1(k_\rho \rho) \sin \phi (C e^{\alpha_1 z} + D e^{-\alpha_1 z}) \tag{2.12}
\]

\[
H_{\phi 1} = \frac{i \omega e_0}{k_1^2 - \alpha_1^2} [(k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho))(C e^{\alpha_1 z} + D e^{-\alpha_1 z}) \cos \phi] \tag{2.13}
\]

where \(\alpha_1\) is the wave propagation constant in region 1.

The field \(E_{\phi 1}\) must vanish at \(z = -L_1\) because of the conducting wall. This results in

\[
D = C e^{-2\alpha_1 L_1}
\]

Therefore the above equations can be rewritten in terms of one unknown, \(C\), as

\[
E_{\rho 1} = \frac{2C \alpha_1 e^{-\alpha_1 L_1}}{k_1^2 - \alpha_1^2} [k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)] \cos \phi \sinh (\alpha_1(z + L_1)) \tag{2.14}
\]

\[
E_{\phi 1} = -\frac{2C \alpha_1 e^{-\alpha_1 L_1}}{\rho(k_1^2 - \alpha_1^2)} J_1(k_\rho \rho) \sin \phi \sinh (\alpha_1(z + L_1)) \tag{2.15}
\]

\[
E_{z 1} = 2C J_1(k_\rho \rho) \cos \phi \cosh (\alpha_1(z + L_1)) \tag{2.16}
\]

\[
H_{\rho 1} = \frac{2C e^{-\alpha_1 L_1} i \omega e_0}{\rho(k_1^2 - \alpha_1^2)} J_1(k_\rho \rho) \sin \phi \cosh (\alpha_1(z + L_1)) \tag{2.17}
\]

\[
H_{\phi 1} = \frac{2C i \omega e_0 e^{-\alpha_1 L_1}}{k_1^2 - \alpha_1^2} [(k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)) \cosh (\alpha_1(z + L_1)) \cos \phi \tag{2.18}
\]

In an analogous manner, by making the substitution \(-L_1 = L_2 + L\), we obtain the following fields for region 2:

\[
E_{\rho 2} = \frac{2F \alpha_2 e^{\alpha_2 (L_2 + L)}}{k_2^2 - \alpha_2^2} [k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)] \cos \phi \sinh (\alpha_2(z - L_2 - L)) \tag{2.19}
\]

\[
E_{\phi 2} = -\frac{2F \alpha_2 e^{\alpha_2 (L_2 + L)}}{\rho(k_2^2 - \alpha_2^2)} J_1(k_\rho \rho) \sin \phi \sinh (\alpha_2(z - L_2 - L)) \tag{2.20}
\]

\[
E_{z 2} = 2F J_1(k_\rho \rho) \cos \phi \cosh (\alpha_2(z - L_2 - L)) \tag{2.21}
\]

\[
H_{\rho 2} = \frac{2F e^{\alpha_2 (L_2 + L)} i \omega e_0}{\rho(k_2^2 - \alpha_2^2)} J_1(k_\rho \rho) \sin \phi \cosh (\alpha_2(z - L_2 - L)) \tag{2.22}
\]

\[
H_{\phi 2} = \frac{2F i \omega e_0 \rho e^{\alpha_2 (L_2 + L)}}{k_2^2 - \alpha_2^2} [J_0(k_\rho \rho) - \frac{1}{k_\rho \rho} J_1(k_\rho \rho)] \cosh (\alpha_2(z - L_2 - L)) \cos \phi \tag{2.23}
\]
where
\[ \alpha_3^2 = k_p^2 - k_2^2 \] (2.24)

Next, we use the boundary condition at \( z=0 \). The boundary condition is that the tangential fields must be continuous. This lead to the following condition,

\[ E_{\psi_1} = E_{\psi_2} \] (2.25)

\[ -i\beta (Ae^{i\beta z} - Be^{-i\beta z}) = -2\epsilon_0 \alpha_1 e^{-\alpha_1 L_1} \sinh (\alpha_1 (z + L_1)) \] (2.26)

and

\[ H_{\rho_1} = H_{\rho_2} \] (2.27)

\[ \epsilon (Ae^{i\beta z} + Be^{-i\beta z}) = 2\epsilon_0 C_{\alpha_1} e^{-\alpha_1 L_1} \cosh (\alpha_1 (z + L_1)) \] (2.28)

Dividing the equation 2.26 by equation 2.28, we get

\[ i\beta \frac{(Ae^{i\beta z} - Be^{-i\beta z})}{(Ae^{i\beta z} + Be^{-i\beta z})} = \alpha_1 \epsilon_0 \tanh (\alpha_1 (z + L_1)) \] (2.29)

Now, we must remember to evaluate \( z \) at 0. Doing so we get from equation 2.29

\[ i\beta \frac{(A - B)}{(A + B)} = \alpha_1 \epsilon_0 \tanh (\alpha_1 L_1) \] (2.30)

The boundary condition at \( z=L \) is the same as at \( z=0 \). Thus we just need to substitute the variable \( L_1 \) with \(-L - L_2\) and evaluate \( z \) at \( L \). Doing so we get,

\[ i\beta \frac{(Ae^{i\beta L} - Be^{-i\beta L})}{(Ae^{i\beta L} + Be^{-i\beta L})} = \alpha_2 \epsilon_0 \tanh (-\alpha_2 L_2) \] (2.31)

Now, for a perfect standing wave, \( A \) and \( B \) must have the same amplitude. Thus

\[ \frac{B}{A} = e^{i\theta} \]

With this, we can evaluate the left hand side terms
\[ \frac{A - B}{A + B} = -i \tan \theta /2 \]

and

\[ \frac{Ae^{i\beta L} - Be^{-i\beta L}}{Ae^{i\beta L} + Be^{-i\beta L}} = i \tan (\beta L - \theta /2) \]

Let

\[ \frac{\theta}{2} = \frac{\theta_1}{2} = \arctan \left( \frac{\alpha_1 \varepsilon_r}{\beta} \tanh (\alpha_1 L_1) \right) \quad (2.32) \]

\[ \frac{\theta_2}{2} = \arctan \left( \frac{-\alpha_2 \varepsilon_r}{\beta} \tanh (-\alpha_2 L_2) \right) \quad (2.33) \]

For an isolated DRA in free space, we let \( L_1 \) and \( L_2 \) approach \( \infty \). Then the above conditions reduce to,

\[ \frac{\theta}{2} = \frac{\theta_1}{2} = \arctan \left( \frac{\alpha_1 \varepsilon_r}{\beta} \right) \quad (2.34) \]

\[ \frac{\theta_2}{2} = \arctan \left( \frac{\alpha_2 \varepsilon_r}{\beta} \right) \quad (2.35) \]

In free space with \( \varepsilon_1 = 1, \varepsilon_2 = 1 \), and \( \alpha_1 = \alpha_2 \) we have,

\[ \beta L = \frac{\theta_1}{2} + \frac{\theta_2}{2} = 2 \arctan \left( \frac{\alpha \varepsilon_r}{\beta} \right) \quad (2.36) \]

where \( \alpha_2 \) is given by equation 2.24 and \( \beta \) is given by equation 2.9, both which are the \( z \)-propagation constants in the respective dielectric permittivities.

Now, we have the resonant frequencies for the \( HE_{11\delta} \) by applying equation 2.36.

### 2.2.1 Summary and physical interpretation

We attempted to solve the problem of an isolated DR in free space by using a simplified model. In this model, we assumed that the curved surface of the DR is a perfect magnetic conductor. When an EM wave propagates from region of high permittivity to low permittivity, it can be shown that the tangential magnetic vanishes in the limit \( \varepsilon_r \) approaches \( \infty \).
What is the motivation of this concept?

Now, we know that the intrinsic impedance is inversely proportional to $\sqrt{\varepsilon_r}$. Thus, in region of high permittivity, the impedance is very low. In transmission line theory, the reflection coefficient is given by $\frac{R_o - R_L}{R_o + R_L}$. For a wave traveling from high to low region of permittivity, $R_o$ is much smaller than $R_L$. Thus, the reflection coefficient approaches 1. This is an open circuit thus we can assume it to be a perfect magnetic conductor. On the other hand, for a wave traveling from region of low permittivity to high permittivity, $R_o$ is much larger than $R_L$. Thus the reflection coefficient approaches -1. This is a short circuit in transmission line, or a perfect electric conductor.

Therefore with a curved surface as a perfect magnetic conductor and the ends as perfect electric conductor at some distance away from the resonator, we derived the dispersion relations. This allows us to find the resonant frequency of the DR.

Mongia and Bartha [26] have an empirical expression for the calculating the resonant frequency for the $HEM_{116}$ mode. Table 2.1 compares the values obtained using the method developed in this section with their formula for a cylindrical dielectric resonator in freespace.

<table>
<thead>
<tr>
<th>$\varepsilon_r$</th>
<th>radius (in)</th>
<th>height (in)</th>
<th>Mongia (GHz)</th>
<th>Magnetic Wall (GHz)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.300</td>
<td>1.92</td>
<td>1.56</td>
<td>19</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.425</td>
<td>1.58</td>
<td>1.30</td>
<td>18</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.800</td>
<td>1.22</td>
<td>1.05</td>
<td>14</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>1.500</td>
<td>1.03</td>
<td>0.95</td>
<td>8</td>
</tr>
<tr>
<td>45</td>
<td>0.40</td>
<td>0.300</td>
<td>2.25</td>
<td>1.58</td>
<td>18</td>
</tr>
<tr>
<td>30</td>
<td>0.57</td>
<td>0.425</td>
<td>1.92</td>
<td>1.58</td>
<td>18</td>
</tr>
</tbody>
</table>

The error between the magnetic wall model and the formula obtained by Mongia is large. The reason is because the magnetic wall model is not a perfect model. The dielectric resonator is not a perfect magnetic conductor, but fields actually penetrate across the curved surface of the resonator.

Figure 2-2 shows the plots of the electric field vector for the $HE_{116}$ mode. The top figure is the electric field plot on the side of the resonator. The lower figure is the cut along the equatorial plane. From both sketches, we can deduce that the dominant multipole expansion term should be the horizontal magnetic dipole term.
2.2.2 Perturbation Correction to the Magnetic Wall Model

The previous model gave a close answer but we can get a more accurate result by including a perturbation model. We know that the magnetic field does penetrate through the surface of the DR because it is not a perfect magnetic conductor. So let’s move the perfect magnetic conductor out to a radius distance of $\infty$. What is the new resonant frequency? We will solve this using the perturbation model.

From cavity theory, when an outward perturbation is made at a place of large magnetic field, the resonant frequency is lowered; if made at a place of large electric field, the resonant frequency is raised. The opposite effect occurs for an inward perturbation [14].

Let us collect our results so far, in region $0 < z < L$ which I will label as region 6 using equations 2.5 to 2.8 after substituting the appropriate constants for $A$ and $B$,

\begin{align}
E_{\rho 6} &= \frac{E_0 \beta}{i \omega \varepsilon_0} [k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)] \cos \phi \sin (\beta z - \frac{\theta_1}{2}) \\
E_{\phi 6} &= -\frac{E_0 \beta}{\rho i \omega \varepsilon_0} J_1(k_\rho \rho) \sin \phi \sin (\beta z - \frac{\theta_1}{2})
\end{align}

(2.37)  (2.38)
\[ E_{z6} = J_1(k_p \rho) \cos \phi \cos (\beta z - \theta_1) \]  
\[ H_{\rho6} = -E_0 \frac{\varepsilon_r}{\rho} J_1(k_p \rho) \sin \phi \cos (\beta z - \theta_1) \]  
\[ H_{\phi6} = -\varepsilon_r E_0 [(k_p J_0(k_p \rho) - \frac{1}{\rho} J_1(k_p \rho)) \cos \phi \cos (\beta z - \theta_1)] \]

where \[ E_0 = \frac{-i\omega \varepsilon_0}{(k^2 - \beta^2)} \]

Now, let us break the region into 6 different regions as shown in the Figure 2-3. The region with the DR is region 6, and each corresponding region's fields will be subscripted with the region number. To use the perturbation model, we must first calculate all the fields in each region because we need to eventually calculate the energy.

Figure 2-3: Region around and at the DRA is separated into 6 regions

For region \(-L_1 < z < 0\), using results from previous section, we can immediately write down the fields ensuring continuity at boundary as,

\[ E_{\rho 1} = E_0 \frac{\sin \theta_1}{i \omega \varepsilon_0 \sinh \alpha_1 L_1} [k_p J_0(k_p \rho) - \frac{1}{\rho} J_1(k_p \rho)] \cos \phi \sinh (\alpha_1(z + L_1)) \]  

(2.42)
\[ E_{\phi 1} = -\frac{E_0 \beta}{\rho \omega_0 \sinh \alpha_1 L_1} J_1(k_{p \rho}) \sin \phi \sinh (\alpha_1(z + L_1)) \] (2.43)

\[ E_{z 1} = \frac{\cos \frac{\theta_1}{2}}{\cosh \alpha_1 L_1} J_1(k_{p \rho}) \cos \phi \cosh (\alpha_1(z + L_1)) \] (2.44)

\[ H_{\rho 1} = -\epsilon_r E_0 \frac{\cos \frac{\theta_1}{2}}{\rho \cosh \alpha_1 L_1} J_1(k_{p \rho}) \sin \phi \cosh (\alpha_1(z + L_1)) \] (2.45)

\[ H_{\phi 1} = -\epsilon_r E_0 \frac{\cos \frac{\theta_1}{2}}{\cosh \alpha_1 L_1} [k_p J_0(k_{p \rho}) - \frac{1}{\rho} J_1(k_{p \rho})] \cos \phi \cosh (\alpha_1(z + L_1)) \] (2.46)

and similarly for region \( L < z < L_2 \),

\[ E_{\phi 2} = E_0 \frac{\beta \sin (\beta L - \frac{\theta_1}{2})}{i \omega \epsilon_0 \sinh (-\alpha_2 L_2)} [k_p J_0(k_{p \rho}) - \frac{1}{\rho} J_1(k_{p \rho})] \cos \phi \sinh (\alpha_2(z - L_2 - L)) \] (2.47)

\[ E_{\phi 2} = -\frac{E_0}{\rho \sinh (-\alpha_2 L_2)} \sin \phi \sinh (\alpha_2(z - L_2 - L)) \] (2.48)

\[ E_{z 2} = \frac{\cos (\beta L - \frac{\theta_1}{2})}{\cosh (-\alpha_2 L_2)} J_1(k_{p \rho}) \cos \phi \cosh (\alpha_2(z - L_2 - L)) \] (2.49)

\[ H_{\rho 2} = -\epsilon_r E_0 \cos (\beta L - \frac{\theta_1}{2}) J_1(k_{p \rho}) \sin \phi \cosh (\alpha_2(z - L_2 - L)) \] (2.50)

\[ H_{\phi 2} = -\epsilon_r E_0 \frac{\cos (\beta L - \frac{\theta_1}{2})}{\cosh (-\alpha_2 L_2)} [k_p J_0(k_{p \rho}) - \frac{1}{\rho} J_1(k_{p \rho})] \cos \phi \cosh (\alpha_2(z - L_2 - L)) \] (2.51)

The fields in regions 3, 4, and 5 are now selected such that the solutions are of the forms of modified Hankel functions which are monotonically decaying with increasing \( \rho \). Thus, the fields in region 3, 4 and 5 are:

In region 3:

\[ E_{\phi 3} = -E_0 \frac{\beta}{i \omega \epsilon_0 \rho \sinh (\alpha_1 L_1) K_1(k_{p \rho} \rho)} K_1(k_{p \rho} \rho) \sin \phi \sinh (\alpha_1(z + L_1)) \] (2.52)

\[ E_{z 3} = \frac{\cos \frac{\theta_1}{2}}{\cosh (\alpha_1 L_1) K_1(k_{p \rho} \rho)} K_1(k_{p \rho} \rho) \cos \phi \cosh (\alpha_1(z + L_1)) \] (2.53)

\[ H_{\rho 3} = -\epsilon_r E_0 \frac{\cos \frac{\theta_1}{2}}{\rho \cosh (\alpha_1 L_1) K_1(k_{p \rho} \rho)} K_1(k_{p \rho} \rho) \sin \phi \cosh (\alpha_1(z + L_1)) \] (2.54)

In region 4:

\[ E_{\phi 4} = -E_0 \frac{\beta}{i \omega \epsilon_0 \rho} \frac{J_1(k_{p \rho} \rho)}{K_1(k_{p \rho} \rho)} K_1(k_{p \rho} \rho) \sin \phi \sin (\beta z - \frac{\theta_1}{2}) \] (2.55)
\[ E_{z4} = \frac{J_1(k_\rho a)}{K_1(k_\rho a)} K_1(k_\rho a) \cos \phi \cos (\beta z - \theta_1) \] (2.56)

\[ H_{\rho 4} = -\epsilon_r E_0 - \frac{J_1(k_\rho a)}{\rho K_1(k_\rho a)} K_1(k_\rho a) \sin \phi \cos (\beta z - \theta_1) \] (2.57)

In region 5:

\[ E_{\phi 5} = -E_0 \frac{i \beta \sin (\beta L - \theta_1)}{\omega_0 \rho \sinh (-\alpha_2 L) K_1(k_\rho a)} K_1(k_\rho a) \sin \phi \sinh (\alpha_2 (z - L_2 - L)) \] (2.58)

\[ E_{z5} = \frac{\cos (\beta L - \theta_1) J_1(k_\rho a)}{\cosh (-\alpha_2 L) K_1(k_\rho a)} K_1(k_\rho a) \cos \phi \cosh (\alpha_2 (z - L_2 - L)) \] (2.59)

\[ H_{\rho 5} = -\epsilon_r E_0 \frac{\cos (\beta L - \theta_1) J_1(k_\rho a)}{\rho \cosh (-\alpha_2 L) K_1(k_\rho a)} K_1(k_\rho a) \sin \phi \cosh (\alpha_2 (z - L_2 - L)) \] (2.60)

where

\[ k_\rho^2 = \beta^2 - k_0^2 \]

Now, we have all the field expressions from region 1 to 6. We can proceed to apply perturbation theory to the problem. We can use the expression given in [14]:

\[ \omega = \omega_0 (1 + \frac{\Delta W_m - \Delta W_e}{W_m + W_e}) \] (2.61)

where \(\omega_0\) is the original frequency obtained using the Magnetic Wall model from the previous section. \(\Delta W_m\) and \(\Delta W_e\) are just the energies in regions 3, 4, and 5.

Written in a different form,

\[ \omega = \omega_0 (1 + \frac{W_{m3} + W_{m4} + W_{m5} - W_{e3} - W_{e4} - W_{e5}}{W_{m1} + W_{m2} + W_{m6} + W_{e1} + W_{e2} + W_{e6}}) \] (2.62)

where

\[ W_{mi} = \int \int \int_{\Delta V_i} dV |H_0|^2 \] (2.63)

\[ W_{ei} = \int \int \int_{\Delta V_i} dV |E_0|^2 \] (2.64)

where \(E_0\) and \(H_0\) are the fields in the region where the fields in region \(i\). Again, we compare the results to the empirical formulas obtained by Mongia. The values are shown in the table below.

From the previous section, we see that a perfect magnetic wall model is not a good model
Table 2.2: Comparison of Resonant Frequency for $HEM_{11\delta}$ mode for a cylindrical resonator in free space using empirical formula by Mongia and the magnetic wall model with the perturbation correction (Perturbation).

<table>
<thead>
<tr>
<th>$\varepsilon_r$</th>
<th>radius (in)</th>
<th>height (in)</th>
<th>Mongia (GHz)</th>
<th>Perturbation (GHz)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.3</td>
<td>1.92</td>
<td>1.90</td>
<td>1.1</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.425</td>
<td>1.58</td>
<td>1.61</td>
<td>1.6</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.8</td>
<td>1.22</td>
<td>1.27</td>
<td>4.1</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>1.5</td>
<td>1.03</td>
<td>1.09</td>
<td>5.8</td>
</tr>
<tr>
<td>45</td>
<td>0.4</td>
<td>0.3</td>
<td>2.25</td>
<td>2.27</td>
<td>0.9</td>
</tr>
<tr>
<td>30</td>
<td>0.57</td>
<td>0.425</td>
<td>1.92</td>
<td>1.95</td>
<td>1.6</td>
</tr>
</tbody>
</table>

because the magnetic field does penetrates across the walls of the DR. Thus, we perturbed our solution by moving the magnetic walls outwards. According to perturbation theory, depending on whether there is an increase in magnetic or electric energy, the resonant frequency would shift either up or down.

The physical interpretation of the perturbation result is in order. This is how I understand it qualitatively. First, our unperturbed system is at resonance. At resonance, the time averaged electric and magnetic field energies are equal. If you perturbed the system such that you take away from it equal quantities of time averaged electric and magnetic field energies, then the system is still at resonance because those two quantities are still equal.

Next, perturb the system again so that you take away more time averaged magnetic power than the time averaged electric power. The system is now not in balance. There is more time averaged electric field energy than the time averaged magnetic field energy. Then, the resonant frequency must change to a new frequency such that the total time averaged electric and magnetic field energy would be equal again. Here is another look at a thought experiment.

Imagine that we have a capacitor. At DC, the electric fields would be non time varying and the field arrows would point from the positive plate to the negative plate as shown in figure 2-4. The energy stored is purely electrical. The polarity of the terminals is now switched. If we change it slowly, the fields would point downwards when the polarity is switched - always pointing from the more positively charged plate to the negatively charged plate. Now, let us change the polarity of the plates at a faster rate. As we increase this
frequency, magnetic fields begin to appear in the space between the plates. This is due to Ampere’s Law. A time varying electric field produces magnetic field. Now, we have some magnetic energy. So, what began as a capacitor is now behaving more like an inductor.

![Capacitor electric field lines at DC.](image)

Let us increase the rate of change even faster. Now, more and more magnetic energy builds up. There would be an interplay between Faraday’s law and Ampere’s law. The time varying magnetic field would create electric field, and the time varying electric field would create a time varying magnetic field. Soon, the amount of time averaged electrical energy and magnetic energy within the space between the two plates would be equal. Our system would then be at resonance. And at this point, we can place conductors so that our original capacitor now becomes a cavity.

Perturb the system now by taking away some magnetic energy. After perturbation, there is more electrical energy than there is magnetic energy. To keep both the energy in balance again, we must increase the frequency to generate more magnetic field. This is what is described by the theory.

In the same way, we could describe the thought experiment with an inductor in which case, the result would be the opposite.

### 2.3 A note on fine tuning

Any derived empirical formula is not expected to be exactly precise. The derived formula could be erroneous and these errors could come from experimental errors or interpolation...
errors and many other sources. The antenna engineer must therefore understand the un-
derlying physics to know which parameter is to be reduced or increased to fine tune the resonant frequency of the antenna.

Even if the antenna engineers fully understand the underlying physics, as the antenna gets manufactured, manufacturing tolerances on design parameters will change the resonant frequency. Examples of such manufacturing tolerances are the radome thickness, and the measurement tolerances for cutting and shaping the antennas.

One must therefore perform a sensitivity and manufacturing tolerance study. I like to present here a method that can be used to solve this problem. The problem to solve is the problem of fine tuning the resonant frequency of the cylindrical DRA and to determine its manufacturing tolerances. It is based on a Taylor Series approximation. The idea is that manufacturing tolerances are expected to be small changes in the design parameters. Therefore a first order Taylor Series approximation about the difference between the manufacturing sample and the ideal sample should be adequate to describe its effect on the resonant frequency.

Let the resonant frequency be a function of the radius a, height L and permittivity \( \varepsilon_r \) or \( f(a,L,\varepsilon_r) \).

Then, the differential change in the resonant frequency is,

\[
df = \frac{\partial f}{\partial a} da + \frac{\partial f}{\partial b} db + \frac{\partial f}{\partial \varepsilon_r} d\varepsilon_r
\]

Making a 1st order Taylor Series approximation,

\[
\Delta f = \frac{\partial f}{\partial a} \Delta a + \frac{\partial f}{\partial b} \Delta b + \frac{\partial f}{\partial \varepsilon_r} \Delta \varepsilon_r
\]  

(2.65)

The goal now is to determine the value of the partial derivatives given by the above equation. At simulation stage, this can be easily done. For example, set \( \Delta b = \Delta \varepsilon_r = 0 \). Then vary a by a small amount, \( \Delta a \). Run the simulation and obtain \( \Delta f \) from the simulation. The value of \( \frac{\partial f}{\partial a} \) is then given by \( \Delta f / \Delta a \). The process can be repeated for the other partial derivatives. The end result is that the above equation is now a design equation. It tells the designer exactly how much he or she needs to vary the parameters to further fine tune the resonant frequency.

At manufacturing stage, it is impossible to determine a priori which parameter is to
be held constant. This does not present any problem in my formulation. Each of the change in the parameter can be measured. The change in the resonant frequency can also be measured. From one manufacturing sample, one would be able to rewrite the above equation as an equation in \( n \) number of unknowns for \( n \) number of parameters that are subjected to manufacturing tolerances. Then one needs \( n \) manufacturing samples to produce \( n \) independent equations. The set of \( n \) equations in \( n \) unknowns are a set of linear equations and they can be conveniently solved using linear algebra techniques where the set of equations are expressed in the form \( Ax = b \). If there are more samples than the number of parameters, then this becomes an over determined system in which the solution can be obtained by using least squares. The linear algebraic method solution to least squares is also very straightforward. It is given by \( A^T Ax = A^T b \). The method can of course be generalized to any arbitrary number of parameters.

This method is so general that it does not even require any understanding of the underlying physics beyond the functional dependence of the resonant frequency. It is also applicable to all forms of manufacturing processes beyond antenna engineering. I believe that this is the power of a Taylor Series approximation. Newton was able to come up with \( F = ma \) - an equation still used today despite lacking a true picture of the actual structure of space-time. His famous equation was so accurate for describing macroscopic motions because it is a first order approximation in \( v/c \) where \( v \) is velocity of some object and \( c \) is the speed of light.

2.4 Degeneracy and its similarities to the Zeeman effect - Symmetry

From the field expressions in the DRA, we had a choice of replacing the azimuthal term \( \cos \phi \) with \( \sin \phi \) and vice versa or the linear sum of both and the equations would still be a solution to Maxwell’s equation. Thus, at the same frequency, there are actually two orthogonal modes that exist for the \( HE_{11\delta} \) mode. What is the source of this degeneracy?

Degeneracy in atomic radiation is a very important subject in Quantum Mechanics. Many of the physics of degeneracy between antenna radiation and atomic radiation share the same explanations. I like to discuss this degeneracy in a more broader setting to show that understanding in an area that is not so well known (DRA) can be achieved via
understanding of the same phenomena from other areas (Quantum Mechanics).

In the hydrogen atom, which consist of just a proton and an electron, degeneracy is caused by the symmetry of the system. The Schroedinger equation, which is also a statement of conservation of energy, would contain a term involving the potential energy of the electron which is a function of the distance away from the nucleus, \( r \). The other spatial variables, the polar angle \( \theta \) and the azimuthal angle \( \phi \) do not come into play at first order. The quantum states of the different angular momentum of the electron are orthogonal states. However, they all have the same energy. So we identify the system as having degeneracy because the energy levels of the different electrons are all at the same level.

Symmetry is also the reason for the existence of degenerate modes in the cylindrical DRA. The DRA is symmetric in the azimuthal direction. There isn’t any azimuthal angle that is preferred. We could place our probe at any azimuthal angle and all the results would still be the same had we placed the probe at some other azimuthal angle. But once we have decided on the azimuthal angle to place our first probe, that would destroy the symmetry and the other degenerate mode would have to be excited by a probe placed at 90° away from the probe. This would then be the only other orthogonal mode at the same frequency.

The detection of degenerate energies and its related experiment in Quantum Physics comes under the name of the Zeeman effect. In the Zeeman effect, magnetic fields are applied that effectively breaks the symmetry of the system. The application of the magnetic field interacts with the angular momentum of the electron. Depending on the quantum number of the angular momentum, electrons at different angular momentum states which all previously share the same energy will now have different energy levels. The degeneracy is broken. With the degeneracy broken, two or more slightly different frequencies are radiated and it can be measured. Circular polarized radiation can be observed in Zeeman’s experiment. The interested reader can read more in [2].

We can do an analogy of the Zeeman effect on the DRA. In Chapter 5, I will show several methods that can be used to break this degeneracy which are simpler to implement. This would turn out to have many engineering applications.
2.5 Formula for resonance frequency of DR

The solution using the above magnetic wall model can be involved as one needs to solve the transcendental equation (2.36). Also, a closed form solution given by a simple formula is not obtainable using this method. However, we will use the magnetic wall model to develop a physical insight into obtaining a simple formula.

A crude approximation of the value of $k_z$ can be approximated by assuming that the upper and lower surface of the cylinder is also a perfect magnetic conductor. Then letting $H = L/2$ we can express $k_z$ as,

$$k_z = \frac{\pi}{H}$$

Using the dispersion relations in a cylindrical DRA, we can express the frequency in functional form:

$$f = F\left(\frac{1}{\sqrt{\varepsilon_r}}\right)G\left(\frac{a}{H}\right)$$

(2.66)

For the case of the non-ideal resonator, we have to relax the $\varepsilon_r$ dependence into an $\varepsilon_r + X$ dependence, where $X$ is a constant that varies for different values of $\varepsilon_r$. We can expect the function $G$ to behave in a non-linear manner. If we assume that the ratio of $\frac{a}{H}$ is small, which typically is, we can approximate the function $G$ using power series and discard the very high order terms. Then, one can perform a few experiments and curve fit some experimental data to such a curve to obtain a formula. The formula for such a relation for the $HE_{11\delta}$ mode was published in [26] and is given by:

$$f = \frac{6.324c}{2\pi a\sqrt{\varepsilon_r} + 2}\left[0.27 + 0.36\left(\frac{a}{2H}\right) + 0.002\left(\frac{a}{2H}\right)^2\right]$$

(2.67)

to second order non-linearity. This formula is known to be accurate in the range $0.4 \leq a/H \leq 6$. From here on, we will use the above formula as our formula for the resonant frequency.

For completeness, the formula for $TE_{01\delta}$ mode is [26]:

36
\[ k_0a = \frac{2.327}{\epsilon_r + 1} \left[ 1 + 0.2123 \left( \frac{a}{H} \right) - 0.00898 \left( \frac{a}{H} \right)^2 \right] \] (2.68)

\[ T E_{011+\delta} : \]

\[ k_0a = \frac{2.208}{\epsilon_r + 1} \left[ 1 + 0.7013 \left( \frac{a}{H} \right) - 0.002713 \left( \frac{a}{H} \right)^2 \right] \] (2.69)

and \( T M_{01\delta} : \)

\[ k_0a = \sqrt{3.83^2 + \left( \frac{\pi a}{2H} \right)^2} \] \sqrt{\epsilon_r + 2} (2.70)

Figure 2-5 is a summary plot for \( \epsilon_r = 45 \) for the resonant frequency of the respective low order modes of the DRA. Depending on the field distribution of the respective modes, the radiation will radiate according to the strongest multipole expansion terms. For example, for \( a/H < 1 \), the \( T E_{104} \) is the lowest order mode. This mode radiates like a vertical magnetic dipole because if the field lines were to be expressed into a sum of multipole terms, then the strongest multipole term is that of a vertical magnetic dipole. This is also obvious from the field lines. The next higher order mode is the \( H E_{11\delta} \) mode. This mode radiates like a horizontal magnetic dipole. Radiation and the field lines will be covered in a more extensive manner in a later chapter.

The choice of the mode for antenna purpose would also depend on its Q-factor. The topic on Q factor is discussed in the next section. Low order modes of the DRA have low Q factors (< 100) and are therefore usually used for antenna applications.
Figure 2-5: Resonant wavenumber of different modes of an isolated cylindrical dielectric resonator with relative epsilon = 45.
2.6 Bandwidth and Q-factor

The choice of the mode to utilize for antenna applications depends on the Q factor and also on the radiation pattern. The radiation pattern will be discussed in the next chapter. The Q factor or quality factor is a measure of the bandwidth of operation. It is used as a figure of merit for assessing the performance or quality of a resonator. The Q factor is defined by:

\[ Q = \frac{\omega_0 U}{P} \]  

(2.71)

where \( \omega_0 \) is the resonant frequency, \( U \) is the stored energy, and \( P \) is the power dissipation.

There are many different Q's that are defined. First there is the \textit{unloaded Q}. This is the Q when the resonator is freely oscillating without being driven externally by an external source. When an external source is connected to the resonator and is continuously provided with energy, the appropriate Q would be the \textit{loaded Q}. These definitions are the same as the definitions that one would apply for solid-state circuit resonators with inductors, capacitors and resistors.

For antennas, the important Q-factor is \( Q_{\text{rad}} \), where here the power dissipated term \( P \) in equation 2.71 is the power radiated. Van Bladel [21] has shown that the \( Q_{\text{rad}} \) is proportional to \( \epsilon_r \) for dielectric resonators and this relationship for high \( \epsilon_r \) (i.e., 100) is given as:

\[ Q_{\text{rad}} \propto \epsilon_r^P \]  

(2.72)

where

\( P = 1.5 \) for modes that radiate like a magnetic dipole
\( P = 2.5 \) for modes that radiate like an electric dipole
\( P = 2.5 \) for modes that radiate like a magnetic quadrupole

What is the physical interpretation of Q and why are antenna engineers interested in this quantity? The Q is directly related to the bandwidth. The bandwidth is defined as the frequency bandwidth in which the input VSWR of the antenna is less than a specified value S. The larger the bandwidth, the more coverage over frequency space that one can
utilize the antenna. This relationship is given by [26] as

\[ BW = \frac{S - 1}{Q_u \sqrt{S}} \]  

(2.73)

where \( Q_u \) is the unloaded Q. Dielectric resonator antennas have negligible dielectric and conductor loss compared to its radiated power. Therefore the radiated Q,

\[ Q_u \approx Q_{rad} \]  

(2.74)

Equation 2.72 and the values of P are only valid for very high \( \epsilon_r \) (> 100). For low \( \epsilon_r \), numerical results from published sources [26] have found that the relationship given by equation 2.72 is still valid, although the values of P have to be adjusted. Another important result is that the values of P are nearly independent of the aspect ratio of the resonator. Table 2.3 summarizes the value of the index P for the lower order modes [26].

<table>
<thead>
<tr>
<th>Mode</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE_{01\delta}</td>
<td>1.27</td>
</tr>
<tr>
<td>HE_{11\delta}</td>
<td>1.30</td>
</tr>
<tr>
<td>HE_{21\delta}</td>
<td>2.49</td>
</tr>
<tr>
<td>EH_{11\delta}</td>
<td>2.71</td>
</tr>
<tr>
<td>TE_{011+\delta}</td>
<td>2.38</td>
</tr>
</tbody>
</table>

Table 2.3: Value of P for various modes of a cylindrical DR

### 2.7 The relationship between the resonant frequency and the Q-factor

In systems theory, a typical resonance curve is given by

\[ H(s) = \frac{s}{s^2 + 2\sigma s + \omega_0^2} \]  

(2.75)

The poles of the system are located at \(-\sigma \pm i\beta\) where \( \beta \approx \omega_0 \) provided that \( \sigma \ll \beta \). The Q-factor is given by

\[ Q \approx \frac{\omega_{peak}}{\Delta \omega} \approx \frac{\omega}{2\sigma} \]  

(2.76)
Numerical techniques can be used to locate the poles. As an example, in the method of moment used in [17] and [25], the final impedance matrix is solved for its homogeneous solution,

\[ ZI = 0 \]  
(2.77)

where the solution to the above equation occurs when the determinant of matrix \( Z \) is zero at some complex frequency \( \sigma + i\omega_0 \). Using equation 2.76, one can then solve for the \( Q \) factor.

### 2.8 Criteria for \( Q_{rad} \)-factor for radiation applications and approximate formulas

The lower order modes typically have lower \( Q_{rad} \) making them more suitable for practical applications. Table 2.4 shows the resonant frequencies for the 5 lowest modes and their corresponding \( Q_{rad} \)-factor [23]. The dimensions for the cylindrical DRA are \( a=5.25 \) mm, \( L=4.6 \) mm, and \( \epsilon_r = 38 \). The \( Q_{rad} \)-factor is important because a low \( Q_{rad} \) would indicate a large bandwidth but a high \( Q_{rad} \) would indicate a small bandwidth of operation. From Table 2.4 we can see that the \( HE_{21\delta} \) mode is not as efficient as a radiator when compared to say the \( HE_{11\delta} \) mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Res. Freq (GHz)</th>
<th>( Q_{rad} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TE_{01\delta} )</td>
<td>4.829</td>
<td>45.8</td>
</tr>
<tr>
<td>( TM_{01\delta} )</td>
<td>7.524</td>
<td>76.8</td>
</tr>
<tr>
<td>( HE_{11\delta} )</td>
<td>6.333</td>
<td>30.7</td>
</tr>
<tr>
<td>( HE_{12\delta} )</td>
<td>6.638</td>
<td>52.1</td>
</tr>
<tr>
<td>( HE_{21\delta} )</td>
<td>7.752</td>
<td>327.1</td>
</tr>
</tbody>
</table>

Table 2.4: Resonant Frequencies and \( Q \) Factors of the Five lowest modes for a cylindrical DRA with dimensions \( a=5.25 \) mm, \( L=4.6 \) mm, and \( \epsilon_r = 38 \).

The following are the CAD formulas for some lower order modes as published in [26]. The formulas are valid between \( 0.5 \leq a/H \leq 5 \).

\( HE_{11\delta} \):

\[ Q_{rad} = 0.01007\epsilon_r^{1.3} \frac{a}{H} (1 + 100e^{-2.05(0.5a/H-0.0125(a/H)^2)}) \]  
(2.78)
$TE_{015}$:

$$Q_{rad} = 0.078192\varepsilon_r^{1.27}(1 + 17.31\frac{H}{a} - 21.57\left(\frac{H}{a}\right)^2 + 10.86\left(\frac{H}{a}\right)^3 - 1.98\left(\frac{H}{a}\right)^4) \quad (2.79)$$

$TE_{011+6}$:

$$Q_{rad} = 0.03628\varepsilon_r^{2.38}\left[-1 + 7.81\left(\frac{H}{a}\right) - 5.858\left(\frac{H}{a}\right)^2 + 1.277\left(\frac{H}{a}\right)^3\right] \quad (2.80)$$

Figure 2-6 is a summary plot for $\varepsilon_r = 45$.

![Figure 2-6](image_url)

Figure 2-6: $Q_{rad}$ of different modes of an isolated cylindrical dielectric resonator with $\varepsilon_r = 45$.

### 2.9 Summary

In this chapter, the cylindrical DRA was analyzed using the magnetic wall model. Perturbation corrections were introduced to obtain more accurate results. Design formulas
were given for the resonant frequency and Q-factor as a function of the radius, height and permittivity of the resonator.
Chapter 3

Radiation for $HE_{11\delta}$ mode

3.1 Introduction

The DRA is usually operated at one of its lower order modes. At its lower order modes, the field lines show a behavior very similar to the lower order terms of the multipole expansion. Therefore, the radiation pattern is expected to show a radiation pattern that is similar to the corresponding multipole term. This effect is more pronounced because DRAs are usually operated at very high $\varepsilon_r$. Thus, the DRA is like a tiny aperture sitting in free space that radiates energy away. This is very much like radiation from a tiny aperture on a waveguide - a problem that was solved by H. A. Bethe [33].

The radiation pattern for the DRA can also be approximated by using the equivalence principle [14]. The equivalence principle replaces the effect of all the fields inside the dielectric with equivalent currents on the surface. That is, we can replace the radiated power due to the fields inside the volume of a DRA by replacing it with currents on the surface of the resonator. A tabulated method of calculating the radiation pattern using equivalence principle can be found in Balanis [1] for rectangular and for cylindrical coordinates. This method of calculating the radiation pattern is very popular in aperture antennas such as horn antennas. However, to obtain an understanding of the nature of the pattern, we can use field lines.
3.2 Field lines and Multipoles

In this section, I will use 3 modes as examples. The modes are the $TE_{016}$, $TM_{016}$ and $HE_{116}$. These correspond to the lower order modes of the cylindrical DRA isolated in freespace.

The field lines for the $TE_{016}$ is shown in Figure 3-1. From the figure, the fields appear similar to the field lines generated by a magnetic dipole with the dipole axis located along the axis of the cylinder. Therefore, the radiation pattern for this mode is similar to radiation from a vertical magnetic dipole.

![E-field lines](image1)

**Figure 3-1:** Top: E-field lines at the equatorial plane of the DRA. Bottom: the H-field lines at $\phi = 0$ for $TE_{016}$.

For the $TM_{016}$ mode, the field lines as they appear in Figure 3-2 appear to be generated mainly by an electric dipole with the dipole axis located along the axis of the cylindrical DRA. Therefore, the radiation pattern for this mode is similar to the radiation from a vertical electric dipole.

![Field lines](image2)

**Figure 2-2** is the field sketch for the $HE_{116}$ mode. The field lines appear to be generated mainly by a magnetic dipole with the dipole axis located horizontally on the equatorial plane of the cylindrical DRA. Therefore the radiation pattern for this mode is similar to a horizontal magnetic dipole.

The final conclusion with respect to radiation pattern is that the radiation pattern
Figure 3-2: Top: H-field lines at the equatorial plane of the DRA. Bottom: the E-field lines at $\phi = 0$ for $TM_{01\delta}$.

depends ultimately on the field distribution inside the DRA. However, this conclusion is drawn disregarding beam-shaping methods or finite ground plane effects.

3.3 A Model for the Radiation Mechanism

I will use the dielectric waveguide to show a model of the radiation mechanism. First, imagine a dielectric waveguide such as an optical fiber that has a very high permittivity or index of refraction. Electromagnetic waves can propagate in such a medium and be guided, due to the waves bouncing off the walls of the waveguide and very little power is actually lost because the criteria for total internal reflection is satisfied. There are fields beyond the walls of the waveguide but these fields do not radiate energy away. This is because they are evanescent [14]. The only place where there is a net flow of energy is along the waveguide, and no energy is lost through the sidewalls. Now, this is still a waveguide, not a cavity yet.

To make a cavity from a waveguide, we can first short one end of the waveguide or leave it as opened and repeat one of it for the other side. If one end is shorted, and the other is open, then this becomes approximately the $\lambda/4$ transmission line resonator. This is shown in Figure 3-3 where one end of the transmission line is left open and the other end is shorted. If both sides are opened or shorted together, then this becomes approximately the $\lambda/2$ transmission line. Figure 3-4 shows a transmission line opened at both ends. The
dielectric waveguide is now a cavity and it would resonate at some frequencies with many possible modes. So far, this is nothing more than the standard transmission line resonator [15].

![Diagram of a \( \lambda/4 \) transmission line cavity](image)

**Figure 3-3: A \( \lambda/4 \) transmission line cavity**

This is the model that I will use for the radiation mechanism of the DRA. We have a dielectric wave guide where one end is left opened, and on the other circular end of the waveguide, it is shorted with a ground plane. However, the open end of the dielectric resonator is not a perfect open circuit. When the wave hits this boundary, some of it will be reflected and some of it will be transmitted. The transmitted energy becomes the radiated energy from the DRA. The energy reflected off the open end of the DR, will travel back to the short circuited end and be reflected off that end and return back to open end and the process repeats.

This concept of making a radiation mechanism from a cavity by making one of the cavity walls slightly lossy is used to make some kinds of lasers. In certain kinds of lasers, a Fabry Perot resonator is used which consist of two mirrors with one of the mirrors slightly lossy to allow some of the energy inside the cavity to escape. The escaped energy becomes the laser beam.

An important point of the model is that it assumes radiation only escapes from the circular surface of the DRA. No radiation is lost from the cylindrical sidewalls, and therefore when using Huygen's Principle, we need to integrate the fields only on the circular surface.
3.4 Calculating the radiation pattern for the DRA using equivalence principle.

We will replace all the fields on the top of the DRA with equivalent current sources. To account for the ground plane, we must increase our equivalent magnetic source by a factor of 2. We can use equation 12-3 and 12-4 from Balanis [1]. The equations are:

\[
\begin{align*}
\vec{J}_s &= \hat{n} \times \vec{H}_1 \\
\vec{M}_s &= \vec{E}_1 \times \hat{n}
\end{align*}
\]

This form of equivalence principle is also known as Love's Equivalence Principle. For the special case of a cylindrical DRA, the \( \hat{n} \) term is \( \hat{z} \). Also, on the DRA's top surface that is assumed to be a magnetic wall, there are no H fields. Thus \( \vec{J}_s \) is zero. We only need to compute \( \vec{M}_s \).
\[ \hat{M}_s = -E_\phi \hat{\rho} + E_\rho \hat{\phi} \]
\[ = \frac{\beta}{\rho(k^2 - \beta^2)} J_1(k_\rho \rho) \sin \phi \hat{\rho} + \frac{\beta}{(k^2 - \beta^2)} [k_\rho J_0(k_\rho \rho) - \frac{1}{\rho} J_1(k_\rho \rho)] \cos \phi \hat{\phi} \]

\( E_\phi \) and \( E_\rho \) are the radial and azimuthal components of the Electric field obtained from chapter 2 from equations 2.37 and 2.38. With the above current source and increasing it by a factor of 2 to take away the effect of an infinite ground plane, one can then use Huygen’s integral and compute the far field. There are a lot of algebraic manipulations involved, but Balanis [1] has simplified most of the algebraic manipulations. Using equations 12-10 from Balanis, which is reproduced below:

\[ E_\theta = -\frac{ie^{-ikr}}{4\pi r} L_\phi \]
\[ E_\phi = \frac{ie^{-ikr}}{4\pi r} L_\phi \]

where
\[ L_\theta = \int_S [M_\rho \cos \theta \cos (\phi - \phi') + M_\phi \cos \theta \sin (\phi - \phi') - M_\rho \sin \theta] e^{ikr' \cos \psi} ds' \]
\[ L_\phi = \int_S [M_\phi \sin (\phi - \phi') + M_\phi \cos (\phi - \phi')] e^{ikr' \cos \psi} ds' \]

The above equations are in Balanis’s notation. Changing to the notation used in this thesis, the variables are:
\[ r' \cos \psi = \rho' \sin \theta \cos (\phi - \phi') \]
\[ ds' = \rho' d\rho' d\phi' \]

With the above, we can evaluate \( L_\theta \) and \( L_\phi \) using the expression for \( \hat{M}_s \), where \( M_\rho, M_\phi \)
and $M_z$ are the respective $\rho$, $\phi$ and $z$ components of $\mathbf{M}_z$. We will then have an expression for the far field $E_\theta$ and $E_\phi$. The respective radiation pattern on the E and H plane can be found by evaluating the fields at appropriate angles.

In [34], this technique was used to predict the radiation pattern for DRA on a large ground plane. It has been shown in the conference paper that there is good agreement between the theory and the experimental result.

### 3.5 Finite Ground Plane Effects

A finite ground plane is more practical for application purposes. However, the finite ground plane would have an impact on nearly all the parameters of the antenna from the resonance frequency to the radiation pattern. This is well known and many physicists and engineers have studied this problem. I managed to trace back some of the following papers that were useful for studying the effects of ground planes.

We can separate the study mainly into two categories. The first is the effect on a large ground plane which is greater than 1 $\lambda$. There are methods to solve this and my research has unveiled that the most straightforward manner is to use the Geometrical Theory of Diffraction (GTD) method. This method has some shortcomings but if combined with equivalent current as explained by Sutzman and Thiele [16] can give pretty good results. All elegant theories except that I was interested in a much smaller ground plane.

My first study brought me back to as early as 1951. Storer [37] published a very nice paper on the effects of a monopole on a circular ground plane. In it, he derived the change in the impedance of a monopole as the ground plane radius is reduced. His answer was in analytical form.

John Bardeen [35], who won two joint Nobel prizes, one for discovery research leading to the invention of the transistor and the other in 1972 Nobel Prize with L.N. Cooper and J.R. Schrieffer for the theory of super conductivity, published an earlier paper in 1930 where he solved the problem theoretically as well. However, his answer was in the form of an integral equation for the currents on the ground plane and he didn’t solve the integral equation.

Another relevant research was done by Leitner and Spense [36] where they obtained the solution in the form of an infinite series of spheroidal functions. The solution to their problem is the same solution that was obtained by Storer if the integral equation in Storer’s
formulation was solved by expanding the unknown parameter in Storer's integral equation using an infinite series of spheroidal functions.

It appears that solutions to these problems that are problems of monopole on finite ground plane were very mathematically involved and extremely challenging. It required a fine art and mathematical maturity and knowledge of very advanced mathematics. For my case, I have a much more complicated structure than a monopole and if I try to do what these great physicists have done, it would be very tough and maybe I'll never finish my thesis. In the end, I decided the best method to solve this kind of problem is through numerical simulation. A numerical solution is accurate and the time it takes to run a numerical simulation is only about 30 minutes because for far field calculations, the number of iterative runs can be reduced while maintaining accurate solutions for the far field. The number of iterative runs if decreased will increase the error in the impedance and other near field parameters, however far field parameters are not as sensitive.

The following are some of the simulations that I have conducted by varying the ground plane sizes. For the following finite element simulations, I have used a circular ground plane. The DRA is a cylindrical DRA with radius 0.57 inch, height of 0.425 inch and $\varepsilon_r$ of 45. The ground plane diameter is varied. The input impedances traces a sinusoidal pattern. The results for the input impedance as a function of ground plane size is discussed in the next chapter.

The radiation patterns as a function of ground plane size are shown in Figures 3-5 to 3-10. In these figures, $D_{\text{max}}$ refers to the maximum directivity, $D_{\text{avg}}$ refers to the average directivity averaged over elevation angles $-80^\circ \leq \theta \leq 80^\circ$ and $D_{\text{min}}$ refers to the minimum directivity for the LHCP wave at $\theta = 80^\circ$. The ideal DRA at $HE_{115}$ mode is expected to radiate like a horizontal magnetic dipole. Therefore we expect to see circular polarized wave at $\theta = 0^\circ$ and linearly polarized wave at $\theta = \pm 90^\circ$. From the figures, if the ground plane is finite, then this isn’t true. We can see this by observing the axial ratio. For a finite ground plane of 3.7 inch, the axial ratio is very good being below 2 dB for elevation angle, $\theta$, from 0 to about $60^\circ$. 
Figure 3-5: Radiation pattern for Ground plane diameter = 1.4 inch

Figure 3-6: Radiation pattern for Ground plane diameter = 2.96 inch
Figure 3-7: Radiation pattern for Ground plane diameter = 3.7 inch

Figure 3-8: Radiation pattern for Ground plane diameter = 6.06 inch
Figure 3-9: Radiation pattern for Ground plane diameter = 9.16 inch
However, as the ground plane is increased, we see that the axial ratio worsens and good CP is only obtainable at near $\theta = 0$ for the ground plane with diameter of 12.28 inch. This is in accordance with the theory that the DRA operating at this mode radiates like a horizontal magnetic dipole.

The reason for the good axial ratio when the ground plane is finite is due to diffraction around the edges of the ground plane. This was observed via an animated simulation. Figure 3-11 shows the sketch of the electric field lines when the ground plane is finite.
Figure 3-11: Electric field lines for the DRA on a finite ground plane for $HE_{116}$ mode
3.6 Summary

The field patterns inside the DRA for the fundamental modes shows behavior similar to low order multipole expansion terms such as the electric and magnetic dipole terms. This is expected because the DRA is small in electrical size when compared to the free space wavelength. The low order modes therefore radiate like their corresponding multipole field terms. The radiation mechanism was explained based on a lossy cavity wall model and this was verified by direct computation via equivalence principle.

An infinite ground plane is not practical. Finite ground plane changes the radiation pattern substantially. The radiation pattern changes substantially when the ground plane is small due to diffracted waves. When the ground plane is small, the fields crept under the ground plane and this required us to drop the assumption that we can neglect the higher multipole terms in the field expansion.
Chapter 4

Input Impedance of the DRA at $HE_{11\delta}$ mode

4.1 Introduction

The input impedance is the ratio of the complex voltage and complex current at the terminal of the antenna or any generic black box. The input impedance is an important quantity for any radiating system in order to obtain a good impedance match. In musical instruments, a good impedance match allows a musician greater control of the instrument. For sound speakers, at one time, it was a widely held believe in the audio industry that one needs a large speaker to produce loud bass. Professor Bose was able to use his understanding in acoustics that this is not necessarily true and its not the size that matters, but only if the impedance is well matched. And if the impedance of a small speaker is not matched properly, the impedance can be transformed by using sound waveguides such that at low frequencies, the impedance match is good. Then the small speaker would be able to produce bass.

The criterion for a perfect match is called conjugate matching. This is achieved when the input impedance of the radiating system is the complex conjugate of the output impedance of the source that is providing the signal to the radiating system. The total power delivered to the radiating element is 100%. In general, this criteria is only true at a specific frequency, which is why small speakers tend to produce high frequency notes, and large speakers tend to produce louder bass. Small speakers are matched well at some frequency at the high end.
of the audio spectrum while large speakers are better matched at some frequency at the low end of the audio spectrum.

Antennas such as the half wave dipole have well known input impedance. At its fundamental mode of operation, the input impedance is about 73Ω. The actual value depends on the aspect ratio - the ratio of length over radius of the dipole. The relation between the comparison of a DRA and a dipole is that the radius of the dipole is usually so small compared to its height that the \( \phi \) variation in the fields are assumed negligible. This corresponds to the DRA modes where there is no \( \phi \) variation. However, the DRA can also support modes with \( \phi \) variation because the aspect ratio span a much wider range than a dipole. Terminal effects are also important. The position of the feed of the DRA has a strong effect on the value of the input impedance.

I must admit that I do not have the answers to many of the questions that I would hoped to be able to answer regarding the input impedance. This is still an active research area today. I would consider the most important study to be the one that appeared in [39] which was written by Shum and Luk. In this study, the input impedance was modeled using Finite Difference Time Domain (FDTD) method as a function of various parameters. The paper was excellent because of the large amount of data. In this chapter, I will quote results from that paper because it is tabulated in such a conclusive manner and offer physical interpretation which would lead to design rules on manipulating the input impedance.

4.2 Numerical Results

Figure 4-1 shows the general set up and the configuration of the DRA and coaxial feed. In the following simulations, the results are for a DRA at \( \varepsilon_r = 45 \). The radius, \( a \), is kept constant at 0.57 inch. The height \( H \) is varied. The probe height is kept at near or at the same height as the height of the DRA and is located at the boundary of the DRA and air. That means \( b = a \). The radius of the ground plane is kept fixed at 1.69 inch. The simulation was done on High Frequency Solid Structure Modeler (HFSS), which is a finite element electromagnetic simulator. HFSS uses a 2nd order absorbing boundary condition as the radiation boundary condition. In all the following simulations, a fast frequency sweep was used rather than solving the problem frequency by frequency. Fast frequency sweep uses asymptotic waveform evaluation to extrapolate solutions for a range of frequencies.
from a single solution at a center frequency. In all the cases, the center frequency has been chosen according to the CAD formulas presented in Chapter 2.

In this section, the aspect ratio will be redefined as \((H/a)\) rather than \((a/H)\) as in previous sections. This is to make a comparison study with results from dipole impedances such as the one by Tai [12]. Table 4.1 shows the aspect ratio \((H/a)\) of the DRA and its input impedance. It is observed that the input impedance of the DRA traces a similar path on the complex \(Z\) plane as the one for a monopole as the probe is varied.

<table>
<thead>
<tr>
<th>(H) (in)</th>
<th>(l) (in)</th>
<th>(l \times 10^\lambda)</th>
<th>Aspect Ratio ((H/a))</th>
<th>Input Resistance ((\Omega))</th>
<th>(f_{res}) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.150</td>
<td>0.100</td>
<td>0.27</td>
<td>0.263</td>
<td>1</td>
<td>3.19</td>
</tr>
<tr>
<td>0.200</td>
<td>0.200</td>
<td>0.43</td>
<td>0.351</td>
<td>3</td>
<td>2.52</td>
</tr>
<tr>
<td>0.300</td>
<td>0.200</td>
<td>0.32</td>
<td>0.526</td>
<td>9</td>
<td>1.89</td>
</tr>
<tr>
<td>0.425</td>
<td>0.425</td>
<td>0.55</td>
<td>0.746</td>
<td>60</td>
<td>1.54</td>
</tr>
<tr>
<td>0.475</td>
<td>0.425</td>
<td>0.52</td>
<td>0.833</td>
<td>76</td>
<td>1.45</td>
</tr>
<tr>
<td>1.500</td>
<td>1.500</td>
<td>1.13</td>
<td>2.63</td>
<td>1200</td>
<td>0.89</td>
</tr>
<tr>
<td>1.500</td>
<td>0.425</td>
<td>0.41</td>
<td>2.63</td>
<td>58</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Table 4.1: Input resistance as a function of Aspect Ratio

When the probe length is increased, the real part of the impedance increases like that of a monopole [12]. From the table and from the corresponding results in Appendix A, the input impedance becomes highly reactive when the probe height is short. Figure 4-2 shows the input impedance calculated using Method of Moments for a dipole as a function of both aspect ratio and the electrical length. From that figure, the dipole is indeed highly reactive.
when its electrical length is short. This behavior is manifested along when the monopole (where the impedance of the monopole is half that of the dipole) is coupled to the DRA where the imaginary part of the input impedance never crosses zero. The reason for this behavior is that we have used a very short and therefore reactive probe as the feed.

![Antenna resistance versus antenna length for various ratio of length to radius](image1)

![Antenna reactance versus antenna length for various ratio of length to radius](image2)

**Figure 4-2**: Input impedance of a dipole as a function of aspect ratio and electrical length

Another interesting note is that the aspect ratio of the DRA has little effect on the input impedance. This is seen by comparing the final 2 data from Table 4.1 where the same aspect ratio of the DRA produces two vastly different input impedance, one which is at 1200Ω and the other at 58Ω.

[39]'s results is hereby referred to. It was observed that when the length of the probe is changed in a progressive manner from 1.51 cm to 3.03 cm, the resonant frequency varies only within ±0.05 GHz. Figure 4-3 shows a reproduction sketch of the results obtained in their study. The input impedance at resonant ranges from 140Ω to 40Ω. This is the same
trend of a monopole or dipole antenna as a function of electrical length [12] and is shown in Figure 4-2.

Figure 4-3: Input impedance of a DRA as a function of probe length, l

A model to interpret the results is given in Figure 4-4. It is given by a circuit where the monopole probe and the dielectric resonator (DR) are in series. Near the region where resonance is expected to occur, the impedance of the DR is nearly purely real. Any effect of high reactance comes from the monopole probe. By properly controlling the length of the monopole probe and thus the overall input impedance, the designer can tune the input impedance of the overall structure.

In general, to obtain a reasonable input impedance, the height of the DRA should be selected to be about λ/4 or more. This gives the designer the freedom to select the value of the input impedance and to fine tune the height of the monopole probe. The monopole antenna should best be operated at an electrical length where there is very little input reactance.

A useful design curve that one can plot is a graph of the input impedance versus probe
Figure 4-4: Model for the input impedance

height for various aspect ratio of the Dielectric Resonator. This result should be useful for most DRA designers. However, even without such design curves, keeping to the rule by choosing the height of the resonator to be at least $\lambda/4$ and by fine tuning it by adjusting the probe height should be enough for most purposes.

4.3 Finite Ground Plane effects on the Input Impedance

The following are some of the simulations that I have conducted by varying the ground plane sizes. For the following finite element simulations, I have used a circular ground plane. The DRA is a cylindrical DRA with radius .57 inch, height of .425 inch and $\varepsilon_r$ of 45. The ground plane diameter is varied.

Table 4.2 summaries the results of how the resonant frequency and input impedance varies for various ground plane sizes.

The table shows that the input impedance does not show a monotonically increasing or decreasing behavior but instead an oscillatory behavior. This trend is similar to the effect of a finite ground plane size for a monopole. To confirm this, Figure 4-5 shows the plot of the results in Table 4.2 for the input impedance as a function of ground plane size measured in its electrical length. The figure shows a decaying oscillatory behavior as expected. The peak of the oscillating curve repeats itself after approximately $1\lambda$. This is exactly the same behavior as reported for the monopole on finite ground plane by Storer [37]. When the
<table>
<thead>
<tr>
<th>Gnd. Plane Diameter (in)</th>
<th>Resonant Frequency (GHz)</th>
<th>Input Impedance (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.40</td>
<td>1.500</td>
<td>109</td>
</tr>
<tr>
<td>1.65</td>
<td>1.527</td>
<td>123</td>
</tr>
<tr>
<td>2.00</td>
<td>1.531</td>
<td>106</td>
</tr>
<tr>
<td>2.50</td>
<td>1.534</td>
<td>71</td>
</tr>
<tr>
<td>2.96</td>
<td>1.538</td>
<td>57</td>
</tr>
<tr>
<td>3.38</td>
<td>1.543</td>
<td>56</td>
</tr>
<tr>
<td>3.70</td>
<td>1.546</td>
<td>58</td>
</tr>
<tr>
<td>6.06</td>
<td>1.552</td>
<td>77</td>
</tr>
<tr>
<td>9.16</td>
<td>1.554</td>
<td>94</td>
</tr>
<tr>
<td>12.28</td>
<td>1.550</td>
<td>84</td>
</tr>
</tbody>
</table>

Table 4.2: The Resonant Frequency and Input Impedance for various Ground plane sizes

When the ground plane is large, any change in the ground plane size has only a small effect on the input impedance. As the size of the ground plane is progressively decreased, the input impedance traces an increasing sinusoidal behavior where the local peak repeats itself approximately after every wavelength. The swing in input impedance is more pronounced when the size of the ground plane is small.

Figure 4-5: Input Impedance as a function of ground plane size
4.4 Method for tuning the real part of the Impedance

Based on the model described in the previous section, we can now list some design rules for tuning the real part of the impedance of the DRA. According to the model, if we desire to operate the DRA at a lower \( Z \), we ought to make the probe shorter. Also, the ratio of height over radius of the dielectric resonator should be chosen to be small.

Figure 4-6 shows the numerical simulation using a probe height of 0.400 inch and 0.425 inch. The other parameters are radius=0.57 inch, height=0.425 inch, and \( \varepsilon_r = 45 \) which is the same as the parameter for the parameters simulated earlier.

![Impedance Matrix vs. Frequency](image)

Figure 4-6: Input Impedance for a cylindrical DRA with \( a=0.57 \) in, \( h=0.425 \) in and probe length 0.400 inch and 0.425 inch
From the figure, the peak of the real part of the impedance is about 61 Ω when the probe height is 0.425 inch. When the probe height is reduced to 0.400 inch, the peak of the real part drops to 51 Ω. At the resonant frequency defined by the frequency when the reactance is zero, the input resistance is 57 Ω when the probe height is 0.425 inch. When the probe height is reduced to 0.400 inch, the input resistance drops to 43 Ω at the resonant frequency. When the probe height is 0.425 inch, the resonant frequency is 1.544 GHz. When the probe height is 0.400 inch, the resonant frequency is 1.540 GHz. This is only a small change. This suggests that a method that can be used to fine tune the real part of the impedance is the probe height. This relation can be written as:

\[ \Delta l = \frac{\partial l}{\partial f} \Delta f + \frac{\partial l}{\partial Z} \Delta Z \]  

(4.1)

By solving the above relation for the partial derivatives, a designer can use it to create a design equation to perform the fine tuning of the input resistance. From the previous results, we expect \( \frac{\partial l}{\partial f} \) to be small compared to \( \frac{\partial l}{\partial Z} \). This is the mathematical statement that the probe height is a more sensitive parameter on the input resistance than is its effect on the resonant frequency.

In summary, for high Z, pick a large height to radius ratio of the dielectric resonator. This will allow you to use a tall probe. To fine tune the impedance, increase the probe height to increase the impedance or decrease the probe height to decrease the impedance. For low Z, pick a low height to radius ratio for the dielectric resonator. Then fine tune it using the same rules as for high Z. Remember that the probe height can also be increased by moving the probe into the dielectric as this increases the electrical length.

### 4.5 Summary

In this chapter, the input impedance of the cylindrical DRA is discussed for the \( HE_{11\delta} \) mode. The input impedance for the DRA is still not a fully understood area and is still a subject of active research today. A model was used to interpret the results by relating them to the dipole impedance behavior. The reason for using the dipole impedance behavior is because the input impedance trend behaves very similar to the monopole probe near the region when the dielectric resonator is at resonance. Based on this model, design rules for tuning the input resistance of the DRA is given. Using this design rule, the designer should
be able to pick the desired input impedance by choosing the right aspect ratio and fine tune
the input impedance by adjusting the probe height. As an added note, the choice of the
aspect ratio does not affect the radiation by any significant amount. This is a characteristic
of a low profile antenna radiating into free space. The effect of finite ground plane on the
input impedance is also discussed. The input impedance traces a trend similar to the study
by Storer [37] where it oscillates like a decaying sinusoid as the ground plane is increased.
Chapter 5

The Ring Resonator and a few DRA-Related Inventions

5.1 Introduction

In this chapter, I will show some of the results on the study of the ring resonator, which is a special case of the cylindrical resonator. The importance of the ring resonator is that the bandwidth of the DRA can be increased by adjusting the size of the hole at the center of the resonator. This technique of perturbing the DRA can be analyzed using material perturbation theory. The method of perturbing the material in order to increase the bandwidth can also be applied to other DRA shapes such as the rectangular DRA or the hemispherical DRA. After the discussion of the ring resonator, I will present some of my own contributions to the development of the DRA. This contribution comes in the form of new inventions related to DRA.

5.2 Ring resonator

Figure 5-1 shows the drawings of a cylindrical and ring dielectric resonators. The only physical difference between the cylindrical and ring dielectric resonators is that the ring dielectric resonator has a hole at the center of the resonator, but not the cylindrical dielectric resonator.

To calculate the resonant frequency of the ring resonator, we will attempt to use perturbation theory on the fields obtained using the magnetic wall model technique. From
Professor Kong’s Electromagnetic Theory book on page 200 equation 33 \[14\], the relationship between the change in the resonant frequency as a result of material perturbation is given by

\[
\frac{\omega - \omega_0}{\omega} = -\frac{\iint_M \left[ (\Delta \mu \cdot H_0^* \cdot H^* + (\Delta \epsilon \cdot E) \cdot E^* \right]}{\iint_M \left[ \mu H \cdot H_0^* + \epsilon E \cdot E_0^* \right]}
\]

If we assume that the perturbation is very tiny that the perturbed field is almost identical to the unperturbed field, then we can simplify the above expression by setting \( H = H_0 \) and \( E = E_0 \). From this we will get

\[
\frac{\omega - \omega_0}{\omega} = -\frac{\Delta W_m + \Delta W_e}{W_m + W_e}
\]

where \( \omega_0 \) is the resonant frequency calculated using the magnetic wall model with wall perturbation. With today’s computer speed, we can calculate the first expression which is an exact formula numerically. It would take about 1 second for a PC to do it numerically. Now, inserting the field values from equations (2.37) to (2.41) into the expression, we would find that the resonant frequency is increased. This is intuitively appealing and should not be surprising.

The result obtained from this method is compared to values obtained using a finite element simulator. The results are tabulated in Table 5.1. FE is the results from a finite element simulation and MP is the results obtained using material perturbation method. The orders of magnitude and the closeness between the two results, all within 5 % suggests that
the perturbation method is indeed a good method to calculate the change in the resonant frequency.

Table 5.1: Comparison of Resonant Frequency for $HE_{11\delta}$ mode between numerical simulation by finite elements (FE) and the material perturbation (MP) model developed in this section.

<table>
<thead>
<tr>
<th>$\varepsilon_r$</th>
<th>a (in)</th>
<th>b (in)</th>
<th>h (in)</th>
<th>FE (GHz)</th>
<th>MP (GHz)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.05</td>
<td>0.425</td>
<td>1.56</td>
<td>1.63</td>
<td>4.3</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.15</td>
<td>0.425</td>
<td>1.65</td>
<td>1.70</td>
<td>3.5</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.25</td>
<td>0.425</td>
<td>1.82</td>
<td>1.83</td>
<td>2.2</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.30</td>
<td>0.425</td>
<td>1.93</td>
<td>1.92</td>
<td>1.0</td>
</tr>
<tr>
<td>45</td>
<td>0.57</td>
<td>0.35</td>
<td>0.425</td>
<td>2.06</td>
<td>2.03</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figures B-1 to B-5 are the data obtained via Finite Element Simulator corresponding to the values tabulated in Table 5.1. The figures show the change in resonant frequency as the inner radius is varied. It also provides the value of the input impedance.

In conclusion, material perturbation technique can be used to estimate the change in the resonant frequency. Ring resonators have higher resonant frequencies than their cylindrical counterpart and the change in this frequency depends on the size of the perturbation. The purpose of introducing this perturbation in DRAs is to increase the bandwidth of operation of the antenna.
5.3 DRA with diagonal slot for Circular Polarization (CP)

There are two degenerate modes in the $HEM_{116}$ mode, one which has a $\cos\phi$ variation, and the other which has a $\sin\phi$ variation. This is shown in the equations below for $E_z$:

$$E_z = J_1(k\rho\rho) \cos \phi \sin (\beta(z - L/2)) \quad (5.1)$$

or

$$E_z = J_1(k\rho\rho) \sin \phi \sin (\beta(z - L/2)) \quad (5.2)$$

The central idea in obtaining CP using a single feed point is to introduce tiny perturbations that would separate the two degenerate modes into 2 slightly different resonant frequencies. Circular polarization can then be obtained and depending on the feed position can be Right Hand Circularly Polarize (RHCP) or Left Hand Circularly Polarize (LHCP).

In the diagonal slot method, a small slot is cut at the center. The introduction of this small slot perturbs the symmetry and splits the two degenerate modes with the respective resonant frequencies spaced slightly apart.

For the DRA, to prove that CP can be obtained, I ran a few simulations with different slot sizes. The geometry is shown in figure 5-2. The DRA has the following parameters:

- radius = 0.570 in
- height = 0.425 in
- ground plane diameter = 2.96 in
- $\epsilon_r = 45$
- probe height, $l = 0.425$ in
- probe distance from center, $b = 0.570$ in

The base case has the following parameters for which the other parameters were varied with respect to the base case.

- $W = 0.420$ inch
- $H = 0.425$ inch
The input impedance is shown in Figure 5-3. We can see two resonance, defined by the peak of the real part of the impedance. One is at 1.56 GHz and another one occurs at 1.65 GHz. The E-field lines are shown in Figure 5-4. The lower resonance at 1.56 GHz corresponds to the field lines approximately normal to the L of the rectangular slot. The field lines for the higher resonance at 1.65 GHz correspond to the field lines normal to the W of the rectangular slot. This observation can be used to tune the axial ratio or the spacing of the two resonances. For example, increasing the length L of the slot would have a larger perturbation effect on the lower resonance field lines than it would on the higher resonance field lines.

The radiation pattern is plotted as shown in Figure 5-5 at 1.60 GHz which corresponds to the center frequency between the two resonance peak. The radiation patterns show a strong LHCP wave for \(-90^\circ \leq \theta \leq 90^\circ\). The maximum directivity (Dmax) is about 5.37 dB. The average directivity (Davg) for elevation angles, \(\theta\) from \(-80^\circ\) to \(80^\circ\) for LHCP is
about 2.22 dB and the minimum directivity (D_{min}) at \( \theta = 80^\circ \) is -2.73 dB for the LHCP component.

The next step in the design of this single feed DRA is to try to improve the axial ratio. The axial ratio is a measure of the circular polarization of the radiation. In general, a wave is elliptically polarized. However, if the major axis of the elliptical polarized wave is equal to the minor axis of the elliptical polarized wave, then we get a very case called the circularly polarized wave. The axial ratio is defined as,

\[
\text{Axial Ratio} = \frac{\text{Major axis}}{\text{Minor axis}} \tag{5.3}
\]
Figure 5-4: E-field lines at the equatorial plane of the DRA with rectangular slot. Top: The field lines at 1.56 GHz (lower resonance) Bottom: The field lines at 1.65 GHz (higher resonance)

such that when it is 1, then we get a perfect circular polarized wave. Ultimately, the goodness of the axial ratio depends on the spacing of the two resonance peak.

My next study was to examine how to control the spacing of the two resonance peak. The first parameter that I varied was $W$. In one case, $W$ is 50% shorter than the base case, and in the other it was 50% longer than the base case. The results are shown in figures 5-6 and 5-7.

By making $W$ shorter, the frequency splitting is narrowed. However, when $W$ was increased by 50% from the base case to 0.530 inch, no double resonance was observed as shown in figure 5-7. This is because to obtain frequency splitting, we must perturb the system. But the application of a huge perturbation destroys the structure completely and perturbation is no longer a valid assumption.
Figure 5-5: Radiation Pattern for W=0.420 in, H=0.425 in, L=0.05 in
Figure 5-6: Input Impedance for W=0.210 in, H=0.425 in, L=0.05 in
Figure 5-7: Input Impedance for W=0.530 in, H=0.425 in, L=0.05 in
The next parameter that I varied was $L$. One has $L$ to be 50% longer than the base case, and the other one has $L$ to be 50% shorter than the base case. The input impedance are shown in Figures 5-8 and 5-9.

![Impedance Matrix vs. Frequency](image)

**Figure 5-8:** Input Impedance for $W=0.420$ in, $H=0.425$ in, $L=0.025$ in

The above results show that when $L$ is decreased, the frequency splitting is narrowed. From figure 5-8, the 2 resonances occurred at 1.55 GHz and 1.63 GHz. When $L$ is increased by 50%, the frequency splitting does appear to have increased, but only by a small amount.

In my final case, I reduced $H$ by half. In one case, the slot is on the bottom of the DRA. In another case, it is at the top. The difference is illustrated in Figure 5-10.

For the case when the slot is at the bottom of the DRA, no double resonances was
observed as shown in 5-11. This suggests that perturbation at the bottom of the DRA has little effect when compared to perturbing it at the upper portion of the DRA 5-12. This makes sense because the fields are stronger near the top of the DRA than at the bottom where there is a null at the origin.

In the end, the design rule for tuning the spacing of the two resonance peaks is very easy. It all depends on the degree of perturbation. The 2 modes are spaced further apart in frequency space if the difference in perturbation for the two frequency is increased. The perturbation is increased by the use of a larger slot or placing the slot in a region where the electrical activity is high.
Figure 5-10: The different parameter for the case run for L

Figure 5-11: Input Impedance for W=0.420 in, H=0.2125 (lower) in, L=0.05 in
Figure 5-12: Input Impedance for \( W = 0.420 \) in, \( H = 0.2125 \) (upper) in, \( L = 0.05 \) in
5.4 Elliptical DRA for Circular Polarization (CP)

The elliptical DRA is a CP antenna that uses only one feed point. It is easy to see how such a DRA would have two closely spaced resonant frequencies. The elliptical DRA can be seen as an overlap of two cylindrical DRA with different radius. Since we know that the resonant frequency is a function of radius, we expect the elliptical structure to perturb the degenerate resonant frequency at $HE_{11\ell}$ into two frequencies spaced slightly apart. To excite both the degenerate modes, the probe must be placed at $\pm45^\circ$ away from the major axis of the elliptical DRA. This is shown in Figure 5-13.

![Figure 5-13: Top view of the schematic for an elliptical Dielectric Resonator Antenna](image)

Figure 5-13: Top view of the schematic for an elliptical Dielectric Resonator Antenna

The field lines are shown in Figure 5-14. The top figure shows the field lines at the lower resonance. The lower resonance field lines are parallel to the major axis of the ellipse. The higher resonance field lines are parallel to the minor axis of the ellipse. From the cylindrical DRA, we know that the radius of the DRA is inversely proportional to the resonant frequency. Therefore the field lines parallel to the major axis have a longer wavelength and
thus correspond to the lower frequency while the field lines parallel to the minor axis have a shorter wavelength and therefore correspond to a higher frequency.

Figure 5-15 shows the input impedance for a DRA with major axis 0.57 inch, minor axis 0.50 inch, height = 0.425 inch, and \( \varepsilon_r = 45 \). There are two resonances, defined by the peak of the real part of the impedance. They are located at 1.575 GHz and 1.62 GHz. Figure 5-16 shows the imaginary part of the input impedance. However, it never crosses zero. Figure 5-17 shows the \( S_{11} \) parameter on a Smith Chart. It exhibits a very small loop at the center frequency suggesting that a good axial ratio is expected from this sample. However, it shows a very reactive input impedance indicating that some form of matching circuit would be necessary to make it operational.

The radiation pattern is shown in Figure 5-18 as a function of the elevation angle, \( \theta \) and in Figure 5-19 as a function of the azimuth angle, \( \phi \) at \( \theta = 80^\circ \). The radiation patterns
shows that the radiated wave for $-90^\circ \leq \theta \leq 90^\circ$ is RHCP. It has a maximum directivity ($D_{\text{max}}$) of 5.39 dB, an average directivity ($D_{\text{avg}}$) for RHCP for elevation angles, $\theta$, from $-80^\circ$ to $80^\circ$ of 2.50 dB and a minimum directivity ($D_{\text{min}}$) for RHCP at $\theta = 80^\circ$ of -2.41 dB.

Figure 5-15: Real part of input impedance for an elliptical DRA with $a=0.5$ in, $b=0.57$ in, and $h=0.425$ in; excited with a monopole of height 0.425 in
Figure 5-16: Imaginary part of input impedance for an elliptical DRA with $a=0.5$ in, $b=0.57$ in, and $h=0.425$ in; excited with a monopole of height 0.425 in
Figure 5-17: Smith chart for $S_{11}$ for an elliptical DRA with $a=0.5$ in, $b=0.57$ in, and $h=0.425$ in; excited with a monopole of height 0.425 in.
Figure 5-18: Radiation Pattern as a function of elevation angle for an elliptical DRA with a=0.5 in, b=0.57 in, and h=0.425 in; excited with a monopole of height 0.425 in.

Figure 5-19: Radiation Pattern as a function of azimuth angle for an elliptical DRA with a=0.5 in, b=0.57 in, and h=0.425 in; excited with a monopole of height 0.425 in.
To get a good axial ratio, we need to tune the spacing of the two peaks in the real part of the impedance by changing the major or minor axis of the DRA. The tuning of the major and minor axis of the elliptical DRA to control the this spacing is easy. The closer are the lengths of the major and minor axis, the closer the two peaks are expected to be. To increase the bandwidth, a hole can be drilled inside the DRA, making it the annular elliptical DRA.

The major problem with the elliptical DRA is the highly reactive impedance. To overcome this problem, an external matching network has to be used or the feed probe can be redesigned. This will be discussed in a later section.

5.5 Triangular DRA for Circular Polarization

The triangular DRA uses a different method to produce CP. It is based on reflection of waves from the sidewalls of the DRA. Figure 5-20 shows the schematics for the triangular DRA. The vertices of the triangle are located at (−.57, 0), (0.38, −0.5) and (0.58, 0.50) where the coordinate system is measured in inches. The triangle is nearly an equilateral triangle. The probe is placed on the side of the shortest vertices and is fed slightly off centered. In my simulation, I have placed it at (0.4165, −0.1800). Figure 5-21 shows the CP mechanism. The wave sets out from the probe and travels to one side of the boundary. It hits this boundary and is reflected off it and it travels to the other boundary and back to where it started.

Figure 5-22 shows the input impedance for the triangular DRA. The input impedance shows two resonances. One is located at 1.94 GHz, and the other is located at 1.98 GHz.

Figures 5-23 and 5-24 show the radiation pattern and prove that CP has been achieved by using a triangular DRA using a single probe feed. The radiated fields are LHCP for −90° ≤ θ ≤ 90° with a maximum directivity (Dmax) of 5.98 dB, an average LHCP directivity (Davg) for elevation angles, θ from −80° to 80° of 2.79 dB and a minimum LHCP directivity (Dmin) at θ = 80° of -2.01 dB.
Figure 5-20: Geometry of the Triangular DRA

Figure 5-21: Reflections of the Wave inside a Triangular DRA
Figure 5-22: Input Impedance of the Triangular DRA
Figure 5-23: Radiation Pattern of the Triangular DRA as a function of elevation angle

Figure 5-24: Radiation Pattern of the Triangular DRA as a function of azimuth angle
5.6 The Paddle Feed

The Paddle Feed was originally designed to tune the reactive part of the input impedance of the elliptical DRA. The traditional monopole antenna has only 1 degree of freedom for the designer - and this is the height of the antenna. As discussed in the chapter on impedance and backed by numerical simulation, this can be used to tune the real part of the input impedance without significantly changing the resonant frequency.

The Paddle Feed increases the degree of freedom to 3. Now the designer can also vary the width and length of the end of the paddle. These quantities are labeled as $\alpha$ and $\beta$ in Figure 5-25.

![Traditional Monopole Feed and The new Paddle Feed](image)

Figure 5-25: Geometry of the Paddle Feed in comparison with the traditional Monopole

The effect of varying both $\alpha$ and $\beta$ is that it will allow the tuning of the reactance by a substantial amount. This method has been used in microstrip circuits to create capacitive-like or inductive-like effects. To prove that it can be used to tune the highly reactive input impedance of the elliptical antenna, I have performed a numerical simulation by using the paddle feed with the elliptical DRA. In the elliptical DRA, there was no resonance because the elliptical DRA had a very reactive component that it never crosses the zero axis. Figure 5-26 shows how the input impedance of the elliptical antenna has been transformed. The elliptical DRA now resonates. For comparison with the input impedance of the elliptical DRA using the traditional monopole feed, refer to figure 5-17.

The radiation patterns for the elliptical DRA fed using a paddle feed is shown in figures 5-27 and 5-28. The figures show that the radiation is not distorted by the introduction of the paddle feed. This is in accordance of the radiation model developed in the earlier
chapter where the side walls do not contribute to any radiation. Other methods of tuning the reactance includes the capped monopole but this will have a significant impact on the radiation pattern because the disk portion of the capped monopole overlaps the radiating portion of the DRA.
Figure 5-26: Smith Chart showing the reflection coefficient parameter $S_{11}$ of the elliptical DRA using a Paddle Feed
Figure 5-27: Radiation Pattern as a function of elevation angle for an elliptical DRA with $a=0.5$ in, $b=0.57$ in, and $h=0.425$ in; excited with a capped probe of height 0.425 in.

Figure 5-28: Radiation Pattern as a function of azimuth angle for an elliptical DRA with $a=0.5$ in, $b=0.57$ in, and $h=0.425$ in; excited with a capped probe of height 0.425 in.
5.7 Dual Band DRA-Patch Antenna

In many wireless applications today, many mobile devices are including GPS as an option. GPS operates at L band and so it becomes necessary to include a second antenna to receive the GPS signal. There are many options to meet this requirement. One option is to use dual band antennas. Dual band antennas are antennas are designed to operate at 2 different frequencies. These can come in the form of excitation of higher order modes. However, I wanted to design a dual band antenna that radiates like fundamental-like modes for both the frequencies.

I have tried the stacking trick where I stacked one DRA on top of the other. The DRAs have different permittivities, the larger the difference, the better. However, I did not see any dual resonance when I thought I would. The reason for me to think so was because the lower DRA would resonate at some frequency. The upper DRA is approximately a cavity as well. If the permittivities of the DRA were chosen carefully then there would be an impedance mismatch between the top and bottom DRA. This causes the fields to reflect off that boundary. Thus the fields would be approximately confined in a cavity. I tried a numerical simulation of this idea but it did not work.

In the end, I decided that the problem was because the discontinuity of permittivity at the boundary between the 2 DRAs was not enough to create a cavity like structure for the lower portion of the DRA. To mend this problem, I decided to separate the 2 DRAs with a metallic plate. Then I know for sure that I would be able to create a cavity for both the top and bottom DRA because the metallic plate is a perfect reflector. This idea worked as expected. However, putting the metallic plate changes the boundary condition for the bottom DRA and technically, I cannot call this a dual band DRA because the bottom DRA behaves more like a patch antenna. Therefore, I think a good name for this dual band antenna is the DRA-Patch.

The schematics for the antenna is shown in figure 5-29. It consists of a dielectric resonator stacked on top of a patch antenna. The probe and is proximately coupled to the patch. At some frequency, the patch would resonate and the DRA acts as a superstrate for the patch. At some other frequency, the DRA would resonate and the metal on the patch acts as the ground plane for the DRA.

In my simulation, I used a DRA height of 0.425 inch with radius 0.27 inch. The patch
has a height of 0.15 inch and a radius of 1 inch. The expected resonant frequency when each of these antennas are analyzed separately is 2.48 GHz for the DRA and 1.6 GHz for the patch antenna but this does not take into account the superstrate for the patch which is the DRA. The numerical simulation results are shown in Figure 5-30 and Figure 5-31 for the input impedance. The simulation shows 2 resonances. The first is the resonance from the patch antenna and the resonant frequency is 1.51 GHz. The second resonance is the $HE_{116}$ mode from the DRA. The resonant frequency for the DRA is at 2.51 GHz.

The input impedances are different, however, tuning these impedances especially for the frequency corresponding to the resonant frequency of the patch should not be a problem because the problem of tuning the patch and the DRA is now a well understood problem.
Figure 5-30: Input Impedance of the DRA patch antenna near the resonant frequency for the patch antenna
Figure 5-31: Input Impedance of the DRA patch antenna near the resonant frequency for the DRA antenna
5.8 Summary

In this chapter, the results of the study on the ring resonator using material perturbation were presented. The ring resonator is similar to the cylindrical resonator as discussed in the previous chapters with the exception that the material inside the cylindrical resonator is replaced with a material with different $\varepsilon_r$, usually air.

The chapter then discusses several new inventions that resulted from the research. These inventions can be categorized as:

- Single fed DRAs for CP: the elliptical DRA, the circular DRA with rectangular slot and the triangular DRA.

- A reactive probe for tuning the input reactance of the DRA called *paddle feed*

- A dual band DRA-Patch antenna
Chapter 6

Conclusion

In this thesis, I presented the general design rules and the interpretation of the numerical results obtained via numerical methods. I have also introduced some theoretical models. These theoretical models are not accurate models but they do provide good insights to interpret numerical results obtained via numerical methods.

In chapter 1, I outlined some of the major characteristics of the DRA. Then in chapter 2, I used the magnetic wall model to calculate the resonant frequency. This is found to be not accurate but the model provides important insights. As an example, the physical phenomenon of degeneracy in the frequency which is associated to the symmetry of the cylindrical DRA. Then, I discussed about the issue of bandwidth and its relation to the total energy stored. This implied a direct relationship between the permittivity of the material to the bandwidth, because the permittivity of the material is related to the energy stored.

In Chapter 3, I moved to the far field parameter which is the far-field radiation. In this section, I introduced a more appealing model to model the radiation from a DRA. It is based on the dielectric waveguide, where the DRA is modeled as a dielectric cavity which is slightly lossy. The input impedance of the DRA is discussed in Chapter 4. Here, I laid down some important design rules on choosing the various parameters to obtain a good value for the input resistance.

The introduction of a cylindrical hole inside the DRA increases the bandwidth. In Chapter 5, I used the perturbation model to compute the effects of this new introduction via material perturbation theory. I also presented alongside with it numerical simulation
based on finite elements for comparison. The later part of chapter 5 ties all the concept together and contains a higher level of creativity. It uses all the concepts that I have researched to invent new DRA-related elements.

Finally, I hope that you have found the material presented in this thesis to be useful for your work.
Appendix A

Simulation Results of Input Impedance for various Aspect Ratio

The following section are the simulation results for a DRA at $\varepsilon_r = 45$. The radius, $a$, is kept constant at 0.57 inch. The height $H$ is varied. The probe height is kept the same height as the height of the DRA. The probe is kept at the boundary of the DRA and air so that means $b = a$. The radius of the ground plane is kept fixed at 1.69 inch. The simulation was done on High Frequency Solid Structure Modeler (HFSS), which is a finite element electromagnetic simulator. HFSS uses a 2nd order absorbing boundary condition as the radiation boundary condition. In all the following simulations, a fast frequency sweep was used rather than solving the problem frequency by frequency. Fast frequency sweep uses asymptotic waveform evaluation to extrapolate solutions for a range of frequencies from a single solution at a center frequency. In all the cases, the center frequency has been chosen according to the CAD formulas presented in Chapter 1.
Figure A-1: Input Impedance for a cylindrical DRA with a=0.57 in, h=0.15 in, l=0.15 in, $\epsilon_r = 45$
Figure A-2: Input Impedance for a cylindrical DRA with $a=0.57 \text{ in}$, $h=0.20 \text{ in}$, $l=0.20 \text{ in}$, $\epsilon_r = 45$
Figure A-3: Input Impedance for a cylindrical DRA with $a=0.57$ in, $h=0.30$ in, $l=0.30$ in, $\varepsilon_r = 45$
Figure A-4: Input Impedance for a cylindrical DRA with $a=0.57$ in, $h=0.425$ in, $l=0.425$ in, $b=0.57$ in, $\varepsilon_r = 45$
Figure A-5: Input Impedance for a cylindrical DRA with $a=0.57$ in, $h=0.475$ in, $l=0.475$ in, $\varepsilon_r = 45$
Figure A-6: Input Impedance for a cylindrical DRA with \( a=0.57 \) in, \( h=1.5 \) in, \( l=1.5 \) in, \( \epsilon_r = 45 \)
Figure A-7: Input Impedance for a cylindrical DRA with $a=0.57$ in, $h=1.5$, $l=0.425$ in, $\epsilon_r = 45$
Appendix B

Results of Input Impedance Simulation for the Ring Resonator

Figures B-1 to B-5 are the data obtained via Finite Element Simulator corresponding to the values tabulated in Table 5.1. The figures show the change in resonant frequency as the inner radius is varied. It also provides the value of the impedance.
Figure B-1: Input Impedance for a ring resonator for a=0.57 in, b=0.05 in, h=0.425 in, $\epsilon_r = 45$
Figure B-2: Input Impedance for a ring resonator for a=0.57 in, b=0.15 in, h=0.425 in, \( \varepsilon_r = 45 \)
Figure B-3: Input Impedance for a ring resonator for a=0.57 in, b=0.25 in, h=0.425 in, \( \epsilon_r = 45 \)
Figure B-4: Input Impedance for a ring resonator for $a=0.57$ in, $b=0.3$ in, $h=0.425$ in, $\epsilon_r = 45$
Figure B-5: Input Impedance for a ring resonator for $a=0.57$ in, $b=0.35$ in, $h=0.425$ in, $\varepsilon_r = 45$
Bibliography


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