

String Field Theory and Tachyon Dynamics

by

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Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

[June 2006]
May 2006

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Author

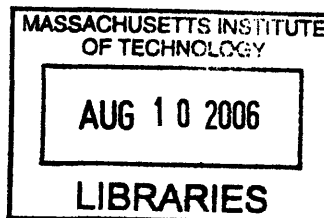
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Abstract

In this thesis we present some works done during my doctoral studies. These results focus on two directions. The first one is motivated by tachyon dynamics in open string theory. We calculate the stress tensors for the p-adic string model and for the pure tachyonic sector of open string field theory (OSFT). We give the energy density of lump solutions and attempt to evaluate the evolution of the pressure in rolling tachyon solutions. We discuss the relevance of the pressure calculation for the identification of the large time solution with a gas of closed strings.

In the second direction, we give some results in closed string field theory (CSFT). We considered marginal deformations in CSFT. The marginal parameter, called a , is that associated with the dimension-zero primary operator $c\bar{c}\partial X\bar{\partial}X$. We use this marginal operator to test the quartic structure of CSFT and the feasibility of level expansion. We check the vanishing of the effective potential for a . In the level expansion the quartic terms generated by the cubic interactions must be cancelled by the elementary quartic interaction of four marginal operators. We confirm this prediction, thus giving evidence that the sign, normalization, and region of integration $\mathcal{V}_{0,4}$ for the quartic vertex are all correct. This is the first calculation of an elementary quartic amplitude for which there is an expectation that can be checked. We also extend the calculation to the case of the four marginal operators associated with two space coordinates.

We then try to search a critical point of the tachyon potential in CSFT. We include the tachyon, the dilaton, and massive fields in the computation. Some evidence is found for the existence of a closed string tachyon vacuum. It seems that this critical point becomes more shallow when higher level contributions are considered. We also relate fields in the sigma model and those in CSFT. Moreover, large dilaton deformations are studied numerically.

Finally, we use the low-energy effective field equations that couple gravity, the dilaton, and the bulk closed string tachyon to study the end result of the physical decay process associated with the instability of closed string tachyon. We establish that whenever the tachyon induces the rolling process, the Einstein metric undergoes collapse while the dilaton rolls to strong coupling. Some more general potentials and the possible cosmological application are discussed.

Thesis Supervisor: Barton Zwiebach
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Acknowledgments

I am especially indebted to my advisor Barton Zwiebach for his invaluable help in my research, for advising me, for his patience, guidance and countless things he has taught me.

I would like to express my thanks to Prof. Washington Taylor and Hong Liu for very helpful instructions and discussions.

I also want to thank Martin Schnabl, Yuji Okawa for useful discussions.

Contents

1	Introduction	8
2	Stress Tensors in p-adic String Theory and Truncated OSFT	14
2.1	p-adic String Theory Case	15
2.1.1	Stress Tensor for p-adic model	15
2.1.2	Energy of The Lump Solution	17
2.2	The Pure Tachyon Field of String Field Theory Case	19
2.2.1	Stress Tensor for the Tachyon field in SFT	19
2.2.2	Energy distribution of the SFT lump solution	21
2.2.3	Pressure evolution of the SFT rolling tachyon solution	22
2.3	Conclusion	23
3	Testing Closed String Field Theory with Marginal Fields	25
3.1	Marginal field potential from cubic interactions	27
3.2	Contributions to a^4 calculated using OSFT	29
3.3	Elementary contribution to a^4	31
3.4	A moduli space of marginal deformations	33
3.5	Conclusion	35
4	A Closed String Tachyon Vacuum ?	37
4.1	Computation of the tachyon potential	41
4.1.1	Tachyon potential universality and the ghost-dilaton	42
4.1.2	The quadratic and cubic terms in the potential	46
4.1.3	Tachyon vacuum with cubic vertices only	47
4.1.4	Tachyon vacuum with cubic and quartic vertices	48
4.2	The sigma model and the string field theory pictures	50
4.2.1	Relating sigma model fields and string fields	51
4.2.2	The many faces of the dilaton	53
4.2.3	Relating the sigma model and string field dilaton and tachyon	54
4.2.4	Dilaton deformations	57
4.3	Conclusions	58

5 Rolling Closed String Tachyons and the Big Crunch	61
5.1 The coupled system of rolling fields	62
5.2 Tachyon-driven rolling and the string metric	64
5.3 Finite-time crunch with negative scalar potentials	66
5.4 Crunching with arbitrary potentials	67
5.5 Conclusions	70
Appendices	71
A Quartic Computations	72
A.1 The setup	72
A.2 Couplings of dilatons and tachyons	74
A.3 Couplings of tachyon to massive fields	75

List of Figures

2.1	The energy distribution of the lump solution (2.13) of the p-adic model for $p = 2$ (solid line, $g_p^2 E(x)$ versus x) and that (2.18) of ordinary ϕ^3 field theory (dashed line, $g_0^2 E(x)$ versus x).	19
2.2	The Energy density of the pure tachyonic lump solution $\phi(x) = t_0 + t_1 \cos \frac{x}{R} + t_2 \cos \frac{2x}{R}$ of OSFT theory with $R = \sqrt{3}$. the plot is $g_0^2 E(x)$ versus x	23
2.3	The pressure evolvement of the rolling tachyon solution $\phi(t) = t_0 + t_1 \cosh t + t_2 \cosh 2t$ of OSFT theory. $g_0^2 p(t)$ versus t . As t becomes larger, $p(t)$ oscillates rapidly.	24
4.1	A sketch of a closed string tachyon potential consistent with present evidence. The perturbative vacuum is at $T = 0$. The closed string tachyon vacuum would be the critical point with zero cosmological term, shown here at $T \rightarrow \infty$ (in CSFT this point corresponds to finite tachyon vev). A critical point with negative cosmological constant cannot provide a spacetime independent tachyon vacuum.	40
4.2	The solid line is the dilaton marginal direction defined by the set of points $(d, t(d))$ where $t(d)$ is the expectation value of t obtained solving the tachyon equation of motion for the given d . The dashed line represents the direction along the sigma model dilaton Φ (thus $T = 0$). It is obtained by setting $T = 0$ in equation (4.77). The two lines agree well even reasonably far from the origin.	56
4.3	Dilaton effective potential. The dashed line arises from $\mathbb{V}_4^{(3)}$, the solid line arises from $\mathbb{V}_8^{(3)}$, and the thick line arises from $\mathbb{V}_8^{(4)}$	58
4.4	A non-critical $(p+1)$ -dimensional string theory would correspond to a solitonic solution of critical string theory in which, far away from the reduced space, the fields approach the values of the closed string tachyon vacuum.	59
5.1	Tachyon histories $T(t)$ in the potential $V(T) = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, starting with $T(t = 0) = 2$. The critical trajectory is dashed, begins with critical velocity $\dot{T}(0) = 2$, and crunches at infinite time. The solid line turning upwards starts with $\dot{T}(0) = 2.01$ and hits $T = \infty$ at $t \simeq 0.9808$. The solid line turning downward starts with initial velocity $\dot{T}(0) = 1.99$, encounters a turning point, and actually reaches minus infinity at $t \simeq 1.1609$	69

5.2 Curves and solutions for $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V}$ with potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, plotted in the (T, \mathcal{E}) plane with vertical axis at $T = 2$. From bottom to top, the three dashed curves are: the turning point curve ($\mathcal{E}(T = 2) = 0$), the inflection curve ($\mathcal{E}(2) = 1$), and the separating curve ($\mathcal{E}(2) = \sqrt{51}/4 \simeq 1.7854$). The three solid curves are solutions. From bottom to top they are: a solution with subcritical $\mathcal{E}(2) = 1.20$ that hits the turning point curve, the critical solution, with $\mathcal{E}(2) = \sqrt{2}$, and a solution above the separating curve, with $\mathcal{E}(2) = 2.0$

List of Tables

3.1	χ_ℓ and $\alpha_4(\ell)$ give the contribution of level ℓ fields and the total contributions up to level ℓ , respectively, to the quartic term in the potential for the Wilson line parameter a_s . The last two columns give the contribution $c(2\ell) = -(\ell - 1)\chi_\ell^2$ of closed string fields of level 2ℓ and the total contributions $\mathcal{C}(2\ell)$ up to level 2ℓ to the quartic term in the potential for the closed string marginal field a . The last row gives the projections from fits.	31
3.2	Contributions from the given level to the coefficients that multiply the invariants U^2 and V^2 in the effective potential for the marginal fields. The row “quartic” gives the contributions from the elementary quartic interactions. The last row is the residual quartic potential, obtained after adding all contributions.	35
4.1	Vacuum solution with cubic vertices only	48
4.2	Vacuum solution with cubic and quartic vertices. We see that the magnitude of the action density becomes smaller as we begin to include the effects of quartic couplings.	49

Chapter 1

Introduction

Born as a candidate to describe the strong interactions, string theory has proven to be the most promising model of unified theory. When quantized, oscillating modes of the relativistic string represent particles with arbitrarily high masses and spins. In order to be anomaly free bosonic string theory requires 26-dimensional spacetime. The supersymmetric string theory, including both bosonic and fermionic excitations, selects 10-dimensional spacetime. There are five consistent supersymmetric string theories in ten dimensions: Type I, Type IIA, Type IIB, Heterotic $SO(32)$ and Heterotic $E_8 \times E_8$. In the mid-1990's, it was realized that these five superstring theories are related to one another through dualities. Furthermore, under some particular limits they arise as effective theories of a 11-dimensional underlying theory, named as M-theory. Moreover, M-theory has a field theory limit: 11-dimensional supergravity theory.

In addition to the fundamental strings, it was discovered that all the five superstring theories and M-theory contain “branes”, extended objects in the spacetime. The branes with different dimensionalities are related through duality transformations. A Dp -brane is defined as a p -dimensional extended object on which open strings end. For example, Type IIA (IIB) theory contains BPS Dp -branes for even (odd) p and a 5-brane. The BPS D-branes preserve half of the supersymmetries of the theory and carry conserved charges associated with the Ramond-Ramond gauge fields of the theory. Their tension is determined in terms of their charge. They are stable and all the modes of the open strings attached on them have positive mass-squared. The BPS D-branes are oriented. Therefore, one can define an anti-BPS D-brane with opposite orientation.

There are also some unstable Dp -brane configurations. When we consider an open string stretched between a pair of coincident BPS Dp -brane and anti-BPS Dp -brane (with opposite orientation), the lowest mode is tachyonic with mass-squared. Moreover, type II string theories also contain unstable non-BPS branes. These non-BPS branes break all the supersymmetries and carry no conserved charge. The open strings that end on non-BPS branes possess tachyonic modes and thus signal the instability of the configurations. In type IIA (IIB) theory, p is odd (even) for non-BPS Dp -branes.

Though the final purpose is to understand the instability of the perturbative vacua in superstring theory, most of results obtained in last years are about bosonic string theory. The reason is that both theories share many common features and Dp -branes issues in bosonic string theory are much

simpler¹. Moreover, studying tachyon dynamics of bosonic D-branes has its own importance. The main goal of this thesis is to discuss these problems in the framework of bosonic string theory.

We will focus on bosonic open string theory first. The Dp -branes are all unstable and p is than or equal to 25. In the classical level, closed strings are decoupled from open string, though closed strings must appear in the loop diagrams in the quantum open string theory. Three conjectures were proposed by Sen:

- The local maximum of the tachyon potential is the perturbative vacuum. There is a locally stable minimum of the potential. The energy density, measured from the perturbative vacuum is exactly cancelled by the tension of the original Dp -brane.
- Lower dimensional $D(p - q)$ -branes are codimension q lump solutions of the string theory on the background of Dp -brane.
- The locally stable vacuum is identified with the closed string vacuum. At this vacuum, the total energy vanishes. Therefore, no D-brane presents. Since open string must attach on D-branes, open string perturbative excitations disappear.

Straightforward generalizations of these conjectures exist for superstring theories. One indirect way to verify these conjectures is to work in boundary conformal field theory(BCFT), using the correspondence between classical solutions of string theory and two dimensional conformal field theory (CFT). The first two conjectures are proved using this formalism in [1, 2, 3].

In the framework of first-quantized string theory, conformal invariance is crucial. One can map the conformally invariant string diagrams to Riemann surfaces and do the calculations. This restricts us on computations of on-shell scattering amplitudes only. However, in tachyon condensation, at the stable vacuum, tachyon and infinite many massive scalars acquire nonzero vacuum expectation values (vev) and their momenta vanish. They are spacetime independent field configurations. Therefore, we are facing off-shell issues. Furthermore, a non-perturbative formalism of string theory would be naturally expected to study the non-perturbative vacuum. From the lessons in usual quantum field theory, string field theory, a second-quantized version of string theory with infinitely many fields, is the solution. The fundamental degree of freedom of string field theory is a string field, which can be expanded on the basis of first quantized modes of string theory. Each mode is associated with a field. These field configurations are off-shell and do not satisfy the physical state conditions. The on-shell conditions come from the linearized equations of motion of the string field theory.

The most widely used model of bosonic OSFT is Witten's cubic action [4]:

$$S = -\frac{1}{g^2} \left[\frac{1}{2} \langle \Phi, Q_B \Phi \rangle + \frac{1}{3} \langle \Phi, \Phi \star \Phi \rangle \right], \quad (1.1)$$

where Q_B is the BRST operator and g is the open string coupling constant. The \star denotes a non-commutative, associative multiplication. Geometrically, it glues the right half of the first string with

¹Another reason is that the action of closed superstring field theory is not known yet except for the Neveu-Schwarz sector of heterotic string field theory.

the left half of the second string. The 2-point vertex $\langle \cdot, \cdot \rangle$ is defined in terms of CFT correlators in the state space. The equation of motion of this action is:

$$Q_B \Phi + \Phi \star \Phi = 0. \quad (1.2)$$

This action can describe the spacetime field theory of any D-brane. For an instance, with a Dp -brane, the underlying conformal field theory is composed of $p + 1$ fields with Neumann boundary conditions and $25 - p$ fields with Dirichlet boundary conditions.

There was considerable effort toward constructing both analytical and numerical solutions of the equation of motion. As early as in 1987, a stable minimum of the effective tachyon potential of bosonic string theory was found [5]. However, since D-brane had not appeared before the eyes of physicists at that time, the physical interpretation of this critical point was unknown until Sen proposed his conjectures. In [5, 6], a computational tool called level truncation, was introduced. Since then, very impressive results have been obtained in [7, 8, 9, 10], where the D-brane tension was reproduced at very high accuracy. Recently, a nontrivial analytical solution of the action was constructed [11, 12, 13]. This solution represents the stable non-perturbative vacuum. Sen's first conjecture was analytically verified with this solution.

Another important issue in tachyon condensation is the dynamical process through which the tachyon rolls from the perturbative unstable vacuum to the stable non-perturbative vacuum. For this we need time dependent solutions. In the classical level where D-branes decouple from the closed string, Sen studied this process in the context of BCFT [14, 15]. It was found that when displaced by some amount towards the direction of the critical point from the local maximum, the tachyon evolves to a pressureless state with positive energy density. It is of interest to study this process in OSFT and p-adic string theory.

The p-adic model is a good laboratory to study tachyon condensation. Its action is given by:

$$S = \int d^d x \mathcal{L} = \frac{1}{g_p^2} \int d^d x \left[-\frac{1}{2} \phi p^{-\frac{1}{2} \square} \phi + \frac{1}{p+1} \phi^{p+1} \right], \quad \frac{1}{g_p^2} = \frac{1}{g^2} \frac{p^2}{p-1}, \quad (1.3)$$

where $\phi(x)$ is a scalar field, $p \geq 2$ is a prime integer and g is the open string coupling constant. This model is a tachyonic scalar field theory with infinitely many derivatives. The existence of some analytical solutions of this model makes it very attractive. It captures many non-perturbative features of OSFT. Besides the perturbative unstable vacuum, the potential also possesses one stable minimum. There do not exist perturbative solutions of the equation of motion at this non-perturbative vacuum. This property is similar to the situation in OSFT where at the true vacuum, no open string excitation exists. Furthermore, solitonic solutions of this model can be identified with lower dimensional D-branes as in OSFT.

It turns out that the behavior of the rolling tachyon in OSFT or p-adic string theory is totally different from that obtained by BCFT method: the tachyon rolls to the critical point and turns around to oscillate wildly rather than approaches the stable vacuum asymptotically [16, 17]. The pressure oscillates with ever increasing amplitude instead of asymptotically vanishing. The reason of

this apparent contradiction is that in OSFT and p-adic model, there are infinitely many spacetime derivatives acting on the fields. In [18], by constructing a complicated field redefinition to map one rolling tachyon solution from OSFT to BCFT, the authors resolved this puzzle. They have shown that the wildly oscillatory trajectory of the rolling tachyon in OSFT is stable when higher level fields are included in the calculation.

To understand what is the end state of an unstable D-brane decay, one needs to consider the coupling of D-branes to closed strings. With semi-classical approach, it was found that D-branes are sources of closed string states [19, 20]. For a time dependent rolling tachyon solution, representing a homogenous decay, all the energy of the D-brane is radiated away into closed string fields. Therefore, the end-result of the physical decay process is an excited state of closed strings that carries the original energy of the unstable D-brane.

Unlike the achieved progress in bosonic OSFT, the tachyon dynamics problem in bosonic CSFT is much more difficult. Some results have been obtained for the instability of localized closed string tachyons that live on subspaces of spacetime [21, 22, 23, 24, 25]. For the bulk bosonic closed string tachyon, the story is very complicated with the nonpolynomial action [26]:

$$S = -\frac{2}{\alpha'} \left(\frac{1}{2} \langle \Psi | c_0^- Q | \Psi \rangle + \sum_{N=3}^{\infty} \frac{\kappa^{N-2}}{N!} \{ \Psi^N \}_{\mathcal{V}_{0,N}} \right), \quad (1.4)$$

$$(L_0 - \bar{L}_0) | \Psi \rangle = 0 \quad \text{and} \quad (b_0 - \bar{b}_0) | \Psi \rangle = 0, \quad (1.5)$$

where κ is the closed string coupling constant. Here the BRST operator is $Q = c_0 L_0 + \bar{c}_0 \bar{L}_0 + \dots$, where the dots denote terms independent of c_0 and of \bar{c}_0 . Moreover, $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$. The fundamental degree of freedom is the closed string field $| \Psi \rangle$, expanded on the basis of first quantized closed string modes:

$$| \Psi \rangle = t c_1 \bar{c}_1 | 0 \rangle + d (c_1 c_{-1} - c_1 \bar{c}_{-1}) | 0 \rangle + h_{\mu\nu} c_1 \bar{c}_1 \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu | 0 \rangle + \dots, \quad (1.6)$$

where t is the closed string tachyon field, d is the dilaton field and $h_{\mu\nu}$ is the graviton. Massive modes are represented by the dots. Since CSFT action is nonpolynomial it is not obvious how level truncation works. If a class of closed-string computations can be done in level expansion, it is then necessary to compute higher-order couplings efficiently. The results of Moeller [27] make this possible for the case of four-point couplings.

It is well known that the effective potential of a marginal field must vanish. The zero-momentum graviton-like primary operator $c\bar{c}\partial X\bar{\partial}X$ (dimension zero) in the closed string spectrum provides a nice laboratory to check the calculation of elementary quartic amplitudes. Denote the field parameter associated with the marginal operator as a . The effective potential can be expressed as power series of the field. The leading term a^4 has two contributions. One is generated from the cubic vertex with all massive fields integrated out. The second one arises from the elementary quartic vertex. It turns out that these two contributions are cancelled with very impressive precision. This confirms correctness of the computation mechanism of the elementary quartic vertex. In [28], the flatness of the zero-momentum ghost dilaton potential was checked. Though the ghost dilaton operator is not

strictly marginal since it is not primary, the dilaton theorem states that a shift in the expectation value of ghost dilaton field corresponds to a change in the string coupling constant. Around the flat spacetime background there is no potential for the dilaton, so it behaves like a marginal field.

From the results in open string theory, two questions arise immediately for the tachyon in CSFT:

- Is there a ground state of the theory without the instability ?
- What is the end-result of the physical decay process associated with the instability ?

It is natural to think that there exists a stable critical point. At this vacuum there would be no closed string excitations. Without gravity excitations spacetime ceases to be dynamical and it would seem that the spacetime has disappeared.

Early computations showed that there is a local minimum analogous to the stable vacuum in OSFT if we ignore the elementary vertices higher than cubic interaction in the CSFT action [29, 30]. However, when the contribution from the quartic elementary vertex is introduced, this critical point is destroyed. It turns out that the above conclusion is not correct. One must include all the fields sourced by the zero momentum tachyon, especially the massless ghost dilaton field in eqn. (1.6), in the computation. Given a nonvanishing vev of the tachyon field, the quartic elementary vertices force the ghost dilaton to be nonzero required by the equations of motion. Once the effects of all the sourced fields are included, the critical point survives. It looks like that the contributions from higher level massive fields make the critical point more shallow. Moreover, the ghost-dilaton has a positive expectation value at the critical point. The string metric in eqn. (1.6) is not sourced and needs not acquire vev. Two other questions arise in turn:

- Does the positive expectation value at the critical point of ghost-dilaton correspond to stronger or weaker string coupling ?
- Is the string metric or the Einstein metric excited?

In order to answer these questions and gain more insights into the critical point, the low energy effective action is studied. It is found that the positive dilaton expectation value corresponds to *stronger* coupling. Furthermore, through studying a tachyon-induced rolling solution of the low energy effective action, qualitatively consistent results are obtained. The string metric remains constant all the time. Both dilaton and the string coupling run to infinity. Therefore, the Einstein metric crumples up and familiar spacetime no longer exists.

Outline

In chapter 2, we construct the stress tensors for the p-adic string model and for the pure tachyonic sector of open string field theory by naive metric covariantization of the action. Then we give the

concrete energy density of a lump solution of the p-adic model. It has similar profile of that of ϕ^3 field theory. In the cubic open bosonic string field theory, we also give the energy density of a lump solution and pressure evolution of a rolling tachyon solution. It turns out that the pressure oscillates with growing amplitude rather than approaches zero, the results obtained by using BCFT method.

In Chapter 3, we study the feasibility of level expansion and test the quartic vertex of closed string field theory by checking the flatness of the potential in marginal directions. The tests, which work out correctly, require the cancellation of two contributions: one from an infinite-level computation with the cubic vertex and the other from a finite-level computation with the quartic vertex. The numerical results suggest that the quartic vertex contributions are comparable or smaller than those of level four fields.

In Chapter 4, we focus on searching the critical point of CSFT. In bosonic closed string field theory the “tachyon potential” is a potential for the tachyon, the dilaton, and an infinite set of massive fields. Earlier computations of the potential did not include the dilaton and the critical point formed by the quadratic and cubic interactions was destroyed by the quartic tachyon term. We include the dilaton contributions to the potential and find that a critical point survives and appears to become more shallow. We are led to consider the existence of a closed string tachyon vacuum, a critical point with zero action that represents a state where space-time ceases to be dynamical. Some evidence for this interpretation is found from the study of the coupled metric-dilaton-tachyon effective field equations, which exhibit rolling solutions in which the dilaton runs to strong coupling and the Einstein metric undergoes collapse.

In Chapter 5, we study the low-energy effective field equations that couple gravity, the dilaton, and the bulk closed string tachyon of bosonic closed string theory. We establish that whenever the tachyon induces the rolling process, the string metric remains fixed while the dilaton rolls to strong coupling. For negative definite potentials we show that this results in an Einstein metric that crunches the universe in finite time. This behavior is shown to be rather generic even if the potentials are not negative definite. The solutions are reminiscent of those in the collapse stage of a cyclic universe cosmology where scalar field potentials with negative energies play a central role.

Chapter 2

Stress Tensors in p-adic String Theory and Truncated OSFT

Much work has been devoted to looking for solutions in string field theory (SFT). Generally speaking, physicists are concerned with two kinds of solutions with different properties. One kind of solutions are the time independent ones which represent the tachyon vacuum or lower dimensional D-branes [31]-[36]. Initiated by Sen [14], time dependent rolling tachyon solutions have recently attracted much attention [15]-[46]. Studying rolling tachyon solutions can give us information about how the tachyon approaches the tachyon vacuum. At the same time, the p-adic model [47], which exhibits a lot of properties of string field theory, is also of interest. In this model, the potential has a stable vacuum and a tachyon. Studying the dynamics of the tachyon may suggest to us what happens in the same situation for the SFT. Furthermore, one also has lump solutions in the p-adic theory which are identified as lower dimensional D-branes [48].

In [16], Moeller and Zwiebach discussed how to construct the stress tensor for the rolling tachyon solution in the p-adic model. They obtained an unambiguous expression for the energy through a generalized Noether procedure. This analysis could not be extended to the pressure calculation, however, as there are ambiguities in that case. Instead, they included the metric in the action and used the definition of stress tensor in general relativity to calculate the pressure. Then they constructed the rolling tachyon solutions for both the p-adic model and open string field theory (OSFT) in the form of series expansions. After that, they calculated the pressure evolution in the p-adic string case.

It is of interest to consider the stress tensor in the case when the scalar field in the p-adic model depends on all the coordinates. Especially, for a lump solution, what is the profile of the energy distribution along the spatial coordinate? Is it the same as what we expect intuitively? Furthermore, in OSFT, it is important to know if the profile of the energy density has the same properties as that in p-adic string theory. Moeller and Zwiebach showed in [16] that the pressure of the rolling solution in p-adic model does not vanish at large times. For the rolling solution in OSFT, it is of interest to test if one gets vanishing pressure asymptotically or not.

In this chapter, we first give the stress tensor in a general form for the p-adic model. When our results are specialized to the time dependent solution in p-adic model, they reproduce the results in

[16]. A nontrivial lump solution in p-adic model was given in [47], [48]. We construct the energy density of this solution and compare it with that of the lump solution of ordinary ϕ^3 field theory. We find that these two energy densities have similar spatial profiles. Section 3 is devoted to the case of the pure tachyon field in OSFT. We again construct the stress tensor in a general form. The energy density of a solitonic solution [34] is then constructed in subsection 3.1. Finally we calculate the pressure evolution of a rolling tachyon solution [16].

2.1 p-adic String Theory Case

In this section, we first construct the stress tensor of the p-adic string theory by varying the metric. We will find that the expression is exactly the same as the one obtained in [16] if we constrain scalar field to only depend on time. We will also consider the case where the tachyon scalar only depends one spatial coordinate. In that situation, one nontrivial solitonic solution was already given [47], [48]. We then calculate the energy density of that solution. The results show that the total energy, integrated over all space, perfectly agrees with the D24 brane tension as expected. The spatial profile of this energy density looks very like the one of the solitonic solution of ordinary ϕ^3 field theory.

2.1.1 Stress Tensor for p-adic model

The p-adic string theory is defined by the action:

$$S = \int d^d x \mathcal{L} = \frac{1}{g_p^2} \int d^d x \left[-\frac{1}{2} \phi p^{-\frac{1}{2}\square} \phi + \frac{1}{p+1} \phi^{p+1} \right], \quad \frac{1}{g_p^2} = \frac{1}{g^2} \frac{p^2}{p-1}, \quad (2.1)$$

where $\phi(x)$ is a scalar field, p is a prime integer and g is the open string coupling constant. Though the theory makes sense even as $p \rightarrow 1$, in most cases, we will consider $p \geq 2$ in this chapter. In this action, there is an infinite number of both time and spatial derivatives. One defines:

$$p^{-\frac{1}{2}\square} \equiv \exp\left(-\frac{1}{2} \ln p \square\right) = \sum_{n=0}^{\infty} \left(-\frac{1}{2} \ln p\right)^n \frac{1}{n!} \square^n, \quad (2.2)$$

and

$$\square = -\frac{\partial^2}{\partial t^2} + \nabla^2. \quad (2.3)$$

Now we include the metric in the action [16]:

$$\begin{aligned} S = S_1 + S_2 &= \frac{1}{g_p^2} \int d^d x \sqrt{-g} \left[-\frac{1}{2} \phi^2 + \frac{1}{p+1} \phi^{p+1} \right] \\ &\quad - \frac{1}{2g_p^2} \sum_{l=1}^{\infty} \left(-\frac{1}{2} \ln p\right)^l \frac{1}{l!} \int d^d x \sqrt{-g} \phi \square^l \phi, \end{aligned} \quad (2.4)$$

where we have split the action into two parts: S_1 represents the potential and S_2 represents the kinetic term. After introduction of the metric \square becomes the covariant D'Alembertian.

$$\begin{aligned}
B_l \equiv \int d^d x \sqrt{-g} \phi \square^l \phi &= \int d^d x \phi \partial_{\mu_1} \sqrt{-g} g^{\mu_1 \nu_1} \partial_{\nu_1} \frac{1}{\sqrt{-g}} \partial_{\mu_2} \sqrt{-g} g^{\mu_2 \nu_2} \partial_{\nu_2} \\
&\cdots \frac{1}{\sqrt{-g}} \partial_{\mu_l} \sqrt{-g} g^{\mu_l \nu_l} \partial_{\nu_l} \phi.
\end{aligned} \tag{2.5}$$

The stress tensor is given by:

$$T_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\alpha\beta}}. \tag{2.6}$$

The variation of the potential S_1 in (2.4) contributes:

$$\frac{2}{\sqrt{-g}} \frac{\delta S_1}{\delta g^{\alpha\beta}} = -\frac{1}{g_p^2} \left(-\frac{1}{2} \phi^2 + \frac{1}{p+1} \phi^{p+1} \right) g_{\alpha\beta}, \tag{2.7}$$

where we have set the metric to be flat with signature $(-, +, + \cdots +)$ after the variation and we will use the same convention in the rest of this chapter. As for the variation of the kinetic term S_2 in (2.4), from (2.5), we need to vary both factors of $\sqrt{-g}$ and $g^{\mu_i \nu_i}$ with respect to $g^{\alpha\beta}$. First consider varying factors of $\sqrt{-g}$ in (2.5) with respect to $g^{\alpha\beta}$:

$$\begin{aligned}
\frac{\delta B_l}{\delta \sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\alpha\beta}} &= g^{\mu_1 \nu_1} g^{\mu_2 \nu_2} \cdots g^{\mu_l \nu_l} (\phi_{\mu_1} \phi_{\nu_1 \mu_2 \nu_2 \cdots \mu_l \nu_l} + \phi_{\mu_1 \nu_1} \phi_{\mu_2 \nu_2 \cdots \mu_l \nu_l} + \cdots \\
&\cdots + \phi_{\mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_l} \phi_{\nu_l}) g_{\alpha\beta},
\end{aligned} \tag{2.8}$$

with the definition:

$$\phi_{\mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_l \nu_l} \equiv \partial_{\mu_1} \partial_{\nu_1} \partial_{\mu_2} \partial_{\nu_2} \cdots \partial_{\mu_l} \partial_{\nu_l} \phi(x).$$

The variation of the factors of $g^{\mu_i \nu_i}$ in (2.5) with respect to $g^{\alpha\beta}$ contributes:

$$\begin{aligned}
\frac{\delta B_l}{\delta g^{\mu_i \nu_i}} \frac{\delta g^{\mu_i \nu_i}}{\delta g^{\alpha\beta}} &= -2g^{\mu_1 \nu_1} g^{\mu_2 \nu_2} \cdots g^{\mu_{l-1} \nu_{l-1}} \left(\phi_{\alpha} \phi_{\beta \mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_{l-1} \nu_{l-1}} \right. \\
&\left. + \phi_{\alpha \mu_1 \nu_1} \phi_{\beta \mu_2 \nu_2 \cdots \mu_{l-1} \nu_{l-1}} + \cdots + \phi_{\alpha \mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_l} \phi_{\beta} \right).
\end{aligned} \tag{2.9}$$

So, we can calculate δS_2 . Finally, the stress tensor is:

$$\begin{aligned}
T_{\alpha\beta} &= -\frac{1}{g_p^2} \left(-\frac{1}{2} \phi^2 + \frac{1}{p+1} \phi^{p+1} \right) g_{\alpha\beta} \\
&\quad - \frac{1}{2g_p^2} \sum_{l=1}^{\infty} \left(-\frac{1}{2} \ln p \right)^l \frac{1}{l!} \left\{ g^{\mu_1 \nu_1} g^{\mu_2 \nu_2} \cdots g^{\mu_l \nu_l} \left(\phi_{\mu_1} \phi_{\nu_1 \mu_2 \nu_2 \cdots \mu_l \nu_l} \right. \right. \\
&\quad \left. \left. + \phi_{\mu_1 \nu_1} \phi_{\mu_2 \nu_2 \cdots \mu_l \nu_l} + \cdots + \phi_{\mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_l} \phi_{\nu_l} \right) g_{\alpha\beta} \right. \\
&\quad \left. - 2g^{\mu_1 \nu_1} g^{\mu_2 \nu_2} \cdots g^{\mu_{l-1} \nu_{l-1}} \left(\phi_{\alpha} \phi_{\beta \mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_{l-1} \nu_{l-1}} \right. \right. \\
&\quad \left. \left. + \phi_{\alpha \mu_1 \nu_1} \phi_{\beta \mu_2 \nu_2 \cdots \mu_{l-1} \nu_{l-1}} + \cdots + \phi_{\alpha \mu_1 \nu_1 \mu_2 \nu_2 \cdots \mu_l} \phi_{\beta} \right) \right\}.
\end{aligned} \tag{2.10}$$

If $\phi(x)$ is only time dependent, in (2.10), each $g^{\mu_i \nu_i}$ contributes one ‘-’ sign and the second term in the sum survives only for the component T_{00} . This gives the same results as in [16].

One can also use the following identity ¹

$$\delta e^A = \int_0^1 dt e^{tA} (\delta A) e^{(1-t)A}$$

to get an alternative “closed” form of the stress tensor, compared with the series expression (2.10):

$$\begin{aligned} T_{\alpha\beta} = & \frac{g_{\alpha\beta}}{2g_p^2} \left\{ \phi e^{-k\Box} \phi - \frac{2}{p+1} \phi^{p+1} + k \int_0^1 dt (e^{-kt\Box} \phi) (\Box e^{-k(1-t)\Box} \phi) \right. \\ & \left. + k \int_0^1 dt (\partial_\mu e^{-kt\Box} \phi) (\partial^\mu e^{-k(1-t)\Box} \phi) \right\} \\ & - \frac{k}{g_p^2} \int_0^1 dt (\partial_\alpha e^{-kt\Box} \phi) (\partial_\beta e^{-k(1-t)\Box} \phi), \end{aligned} \quad (2.11)$$

where $k \equiv \frac{1}{2} \ln p$.

In the case that $\phi(x)$ only depends on one spatial coordinate, say $x \equiv x^{25}$, the last term in the right hand side of (2.11) vanishes for all the components except for $T_{25,25}$. The energy density is

$$\begin{aligned} E(x) = T_0^0 = & \frac{1}{2g_p^2} \left\{ \phi e^{-k\partial^2} \phi - \frac{2}{p+1} \phi^{p+1} + k \int_0^1 dt (e^{-kt\partial^2} \phi) (\partial^2 e^{-k(1-t)\partial^2} \phi) \right. \\ & \left. + k \int_0^1 dt (\partial e^{-kt\partial^2} \phi) (\partial e^{-k(1-t)\partial^2} \phi) \right\}, \end{aligned} \quad (2.12)$$

where $\partial^2 \equiv \frac{\partial^2}{\partial x^2}$.

2.1.2 Energy of The Lump Solution

There are some previously known solutions for the p-adic model [47], [48]. One of them is the lump solution:

$$\phi(x) = p^{\frac{1}{2(p-1)}} \exp\left(-\frac{1}{2} \frac{p-1}{p \ln p} x^2\right). \quad (2.13)$$

This solution is interpreted as a D24-brane, where x is the coordinate transverse to the brane. This solution can be generalized to lower dimensional branes [48]. The D-brane tension of this solution is:

$$\begin{aligned} T_{24} = & - \int dx \mathcal{L}(\phi(x)) = - \int dx \frac{1}{2g_p^2} \frac{1-p}{1+p} \phi^{(p+1)}(x) \\ = & \frac{1}{g_p^2} \frac{p-1}{2(p+1)} p^{\frac{p}{p-1}} \sqrt{\frac{2\pi \ln p}{p^2-1}}. \end{aligned} \quad (2.14)$$

¹I thank M. Schnabl for suggesting the use of this identity.

Using the identity

$$\exp\left(-a\frac{d^2}{dx^2}\right)\exp(-bx^2) = \frac{1}{\sqrt{1-4ab}}\exp\left(-\frac{bx^2}{1-4ab}\right),$$

from (2.12), we can write down the energy density:

$$E(x) = \frac{p-1}{p+1}\sqrt{\frac{2\pi}{(p^2-1)\ln p}}p^{\frac{p}{p-1}}|x|\operatorname{Erf}\left[\frac{p-1}{p+1}\sqrt{\frac{p^2-1}{2p\ln p}}|x|\right]e^{-\frac{2(p-1)x^2}{(p+1)\ln p}}, \quad (2.15)$$

where $\operatorname{Erf}[x] \equiv \frac{2}{\sqrt{\pi}}\int_0^x dt \exp(-t^2)$ is the error function. In Figure 2.1, we plot this energy density (the solid line) for $p = 2$. At $x = 0$ and $x \rightarrow \pm\infty$, this energy density vanishes. By solving $\frac{d}{dx}E(x) = 0$ numerically as $p = 2$, one can see the energy reaches its maxima at $x \approx \pm 0.9997$. From (2.1), the potential is

$$\frac{1}{2}\phi^2 - \frac{1}{p+1}\phi^{p+1},$$

so, the D-brane vacuum is at $\phi = 1$. Moreover, from (2.13), one gets $\phi = 1$ at $x = \pm\sqrt{2}\ln 2 \approx \pm 0.9803$, which are close to the locations where the energy gets its maxima.

The lump solution (2.13) we are considering here, as we mentioned at the beginning of this subsection, is interpreted as a D-24 brane sharply localized on the hyperplane $x = 0$. Therefore, intuitively one may expect the energy to be sharply localized around $x = 0$. But from figure 2.1, one can see that the energy is somewhat localised around $x \approx \pm 0.9997$ and reaches a local minimum at $x = 0$.

The total energy is:

$$\int_{-\infty}^{\infty} dx E(x) = \frac{1}{g^2}\frac{p-1}{2(p+1)}p^{\frac{p}{p-1}}\sqrt{\frac{2\pi\ln p}{p^2-1}} \quad (2.16)$$

which is exactly the same as (2.14). In the limit $p \rightarrow 1$, $E(x)$ becomes:

$$\lim_{p \rightarrow 1} E(x) = \frac{1}{2g^2}x^2 \exp(1-x^2). \quad (2.17)$$

On the other hand, from (2.1), as $p \rightarrow 1$, the action becomes:

$$S = \frac{1}{2g^2} \int d^d x \left(\frac{1}{2}\phi\Box\phi - \frac{1}{2}\phi^2 + \phi^2 \ln \phi \right).$$

This action has a lump solution:

$$\phi(x) = \exp\left(\frac{1}{2}(1-x^2)\right),$$

whose energy density is exactly the same as (2.17).

This energy density looks very similar to that of the ordinary ϕ^3 field theory with coupling constant g_0 and unit mass [49]:

$$S = \frac{1}{g_0^2} \int d^d x \left\{ \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + \frac{1}{3}\phi^3 \right\},$$

which has the lump solution:

$$\phi(x) = \frac{3}{2}(1 - \tanh^2 \frac{x}{2}), \quad (2.18)$$

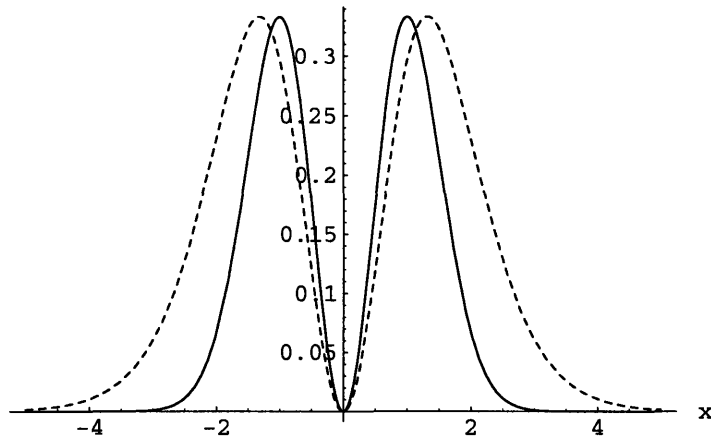


Figure 2.1: The energy distribution of the lump solution (2.13) of the p-adic model for $p = 2$ (solid line, $g_p^2 E(x)$ versus x) and that (2.18) of ordinary ϕ^3 field theory (dashed line, $g_0^2 E(x)$ versus x).

with energy density

$$E(x) = \frac{1}{g_0^2} \frac{9}{4} \operatorname{sech}^4 \frac{x}{2} \tanh^2 \frac{x}{2}, \quad (2.19)$$

which is plotted in Figure 1 (dashed line).

2.2 The Pure Tachyon Field of String Field Theory Case

When we expand the string field in the Hilbert space of the first quantized string theory, we can read off the action of the pure tachyonic cubic string field theory. As in the last section, we include the metric in the action and convert all the ordinary derivatives to covariant ones. Variations of the metric again give the stress tensor. Then we calculate the energy density of the lump solution given in [34] and the pressure of the rolling tachyon solution given in [16].

2.2.1 Stress Tensor for the Tachyon field in SFT

Firstly, we write down the pure tachyonic action of the cubic SFT. From Sen's conjecture [31], we should add the D-brane tension into the SFT action to cancel the negative energy due to the tachyon. We know that after adding the D-brane tension term to the potential of the cubic SFT, the local minimum of the new potential vanishes [7]. In the same spirit, here we should add a term $\frac{1}{6}K^{-6}$ to the potential to set the local minimum of the potential to zero.

$$S = \frac{1}{g_0^2} \int d^d x \left(\frac{1}{2} \phi^2 - \frac{1}{2} (\partial \phi)^2 - \frac{1}{3} K^3 \tilde{\phi}^3 - \frac{1}{6} K^{-6} \right), \quad (2.20)$$

where

$$\tilde{\phi} = \exp(\ln K \square) \phi(x) = K^\square \phi(x). \quad (2.21)$$

g_0 is the open bosonic string coupling constant and $K = 3\sqrt{3}/4$. \square is defined as in the last section. The equation of motion from this action is:

$$K^{-2\square}(1 + \square)\tilde{\phi} = K^3\tilde{\phi}^2.$$

In order to separate the term without derivatives from $\tilde{\phi}(x)$, we define:

$$\begin{aligned}\psi(x) &= \tilde{\phi}(x) - \phi(x) = \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} \square^l \phi(x) \\ &= \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} \frac{1}{\sqrt{-g}} \partial_{\mu_1} \sqrt{-g} g^{\mu_1 \nu_1} \partial_{\nu_1} \frac{1}{\sqrt{-g}} \partial_{\mu_2} \sqrt{-g} g^{\mu_2 \nu_2} \partial_{\nu_2} \dots \\ &\quad \dots \frac{1}{\sqrt{-g}} \partial_{\mu_l} \sqrt{-g} g^{\mu_l \nu_l} \partial_{\nu_l} \phi(x),\end{aligned}\tag{2.22}$$

where in the last step, we have written the expression in the covariant form. For an arbitrary differentiable function $f(x)$,

$$\int d^d x f(x) \frac{\delta \psi(x)}{\delta g^{\alpha\beta}} = \frac{1}{2} f \psi g_{\alpha\beta} + A_{\alpha\beta}(f)\tag{2.23}$$

where

$$\begin{aligned}A_{\alpha\beta}(f) &= \frac{1}{2} g_{\alpha\beta} \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} g^{\mu_1 \nu_1} \dots g^{\mu_l \nu_l} \\ &\quad \cdot \left(f_{\mu_1 \nu_1 \mu_2 \nu_2 \dots \mu_l \nu_l} + f_{\mu_1 \nu_1 \mu_2 \nu_2 \dots \mu_l \nu_l} + \dots + f_{\mu_1 \nu_1 \dots \mu_l \nu_l} \right) \\ &\quad - \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} g^{\mu_1 \nu_1} \dots g^{\mu_{l-1} \nu_{l-1}} \left(f_{\alpha \phi \beta \mu_1 \nu_1 \dots \mu_{l-1} \nu_{l-1}} + \right. \\ &\quad \left. f_{\alpha \mu_1 \nu_1 \phi \beta \mu_2 \nu_2 \dots \mu_{l-1} \nu_{l-1}} + \dots + f_{\alpha \mu_1 \nu_1 \dots \mu_{l-1} \nu_{l-1} \phi \beta} \right).\end{aligned}\tag{2.24}$$

Again, we set the metric to be flat with signature $(-1, 1, 1 \dots 1)$ after the variation. Replace $\tilde{\phi}$ by $\phi + \psi$ in (2.20), expanding and coupling to the metric:

$$\begin{aligned}S &= \frac{1}{g_0^2} \int d^d x \sqrt{-g} \left\{ \left(\frac{1}{2} \phi^2 - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{3} K^3 \phi^3 - \frac{1}{6} K^{-6} \right) \right. \\ &\quad \left. - K^3 \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3} \psi^3 \right) \right\}.\end{aligned}\tag{2.25}$$

Varying the first term in the last right hand side of (2.25) with respect to $\delta g^{\alpha\beta}$ gives

$$\begin{aligned}&\frac{1}{g_0^2} \delta g^{\alpha\beta} \int d^d x \sqrt{-g} \left(\frac{1}{2} \phi^2 - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{3} K^3 \phi^3 - \frac{1}{6} K^{-6} \right) \\ &= -\frac{g_{\alpha\beta}}{2g_0^2} \left(\frac{1}{2} \phi^2 - \frac{1}{2} (\partial\phi)^2 - \frac{1}{3} K^3 \phi^3 - \frac{1}{6} K^{-6} \right) - \frac{1}{2g_0^2} \partial_{\alpha} \phi \partial_{\beta} \phi \\ &\equiv -\frac{1}{g_0^2} C_{\alpha\beta},\end{aligned}\tag{2.26}$$

where we have defined $C_{\alpha\beta}$ to simplify our notation. As for the second term in the last right hand side of (2.25), note

$$\begin{aligned} & -K^3 \delta_{g^{\alpha\beta}} \int d^d x \sqrt{-g} \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3} \psi^3 \right) \\ & = \frac{1}{2} K^3 \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3} \psi^3 \right) g_{\alpha\beta} - K^3 \int d^d x \sqrt{-g} \tilde{\phi}^2 \delta_{g^{\alpha\beta}} \psi. \end{aligned}$$

So, from (2.23) and (2.24) the variation of the second term in the last step of (2.25) contributes:

$$\begin{aligned} & \left(\frac{-K^3}{g_0^2} \right) \delta_{g^{\alpha\beta}} \int d^d x \sqrt{-g} \left(\phi^2 \psi + \phi \psi^2 + \frac{1}{3} \psi^3 \right) \\ & = \frac{-K^3}{g_0^2} \left\{ A_{\alpha\beta}(\tilde{\phi}^2) + \frac{1}{2} \left(\phi \psi^2 + \frac{2}{3} \psi^3 \right) g_{\alpha\beta} \right\}, \end{aligned} \quad (2.27)$$

Finally, from (2.24), (2.26) and (2.27), the stress tensor is:

$$T_{\alpha\beta} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\alpha\beta}} = -\frac{2K^{-3}}{g_0^2} A_{\alpha\beta}(\tilde{\phi}^2) - \frac{2}{g_0^2} C_{\alpha\beta}. \quad (2.28)$$

In the case that $\phi(x)$ only depends on one spatial coordinate, say x^{25} , from (2.24),

$$A_{\alpha\beta}(\tilde{\phi}^2) = \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} \left\{ \frac{1}{2} g_{\alpha\beta} \sum_{m=1}^{2l-1} \tilde{\phi}_m^2 \phi_{2l-m} - \delta_{\alpha,25} \delta_{\beta,25} \sum_{m=1}^l \tilde{\phi}_{2m-1}^2 \phi_{2l-2m+1} \right\}. \quad (2.29)$$

Plug it into (2.28), we obtain the stress tensor for lump solutions. Similarly, if $\phi(x)$ only depends on time, we can write:

$$A_{\alpha\beta}(\tilde{\phi}^2) = -K^3 \sum_{l=1}^{\infty} \frac{(-\ln K)^l}{l!} \left\{ \frac{1}{2} g_{\alpha\beta} \sum_{m=1}^{2l-1} \tilde{\phi}_m^2 \phi_{2l-m} + \delta_{\alpha,0} \delta_{\beta,0} \sum_{m=1}^l \tilde{\phi}_{2m-1}^2 \phi_{2l-2m+1} \right\}. \quad (2.30)$$

Plug it into (2.28), we obtain the stress tensor for rolling solutions

2.2.2 Energy distribution of the SFT lump solution

In [34], a lump solution of OSFT has been given in the form of an expansion in terms of cosines. We are only concerned with the pure tachyonic mode here, so drop the higher modes:

$$\phi(x) = t_0 + t_1 \cos\left(\frac{x}{R}\right) + t_2 \cos\left(\frac{2x}{R}\right) + \dots, \quad (2.31)$$

where R is the radius of the circle on which the coordinate x is compactified. We can calculate the energy distribution of this solution, from (2.21), (2.22), (2.28) and (2.29):

$$\begin{aligned} \tilde{\phi}(x) & = K^{\partial_x^2} \phi(x) = t_0 + t_1 K^{-\frac{1}{R^2}} \cos\left(\frac{x}{R}\right) + t_2 K^{-\frac{4}{R^2}} \cos\left(\frac{2x}{R}\right) + \dots, \\ \psi(x) & = \tilde{\phi}(x) - \phi(x) = t_1 \left(K^{-\frac{1}{R^2}} - 1 \right) \cos\left(\frac{x}{R}\right) + t_2 \left(K^{-\frac{4}{R^2}} - 1 \right) \cos\left(\frac{2x}{R}\right) + \dots, \end{aligned}$$

$$\begin{aligned}
E(x) &= T_0^0 = -T_{00} \\
&= -\frac{1}{g_0^2} \left(\frac{1}{2} \phi^2 - \frac{1}{3} K^3 \phi^3 - \frac{1}{2} (\partial_x \phi)^2 - \frac{1}{6} K^{-6} + K^3 \phi \psi^2 + \frac{2}{3} K^3 \psi^3 \right) \\
&\quad - \frac{K^3}{g_0^2} \sum_{l=1}^{\infty} \frac{(\ln K)^l}{l!} \sum_{m=1}^{2l-1} \left(\tilde{\phi}^2 \right)_m \phi_{2l-m}.
\end{aligned} \tag{2.32}$$

In $R = \sqrt{3}$ case, using the method introduced in [34], one can obtain:

$$t_0 = 0.216046, \quad t_1 = -0.343268, \quad t_2 = -0.0978441,$$

when we plug these values into (2.32), we find:

$$\begin{aligned}
E(x) &= \frac{1}{g_0^2} \left(0.0206937 + 0.0242345 \cos \frac{x}{R} \right. \\
&\quad \left. - 0.00780954 \cos \frac{2x}{R} - 0.0204855 \cos \frac{3x}{R} \right. \\
&\quad \left. - 0.0111187 \cos \frac{4x}{R} - 0.00218278 \cos \frac{5x}{R} - 0.000177055 \cos \frac{6x}{R} \right).
\end{aligned}$$

This lump solution has the interpretation of D24 brane, the tension is:

$$\mathcal{T}_{24} = \int_{-\pi R}^{\pi R} dx E(x) \simeq 0.225206 \frac{1}{g_0^2}.$$

On the other hand, $\phi = 0$ is supposed to represent the D25 brane. We have $\mathcal{T}_{25} = -V(\phi = 0) = \frac{1}{6} \frac{K^{-6}}{g_0^2} \simeq 0.0346831 \frac{1}{g_0^2}$. Therefore,

$$\frac{1}{2\pi} \frac{\mathcal{T}_{24}}{\mathcal{T}_{25}} \simeq 1.03343$$

a ratio that is unity in string theory.

Figure 2 shows the energy density $E(x)$. As the lump solutions in the p-adic string theory, the energy density is not localised around the hyperplane $x = 0$. Instead, $E(x = 0)$ is a local minimum. A difference from the p-adic model is that $E(0)$ does not vanish here.

2.2.3 Pressure evolution of the SFT rolling tachyon solution

In [16], a rolling tachyon solution of OSFT is expressed as a series expansion in $\cosh(nt)$:

$$\phi(t) = t_0 + t_1 \cosh t + t_2 \cosh 2t + \dots.$$

From (2.21), (2.22), (2.28) and (2.30):

$$\begin{aligned}
\tilde{\phi}(t) &= K^{-\partial_t^2} \phi(t) = t_0 + t_1 K^{-1} \cosh t + t_2 K^{-4} \cosh 2t + \dots, \\
\psi(t) &= \tilde{\phi}(t) - \phi(t) = t_1 (K^{-1} - 1) \cosh t + t_2 (K^{-4} - 1) \cosh 2t + \dots,
\end{aligned}$$

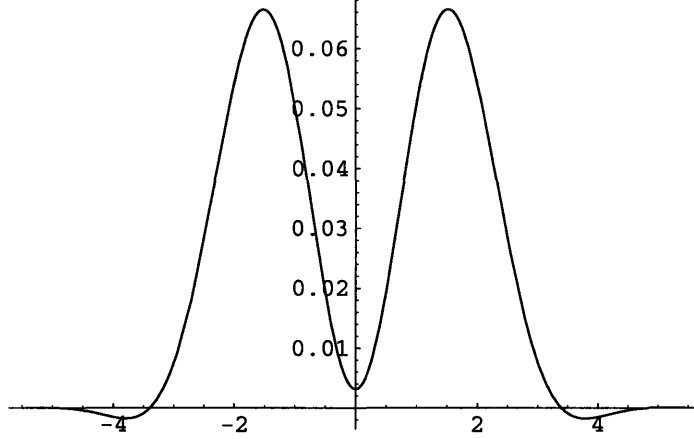


Figure 2.2: The Energy density of the pure tachyonic lump solution $\phi(x) = t_0 + t_1 \cos \frac{x}{R} + t_2 \cos \frac{2x}{R}$ of OSFT theory with $R = \sqrt{3}$. the plot is $g_0^2 E(x)$ versus x .

$$\begin{aligned}
p(t) &= -T_{11} \\
&= \frac{1}{g_0^2} \left(\frac{1}{2} \phi^2 - \frac{1}{3} K^3 \phi^3 + \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{6} K^{-6} + K^3 \phi \psi^2 + \frac{2}{3} K^3 \psi^3 \right) \\
&\quad + \frac{K^3}{g_0^2} \sum_{l=1}^{\infty} \frac{(-\ln K)^l}{l!} \sum_{m=1}^{2l-1} \left(\tilde{\phi}^2 \right)_m \phi_{2l-m}.
\end{aligned} \tag{2.33}$$

From section 7 in [16],

$$t_0 = 0.00162997, \quad t_1 = 0.05, \quad t_2 = -0.000189714,$$

and therefore,

$$\begin{aligned}
p(t) &= \frac{1}{g_0^2} \left(-0.0346844 + 0.0000416895 \cosh t + 0.00124462 \cosh 2t \right. \\
&\quad \left. - 0.0000416042 \cosh 3t + 2.59666 \times 10^{-7} \cosh 4t \right. \\
&\quad \left. - 3.97466 \times 10^{-10} \cosh 5t + 2.09045 \times 10^{-13} \cosh 6t \right).
\end{aligned}$$

Figure 2.3 shows the pressure evolution. It has the same property as the pressure in p-adic theory (Figure 10 in [16]). The pressure starts from negative value at time $t = 0$ to force the tachyon roll to the vacuum. But instead of decreasing to zero as $t \rightarrow \infty$, it oscillates without bound at large times. So, this solution does not seem to represent tachyon matter.

2.3 Conclusion

By introducing the metric, we have obtained general expressions for the stress tensors both for the p-adic model and for the pure tachyonic sector of open bosonic string field theory [31], [7], [8], [34].

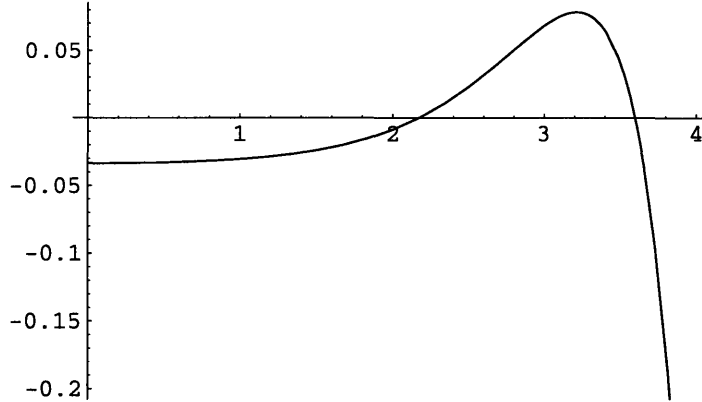


Figure 2.3: The pressure evolution of the rolling tachyon solution $\phi(t) = t_0 + t_1 \cosh t + t_2 \cosh 2t$ of OSFT theory. $g_0^2 p(t)$ versus t . As t becomes larger, $p(t)$ oscillates rapidly.

Furthermore, we considered some available solutions and wrote down the corresponding energy densities for space dependent ones and pressure evolutions for time dependent ones. In conformal field theory, D-branes are boundary conditions and one could expect the energy to be sharply localized at the D-brane position. It was not clear whether or not the lumps of the p-adic string theory would have this property. Our results show that they do not. The energy density vanishes at $x = 0, \pm\infty$. It has two maxima. These two maxima are symmetrically localized with respect to $x = 0$. In the pure tachyonic sector of OSFT, the energy density for the lump solution reaches a local minimum at $x = 0$. For the rolling tachyon solution, the pressure oscillates with growing amplitude instead of asymptotically vanishing. Therefore, as in the p-adic model, the rolling solution we considered in this chapter does not seem to represent tachyon matter.

There are two shortcomings of the calculations in OSFT. The first is not including the massive fields. The second is that the coupling of open strings to the metric could have additional terms that vanish in the flat space limit but contribute to the stress tensor. Such phenomena happens in noncommutative field theory [50]. Open-closed string field theory [51] might be needed to calculate the stress tensor with complete confidence. I thank M. Schnabl for bringing this point to my attention.

Chapter 3

Testing Closed String Field Theory with Marginal Fields

Marginal deformations have provided a useful laboratory to deepen our understanding of open string field theory. The effective potential for a marginal field must vanish, but in the level expansion one sees a potential that becomes progressively flatter as the level ℓ is increased [52, 53, 54, 55]. The marginal operator was taken to be $c\partial X$ and corresponds to a constant deformation of the $U(1)$ gauge field in open string theory. The associated spacetime field a_s can be viewed as a Wilson line parameter. For small a_s the potential can be expanded in the form

$$g^2 V_\ell(a_s) = \alpha_4(\ell) a_s^4 + \mathcal{O}(a_s^6). \quad (3.1)$$

Numerical evidence was found that the coefficient $\alpha_4(\ell)$ decreases as ℓ increases. Eventually, $\alpha_4(\ell)$ was elegantly shown to be exactly zero as ℓ goes to infinity [56]. This is, of course, a necessary condition for the potential to vanish completely at infinite level. One can also study large marginal deformations and the relationship between the string field marginal parameter a_s and the conformal field theory marginal parameter [57, 52].

In this chapter we use the closed string marginal operator $c\bar{c}\partial X\bar{\partial}X$ to test closed string field theory [26, 58] and to study the feasibility of level expansion in this theory. In order to do this we compute the effective potential for the associated marginal parameter, which we denote as a . We focus on the leading a^4 term in the expansion of this potential for small a . This term receives two contributions. The first one, $\mathcal{C}(\ell)$, arises from the cubic vertex by integration of massive fields of level less than or equal to ℓ . The second contribution, I_4 , arises from the elementary quartic vertex of closed string field theory and it has no open string field theory analog. General computations with the quartic vertex are now possible thanks to the work of Moeller [27]. If we denote by ℓ the maximum level for the massive closed string states that are being integrated, the total potential is

$$\kappa^2 \mathbb{V}_{(\ell)}^{tot}(a) = (\mathcal{C}(\ell) + I_4) a^4 + \mathcal{O}(a^6). \quad (3.2)$$

It is natural to write the coefficient $\mathcal{C}(\ell)$ as

$$\mathcal{C}(\ell) = \sum_{\ell'=0}^{\ell} c(\ell'), \quad (3.3)$$

where $c(\ell')$ is the contribution from the massive fields of level ℓ' . Marginality of a requires that the term in parenthesis in (3.2) vanishes as $\ell \rightarrow \infty$, or equivalently, that

$$\mathcal{C}(\infty) + I_4 = \sum_{\ell'=0}^{\infty} c(\ell') + I_4 = 0. \quad (3.4)$$

We find strong evidence for this cancellation by computing I_4 and the coefficients $c(\ell)$ to high level. This provides a test of the quartic structure of closed string theory. It is, in fact, the first computation with the quartic vertex in which there is a clear expectation that can be checked. Quartic terms have been computed earlier, most notably the quartic term in the (bulk) tachyon potential [30, 59]. In that case, however, there was no prediction for the magnitude or the sign of the result. Our present work gives us confidence that these early computations are correct.

In open string field theory the level of a cubic interaction is defined to be the sum of the levels of the three states that are coupled. It seems likely that the level of cubic closed string interactions should be defined in the same way. It is less clear how to define a level for quartic interactions in such a way that cubic and quartic contributions may be compared. Equation (3.4) allows us to do such comparison. In particular we can determine the level ℓ_* for which $c(\ell_*) \sim I_4$. Since $|c(\ell)|$ decreases with level, ℓ_* is the level at which inclusion of the quartic interaction seems appropriate. Our results suggest that $\ell_* \gtrsim 4$.

A puzzle arises in the computations. The value of $\mathcal{C}(\infty)$ depends only on the cubic vertex of the string field theory. The value of I_4 , which must cancel against $\mathcal{C}(\infty)$, depends on the quartic vertex. It is well known that the quartic vertex is not fully determined by the cubic vertex (although there is a canonical choice). How is it then possible for the cancellation to work for all four-string vertices consistent with the cubic vertex? This happens because of two facts: first, the cubic vertex determines the boundary of the region $\mathcal{V}_{0,4}$ of moduli space that defines the quartic vertex and, second, the integrand for I_4 is a total derivative and the integral reduces to the boundary of $\mathcal{V}_{0,4}$.

Let's review the organization of this chapter. In section 2 we state our conventions and carry out the computation of the coefficients $c(\ell)$ for $\ell \leq 4$. In section 3 we obtain a simple relation between the coefficients $c(\ell)$ and the analogous coefficients in the open string potential for the marginal Wilson line parameter. Using this relation and the results in [52, 55] we obtain $c(\ell)$ for $\ell \leq 20$. With this data we find a fit for $\mathcal{C}(\ell)$ and extrapolate to find $\mathcal{C}(\infty)$. This projected value gives an accurate cancellation against I_4 , the value of which is calculated in section 4. In fact, using the unpublished numerical work of [60, 61] the cancellation works to five significant digits. In section 5, we extend our discussion to the case of the four marginal operators associated with two spacetime directions. The $O(2)$ rotational symmetry implies the existence of two independent structures that can enter into the effective potential to leading (quartic) order in the fields. We compute the contributions to these

structures from the cubic and quartic string field vertices and again find convincing cancellations. We offer a discussion of our results in section 6.

3.1 Marginal field potential from cubic interactions

The bosonic closed string field theory action [26, 58] takes the form

$$S = -\frac{2}{\alpha'} \left(\frac{1}{2} \langle \Psi | c_0^- Q | \Psi \rangle + \frac{\kappa}{3!} \{ \Psi, \Psi, \Psi \} + \frac{\kappa^2}{4!} \{ \Psi, \Psi, \Psi, \Psi \} + \dots \right). \quad (3.5)$$

Here Q is the BRST operator, $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$, and the string field $|\Psi\rangle$ is a ghost number two state that satisfies $(L_0 - \bar{L}_0)|\Psi\rangle = 0$ and $(b_0 - \bar{b}_0)|\Psi\rangle = 0$. In this chapter we only consider states with vanishing momentum. After setting $\alpha' = 2$ and rescaling $\Psi \rightarrow \kappa^{-1}\Psi$, the potential $V = -S$ is given by

$$\kappa^2 V = \frac{1}{2} \langle \Psi | c_0^- Q | \Psi \rangle + \frac{1}{3!} \{ \Psi, \Psi, \Psi \} + \frac{1}{4!} \{ \Psi, \Psi, \Psi, \Psi \} + \dots \quad (3.6)$$

We fix the gauge invariance of the theory using the Siegel gauge $(b_0 + \bar{b}_0)|\Psi\rangle = 0$. The level ℓ of a state is defined as $\ell = L_0 + \bar{L}_0 + 2$. The tachyon state $c_1 \bar{c}_1 |0\rangle$ has level zero and marginal fields have level two. For a convenient normalization we assume that all spacetime coordinates have been compactified and the volume of spacetime is equal to one. We then use $\langle 0 | c_{-1} \bar{c}_{-1} c_0^- c_0^+ c_1 \bar{c}_1 | 0 \rangle = 1$, or equivalently,

$$\langle c(z_1) \bar{c}(\bar{w}_1) c(z_2) \bar{c}(\bar{w}_2) c(z_3) \bar{c}(\bar{w}_3) \rangle = 2(z_1 - z_2)(\bar{w}_1 - \bar{w}_2)(z_1 - z_3)(\bar{w}_1 - \bar{w}_3)(z_2 - z_3)(\bar{w}_2 - \bar{w}_3). \quad (3.7)$$

Since open string field theory uses $\langle c(z_1)c(z_2)c(z_3) \rangle_o = (z_1 - z_2)(z_1 - z_3)(z_2 - z_3)$ we can write

$$\langle c(z_1)c(z_2)c(z_3) \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3) \rangle = -2 \langle c(z_1)c(z_2)c(z_3) \rangle_o \cdot \langle \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3) \rangle_o. \quad (3.8)$$

This closed/open relation can be used to calculate the cubic coupling of three closed string tachyons:

$$\{ c_1 \bar{c}_1, c_1 \bar{c}_1, c_1 \bar{c}_1 \} = 2 \cdot \langle c_1, c_1, c_1 \rangle_o \cdot \langle \bar{c}_1, \bar{c}_1, \bar{c}_1 \rangle_o = 2 \cdot \mathcal{R}^3 \cdot \mathcal{R}^3 = 2\mathcal{R}^6, \quad (3.9)$$

where $\mathcal{R} \equiv \frac{1}{\rho} = \frac{3\sqrt{3}}{4} \simeq 1.2990$, ρ is the (common) mapping radius of the disks that define the three-string vertex, and $\langle c_1, c_1, c_1 \rangle_o = \mathcal{R}^3$ is the coupling of three tachyons in open string field theory (see, for example, [35], eqn. (5.6)).

In this section we only examine quadratic and cubic interactions. We begin by considering the effects of the level zero tachyon t on the potential for the (level two) marginal field a . The string field is therefore

$$|\Psi_0\rangle = t c_1 \bar{c}_1 |0\rangle + a \alpha_{-1} \bar{\alpha}_{-1} c_1 \bar{c}_1 |0\rangle. \quad (3.10)$$

The subscript on the string field indicates the level of the highest-level *massive* field – in this case zero, because the tachyon is the only massive state. The kinetic term and cubic vertex give the following potential:

$$\kappa^2 V_{(0)} = -t^2 + \frac{1}{3} \mathcal{R}^6 t^3 + \mathcal{R}^2 t a^2 = -t^2 + \frac{6561}{4096} t^3 + \frac{27}{16} t a^2. \quad (3.11)$$

To find an effective potential for a we fix values of a , solve for the tachyon field, and substitute back in the potential. For each value of a there are two solutions for the tachyon. One gives the vacuum branch V while the other one gives the marginal branch M . The tachyon values are

$$t^{V/M} = \frac{8192 \pm \sqrt{67108864 - 544195584 a^2}}{39366}. \quad (3.12)$$

As in open string field theory, the marginal parameter is bounded $|a| \leq 0.3512$. It is not clear how higher level and higher order interactions will affect this bound. In the marginal branch we can expand the potential for small a and find $\kappa^2 V_{(0)} \simeq 0.7119 a^4 + 0.9622 a^6 + \dots$. The quartic coefficient can be computed directly using the potential in (3.11) without including the t^3 term. The equation for the tachyon becomes linear and we get

$$\kappa^2 V_{(0)} = \frac{3^6}{2^{10}} a^4 \simeq 0.71191 a^4 \quad \rightarrow \quad C(0) = c(0) = 0.71191, \quad (3.13)$$

using the notation described in the introduction. In general, to find the contribution to a^4 from a massive field M we only need the kinetic term for M and the coupling $a^2 M$. In terms of Feynman diagrams we are simply computing a tree graph with four external a 's, two cubic vertices, and an intermediate massive field.

The string states needed for higher-level computations are built with oscillators $\alpha_{n \leq -1}, \bar{\alpha}_{n \leq -1}$ of the coordinate X , Virasoro operators $L'_{m \leq -2}, \bar{L}'_{m \leq -2}$ for the remaining coordinates (thus $c = 25$), and ghost/antighost oscillators. We can list such fields systematically using the generating function:

$$f(x, \bar{x}, y, \bar{y}) = \prod_{n=1}^{\infty} \frac{1}{1 - \alpha_{-n} x^n} \frac{1}{1 - \bar{\alpha}_{-n} \bar{x}^n} \prod_{m=2}^{\infty} \frac{1}{1 - L'_{-m} x^m} \frac{1}{1 - \bar{L}'_{-m} \bar{x}^m} \\ \cdot \prod_{\substack{k=-1 \\ k \neq 0}}^{\infty} (1 + c_{-k} x^k y) (1 + \bar{c}_{-k} \bar{x}^k \bar{y}) \prod_{l=2}^{\infty} (1 + b_{-l} x^l y^{-1}) (1 + \bar{b}_{-l} \bar{x}^l \bar{y}^{-1}). \quad (3.14)$$

A term of the form $x^n \bar{x}^{\bar{n}} y^m \bar{y}^{\bar{m}}$ corresponds to a state with $(L_0, \bar{L}_0) = (n, \bar{n})$ and ghost numbers $(G, \bar{G}) = (m, \bar{m})$. A massive field M is relevant to our calculation if the coupling $M a^2$ does not vanish. This requires that M have $(G, \bar{G}) = (1, 1)$, an even number of α 's, and an even number of $\bar{\alpha}$'s.

At level two we get three states: the marginal field itself, $c_{-1} c_1 |0\rangle$, and $\bar{c}_{-1} \bar{c}_1 |0\rangle$. One linear combination of the last two is the ghost dilaton and the other is pure gauge. Since none of the three states couples to a^2 , we have $c(2) = 0$. At level four $L_0 = \bar{L}_0 = 1$ and the coefficients of $(x \bar{x} y \bar{y})$ give all possible terms. With the above rule the set is reduced to

$$|\Psi_4\rangle = f_1 c_{-1} \bar{c}_{-1} + f_2 L'_{-2} \bar{L}'_{-2} c_1 \bar{c}_1 + (f_3 L'_{-2} c_1 \bar{c}_{-1} + \tilde{f}_3 \bar{L}'_{-2} c_{-1} \bar{c}_1) + r_1 \alpha_{-1}^2 \bar{\alpha}_{-1}^2 c_1 \bar{c}_1 \\ + (r_2 \alpha_{-1}^2 c_1 \bar{c}_{-1} + \tilde{r}_2 \bar{\alpha}_{-1}^2 c_{-1} \bar{c}_1) + (r_3 \alpha_{-1}^2 \bar{L}'_{-2} c_1 \bar{c}_1 + \tilde{r}_3 L'_{-2} \bar{\alpha}_{-1}^2 c_1 \bar{c}_1). \quad (3.15)$$

The corresponding terms in the potential are

$$\kappa^2 V_{(4)} = f_1^2 + \frac{121}{432} a^2 f_1 + \frac{625}{4} f_2^2 + \frac{15625}{1728} a^2 f_2 - \frac{25}{2} [f_3^2 + \tilde{f}_3^2] - \frac{1375}{864} a^2 (f_3 + \tilde{f}_3) \\ + 4r_1^2 + \frac{27}{16} a^2 r_1 - 2[r_2^2 + \tilde{r}_2^2] + \frac{11}{16} a^2 (r_2 + \tilde{r}_2) + 25[r_3^2 + \tilde{r}_3^2] - \frac{125}{32} a^2 (r_3 + \tilde{r}_3), \quad (3.16)$$

where we used the conservation laws in [35] to evaluate the cubic interactions. Solving for all the massive fields and substituting back into $V_{(4)}$ we obtain

$$\kappa^2 V_{(4)} = -\frac{19321}{46656} a^4 \simeq -0.41412 a^4 \quad \rightarrow \quad c(4) = -0.41412. \quad (3.17)$$

To get the total contribution up to level four we add the above to the result in (3.13):

$$\kappa^2 \mathbb{V}_{(4)} = \frac{222305}{746496} a^4 \simeq 0.29780 a^4 \quad \rightarrow \quad \mathcal{C}(4) = 0.29780. \quad (3.18)$$

The contribution from level six string fields vanishes because none of the string fields has even number of α 's as well as even number of $\bar{\alpha}$'s and satisfies the condition that $(G, \bar{G}) = (1, 1)$. Therefore $c(6) = 0$. We note that $\mathcal{C}(4) < \mathcal{C}(0)$. To get additional information we turn to open string field theory.

3.2 Contributions to a^4 calculated using OSFT

As long as we consider closed string states of ghost number $(1, 1)$, work in the Siegel gauge, and restrict ourselves to quadratic and cubic interactions, closed string field theory functions as a kind of product of two copies of open string field theory. This will enable us to relate the contributions to the a^4 term in the effective potential to the similar contributions to a_s^4 in the case of open string field theory.

In classical open string field theory the marginal state is $|\phi_a\rangle = \alpha_{-1}^X c_1 |0\rangle$ and the marginal field is called a_s [52]. To calculate the quartic potential a_s^4 it suffices to consider

$$g^2 V_O(\ell) = \sum_{\ell'=0,2,\dots}^{\ell} -g^2 S_O^{(\ell')}, \quad -g^2 S_O^{(\ell)} = \frac{1}{2} \langle \Phi^{(\ell)} | Q_B | \Phi^{(\ell)} \rangle + \langle \Phi^{(\ell)}, \phi_a, \phi_a \rangle a_s^2, \quad (3.19)$$

where O is for open string, g is the open string coupling, Q_B is the open string BRST operator, and ℓ is open string level: $(L_0 + 1)|\Phi^{(\ell)}\rangle = \ell|\Phi^{(\ell)}\rangle$. Only even levels contribute because states of odd level are twist odd and their coupling to a_s^2 vanishes. For each ℓ we sum over all basis states of ghost number one in the Siegel gauge:

$$|\Phi^{(\ell)}\rangle \equiv \sum_i \phi_i^{(\ell)} |\mathcal{O}_i^{(\ell)}\rangle, \quad L_0 |\mathcal{O}_i^{(\ell)}\rangle = (\ell - 1) |\mathcal{O}_i^{(\ell)}\rangle. \quad (3.20)$$

We will leave out the superscript ℓ whenever possible and define

$$m_{ij} \equiv \langle \mathcal{O}_i | c_0 | \mathcal{O}_j \rangle, \quad K_i \equiv \langle \mathcal{O}_i, \phi_a, \phi_a \rangle, \quad (3.21)$$

where m_{ij} is a symmetric nondegenerate matrix. In Siegel gauge $Q_B = c_0 L_0$ and therefore

$$-g^2 S_O^{(\ell)} = \frac{\ell - 1}{2} \phi_i m_{ij} \phi_j + K_i \phi_i a_s^2, \quad (3.22)$$

where summation over repeated i and j indices is implicit. Using matrix notation, $[M]_{ij} = m_{ij}$, $[K]_i = K_i$, $[\phi]_i = \phi_i$, we readily find the solution for ϕ and the value of the action:

$$\phi = -\frac{1}{\ell - 1} (M^{-1} K) a_s^2 \quad \rightarrow \quad -g^2 S_O^{(\ell)} = -\frac{1}{2(\ell - 1)} K^T M^{-1} K a_s^4. \quad (3.23)$$

Back in (3.19) we have

$$g^2 V_O(\ell) = \alpha_4(\ell) a_s^4 = a_s^4 \sum_{\ell'=0,2,\dots}^{\ell} \chi_{\ell'}, \quad \text{with} \quad \chi_{\ell'} = -\frac{1}{2(\ell'-1)} K^T M^{-1} K. \quad (3.24)$$

Let us now turn to closed strings. Because of level matching and the constraints $(b_0 \pm \bar{b}_0)\Psi = 0$, a closed string field of level $L_0 + \bar{L}_0 + 2 = 2\ell$ in the Siegel gauge can be written as a sum of factors: $\Psi^{(2\ell)} = \psi_{ij} |\mathcal{O}_i^{(\ell)}\rangle \otimes |\bar{\mathcal{O}}_j^{(\ell)}\rangle$ where the open string states are those in (3.20). Therefore

$$\langle \mathcal{O}_i \otimes \bar{\mathcal{O}}_j | c_0^- Q_B | \mathcal{O}_{i'} \otimes \bar{\mathcal{O}}_{j'} \rangle = \frac{1}{2} \langle \mathcal{O}_i \otimes \bar{\mathcal{O}}_j | c_0 \bar{c}_0 (L_0 + \bar{L}_0) | \mathcal{O}_{i'} \otimes \bar{\mathcal{O}}_{j'} \rangle = 2(\ell-1) m_{ii'} m_{jj'}, \quad (3.25)$$

where the factor of two in the last step is from the normalization (A.1). For closed strings the marginal state is $G = \alpha_{-1}^X \bar{\alpha}_{-1}^X c_1 \bar{c}_1 |0\rangle$. Since $G = \phi_a \otimes \bar{\phi}_a$, the cubic interaction factorizes: $\{\mathcal{O}_i \otimes \bar{\mathcal{O}}_j, G, G\} = 2 K_i K_j$. Therefore, up to the order a^4 , the potential is calculated from

$$\kappa^2 \mathbb{V}_{(2\ell)} = \sum_{\ell'=0,2,\dots}^{\ell} -\kappa^2 S^{(2\ell')}, \quad -\kappa^2 S^{(2\ell)} = \frac{1}{2} \langle \Psi^{(2\ell)} | c_0^- Q_B | \Psi^{(2\ell)} \rangle + \frac{1}{2} \{\Psi^{(2\ell)}, G, G\} a^2. \quad (3.26)$$

Our earlier comments allow explicit evaluation:

$$-\kappa^2 S^{(2\ell)} = (\ell-1) \psi_{ij} \psi_{i'j'} m_{ii'} m_{jj'} + a^2 \psi_{ij} K_i K_j. \quad (3.27)$$

The equation of motion for ψ_{ij} is readily solved:

$$\psi_{ij} = -\frac{1}{2(\ell-1)} m_{ii'}^{-1} m_{jj'}^{-1} K_{i'} K_{j'} a^2. \quad (3.28)$$

Substituting back into $S^{(2\ell)}$ and using (3.24) we find

$$-\kappa^2 S^{(2\ell)} = -(\ell-1) \left(-\frac{1}{2(\ell-1)} K^T M^{-1} K \right)^2 a^4 = -(\ell-1) \chi_{\ell}^2 a^4. \quad (3.29)$$

We recognize that the contribution to a_s^4 from the open string fields of level ℓ determines the contribution to a^4 from the closed string fields of level 2ℓ . With the notation described in the introduction,

$$\kappa^2 \mathbb{V}_{(2\ell)} = \mathcal{C}(2\ell) a^4 = a^4 \sum_{\ell'=0,2,\dots}^{\ell} c(2\ell'), \quad \text{with} \quad c(2\ell) = -(\ell-1) \chi_{\ell}^2. \quad (3.30)$$

The values of $\alpha_4(\ell)$ (recall (3.24)) for $\ell = 0, 2$, and 4 can be read from Table 1 of [52], and values up to $\ell = 10$ from Table 1 of [55] (with extra digits provided by [61]). We reproduce them in Table 3.1, along with the corresponding values of χ_{ℓ} . For $\ell = 0, 2$, we confirm the closed string results of section 2. Since fits in powers of $1/\ell$, where ℓ is open string level, accurately describe the behavior of coefficients in open string effective potentials [9], we use the data for $\ell = 4, 6, 8$, and 10 to fit α_4 to $b_0 + b_1/\ell + b_2/\ell^2$:

$$\alpha_4(\ell) \simeq -0.00026 + \frac{0.35681}{\ell} + \frac{0.12893}{\ell^2}. \quad (3.31)$$

This is a good fit since $\alpha_4(\ell)$ must vanish for infinite level. We now use this fit and (3.30) to predict the behavior of $\mathcal{C}(\ell)$ as a function of the closed string level ℓ . It follows from (3.24) and (3.31) that

$$\chi_\ell = \alpha_4(\ell) - \alpha_4(\ell - 2) \simeq -\frac{0.71361}{\ell^2}. \quad (3.32)$$

Equation (3.30) then gives

$$\mathcal{C}(2\ell) - \mathcal{C}(2\ell - 4) = -(\ell - 1)\chi_\ell^2 \simeq -\frac{0.50925}{\ell^3}. \quad (3.33)$$

This equation is consistent with the extrapolation

$$\mathcal{C}(2\ell) \simeq f_0 + \frac{0.50925}{(2\ell)^2}. \quad (3.34)$$

Comparing with the open string result (3.31) we see that the potential converges faster in closed string theory. Given (3.34) we now make a direct fit of \mathcal{C} to $d_0 + d_2/\ell^2 + d_3/\ell^3$ using the closed string data in the table for $\ell = 4, 6, 8$, and 10:

$$\mathcal{C}(2\ell) \simeq 0.25585 + \frac{0.50581}{(2\ell)^2} + \frac{1.06366}{(2\ell)^3}, \quad (3.35)$$

From this projection we find

$$\mathcal{C}(\infty) \simeq 0.25585. \quad (3.36)$$

Recalling (3.4), this number must be cancelled by the elementary quartic contribution I_4 .

ℓ	χ_ℓ	$\alpha_4(\ell)$	$c(2\ell)$	$\mathcal{C}(2\ell)$
0	0.84375	0.84375	0.71191	0.71191
2	-0.64352	0.20023	-0.41412	0.29780
4	-0.10323	0.09700	-0.03197	0.26583
6	-0.03420	0.06280	-0.00585	0.25998
8	-0.01646	0.04634	-0.00190	0.25808
10	-0.00962	0.03672	-0.00083	0.25725
∞	-	-0.00026	-	0.25585

Table 3.1: χ_ℓ and $\alpha_4(\ell)$ give the contribution of level ℓ fields and the total contributions up to level ℓ , respectively, to the quartic term in the potential for the Wilson line parameter a_s . The last two columns give the contribution $c(2\ell) = -(\ell - 1)\chi_\ell^2$ of closed string fields of level 2ℓ and the total contributions $\mathcal{C}(2\ell)$ up to level 2ℓ to the quartic term in the potential for the closed string marginal field a . The last row gives the projections from fits.

3.3 Elementary contribution to a^4

We now compute the coupling of four marginal operators through the four-string elementary vertex of closed string field theory. If all fields have the simple ghost structure $\Psi_i = \mathcal{O}_i c_1 \bar{c}_1 |0\rangle$, with \mathcal{O}_i a

primary matter operator of conformal dimension (h_i, h_i) , the elementary quartic amplitude is [30]:

$$\{\Psi_1, \Psi_2, \Psi_3, \Psi_4\} = -\frac{2}{\pi} \int_{\mathcal{V}_{0,4}} \frac{dx dy \langle \mathcal{O}_1(0) \mathcal{O}_2(1) \mathcal{O}_3(\xi) \mathcal{O}_4(t=0) \rangle}{\rho_1^{2-2h_1} \rho_2^{2-2h_2} \rho_3^{2-2h_3} \rho_4^{2-2h_4}}. \quad (3.37)$$

Here ρ_i 's are the mapping radii and the correlator has the operators inserted at $z = 0, 1, \xi = x + iy$, and $t = 1/z = 0$. In this chapter all matter operators have dimension $(1, 1)$ and the mapping radii drop out. For the marginal field a the corresponding operator is $G = c\bar{c}\mathcal{O}_{xx}$ with $\mathcal{O}_{xx} = -\partial X \bar{\partial} X$. Using $\langle \partial X(z_1) \partial X(z_2) \rangle = 1/(z_1 - z_2)^2$, as well as the antiholomorphic analog we find

$$\langle \mathcal{O}_{xx}(0) \mathcal{O}_{xx}(1) \mathcal{O}_{xx}(\xi) \mathcal{O}_{xx}(t=0) \rangle = \left| 1 + \frac{1}{\xi^2} + \frac{1}{(1-\xi)^2} \right|^2. \quad (3.38)$$

Therefore, the amplitude $\{G^4\} \equiv \{G, G, G, G\}$ is

$$\{G^4\} = -\frac{2}{\pi} I_{0,4}, \quad \text{with } I_{0,4} \equiv \int_{\mathcal{V}_{0,4}} dx dy \left| 1 + \frac{1}{\xi^2} + \frac{1}{(1-\xi)^2} \right|^2. \quad (3.39)$$

The moduli $\mathcal{V}_{0,4}$ space is comprised of twelve regions, a region \mathcal{A} ([27], Fig. 3) and eleven regions obtained by acting on \mathcal{A} with the transformations $\xi \rightarrow 1-\xi$, $\xi \rightarrow \frac{1}{\xi}$, $\xi \rightarrow \bar{\xi}$, and their compositions [27]. The integrand in (3.39) is invariant under these transformations, so we integrate numerically over \mathcal{A} using the quintic fit provided by Moeller ([27], eqn. (6.5)) and multiply the result by twelve:

$$I_{0,4} = 12 \int_{\mathcal{A}} dx dy \left| 1 + \frac{1}{\xi^2} + \frac{1}{(1-\xi)^2} \right|^2 \simeq 9.65029. \quad (3.40)$$

The contribution to the potential from the elementary quartic interaction is then

$$\kappa^2 V_4 = \frac{1}{4!} \{G^4\} a^4 = -\frac{1}{12\pi} I_{0,4} a^4 \simeq -0.25598 a^4 \quad \rightarrow \quad I_4 = -0.25598. \quad (3.41)$$

Recalling (3.36), the test indicated in (3.4) gives

$$\mathcal{C}(\infty) + I_4 = 0.25585 - 0.25598 = -0.00013. \quad (3.42)$$

The cancellation is impressive: the residue is about 0.05% of I_4 .

Best estimates: With the most accurate description of \mathcal{A} , Moeller [60] has calculated the integral $I_{0,4}$ and his result gives $I_4 = -0.255872(\pm 2)$. Coletti, Sigalov, and Taylor [61] provided us with the χ_ℓ for $\ell \leq 150$. With this data we found $\mathcal{C}(300) = 0.2558765752$, a good estimate of $\mathcal{C}(\infty)$. Fitting \mathcal{C} to $d_0 + d_2/\ell^2 + d_3/\ell^3$ using $2\ell = 204$ to $2\ell = 300$ gives $\mathcal{C}(\infty) = d_0 = 0.2558708731$, which agrees with I_4 to *five* significant digits. With the data for $\ell \leq 78$, M. Beccaria obtained $\mathcal{C}(\infty) = 0.2558708706(3)$ using Levin acceleration and the BST algorithm [62].

The cancellation confirms that the sign and the normalization in (3.37) are correct. This is the same sign that implies that the quartic tachyon self-coupling is negative [59, 27]. We have thus extra confidence of the correctness of the early calculation of the quartic term in the tachyon potential.

One can readily see that the integrand in the amplitude $\{G^4\}$ is a total derivative. It is of the form $f(\xi)f(\bar{\xi})d\xi \wedge d\bar{\xi}$, with $f(\xi) = 1 + \frac{1}{\xi^2} + \frac{1}{(1-\xi)^2}$. We then note that $f(\xi) = \partial g(\xi)$ with $g(\xi) = \xi - \frac{1}{\xi} + \frac{1}{(1-\xi)}$, well defined in $\mathcal{V}_{0,4}$ since this region excludes $\xi = 0, 1$, and $\xi = \infty$. Finally, $f(\xi)f(\bar{\xi})d\xi \wedge d\bar{\xi} = \frac{1}{2} d(g(\xi)f(\bar{\xi})d\bar{\xi} - f(\xi)g(\bar{\xi})d\xi)$, which establishes the claim.

3.4 A moduli space of marginal deformations

If multiple marginal operators define a moduli space the potential for the corresponding fields must vanish identically. An instructive example is provided by the four marginal operators that can be built using the fields X and Y associated with the spacetime coordinates x and y . We will study the potential for the string field

$$|\Psi\rangle = (a_{xx}\alpha_{-1}^X\bar{\alpha}_{-1}^X + a_{yy}\alpha_{-1}^Y\bar{\alpha}_{-1}^Y + a_{xy}\alpha_{-1}^X\bar{\alpha}_{-1}^Y + a_{yx}\alpha_{-1}^Y\bar{\alpha}_{-1}^X) c_1\bar{c}_1|0\rangle. \quad (3.43)$$

The marginal fields a_{xx} , a_{yy} , and $a_{xy} + a_{yx}$ are metric deformations while $a_{xy} - a_{yx}$ is a Kalb-Ramond deformation. The marginal fields are conveniently assembled into the two-by-two matrix M :

$$M = \begin{pmatrix} a_{xx} & a_{yx} \\ a_{xy} & a_{yy} \end{pmatrix}. \quad (3.44)$$

It is useful to consider the global $O(2)$ rotational symmetry of the (x, y) plane. The potential for M should be invariant under an $O(2) \times O(2)$ symmetry where the first $O(2)$ rotates the $(\partial X, \partial Y)$ and the second rotates $(\bar{\partial} X, \bar{\partial} Y)$. Consider two rotation matrices R and S ($R^T R = S^T S = 1$). Together they define an element of $O(2) \times O(2)$ which acts on M as $M \rightarrow R M S^T$. To quadratic order in M there is an invariant U and a *quasi*-invariant V :

$$U = \text{Tr}(M^T M), \quad V = \det M. \quad (3.45)$$

In general $V \rightarrow \pm V$, since R and/or S may have determinant minus one. An example is provided by the parity transformation $S = \text{diag}(1, -1)$. In fact, the Z_2 symmetries that arise because correlators must have even numbers of appearances of holomorphic and antiholomorphic derivatives of each coordinate are taken into account by the various parity transformations. It follows that to quartic order in the fields we have two invariants:

$$U^2 \quad \text{and} \quad V^2. \quad (3.46)$$

There are no more independent invariants: the candidate $\text{Tr}(M^T M M^T M)$ is equal to $U^2 - 2V^2$.

The lowest level potential involves the tachyon and M and requires no new computation. Since U contains a_{xx}^2 the coefficient coupling t to U is the same as that coupling t to a^2 in (3.11). We thus have, as in (3.13),

$$\kappa^2 V_{(0)} = \frac{3^6}{2^{10}} U^2 \simeq 0.7119 U^2. \quad (3.47)$$

At level four 25 states enter the computation. We calculated the effective potential, solved the equations of motion, and verified that all terms assemble into the two anticipated invariants, giving

$$\kappa^2 V_{(4)} = -\frac{19321}{46656} U^2 + \frac{344}{729} V^2 \simeq -0.4141 U^2 + 0.4719 V^2. \quad (3.48)$$

The total effective potential up to level four from the cubic interactions is therefore:

$$\kappa^2 \mathbb{V}_{(4)} = \frac{222305}{746496} U^2 + \frac{344}{729} V^2 \simeq 0.2978 U^2 + 0.4719 V^2. \quad (3.49)$$

At infinite level the coefficients of U^2 and V^2 must be cancelled by elementary quartic interactions. The quartic interactions contribute

$$\kappa^2 V_4 = \gamma_1 U^2 + \gamma_2 V^2 = \gamma_1 (a_{xx}^4 + 2a_{xx}^2 a_{yy}^2 + \dots) + \gamma_2 (a_{xx}^2 a_{yy}^2 + \dots). \quad (3.50)$$

where γ_1 and γ_2 are constants to be determined. The value of γ_1 is determined by our earlier calculation of the quartic amplitude for a . Therefore (3.41) gives $\gamma_1 = -I_{0,4}/(12\pi)$. The coefficient of $a_{xx}^2 a_{yy}^2$ in the potential, to be calculated next, will give us the value of $2\gamma_1 + \gamma_2$, from which we find γ_2 .

To compute the elementary quartic amplitude $a_{xx}^2 a_{yy}^2$, we put the operator \mathcal{O}_{xx} associated with a_{xx} at 0 and 1 and the operator \mathcal{O}_{yy} associated with a_{yy} at ξ and ∞ . This choice is arbitrary and does not affect the value of the *integrated* correlator; this is not manifest but is guaranteed by the symmetry of the four-string vertex and can be checked explicitly. The matter correlator is:

$$\langle \mathcal{O}_{xx}(0) \mathcal{O}_{xx}(1) \mathcal{O}_{yy}(\xi) \mathcal{O}_{yy}(t=0) \rangle = \left\langle \partial X \bar{\partial} X(0) \partial X \bar{\partial} X(1) \right\rangle \left\langle \partial Y \bar{\partial} Y(\xi) \partial Y \bar{\partial} Y(t=0) \right\rangle = 1. \quad (3.51)$$

Since the correlator is just one, the amplitude is proportional to the area $A_{0,4}$ of the region $\mathcal{V}_{0,4}$ viewed as a subset of the z plane (with metric $dzd\bar{z}$):

$$\{\mathcal{O}_{xx}^2 \mathcal{O}_{yy}^2\} = -\frac{2}{\pi} \int_{\mathcal{V}_{0,4}} dx dy = -\frac{2}{\pi} A_{0,4}. \quad (3.52)$$

Since the contribution of a region \mathcal{S} is the same as that of $1 - \mathcal{S}$, $\bar{\mathcal{S}}$, and $1 - \bar{\mathcal{S}}$ we have

$$A_{0,4} = \int_{\mathcal{V}_{0,4}} dx dy = 4 \left(\int_{\mathcal{A}} + \int_{\frac{1}{\mathcal{A}}} + \int_{\frac{1}{1-\mathcal{A}}} \right) dx dy = 4 \int_{\mathcal{A}} dx dy \left(1 + \frac{1}{|\xi|^4} + \frac{1}{|1-\xi|^4} \right) \simeq 6.0774. \quad (3.53)$$

Of course, the integrand for area is a total derivative: $d\xi \wedge d\bar{\xi} = \frac{1}{2} d(\xi d\bar{\xi} - \bar{\xi} d\xi)$. Back to the amplitude in question,

$$\kappa^2 V = \frac{6}{4!} \{\mathcal{O}_{xx}^2 \mathcal{O}_{yy}^2\} a_{xx}^2 a_{yy}^2 = -\frac{1}{2\pi} A_{0,4} a_{xx}^2 a_{yy}^2. \quad (3.54)$$

We thus find:

$$\gamma_2 = -\frac{1}{2\pi} \left(A_{0,4} - \frac{1}{3} I_{0,4} \right). \quad (3.55)$$

Collecting our results, equation (3.50) gives

$$\kappa^2 V_4 = -\frac{1}{12\pi} I_{0,4} U^2 - \frac{1}{2\pi} \left(A_{0,4} - \frac{1}{3} I_{0,4} \right) V^2 \simeq -0.2560 U^2 - 0.4552 V^2. \quad (3.56)$$

This quartic contribution cancels most of the potential in (3.49). The small residual potential is

$$\kappa^2 V_4^{res} = 0.0418 U^2 + 0.0167 V^2. \quad (3.57)$$

The data is collected in Table 3.2. The data for U^2 does not represent a new test, higher level computations would reproduce the result of section 4. The residual coefficient for V^2 is 4% of the original contribution. This is evidence that the infinite-level computation would give the expected cancellation.

level	U^2	V^2
0	0.7119	-
4	-0.4141	0.4719
quartic	-0.2560	-0.4552
residual	0.0418	0.0167

Table 3.2: Contributions from the given level to the coefficients that multiply the invariants U^2 and V^2 in the effective potential for the marginal fields. The row “quartic” gives the contributions from the elementary quartic interactions. The last row is the residual quartic potential, obtained after adding all contributions.

3.5 Conclusion

In this chapter we have tested the quartic vertex of bosonic closed string field theory and the concrete description of it provided by Moeller [27]. The sign, normalization, and region of integration $\mathcal{V}_{0,4}$ of the quartic interaction were all confirmed. This region comprises the set of four-punctured spheres that are not produced by Feynman graphs built with two cubic vertices and a propagator. Our calculations checked the flatness of the effective potential for marginal parameters; this required the cancellation of cubic contributions of all levels against a finite set of quartic contributions. We examined this cancellation in two examples, one with one marginal direction and one with four marginal directions. In the first one, which we could carry to high level, the cancellation was very accurate and became almost perfect once we used additional numerical data provided by [60, 61]. In the second example, carried to low level, the cancellation was less accurate but still convincing. Amusingly, one of the quartic couplings is equal to the area of $\mathcal{V}_{0,4}$ in the canonical presentation.

The cancellations were guaranteed to happen if closed string field theory reproduces a familiar on-shell fact: the S-matrix element coupling four marginal operators vanishes. Closed string field theory breaks this computation into two pieces, one from Feynman graphs and one from an elementary interaction, thus giving us a consistency test. Our test has verified that the moduli space $\mathcal{M}_{0,4}$ of four punctured spheres is correctly generated by the Feynman graphs and the region $\mathcal{V}_{0,4}$.

We found a simple relation between the quartic terms in the closed string potential for the marginal parameter a and those in the open string potential for the marginal parameter a_s : the contribution to a^4 from closed string fields of level 2ℓ is given by $c(2\ell) = -(\ell - 1)\chi_\ell^2$, where χ_ℓ is the contribution to a_s^4 from massive open string fields of level ℓ . Since $\chi_\ell \sim 1/\ell^2$, we have $c(\ell) \sim 1/\ell^3$. Convergence is faster in closed string field theory.

We have gleaned some information about level expansion in closed string field theory by comparing contributions obtained from the cubic and quartic vertices. The natural counter here is the level of the massive fields that are integrated using the cubic vertex and the propagator. Recalling that the quartic vertex contribution is $I_4 \simeq -0.2560$, the column for c in Table 3.1 shows that $|c(8)| < |I_4| < |c(4)|$, namely, the quartic contribution is smaller than that of level four fields and larger than that of level eight fields. For the case of the invariant V^2 in Table 3.2, the quartic contribution is only slightly smaller than that from level four fields. These results indicate that the quartic elementary vertex

should be included once the level of fields reaches or exceeds four. It remains to be seen if this result holds for other types of computations.

It has been suggested (see [22], for example) that quartic interactions may carry an intrinsic level. The level L_4 of a quartic coupling could be given by $L_4 = \alpha + \beta \sum_{i=1}^4 \ell_i$, where α and β are constants to be determined. There is scant evidence for any such relation, but we might assume $\beta = 1$ and attempt to estimate α as follows. We learned that $|I_4|$ was bounded by the contributions from level four and level eight massive fields. Since the cubic couplings involve one massive field and two marginal (level two) fields, I_4 is bounded by contributions from level 8 and level 12 interactions. It would be plausible to say that I_4 carries level 10, in which case $\alpha \sim 2$. The same logic applied to the computation of the invariant V^2 would suggest $\alpha \sim 0$. More work will be necessary to uncover a reliable formula for the level of the quartic interaction in closed string field theory.

There are some obvious questions we have not tried to answer. Is the range of a finite or infinite? The cubic tachyon contribution suggests the range is finite, but higher level and higher order interactions could change this result. There are also questions related to the zero-momentum dilaton, a physical, dimension-zero state that fails to satisfy the CFT definition of marginal state because it is not primary. The dilaton theorem, however, implies that the dilaton has a flat potential. This potential is hard to compute because the dilaton is not primary. This computation, which will appear in a separate paper [28], provides new stringent tests of the quartic string vertex, in particular, of the Strebel quadratic differential that determines local coordinates at the punctures. Since the dilaton state exists for general backgrounds its potential is part of the universal structure of string field theory. The dilaton potential is also an important ingredient in any complete computation of the potential for the bulk closed string tachyon.

Chapter 4

A Closed String Tachyon Vacuum ?

In the last few years the instabilities associated with open string tachyons have been studied extensively and have become reasonably well understood [81]. The instabilities associated with closed string tachyons have proven to be harder to understand. For the case of localized closed string tachyons – tachyons that live on subspaces of spacetime – there are now plausible conjectures for the associated instabilities and a fair amount of circumstantial evidence for them [21, 22, 23, 24, 25].

The bulk tachyon of the closed bosonic string is the oldest known closed string tachyon. It remains the most mysterious one and there is no convincing analysis of the associated instability. The analogy with open strings, however, suggests a fairly dramatic possibility. In open bosonic string in the background of a spacefilling D-brane, the tachyon potential has a critical point that represents spacetime without the D-brane and thus without physical open string excitations. In an analogous closed string tachyon vacuum one would expect no closed string excitations. Without gravity excitations spacetime ceases to be dynamical and it would seem that, for all intents and purposes, it has disappeared.

There has been no consensus that such a closed string tachyon vacuum exists. In fact, no analysis of the closed string tachyon potential (either in the CFT approach or in the SFT approach) has provided concrete evidence of a vacuum with non-dynamical spacetime. Since the analogous open string tachyon vacuum shows up quite clearly in the open string field theory computation of the potential it is natural to consider the corresponding calculation in closed string field theory (CSFT) [26, 58].

The quadratic and cubic terms in the closed string tachyon potential are well known [29, 30]:

$$\kappa^2 \mathbb{V}_0^{(3)} = -t^2 + \frac{6561}{4096} t^3, \quad (\alpha' = 2). \quad (4.1)$$

These terms define a critical point analogous to the one that turns out to represent the tachyon vacuum in the open string field theory. In open string field theory higher level computations make the vacuum about 46% deeper. Since CSFT is nonpolynomial, it is natural to investigate the effect of the quartic term in the potential. This term was found to be [59, 27]

$$\kappa^2 V_0^{(4)} = -3.0172 t^4. \quad (4.2)$$

This term is so large and negative that $\mathbb{V}_0^{(3)} + V_0^{(4)}$ has no critical point. In fact, the quartic term in

the *effective* tachyon potential (obtained by integrating out massive fields) is even a bit larger [59]. The hopes of identifying a reliable critical point in the closed string tachyon potential were dashed¹.

Recent developments inform our present analysis. The tachyon potential must include all fields that are sourced by the zero-momentum tachyon. As discussed in [64], this includes massless closed string states that are built from ghost oscillators, in particular, the zero-momentum ghost-dilaton state $(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})|0\rangle$. The search for a critical point cannot be carried out consistently without including the ghost dilaton. Computations of quartic vertices coupling dilatons, tachyons, and other massive fields are now possible due to the work of Moeller [27] and have been done to test the marginality of matter and dilaton operators [65, 28].

As we explain now, ghost-dilaton couplings to the tachyon restore the critical point in the potential. The key effect can be understood from the cubic and quartic couplings

$$\kappa^2 V(t, d) = -\frac{27}{32} t d^2 + 3.8721 t^3 d + \dots \quad (4.3)$$

The cubic coupling plays no role as long as we only consider cubic interactions: d can be set consistently to zero. The quartic coupling is linear in d . Once included, the equation of motion for the dilaton can only be satisfied if the dilaton acquires an expectation value. Solving for the dilaton one finds $d = 2.2944 t^2$ and substituting back,

$$\kappa^2 V(t, d) = 4.4422 t^5 + \dots \quad (4.4)$$

This positive quintic term suffices to compensate the effects of (4.2) and restores the critical point. Our computations include additional couplings and the effect of massive fields as well. The critical point persists and may be reliable, although more work is needed to establish this convincingly.

In order to interpret the critical point we raise and answer a pair of questions. The ghost-dilaton has a positive expectation value at the critical point. Does this correspond to stronger or weaker string coupling? We do a detailed comparison of quadratic and cubic terms in the closed string field theory action and in the low-energy effective field theory action. The conclusion is that the positive dilaton expectation value corresponds to *stronger* coupling. In our solution the ghost-dilaton is excited but the scalar operator $c\bar{c}\partial X \cdot \bar{\partial} X$, sometimes included in the dilaton vertex operator, is not. We ask: Is the string metric excited? Is the Einstein metric excited? These questions are only well-defined at the linearized level, but the answers are clear: the string metric does *not* change, but the Einstein metric does. We take the opportunity to explain the relations between the four kinds of “dilatons” that are used in the literature: the ghost-dilaton, the matter-dilaton, the dilaton, and the dilaton of the older literature. It is noted that one cannot define unambiguously a dilaton vertex operator unless one specifies which metric is left invariant; conversely, the metric vertex operator is only determined once one specifies which dilaton is left invariant.

¹In the effective open string tachyon potential a negative quartic term also destroys the cubic critical point. Nevertheless, the critical point can be gleaned using Padé-approximants [9]. For closed strings, however, the quartic term is too large: for a potential $v(t) = v_2 t^2 + v_3 t^3 + v_4 t^4$, with $v_2, v_4 < 0$, the approximant formed by the ratio of a cubic and a linear polynomial fails to give a critical point when $v_2 v_4 \geq v_3^2$.

In a companion paper [66] we attempted to gain insight into the tachyon vacuum by considering the rolling solutions² of a low-energy effective action for the string metric $g_{\mu\nu}$, the tachyon T , and the dilaton Φ :

$$S_\sigma = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} e^{-2\Phi} \left(R + 4(\partial_\mu \Phi)^2 - (\partial_\mu T)^2 - 2V(T) \right). \quad (4.5)$$

This action, suggested by the beta functions of sigma models with background fields [71], is expected to capture at least some of the features of string theory solutions. The potential is tachyonic: $V(T) = -\frac{1}{2}m^2 T^2 + \mathcal{O}(T^3)$, but is otherwise left undetermined. We found that solutions in which the tachyon begins the rolling process always have *constant* string metric for all times – consistent with the type of the SFT critical point. The dilaton, moreover, grows in time throughout the evolution – consistent with the larger dilaton vev in the SFT critical point. Rather generally, the solution becomes singular in finite time: the dilaton runs to infinity and the string coupling becomes infinite. Alternatively, the Einstein metric crunches up and familiar spacetime no longer exists. This seems roughly consistent with the idea that the tachyon vacuum does not have a fluctuating spacetime.

Perhaps the most subtle point concerns the value of the on-shell action. In the open string field theory computation of the tachyon potential, the value of the action (per unit spacetime volume) is energy density. The tachyon conjectures are in fact formulated in terms of energy densities at the perturbative and the non-perturbative vacuum [81]. Since the tree-level cosmological constant in closed string theory is zero, the value of the action at the perturbative closed string vacuum is zero. We ask: What is the value of the potential, or action (per unit volume) at the critical point? The low-energy action (5.1) suggests a surprising answer. Consider the associated equations of motion:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - (\partial_\mu T)(\partial_\nu T) &= 0, \\ \nabla^2 T - 2(\partial_\mu \Phi)(\partial^\mu T) - V'(T) &= 0, \\ \nabla^2 \Phi - 2(\partial_\mu \Phi)^2 - V(T) &= 0. \end{aligned} \quad (4.6)$$

If the fields acquire *constant* expectation values we can satisfy the tachyon equation if the expectation value T_* is a critical point of the potential: $V'(T_*) = 0$. The dilaton equation imposes an additional constraint: $V(T_*) = 0$, the potential must itself vanish. This is a reliable constraint that follows from a simple fact: in the action the dilaton appears without derivatives only as a multiplicative factor. This fact remains true after addition of α' corrections of all orders. It may be that $V(T)$ has a critical point T_0 with $V(T_0) < 0$, but this cannot be the tachyon vacuum. The effective field equations imply that a vacuum with spacetime independent expectation values has *zero* action.

The action (5.1) can be evaluated on-shell using the equations of motion. One finds

$$S_{on-shell} = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} e^{-2\Phi} (-4V(T)). \quad (4.7)$$

In rolling solutions the action density changes in time but, as $\Phi \rightarrow \infty$ at late times the action density goes to zero [66]. This also suggests that the tachyon vacuum is a critical point with zero action.

²Rolling solutions have long been considered using Liouville field theory to provide conformal invariant sigma model with spacetime background fields that typically include a linear dilaton and a constant string metric [67, 68, 89, 70].

In Figure 4.1 we present the likely features of the tachyon potential. The unstable perturbative vacuum $T = 0$ has zero cosmological constant, and so does the tachyon vacuum $T = \infty$. The infinite value of T is suggested by the analogous result in the effective open string theory tachyon potential (see conclusions). In SFT the tachyon vacuum appears for finite values of the fields, but the qualitative features would persist. The potential is qualitatively in the class used in cyclic universe models [72].

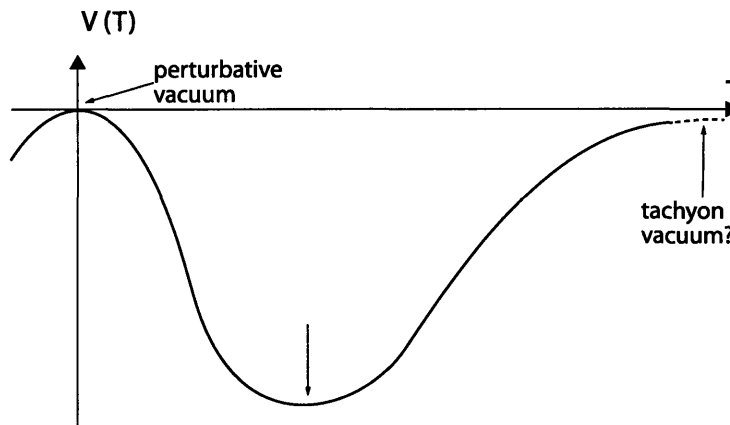


Figure 4.1: A sketch of a closed string tachyon potential consistent with present evidence. The perturbative vacuum is at $T = 0$. The closed string tachyon vacuum would be the critical point with zero cosmological term, shown here at $T \rightarrow \infty$ (in CSFT this point corresponds to finite tachyon vev). A critical point with negative cosmological constant cannot provide a spacetime independent tachyon vacuum.

In our calculations we find some evidence that the action density, which is negative, may go to zero as we increase the accuracy of the calculation. To begin with, the value Λ_0 of the action density at the critical point of the cubic tachyon potential (4.1) may be argued to be rather small. It is a cosmological term about seventy times smaller than the “canonical” one associated with $D = 2$ non-critical string theory (see [22], footnote 5). Alternatively, Λ_0 is only about 4% of the value that would be obtained using the on-shell coupling of three tachyons to calculate the cubic term. The inclusion of cubic interactions of massive fields makes the action density about 10% more negative. This shift, smaller than the corresponding one in open string field theory, is reversed once we include the dilaton quartic terms. In the most accurate computation we have done, the action density is down to 60% of Λ_0 . Additional computations are clearly in order.

As a by-product of our work, we investigate large dilaton deformations in CSFT. For ordinary marginal deformations the description reaches an obstruction for some finite critical value of the string field marginal parameter [52, 57]. The critical value is stable under level expansion, and the potential for the marginal field (which should vanish for infinite level) is small. For the dilaton, however, the lowest-order obstruction is not present [28]. We carry this analysis to higher order and no reliable obstructions are found: critical values of the dilaton jump wildly with level and appear where the dilaton potential is large and cannot be trusted. This result strengthens the evidence that CSFT

can describe backgrounds with arbitrarily large variations in the string coupling. If the infinite string coupling limit is also contained in the configuration space it may be possible to define M-theory using type IIA superstring field theory.

Let us briefly describe the contents of this chapter. In section 2 we reconsider the universality arguments [64] that require the inclusion of the ghost-dilaton, exhibit a world-sheet parity symmetry that allows a sizable truncation of the universal space, and note that universality may apply in circumstances significantly more general than originally envisioned [73]. Our computational strategy for the tachyon potential, motivated by the results of [65, 28], goes as follows. We compute *all* quadratic and cubic terms in the potential including fields up to level four. We then begin the inclusion of quartic terms and obtain complete results up to quartic interactions of total level four. The results make it plausible that a critical point exists and that the value of the action density decreases in magnitude as the accuracy improves. In section 3 we find the linearized relations between the metric, dilaton, and tachyon closed string fields and the corresponding fields in the sigma-model approach to string theory. These relations allow us to establish that the dilaton vev at the critical point represents an increased string coupling and that the string field at the critical point does not have a component along the vertex operator for the string metric. We discuss the vertex operators associated with the various definitions of the dilaton, determine the nonlinear field relations between the string field theory and effective field theory dilatons and tachyons to quadratic order and at zero-momentum, and examine large dilaton deformations. In the concluding section we discuss additional considerations that suggest the existence of the tachyon vacuum. These come from non-critical string theory, p-adic strings, and sigma model arguments. Finally, the details of the nontrivial computations of quartic couplings are given in the Appendix.

4.1 Computation of the tachyon potential

In this section we present the main computations of this chapter. We begin by introducing the string field relevant for the calculation of the tachyon potential, giving a detailed discussion of universality. This string field contains the tachyon, at level zero, the ghost-dilaton, at level two, and massive fields at higher even levels. We then give the quadratic and cubic couplings for the string field restricted to level four and calculate the critical point. Finally, we give the quartic couplings at level zero, two, and four. The critical point survives the inclusion of quartic interactions and becomes more shallow – consistent with the conjecture that the tachyon vacuum has zero action.

The computations use the closed string field action [26, 58, 22], which takes the form

$$S = -\frac{2}{\alpha'} \left(\frac{1}{2} \langle \Psi | c_0^- Q | \Psi \rangle + \frac{\kappa}{3!} \{ \Psi, \Psi, \Psi \} + \frac{\kappa^2}{4!} \{ \Psi, \Psi, \Psi, \Psi \} + \dots \right). \quad (4.8)$$

The string field Ψ lives on \mathcal{H} , the ghost number two state space of the full CFT restricted to the subspace of states that satisfy

$$(L_0 - \bar{L}_0) | \Psi \rangle = 0 \quad \text{and} \quad (b_0 - \bar{b}_0) | \Psi \rangle = 0. \quad (4.9)$$

The BRST operator is $Q = c_0 L_0 + \bar{c}_0 \bar{L}_0 + \dots$, where the dots denote terms independent of c_0 and of \bar{c}_0 . Moreover, $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$, and we normalize correlators using $\langle 0 | c_{-1} \bar{c}_{-1} c_0^- c_0^+ c_1 \bar{c}_1 | 0 \rangle = 1$. All spacetime coordinates are imagined compactified with the volume of spacetime set equal to one.

4.1.1 Tachyon potential universality and the ghost-dilaton

The universality of the closed string tachyon potential was briefly discussed in [64], where it was also noted that the ghost number two universal string field that contains the tachyon should include the zero-momentum ghost-dilaton state $(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})|0\rangle$. In here we review the universality argument and extend it slightly, offering the following observations:

- The ghost-dilaton must be included because closed string field theory is not cubic.
- A world-sheet parity symmetry of closed string field theory can be used to restrict the universal subspace.
- The arguments of [64] do not apply directly to general CFT's, linear dilaton backgrounds, for example. If the closed string background is defined by a general matter CFT, solutions on the universal subspace may still be solutions, but there is no tachyon potential [73].

The original idea in universality is to produce a subdivision of all the component fields of the string field theory into two disjoint sets, a set $\{t_i\}$ that contains the zero-momentum tachyon and a set $\{u_a\}$ such that the string field action $S(t_i, u_a)$ contains no term with a single u -type field. It is then consistent to search for a solution of the equations of motion that assumes $u_a = 0$ for all a .

To produce the desired set $\{t_i\}$ we assume that the matter CFT is such that X^0 is the usual negative-metric field with associated conserved momentum k_0 and the rest of the matter CFT is unitary. The state space \mathcal{H} (see (4.9)) is then divided into three disjoint vector subspaces $\mathcal{H}_1, \mathcal{H}_2$, and \mathcal{H}_3 . One has $\mathcal{H}_i = \mathcal{M}_i \otimes |\mathcal{G}\rangle$, where $|\mathcal{G}\rangle$ denotes a state built with ghost and antighost oscillators only and $\mathcal{M}_1, \mathcal{M}_2$, and \mathcal{M}_3 are disjoint subspaces of the matter CFT whose union gives the total matter CFT state space:

$$\begin{aligned}
\mathcal{M}_1 : & \quad \text{the } SL(2, C) \text{ vacuum } |0\rangle \text{ and descendents,} \\
\mathcal{M}_2 : & \quad \text{states with } k_0 \neq 0, \\
\mathcal{M}_3 : & \quad \text{primaries with } k_0 = 0 \text{ but different from } |0\rangle \text{ and descendents.}
\end{aligned} \tag{4.10}$$

In the above, primary and descendent refers to the matter Virasoro operators. Note that the primaries in \mathcal{M}_3 have positive conformal dimension. The BRST operator preserves the conditions (4.9), and since it is composed of ghost oscillators and matter Virasoro operators, it maps each \mathcal{H}_i into itself. Finally, the spaces \mathcal{H}_i are orthogonal under the BPZ inner product; they only couple to themselves.

The claim is that the set $\{t_i\}$ is in fact \mathcal{H}_1 , the states built upon the zero momentum vacuum. The ‘‘tachyon potential’’ is the string action evaluated for \mathcal{H}_1 .

We first note that because of momentum conservation fields in \mathcal{H}_2 cannot couple linearly to fields in \mathcal{H}_1 . The fields in \mathcal{H}_3 cannot couple linearly to the fields in \mathcal{H}_1 either. They cannot do so through the kinetic term because the BRST operator preserves the space and \mathcal{H}_1 and \mathcal{H}_3 are BPZ orthogonal. We also note that the matter correlator in the n -string vertex does not couple $n - 1$ vacua $|0\rangle$ from \mathcal{H}_1 to a matter primary from \mathcal{H}_3 : this is just the one-point function of the primary in \mathcal{H}_3 , which vanishes because the state has non-zero dimension. The (matter) Virasoro conservation laws on the vertex then imply that the coupling of any $(n - 1)$ states in \mathcal{H}_1 to a state in \mathcal{H}_3 must vanish. This completes the proof that \mathcal{H}_1 is the subspace for tachyon condensation.

The space \mathcal{H}_1 can be written as

$$\text{Span}\left\{L_{-j_1}^m \dots L_{-j_p}^m \bar{L}_{-j_1}^m \dots \bar{L}_{-j_p}^m b_{-k_1} \dots b_{-k_q} \bar{b}_{-\bar{k}_1} \dots \bar{b}_{-\bar{k}_q} c_{-l_1} \dots c_{-l_r} \bar{c}_{-\bar{l}_1} \dots \bar{c}_{-\bar{l}_r} |0\rangle\right\}, \quad (4.11)$$

where

$$j_1 \geq j_2 \geq \dots \geq j_p, \quad j_i \geq 2, \quad \bar{j}_1 \geq \bar{j}_2 \geq \dots \geq \bar{j}_p, \quad \bar{j}_i \geq 2, \quad (4.12)$$

as well as

$$k_i, \bar{k}_i \geq 2, \quad l_i, \bar{l}_i \geq -1, \quad \text{and} \quad r + \bar{r} - q - \bar{q} = 2. \quad (4.13)$$

Finally, the states above must also be annihilated by $L_0 - \bar{L}_0$ as well as $b_0 - \bar{b}_0$.

There is a reality condition on the string field [26]: its BPZ and hermitian conjugates must differ by a sign. We show now that this condition is satisfied by all the states in (4.11), so the coefficients by which they are multiplied in the universal string field (the zero-momentum spacetime fields) must be real. Suppose a state is built with p ghost oscillators and $p - 2$ antighost oscillators. The BPZ and hermitian conjugates differ by the product of two factors: a $(-1)^p$ from the BPZ conjugation of the ghost oscillators and a $(-1)^{(2p-2)(2p-1)/2} = (-1)^{p-1}$ from the reordering of oscillators in the hermitian conjugate. The product of these two factors is minus one, as we wanted to show.

In open string theory twist symmetry, which arises from world-sheet parity, can be used to further restrict the universal subspace constructed from matter Virasoro and ghost oscillators. In the case of closed string theory the world-sheet parity transformation that exchanges holomorphic and antiholomorphic sectors is the relevant symmetry.³ World-sheet parity is not necessarily a symmetry of arbitrary matter CFT's, but it is a symmetry in the universal subspace: correlators are complex conjugated when we exchange holomorphic and antiholomorphic Virasoro operators as $T(z) \leftrightarrow \bar{T}(\bar{z})$. More precisely, we introduce a \star -conjugation, a map of \mathcal{H}_1 to \mathcal{H}_1 that is an involution. In a basis of Virasoro modes \star can be written explicitly as the map of states

$$\star : A L_{-i_1} \dots L_{-i_n} \bar{L}_{-j_1} \dots \bar{L}_{-j_n} |0\rangle \rightarrow A^* \bar{L}_{-i_1} \dots \bar{L}_{-i_n} L_{-j_1} \dots L_{-j_n} |0\rangle, \quad (4.14)$$

where A is a constant and A^* denotes its complex conjugate. Given the operator/state correspondence, the above defines completely the star operation $\star : \mathcal{O} \rightarrow \mathcal{O}^*$ on vertex operators for vacuum

³We thank A. Sen for discussions that led us to construct the arguments presented below.

descendants. It results in the following property for the correlator of n such operators placed at n points on a Riemann surface:

$$\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \langle \mathcal{O}_1^* \dots \mathcal{O}_n^* \rangle^*. \quad (4.15)$$

In the ghost sector of the CFT a small complication with signs arises because the basic correlator is odd under the exchange of holomorphic and anti-holomorphic sectors:

$$\langle c(z_1)c(z_2)c(z_3) \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3) \rangle = -\langle \bar{c}(\bar{z}_1)\bar{c}(\bar{z}_2)\bar{c}(\bar{z}_3) c(w_1)c(w_2)c(w_3) \rangle^*. \quad (4.16)$$

Since two-point functions of the ghost fields are complex conjugated by the exchanges $c(z) \leftrightarrow \bar{c}(\bar{z})$ and $b(z) \leftrightarrow \bar{b}(\bar{z})$, it follows from (4.16) that performing these exchanges on an *arbitrary* correlator of ghost and antighost fields will give minus the complex conjugate of the original correlator. We will define \star -conjugation in the ghost sector by:

$$\star : A c_{i_1} \dots c_{i_n} b_{j_1} \dots b_{j_m} \bar{c}_{k_1} \dots \bar{c}_{k_r} \bar{b}_{l_1} \dots \bar{b}_{l_s} |0\rangle \rightarrow A^* \bar{c}_{i_1} \dots \bar{c}_{i_n} \bar{b}_{j_1} \dots \bar{b}_{j_m} c_{k_1} \dots c_{k_r} b_{l_1} \dots b_{l_s} |0\rangle. \quad (4.17)$$

For a general state Ψ of the universal subspace we define Ψ^* to be the state obtained by the simultaneous application of (4.14) and (4.17). It is clear from the above discussion that the correlators satisfy

$$\langle \Psi_1 \Psi_2 \dots \Psi_n \rangle = -\langle \Psi_1^* \Psi_2^* \dots \Psi_n^* \rangle^*, \quad \Psi_i \in \mathcal{H}_1. \quad (4.18)$$

We now define the action of the world-sheet parity operation \mathcal{P} on arbitrary states of the universal subspace:

$$\mathcal{P}\Psi \equiv -\Psi^*, \quad \Psi \in \mathcal{H}_1. \quad (4.19)$$

We claim that the string field theory action, restricted to \mathcal{H}_1 , is \mathcal{P} invariant:

$$S(\Psi) = S(\mathcal{P}\Psi), \quad \text{for } \Psi \in \mathcal{H}_1. \quad (4.20)$$

First consider the invariance of the cubic term. Using (4.19) and (4.18) we have

$$\langle \mathcal{P}\Psi, \mathcal{P}\Psi, \mathcal{P}\Psi \rangle = -\langle \Psi^*, \Psi^*, \Psi^* \rangle = \langle \Psi, \Psi, \Psi \rangle^* = \langle \Psi, \Psi, \Psi \rangle, \quad (4.21)$$

where in the last step we used the reality of the string field action. The kinetic term of the action is also invariant. First note that $(c_0^- Q\Psi)^* = -c_0^- Q\Psi^*$. It then follows that

$$\langle \mathcal{P}\Psi, c_0^- Q\mathcal{P}\Psi \rangle = \langle \Psi^*, c_0^- Q\Psi^* \rangle = -\langle \Psi^*, (c_0^- Q\Psi)^* \rangle = \langle \Psi, c_0^- Q\Psi \rangle^* = \langle \Psi, c_0^- Q\Psi \rangle. \quad (4.22)$$

For higher point interactions, the invariance follows because the antighost insertions have the appropriate structure. Each time we add a new string field we must add two antighost insertions. For the case of quartic interactions they take the form of two factors $\mathcal{B}\mathcal{B}^*$ (see eqn. (A.3)). Since $(\mathcal{B}\mathcal{B}^*)^* = -\mathcal{B}\mathcal{B}^*$, the extra minus sign cancels against the minus sign from the extra string field. This can be seen to generalize to higher order interactions using the forms of the off-shell amplitudes discussed in section 6 of [30]. This completes our proof of (4.20).

Since $\mathcal{P}^2 = 1$ the space \mathcal{H}_1 can be divided into two disjoint subspaces: the space \mathcal{H}_1^+ of states with $\mathcal{P} = 1$ and the space \mathcal{H}_1^- of states with $\mathcal{P} = -1$:

$$\begin{aligned}\mathcal{P}(\Psi_+) &= +\Psi_+, & \Psi_+ &\in \mathcal{H}_1^+, \\ \mathcal{P}(\Psi_-) &= -\Psi_-, & \Psi_- &\in \mathcal{H}_1^-.\end{aligned}\tag{4.23}$$

It follows from the invariance of the action that no term in the action can contain just one state in \mathcal{H}_1^- . We can therefore restrict ourselves to the subspace \mathcal{H}_1^+ with positive parity.

The string field is further restricted by using a gauge fixing condition. The computation of the potential is done in the Siegel gauge, which requires states to be annihilated by $b_0 + \bar{b}_0$. To restrict ourselves to the Siegel gauge we take the states in (4.11) that have neither a c_0 nor a \bar{c}_0 .

The Siegel gauge fixes the gauge symmetry completely for the massive levels, but does not quite do the job at the massless level. There are two states with $L_0 = \bar{L}_0 = 0$ in \mathcal{H}_1 that are in the Siegel gauge:

$$(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})|0\rangle \quad \text{and} \quad (c_1 c_{-1} + \bar{c}_1 \bar{c}_{-1})|0\rangle.\tag{4.24}$$

The first state is the ghost dilaton and it is proportional to $Q(c_0 - \bar{c}_0)|0\rangle$. Since $(c_0 - \bar{c}_0)|0\rangle$ is not annihilated by $b_0 - \bar{b}_0$ the gauge parameter is illegal and the ghost dilaton is not trivial. The second state is proportional to $Q(c_0 + \bar{c}_0)|0\rangle$, so it is thus trivial at the linearized level. Although trivial at the linearized level, one may wonder if the triviality holds for large fields. Happily, we need not worry: the state is \mathcal{P} odd, so it need not be included in the calculation. The ghost-dilaton, because of the relative minus sign between the two terms, is \mathcal{P} even and it is included.

Had the closed string field theory been cubic we could have discarded the ghost-dilaton state and all other states with asymmetric left and right ghost numbers. We could restrict \mathcal{H}_1^+ to fields of ghost number $(G, \bar{G}) = (1, 1)$. Indeed, the cubic vertex cannot couple two $(1, 1)$ fields to anything except another $(1, 1)$ field. Moreover, in the Siegel gauge $c_0^- Q$ acts as an operator of ghost number $(1, 1)$, so again, no field with asymmetric ghost numbers can couple linearly. The quartic and higher order interactions in CSFT have antighost insertions that do not have equal left and right ghost numbers. It follows that these higher order vertices can couple the ghost-dilaton to $(1, 1)$ fields. Indeed, the coupling of a dilaton to three tachyons does not vanish. We *cannot* remove from \mathcal{H}_1^+ the dilaton, nor other states with asymmetric left and right ghost numbers.

The construction of the universal string field and action presented here does not work fully if the matter CFT contains a linear dilaton background. Momentum conservation along the corresponding coordinate is anomalous and one cannot build an action with states of zero momentum only: the action restricted to \mathcal{H}_1 is identically zero. There would be no universal ‘‘potential’’ in \mathcal{H}_1 . It appears rather likely, however, that any *solution* in the universal subspace would still be a solution in a linear dilaton background. In fact, any solution in the universal subspace may be a solution for string field theory formulated with a general matter CFT [73].

We conclude this section by writing out the string field for the first few levels. The level ℓ of a state is defined by $\ell = L_0 + \bar{L}_0 + 2$. The level zero part of the string field is

$$|\Psi_0\rangle = t c_1 \bar{c}_1 |0\rangle. \quad (4.25)$$

Here t is the zero-momentum tachyon. The level two part of the string field is

$$|\Psi_2\rangle = d(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})|0\rangle. \quad (4.26)$$

Here d is the zero momentum ghost-dilaton. It multiplies the only state of $\mathcal{P} = +1$ at this level. At level four there are four component fields:

$$\begin{aligned} |\Psi_4\rangle = & \left(f_1 c_{-1} \bar{c}_{-1} + f_2 L_{-2} c_1 \bar{L}_{-2} \bar{c}_1 + f_3 (L_{-2} c_1 \bar{c}_{-1} + c_{-1} \bar{L}_{-2} \bar{c}_1) \right. \\ & \left. + g_1 (b_{-2} c_1 \bar{c}_{-2} \bar{c}_1 - c_{-2} c_1 \bar{b}_{-2} \bar{c}_1) \right) |0\rangle. \end{aligned} \quad (4.27)$$

Note that the states coupling to the component fields all have $\mathcal{P} = +1$ and that g_1 couples to a state with asymmetric left and right ghost numbers. In this chapter we will not use higher level terms in the string field.

With $\alpha' = 2$ the closed string field potential V associated with the action in (4.8) is

$$\kappa^2 V = \frac{1}{2} \langle \Psi | c_0^- Q | \Psi \rangle + \frac{1}{3!} \{ \Psi, \Psi, \Psi \} + \frac{1}{4!} \{ \Psi, \Psi, \Psi, \Psi \} + \dots. \quad (4.28)$$

Here $|\Psi\rangle = |\Psi_0\rangle + |\Psi_2\rangle + |\Psi_4\rangle + \dots$. Our computations will not include quintic and higher order interactions in the string action.

4.1.2 The quadratic and cubic terms in the potential

Let us now consider the potential including only the kinetic and cubic terms in (4.28). To level zero:

$$\kappa^2 V_0^{(2)} = -t^2, \quad \kappa^2 V_0^{(3)} = \frac{6561}{4096} t^3. \quad (4.29)$$

All potentials introduced in this subsection have a superscript that gives the order of the interaction (two for quadratic, three for cubic, and so on), and a subscript that gives the level (defined by the sum of levels of fields in the interaction). The next terms arise at level four, where we have couplings of the tachyon to the square of the dilaton and couplings of the level four fields to the tachyon squared:

$$\kappa^2 V_4^{(3)} = -\frac{27}{32} d^2 t + \left(\frac{3267}{4096} f_1 + \frac{114075}{4096} f_2 - \frac{19305}{2048} f_3 \right) t^2. \quad (4.30)$$

At level six we can couple a level four field, a dilaton, and a tachyon. Only level four fields with $G \neq \bar{G}$ can have such coupling, so we find:

$$\kappa^2 V_6^{(3)} = -\frac{25}{8} g_1 t d. \quad (4.31)$$

At level eight there are two kinds of terms. First, we have the kinetic terms for the level four fields:

$$\kappa^2 V_8^{(2)} = f_1^2 + 169 f_2^2 - 26 f_3^2 - 2 g_1^2. \quad (4.32)$$

Second, we have the cubic interactions:

$$\begin{aligned}
\kappa^2 V_8^{(3)} = & -\frac{1}{96} f_1 d^2 - \frac{4225}{864} f_2 d^2 + \frac{65}{144} f_3 d^2 \\
& + \frac{361}{12288} f_1^2 t + \frac{511225}{55296} f_1 f_2 t + \frac{57047809}{110592} f_2^2 t + \frac{470873}{27648} f_3^2 t - \frac{49}{24} g_1^2 t \\
& - \frac{13585}{9216} f_1 f_3 t - \frac{5400395}{27648} f_2 f_3 t.
\end{aligned} \tag{4.33}$$

As we can see, these are of two types: couplings of a level four field to two dilatons (first line) and couplings of two level four fields to a tachyon (second and third lines).

The terms at level 10 couple two level four fields and a dilaton. Because of ghost number conservation, one of the level four fields must have $G \neq \bar{G}$:

$$\kappa^2 V_{10}^{(3)} = -\frac{25}{5832} (361 f_1 + 4225 f_2 - 2470 f_3) d g_1. \tag{4.34}$$

Finally, at level 12 we have the cubic couplings of three level-four fields:

$$\begin{aligned}
\kappa^2 V_{12}^{(3)} = & \frac{1}{4096} f_1^3 + \frac{1525225}{8957952} f_1^2 f_2 - \frac{1235}{55296} f_1^2 f_3 + \frac{6902784889}{80621568} f_1 f_2^2 \\
& - \frac{102607505}{6718464} f_1 f_2 f_3 + \frac{1884233}{2239488} f_1 f_3^2 \\
& + \frac{74181603769}{26873856} f_2^3 - \frac{22628735129}{13436928} f_2^2 f_3 + \frac{4965049817}{20155392} f_2 f_3^2 \\
& - \frac{31167227}{3359232} f_3^3 - \frac{961}{157464} f_1 g_1^2 - \frac{207025}{17496} f_2 g_1^2 + \frac{14105}{26244} f_3 g_1^2.
\end{aligned} \tag{4.35}$$

4.1.3 Tachyon vacuum with cubic vertices only

With cubic vertices only the dilaton expectation value is zero. In fact, only fields with $G = \bar{G} = 1$ can acquire nonvanishing expectation values. To examine the tachyon vacuum we define a series of potentials:

$$\begin{aligned}
\mathbb{V}_0^{(3)} & \equiv V_0^{(2)} + V_0^{(3)}, \\
\mathbb{V}_8^{(3)} & \equiv \mathbb{V}_0^{(3)} + V_4^{(3)} + V_6^{(3)} + V_8^{(2)} + V_8^{(3)}, \\
\mathbb{V}_{12}^{(3)} & \equiv \mathbb{V}_8^{(3)} + V_{10}^{(3)} + V_{12}^{(3)}.
\end{aligned} \tag{4.36}$$

A few observations are in order. In all of the above potentials we can set $d = g_1 = 0$. As a consequence, $V_6^{(3)}$ and $V_{10}^{(3)}$ do not contribute. Since the level-two dilaton plays no role, once we go beyond the tachyon we must include level four fields. The kinetic terms for these fields are of level eight, so $\mathbb{V}_8^{(3)}$ is the simplest potential beyond level zero. With level-four fields the next potential is $\mathbb{V}_{12}^{(3)}$.

The critical points obtained with the potentials $\mathbb{V}_0^{(3)}$, $\mathbb{V}_8^{(3)}$, and $\mathbb{V}_{12}^{(3)}$ are given in Table 4.1. We call the value of the potential $\kappa^2 \mathbb{V}$ at the critical point the action density. The values of the action density follow the pattern of open string theory. The original cubic critical point becomes deeper. It does so by about 10%, a value significantly smaller than the corresponding one in open string field theory.

Potential	t	f_1	f_2	f_3	Action density
$V_0^{(3)}$	0.41620	--	--	--	-0.05774
$V_8^{(3)}$	0.43678	-0.06502	-0.00923	-0.02611	-0.06329
$V_{12}^{(3)}$	0.43709	-0.06709	-0.00950	-0.02693	-0.06338

Table 4.1: Vacuum solution with cubic vertices only

4.1.4 Tachyon vacuum with cubic and quartic vertices

We can now examine the quartic terms in the potential. The associated potentials are denoted with a superscript (4) for quartic and a subscript that gives the sum of levels of the fields that enter the term. The quartic self-coupling of tachyons has been calculated in [59, 27]:

$$\kappa^2 V_0^{(4)} = -3.0172 t^4. \quad (4.37)$$

With total level two we have a coupling of three tachyons and one dilaton. This is calculated in Appendix A.2 and the result is

$$\kappa^2 V_2^{(4)} = 3.8721 t^3 d. \quad (4.38)$$

With total level four there is the coupling of two tachyons to two dilatons (Appendix A.2) and the coupling of three tachyons to any of the level-four fields (Appendix A.3):

$$\kappa^2 V_4^{(4)} = 1.3682 t^2 d^2 + t^3 (-0.4377 f_1 - 56.262 f_2 + 13.024 f_3 + 0.2725 g_1). \quad (4.39)$$

With total level six there are three types of interactions: a tachyon coupled to three dilatons, two tachyons coupled to a dilaton and a level-four field, and three tachyons coupled to a level-six field. We have only computed the first one (Appendix A.2):

$$\kappa^2 V_6^{(4)} = -0.9528 t d^3 + \dots \quad (4.40)$$

The terms that have not been computed are indicated by the dots. Finally, the quartic self-coupling of dilatons was computed in [28], where it played a central role in the demonstration that the effective dilaton potential has no quartic term:

$$\kappa^2 V_8^{(4)} = -0.1056 d^4 + \dots \quad (4.41)$$

We use the dots to indicate the additional level eight interactions that should be computed.

Let us now consider the potentials that can be assembled using the above contributions. We use the following strategy: we include cubic vertices to the highest possible level and then begin to introduce the quartic couplings level by level. The most accurate potential with quadratic and cubic terms that

we have is $\mathbb{V}_{12}^{(3)}$ and the tachyon vacuum it contains appears in the last line of Table 4.1. The lowest order quartic potential that we use is therefore:

$$\mathbb{V}_0^{(4)} \equiv \mathbb{V}_{12}^{(3)} + V_0^{(4)}. \quad (4.42)$$

This potential has a familiar difficulty: the quartic self-coupling of the tachyon is so strong that the critical point in the potential disappears. As we have argued, once additional terms are included the critical point in the potential reappears. The higher level potentials are defined by including progressively higher level quartic interactions:

$$\begin{aligned} \mathbb{V}_2^{(4)} &\equiv \mathbb{V}_0^{(4)} + V_2^{(4)}, \\ \mathbb{V}_4^{(4)} &\equiv \mathbb{V}_2^{(4)} + V_4^{(4)}. \end{aligned} \quad (4.43)$$

Since our computations of $V_6^{(4)}$ and $V_8^{(4)}$ are incomplete, the results that follow from $\mathbb{V}_6^{(4)} \equiv \mathbb{V}_4^{(4)} + V_6^{(4)}$ and $\mathbb{V}_8^{(4)} \equiv \mathbb{V}_6^{(4)} + V_8^{(4)}$ cannot be trusted.

We are now in a position to calculate the critical points of the potentials $\mathbb{V}^{(4)}$. In our numerical work we input the cubic coefficients as fractions and the quartic coefficients as the exact decimals given above (so the t^4 coefficient is treated as exactly equal to 3.0172.) Our results are given in Table 4.2. For ease of comparison, we have included the cubic results for $\mathbb{V}_{12}^{(3)}$ as the first line. Furthermore, we include a line for $\mathbb{V}_0^{(4)}$ even though there is no critical point. The next potential is $\mathbb{V}_2^{(4)}$ which contains only the additional coupling t^3d . The significant result is that the critical point reappears and can be considered to be a (moderate) deformation of the critical point obtained with $\mathbb{V}_{12}^{(3)}$. Indeed, while there is a new expectation value for the dilaton (and for g_1), the expectation value of the tachyon does not change dramatically, nor do the expectation values for f_1 , f_2 , and f_3 . The critical point becomes somewhat shallower, despite the destabilizing effects of the tachyon quartic self-couplings.

Potential	t	d	f_1	f_2	f_3	g_1	Action density
$\mathbb{V}_{12}^{(3)}$	0.43709	0	-0.06709	-0.00950	-0.02693	---	-0.06338
$\mathbb{V}_0^{(4)}$	---	---	---	---	---	---	---
$\mathbb{V}_2^{(4)}$	0.33783	0.49243	-0.08007	-0.00619	-0.02607	-0.10258	-0.05806
$\mathbb{V}_4^{(4)}$	0.24225	0.45960	-0.04528	-0.00140	-0.01233	-0.07249	-0.03382

Table 4.2: Vacuum solution with cubic and quartic vertices. We see that the magnitude of the action density becomes smaller as we begin to include the effects of quartic couplings.

At the next level, where t^2d^2 and t^3M_4 (M_4 denotes a level-four field) terms appear, the critical point experiences some significant change. First of all, it becomes about 40% more shallow; the change is large and probably significant, given the expectation that the action density should eventually reach zero. The tachyon expectation changes considerably but the dilaton expectation value changes little. Due to the t^3M_4 terms the expectation values of some of the level four fields change dramatically.

Glancing at Table 4.2, one notices that the tachyon expectation value is becoming smaller so one might worry that the critical point is approaching the perturbative vacuum. This is, of course, a possibility. If realized, it would imply that the critical point we have encountered is an artifact of level expansion. We think this is unlikely. Since the dilaton seems to be relatively stable, a trivial critical point would have to be a dilaton deformation of the perturbative vacuum, but such deformations have negative tachyon expectation values (see Figure 4.2).

At this moment we do not have full results for higher levels. The computation of $\mathbb{V}_6^{(4)}$ would require the evaluation of couplings of the form $t^2 dM_4$ and, in principle, couplings $t^3 M_6$ of level-six fields, which we have not even introduced in this chapter. The only additional couplings we know at present are td^3 , which enters in $\mathbb{V}_6^{(4)}$ and d^4 , which enters in $\mathbb{V}_8^{(4)}$ (see eqns. (4.40) and (4.41)). Despite lacking terms, we calculated the resulting vacua to test that no wild effects take place. The incomplete $\mathbb{V}_6^{(4)}$ leads to $t = 0.35426$, $d = 0.40763$ and an action density of -0.05553 . The incomplete $\mathbb{V}_8^{(4)}$ leads to $t = 0.36853$, $d = 0.40222$ and an action density of -0.05836 . In these results the action density has become more negative. Given the conjectured value of the action, it would be encouraging if the full results at those levels show an action density whose magnitude does not become larger.

One may also wonder what happens if terms of order higher than quartic are included in the potential. Since the tachyon terms in the CSFT potential alternate signs [30], the quintic term is positive and will help reduce the value of the action at the critical point. The coefficient of this coupling will be eventually needed as computations become more accurate. The sextic term will have a destabilizing effect. Having survived the destabilizing effects of the quartic term, we can hope that those of the sextic term will prove harmless. If, in general, even power terms do not have catastrophic effects, it may be better to work always with truncations of odd power.

4.2 The sigma model and the string field theory pictures

In this section we study the relations between the string field metric $h_{\mu\nu}$ and the ghost-dilaton d and the corresponding sigma model fields, the string metric $\tilde{h}_{\mu\nu}$ and dilaton Φ . These relations are needed to interpret the tachyon vacuum solution and to discuss the possible relation to the rolling solutions.

We begin by finding the precise linearized relations between the string field dilaton and the sigma model dilaton. The linearized relations confirm that the CSFT metric $h_{\mu\nu}$, which does not acquire an expectation value in the tachyon vacuum, coincides with the string metric of the sigma model, which does not change in the rolling solutions. Moreover, the relation (4.57), together with $h_{\mu\nu} = 0$, implies that our $d > 0$ in the tachyon vacuum corresponds to $\Phi > 0$, thus larger string coupling. This is also consistent with what we obtained in the rolling solutions.

Our discussion of the linearized relations also allows us to examine the various vertex operators associated with the various dilaton fields used in the literature (section 3.2.). In section 3.3 we examine the nonlinear relations between the CSFT tachyon and dilaton and the effective field theory ones. We work at zero momentum and up to quadratic order. Finally, in section 3.4, we present evidence that CSFT can describe arbitrarily large dilaton deformations.

4.2.1 Relating sigma model fields and string fields

Consider first the effective action (5.1), suggested by the conditions of conformal invariance of a sigma model with gravity, dilaton and tachyon background fields. If we set the tachyon to zero, this action reduces to the effective action for massless fields, in the conventions of [78]. In this action $g_{\mu\nu}$ is the string metric, Φ is the diffeomorphism invariant dilaton, and T , with potential $V(T) = -\frac{2}{\alpha'}T^2 + \dots$, is the tachyon. In order to compare with the string field action we expand the effective action in powers of small fluctuations using

$$g_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}, \quad (4.44)$$

where we use a tilde in the fluctuation to distinguish it from the metric fluctuation in the string field. The result is

$$\begin{aligned} S_\sigma = \frac{1}{2\kappa^2} \int d^D x \left(\frac{1}{4} \tilde{h}_{\mu\nu} \partial^2 \tilde{h}^{\mu\nu} - \frac{1}{4} \tilde{h} \partial^2 \tilde{h} + \frac{1}{2} (\partial^\nu \tilde{h}_{\mu\nu})^2 + \frac{1}{2} \tilde{h} \partial_\mu \partial_\nu \tilde{h}^{\mu\nu} \right. \\ \left. + 2\tilde{h} \partial^2 \Phi - 2\Phi \partial_\mu \partial_\nu \tilde{h}^{\mu\nu} - 4\Phi \partial^2 \Phi \right. \\ \left. - (\partial T)^2 + \frac{4}{\alpha'} T^2 + \tilde{h}^{\mu\nu} \partial_\mu T \partial_\nu T + \left(\frac{\tilde{h}}{2} - 2\Phi \right) (\partial T)^2 + \dots \right), \end{aligned} \quad (4.45)$$

where we have kept cubic terms coupling the dilaton and metric to the tachyon. Such terms are needed to fix signs in the relations between the fields in the sigma model and the string fields.

Let us now consider the string field action. The string field needed to describe the tachyon, the metric fluctuations, and the dilaton is

$$\begin{aligned} |\Psi\rangle = \int \frac{d^D k}{(2\pi)^D} \left(t(k) c_1 \bar{c}_1 - \frac{1}{2} h_{\mu\nu}(k) \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu c_1 \bar{c}_1 + d(k) (c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1}) \right. \\ \left. + i \sqrt{\frac{\alpha'}{2}} B_\mu(k) c_0^+ (c_1 \alpha_{-1}^\mu - \bar{c}_1 \bar{\alpha}_{-1}^\mu) \right) |k\rangle. \end{aligned} \quad (4.46)$$

Here $t(k)$ is the tachyon, $h_{\mu\nu}(k) = h_{\nu\mu}(k)$ is a metric fluctuation, $d(k)$ is the ghost-dilaton, and $B_\mu(k)$ is an auxiliary field. The sign and coefficient of $h_{\mu\nu}$ have been chosen for future convenience. The linearized gauge transformations of the component fields can be obtained from $\delta|\Psi\rangle = Q_B|\Lambda\rangle$ with

$$|\Lambda\rangle = \frac{i}{\sqrt{2\alpha'}} \epsilon_\mu (c_1 \alpha_{-1}^\mu - \bar{c}_1 \bar{\alpha}_{-1}^\mu) |p\rangle. \quad (4.47)$$

The resulting coordinate-space gauge transformations are:

$$\delta h_{\mu\nu} = \partial_\nu \epsilon_\mu + \partial_\mu \epsilon_\nu, \quad \delta d = -\frac{1}{2} \partial \cdot \epsilon, \quad \delta B_\mu = -\frac{1}{2} \partial^2 \epsilon_\mu, \quad \delta t = 0. \quad (4.48)$$

We now calculate the quadratic part of the closed string field action, finding

$$\begin{aligned} S^{(2)} &= -\frac{1}{\kappa^2 \alpha'} \langle \Psi | c_0^- Q_B | \Psi \rangle, \\ &= \frac{1}{2\kappa^2} \int d^D x \left(\frac{1}{4} h_{\mu\nu} \partial^2 h^{\mu\nu} - 2d \partial^2 d - 2B_\mu (\partial_\nu h^{\mu\nu} + 2\partial^\mu d) - 2B^2 - (\partial t)^2 + \frac{4}{\alpha'} t^2 \right), \\ &= \frac{1}{2\kappa^2} \int d^D x \left(\frac{1}{4} h_{\mu\nu} \partial^2 h^{\mu\nu} + \frac{1}{2} (\partial^\nu h_{\mu\nu})^2 - 4d \partial^2 d - 2d \partial_\mu \partial_\nu h^{\mu\nu} - (\partial t)^2 + \frac{4}{\alpha'} t^2 \right). \end{aligned} \quad (4.49)$$

In the last step we eliminated the auxiliary field B_μ using its algebraic equation of motion.

The gauge transformations (4.48) imply that the linear combination $d + \frac{h}{4}$ is gauge invariant. It follows that the sigma model dilaton must take the form

$$\lambda \Phi = d + \frac{h}{4}, \quad (4.50)$$

where λ is a number to be determined. Using (4.50) to eliminate the ghost-dilaton from the action (4.49) we find

$$S^{(2)} = \frac{1}{2\kappa^2} \int d^D x \left(\frac{1}{4} h_{\mu\nu} \partial^2 h^{\mu\nu} - \frac{1}{4} h \partial^2 h + \frac{1}{2} (\partial^\nu h_{\mu\nu})^2 + \frac{1}{2} h \partial_\mu \partial_\nu h^{\mu\nu} \right. \\ \left. + 2\lambda h \partial^2 \Phi - 2\lambda \Phi \partial_\mu \partial_\nu h^{\mu\nu} - 4\lambda^2 \Phi \partial^2 \Phi - (\partial t)^2 + \frac{4}{\alpha'} \right). \quad (4.51)$$

We also use the string field theory to calculate the on-shell coupling of $h_{\mu\nu}$ to two tachyons. This coupling arises from the term

$$S^{(3)} = -\frac{1}{\alpha' \kappa^2} \langle \mathcal{T}, \mathcal{H}, \mathcal{T} \rangle, \quad (4.52)$$

where \mathcal{T} and \mathcal{H} denote the parts of the string field (4.46) that contain $t(k)$ and $h_{\mu\nu}(k)$, respectively. We thus have

$$S^{(3)} = \frac{1}{2\alpha' \kappa^2} \left(\prod_{i=1}^3 \int \frac{d^D k_i}{(2\pi)^D} \right) \langle c_1 \bar{c}_1 e^{ik_1 \cdot X}, c_1 \bar{c}_1 \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu e^{ik_2 \cdot X}, c_1 \bar{c}_1 e^{ik_3 \cdot X} \rangle t(k_1) t(k_3) h_{\mu\nu}(k_2). \quad (4.53)$$

The on-shell evaluation is readily carried out using $k^\mu h_{\mu\nu}(k) = 0$. We obtain

$$S^{(3)} = -\frac{1}{2\kappa^2} \int \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_3}{(2\pi)^D} k_1^\mu k_3^\nu t(k_1) t(k_3) h_{\mu\nu}(-k_1 - k_3) = \frac{1}{2\kappa^2} \int d^D x h^{\mu\nu} \partial_\mu \partial_\nu t. \quad (4.54)$$

Combining this result with (4.51) we obtain the closed string field theory action

$$S_{csft} = \frac{1}{2\kappa^2} \int d^D x \left(\frac{1}{4} h_{\mu\nu} \partial^2 h^{\mu\nu} - \frac{1}{4} h \partial^2 h + \frac{1}{2} (\partial^\nu h_{\mu\nu})^2 + \frac{1}{2} h \partial_\mu \partial_\nu h^{\mu\nu} \right. \\ \left. + 2\lambda h \partial^2 \Phi - 2\lambda \Phi \partial_\mu \partial_\nu h^{\mu\nu} - 4\lambda^2 \Phi \partial^2 \Phi \right. \\ \left. - (\partial t)^2 + \frac{4}{\alpha'} t^2 + h^{\mu\nu} \partial_\mu t \partial_\nu t + \dots \right). \quad (4.55)$$

We are finally in a position to identify the sigma model action (4.45) and the string field action (4.55). Comparing the quadratic terms in $\tilde{h}_{\mu\nu}$ and those in $h_{\mu\nu}$ we see that $\tilde{h}_{\mu\nu} = \pm h_{\mu\nu}$. We also note that $T = \pm t$. The coupling $\tilde{h}^{\mu\nu} \partial_\mu T \partial_\nu T$ in (4.45) coincides with the corresponding coupling in (4.55) if and only if

$$\tilde{h}_{\mu\nu} = h_{\mu\nu}. \quad (4.56)$$

This simple equality justifies the multiplicative factor of $(-1/2)$ introduced for $h_{\mu\nu}$ in the string field (4.46). The string field $h_{\mu\nu}$ so normalized is the fluctuation of the string metric. Comparing the

couplings of metric and dilaton in both actions we also conclude that $\lambda = +1$ and, therefore, equation (4.50) gives

$$\Phi = d + \frac{h}{4}. \quad (4.57)$$

This expresses the sigma model dilaton Φ in terms of the string field metric trace and the ghost dilaton d . It is important to note that when we give a positive expectation value to d (and no expectation value to h) we are increasing the value of Φ and therefore increasing the value of the string coupling.

4.2.2 The many faces of the dilaton

Equipped with the precise relations between string fields and sigma-model fields we digress on the various dilaton fields used in the literature. Of particular interest are the corresponding vertex operators, which are determined by the CFT states that multiply the component fields in the closed string field.

We introduce the states

$$|\mathcal{O}^{\mu\nu}(p)\rangle = -\frac{1}{4}(\alpha_{-1}^{\mu}\bar{\alpha}_{-1}^{\nu} + \alpha_{-1}^{\nu}\bar{\alpha}_{-1}^{\mu})|p\rangle, \quad |\mathcal{O}^d(p)\rangle = (c_1c_{-1} - \bar{c}_1\bar{c}_{-1})|p\rangle. \quad (4.58)$$

The corresponding vertex operators are

$$\mathcal{O}^{\mu\nu}(p) = \frac{1}{2\alpha'}(\partial X^{\mu}\bar{\partial}X^{\nu} + \partial X^{\nu}\bar{\partial}X^{\mu})e^{ipX}, \quad \mathcal{O}^d(p) = \frac{1}{2}(c\partial^2c - \bar{c}\bar{\partial}^2c)e^{ipX}. \quad (4.59)$$

Working for fixed momentum, the string field (4.46) restricted to metric and dilaton fluctuations is

$$|\Psi\rangle = h_{\mu\nu}|\mathcal{O}^{\mu\nu}\rangle + d|\mathcal{O}^d\rangle. \quad (4.60)$$

This equation states that \mathcal{O}^d is the vertex operator associated with the ghost-dilaton field d . An excitation by this vertex operator does not change the metric $h_{\mu\nu}$. Our transformation to a gauge invariant dilaton gives

$$\Phi = d + \frac{1}{4}h, \quad \tilde{h}_{\mu\nu} = h_{\mu\nu}. \quad (4.61)$$

Here $\tilde{h}_{\mu\nu}$ is the fluctuation of the string metric. Inverting these relations

$$d = \Phi - \frac{1}{4}\tilde{h}, \quad h_{\mu\nu} = \tilde{h}_{\mu\nu}. \quad (4.62)$$

Substituting into the string field (4.60) we obtain

$$|\Psi\rangle = \tilde{h}_{\mu\nu}\left(|\mathcal{O}^{\mu\nu}\rangle - \frac{1}{4}\eta^{\mu\nu}|\mathcal{O}^d\rangle\right) + \Phi|\mathcal{O}^d\rangle. \quad (4.63)$$

It is interesting to note that \mathcal{O}^d is the vertex operator associated with a variation of the gauge-invariant dilaton Φ and no variation of the string metric. On the other hand, $\mathcal{O}^{\mu\nu} - \frac{1}{4}\eta^{\mu\nu}\mathcal{O}^d$ varies the string metric and does not vary the gauge-invariant dilaton (although it varies the ghost-dilaton).

Finally, we consider the formulation that uses the Einstein metric $g_{\mu\nu}^E$ and the dilaton Φ . The field redefinition is

$$g_{\mu\nu}^E = \exp(2\omega)g_{\mu\nu}, \quad \text{with } \omega = -\frac{2}{D-2}\Phi. \quad (4.64)$$

Expanding in fluctuation fields we obtain

$$h_{\mu\nu}^E = \tilde{h}_{\mu\nu} - \frac{4}{D-2} \eta_{\mu\nu} \Phi. \quad (4.65)$$

Solving for d and h_μ in terms of Φ and $h_{\mu\nu}^E$ we get

$$d = -\frac{2}{D-2} \Phi - \frac{1}{4} h^E, \quad h_{\mu\nu} = h_{\mu\nu}^E + \frac{4}{D-2} \eta_{\mu\nu} \Phi. \quad (4.66)$$

Substituting into the string field (4.60) we obtain

$$|\Psi\rangle = h_{\mu\nu}^E \left(|\mathcal{O}^{\mu\nu}\rangle - \frac{1}{4} \eta^{\mu\nu} |\mathcal{O}^d\rangle \right) + \frac{2}{D-2} \Phi \left(2\eta_{\mu\nu} |\mathcal{O}^{\mu\nu}\rangle - |\mathcal{O}^d\rangle \right). \quad (4.67)$$

Interestingly, the vertex operator that varies the Einstein metric (without variation of the dilaton) is the same as that for the string metric (see (4.63)). It is the dilaton operator that changes this time. The vertex operator

$$\mathcal{D} = 2\eta_{\mu\nu} \mathcal{O}^{\mu\nu} - \mathcal{O}^d = \left(\frac{2}{\alpha'} \partial X \cdot \bar{\partial} X - \frac{1}{2} (c \partial^2 c - \bar{c} \bar{\partial}^2 c) \right) e^{ipX}, \quad (4.68)$$

varies the dilaton without varying the Einstein metric. This is the dilaton vertex operator used almost exclusively in the early literature – it is naturally associated with the Einstein metric. The corresponding state $|\mathcal{D}(p)\rangle$ has a particularly nice property: it is annihilated by the BRST operator when $p^2 = 0$. Indeed,

$$Q_B |\mathcal{D}(p)\rangle = \frac{\alpha'}{2} p^2 c_0^+ |\mathcal{D}(p)\rangle. \quad (4.69)$$

The dilaton \mathcal{D} is in fact the unique linear combination of the matter and ghost dilatons that has this property. For other combinations, terms linear in the momentum p (such as $(p \cdot \alpha_{-1}) c_1 \bar{c}_1 \bar{c}_{-1} |p\rangle$), survive.

4.2.3 Relating the sigma model and string field dilaton and tachyon

The closed string theory potential V , as read from the effective action (5.1) is

$$\kappa^2 V = e^{-2\Phi} (V(T) + \dots), \quad \text{with } V(T) = -T^2 + \dots. \quad (4.70)$$

Here Φ and T are the zero momentum dilaton and tachyon fields in the effective field theory. The purpose of this section is to discuss the relation between Φ and T and the corresponding string fields d and t , both sets at zero-momentum. To do this we must consider the effective potential for d and t calculated in string field theory. We only have the potential itself. Collecting our previous results, we write

$$\begin{aligned} \kappa^2 V &= -t^2 + 1.6018 t^3 - 3.0172 t^4 \\ &+ 3.8721 t^3 d + (-0.8438 t + 1.3682 t^2) d^2 - 0.9528 t d^3 - 0.1056 d^4. \end{aligned} \quad (4.71)$$

The contributions from massive fields affect quartic and higher order terms. In our setup, the relevant terms arise when we eliminate the level-four massive fields using their kinetic terms in (4.32) and their linear couplings to t^2 in (4.30), to td in (4.31), and to d^2 in (4.33). We find

$$\Delta V = -\frac{6241}{186624} d^4 + \frac{25329}{16384} d^2 t^2 - \frac{1896129}{4194304} t^4 \simeq -0.0334 d^4 + 1.5460 d^2 t^2 - 0.4521 t^4. \quad (4.72)$$

It follows that the effective potential for the tachyon and the dilaton, calculated up to terms quartic in the fields and including massive fields of level four only, is given by:

$$\begin{aligned} \kappa^2 V_{eff} = & -t^2 + 1.6018 t^3 - 3.4693 t^4 \\ & + 3.8721 t^3 d + (-0.8438 t + 2.9142 t^2) d^2 - 0.9528 t d^3 - 0.1390 d^4 + \dots \end{aligned} \quad (4.73)$$

The dots represent quintic and higher terms, which receive contributions both from elementary interactions and some integration of massive fields. We write, more generically

$$\begin{aligned} \kappa^2 V_{eff} = & -t^2 + a_{3,0} t^3 + a_{4,0} t^4 \\ & + a_{3,1} t^3 d + (a_{1,2} t + a_{2,2} t^2) d^2 + a_{1,3} t d^3 + a_{0,4} d^4 + \dots \end{aligned} \quad (4.74)$$

The values of the coefficients $a_{i,j}$ can be read comparing this equation with (4.73).

There are two facts about V_{eff} that make it clear it is not in the form of a ghost-dilaton exponential times a tachyon potential. First, it does not have a term of the form $t^2 d$ that would arise from the tachyon mass term and the expansion of the exponential. Second, it contains a term linear in the tachyon; those terms should be absent since the tachyon potential does not have a linear term. Nontrivial field redefinitions are necessary to relate string fields and sigma model fields.

To linearized order the fields are the same, so we write relations of the form :

$$\begin{aligned} t &= T + \alpha_1 T \Phi + \alpha_2 \Phi^2 + \dots, \\ d &= \Phi + \beta_0 T^2 + \beta_1 T \Phi + \beta_2 \Phi^2 + \dots, \end{aligned} \quad (4.75)$$

where the dots indicate terms of higher order in the sigma model fields. We found no *need* for a T^2 term in the redefinition of tachyon field, such a term would change the cubic and quartic self-couplings of the tachyon in $V(T)$. Since d gives rise to pure tachyon terms that are quadratic or higher, only at quintic and higher order in T will $V(T)$ differ from the potential obtained by replacing $t \rightarrow T$ in the first line of (4.73). We thus expect that after the field redefinition (4.73) becomes

$$\kappa^2 V = e^{-2\Phi} (-T^2 + 1.6018 T^3 - 3.4693 T^4 + \dots), \quad (4.76)$$

at least to quartic order in the fields. We now plug the substitutions (4.75) into the potential (4.73) and compare with (4.76). A number of conditions emerge.

- In order to get the requisite $T^2 \Phi$ term we need $\alpha_1 = -1$.
- In order to have a vanishing $T \Phi^2$ term $\alpha_2 = \frac{1}{2} a_{1,2}$ must be half the coefficient of td^2 in (4.73).

- Getting the correct $T^3\Phi$ coupling then fixes $\beta_0 = (a_{3,0} - a_{3,1})/(2a_{1,2})$.
- Getting the correct value of $T^2\Phi^2$ fixes $\beta_1 = -(1 + \frac{3}{2}a_{3,0}a_{1,2} + a_{2,2})/(2a_{1,2})$. The vanishing of $T\Phi^3$ fixes $\beta_2 = -a_{1,3}/(2a_{1,2})$. All coefficients in (4.75) are now fixed.
- The coefficient of Φ^4 , which should be zero, turns out to be $(a_{0,4} + \frac{1}{4}a_{1,2}^2) \simeq 0.0389$, which is small, but does not vanish.

Our inability to adjust the coefficient of Φ^4 was to be expected. The potential (4.73) contains the terms $-t^2 + a_{1,2}td^2 + a_{0,4}d^4$ and, to this order, integrating out the tachyon gives an effective dilaton quartic term of $(a_{0,4} + \frac{1}{4}a_{1,2}^2)$. With the contribution of the massive fields beyond level four this coefficient in the dilaton effective potential would vanish. This is, in fact, the statement that was verified in [28]. It follows that we need not worry that the quartic term in Φ do not vanish exactly. Following the steps detailed before we find

$$\begin{aligned}
t &= T - T\Phi - 0.4219\Phi^2 + \dots, \\
d &= \Phi + 1.3453T^2 + 1.1180T\Phi - 0.5646\Phi^2 + \dots.
\end{aligned}
\tag{4.77}$$

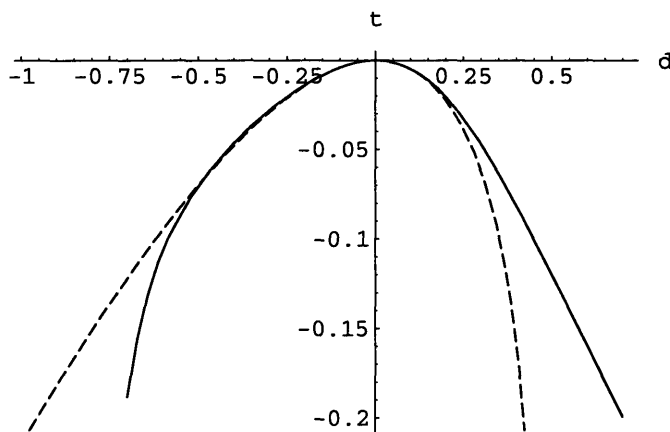


Figure 4.2: The solid line is the dilaton marginal direction defined by the set of points $(d, t(d))$ where $t(d)$ is the expectation value of t obtained solving the tachyon equation of motion for the given d . The dashed line represents the direction along the sigma model dilaton Φ (thus $T = 0$). It is obtained by setting $T = 0$ in equation (4.77). The two lines agree well even reasonably far from the origin.

In string field theory the dilaton deformation is represented in the (d, t) plane by the curve $(d, t(d))$, where $t(d)$ is the expectation value of the tachyon when the dilaton is set equal to d . This curve, calculated using the action (4.73), is shown as a solid line in Figure 4.2. On the other hand, it is clear that Φ (with $T = 0$) defines the marginal direction in the effective field theory. Setting $T = 0$ in (4.77) we find the pair $(d(\Phi), t(\Phi))$, which must be a parameterization of the flat direction in terms of Φ . This curve is shown as a dashed line in Figure 4.2. It is a good consistency check that these two curves agree well with each other over a significant fraction of the plot.

4.2.4 Dilaton deformations

In Ref. [28] we computed the effective dilaton potential that arises when we integrate out the tachyon from a potential that includes only quadratic and cubic terms. We found that the domain of definition of this potential is the full real d line. This happens because the (marginal) branch $t(d)$ that gives the expectation value of t for a given value of d is well defined for all values of d . In this section we extend this computation by including higher level fields and higher order interactions. As we will demonstrate, it appears plausible that the domain of definition for the effective dilaton potential remains $d \in (-\infty, \infty)$.

The marginal branch is easily identified for small values of the dilaton: as the dilaton expectation value goes to zero all expectation values go to zero. For large enough values of the dilaton the marginal branch may cease to exist, or it may meet another solution branch. If so, we obtain limits on the value of d . Since the dilaton effective potential is supposed to be flat in the limit of high level, we propose the following criterion. If we encounter a limit value of d , this value is deemed reliable only if the dilaton potential at this point is not very large. A large value for the potential indicates that the calculation is not reliable because the same terms that are needed to make the potential small could well affect the limit value. In open string field theory a reliable limit value was obtained for the Wilson line parameter: at the limit point the potential energy density was a relatively small fraction of the D-brane energy density. The purely cubic potential for t gives a critical point with $\kappa^2 V \sim -0.05774$. We define $\mathcal{R}(d) \equiv \frac{|\kappa^2 V(d)|}{0.05774}$, where $V(d)$ is the effective dilaton potential. A critical value of d for which $\mathcal{R} > 1$ will be considered unreliable.

We start with cubic potentials and then include the elementary quartic interactions level by level. With cubic potentials, the effective dilaton potential is invariant under $d \rightarrow -d$. With $\mathbb{V}_4^{(3)}$ dilaton deformations can be arbitrarily large [28]. We then find

- The dilaton potential derived from $\mathbb{V}_8^{(3)}$ is defined for $|d| \leq 624$. This is plausible since, at this level, the equations of motion for the level-four fields are linear.
- The dilaton potential derived from $\mathbb{V}_{12}^{(3)}$ is defined for $|d| \leq 1.71$. Since $\mathcal{R}(\pm 1.71) = 42.4$, there is no reliable limit value.
- The dilaton potential derived from $\mathbb{V}_0^{(4)}$ is defined for $|d| \leq 4.67$, where $\mathcal{R}(\pm 4.67) = 49.5$. The large value of \mathcal{R} indicates that there is no evidence of a limit value.
- The dilaton potential derived from $\mathbb{V}_2^{(4)}$ is not invariant under $d \rightarrow -d$. We find a range $d \in (-\infty, 3.124)$. Although $\mathcal{R}(3.124) = 0.387$, the potential has a maximum with $\mathcal{R} = 3.325$ at $d = 1.92$. This fact makes the limit point $d = 3.124$ unreliable.
- The dilaton potential derived from $\mathbb{V}_4^{(4)}$, the highest level potential we have computed fully, is regular for $d \in (-2.643, 6.415)$. Since $\mathcal{R}(6.415) = 1502.4$ and $\mathcal{R}(-2.643) = 89.2$, there is no branch cut in the reliable region.

We have also computed the higher level quartic interactions td^3 and d^4 . We have checked that $\mathbb{V}_4^{(4)}$, supplemented by those interactions does not lead to branch cuts in the potential for the dilaton. This result, however, is not conclusive. Additional interactions must be included at level six (the level of td^3) and at level eight (the level of d^4).

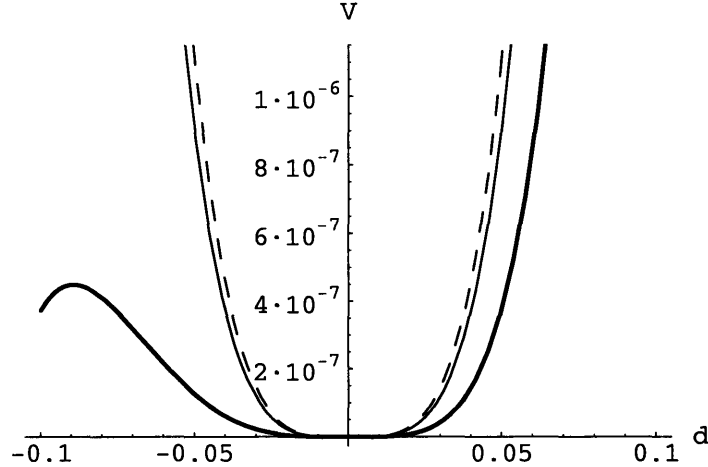


Figure 4.3: Dilaton effective potential. The dashed line arises from $\mathbb{V}_4^{(3)}$, the solid line arises from $\mathbb{V}_8^{(3)}$, and the thick line arises from $\mathbb{V}_8^{(4)}$.

We tested in [28] that cubic and quartic interactions combine to give a vanishing quartic term in the dilaton effective potential. We can ask if the potential for the dilaton becomes flatter as the level of the calculation is increased. We find that it roughly does, but the major changes in the potential are due to the elementary quartic term in the dilaton. For the cubic vertex, the interactions of the type d^2M , with M massive give rise to terms quartic on the dilaton. Other cubic couplings that do not involve the dilaton typically induce d^6 (and higher order) terms, which play a secondary role in flattening the potential if the quartic terms have not cancelled completely. Therefore, the potentials that arise from $\mathbb{V}_8^{(3)}$, $\mathbb{V}_{10}^{(3)}$ and $\mathbb{V}_{12}^{(3)}$ (without the contribution from level six massive fields) have no obvious difference. The potentials obtained at various levels are shown in Figure 4.3. The dashed line arises from $\mathbb{V}_4^{(3)}$, the solid line arises from $\mathbb{V}_8^{(3)}$, and the thick line arises from $\mathbb{V}_8^{(4)}$.

4.3 Conclusions

In this chapter we have presented some calculations that suggest the existence of a tachyon vacuum for the bulk closed string tachyon of bosonic string theory. We have discussed the physical interpretation using the effective field theory both to suggest the value of the action density at the critical point (zero!) and to obtain rolling solutions [66] that seem consistent with the interpretation of the tachyon vacuum as a state in which there are no closed string states.

The numerical evidence presented is still far from conclusive. A critical point seems to exist and

appears to be robust, but it is not all that clear what will happen when the accuracy of the computation is increased. If the action density at the critical point goes to zero it may indeed define a new and nontrivial tachyon vacuum. Conceivably, however, the critical point could approach the perturbative vacuum, in which case there would be no evidence for a new vacuum. Alternatively, if the action density at the critical point remains finite, we would have no interpretation for the result.

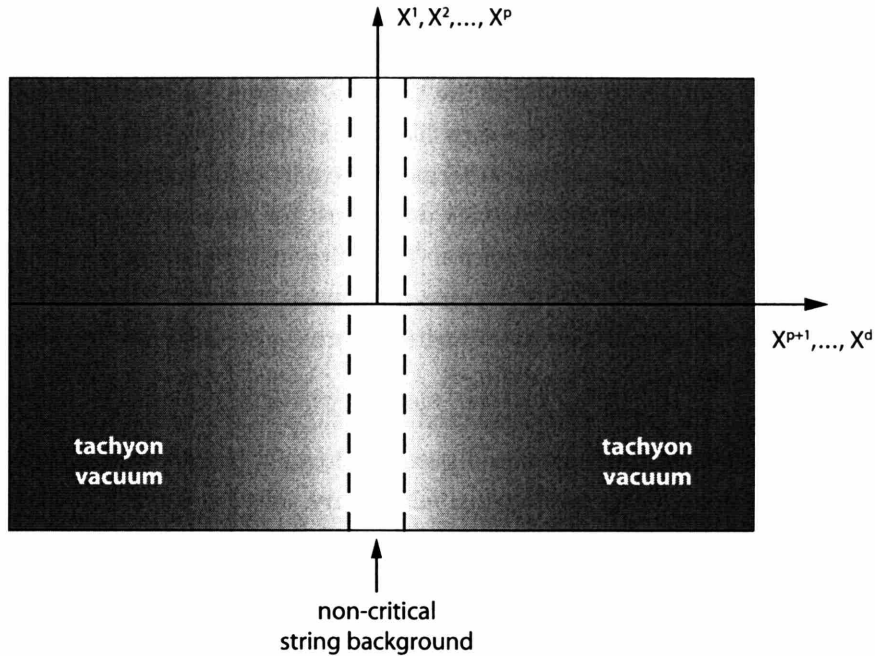


Figure 4.4: A non-critical $(p + 1)$ -dimensional string theory would correspond to a solitonic solution of critical string theory in which, far away from the reduced space, the fields approach the values of the closed string tachyon vacuum.

Let us consider some additional indirect arguments that support the existence of a closed string tachyon vacuum. The first one arises from the existence of sub-critical bosonic string theories. The evidence in string theory is that most string theories are related by compactifications and/or deformations. It seems very likely that non-critical string theories are also related to critical string theory. It should then be possible to obtain a non-critical string theory as a solution of critical string theory. Certainly the view that $D = 2$ bosonic string theory is a ground state of the bosonic string has been held as likely [80]. In non-critical string theory the number of space dimensions is reduced (at the expense of a linear dilaton background). The analogy with lower-dimensional D-branes in open string theory seems apt: the branes are solitons of the open string field theory tachyon in which far away from the branes the tachyon sits at the vacuum. It seems plausible that non-critical string theories are solitonic solutions of the *closed* string theory tachyon. As sketched in Figure 4.4, far away along the coordinates transverse to the non-critical world-volume, the background would approach the closed

string tachyon vacuum. The universality of the tachyon vacuum would imply that a noncritical string theory could be further reduced using the same background configuration used to reduce the original critical theory.

In fact, in the p -adic open/closed string theory lump solutions of the closed string sector appear to describe spacetimes of lower dimensionality, as explained by Moeller and Schnabl [77]. Indeed, far away from the lump the open string tachyon must be at its vacuum and therefore there are no D-brane solutions with more space dimensions than those of the lump. Away from the lump the closed string tachyon is at its vacuum, and no linearized solutions of the equations of motion exist.

A suggestive argument for zero action at the tachyon vacuum follows from the sigma model approach. As discussed by Tseytlin [74], it seems likely that the closed string effective action for the spacetime background fields may be written in terms of the partition function Z of the two-dimensional sigma model as well as derivatives thereof (this does work for open strings [76]). The conventional coupling of the world-sheet area to the tachyon T results in a partition function and an effective action with a prefactor of e^{-T} . Thus one expects a tachyon potential of the form $e^{-T}g(T)$ where g is a polynomial that begins with a negative quadratic term⁴. In this case, for a tachyon vacuum at $T \rightarrow \infty$ the action goes to zero.

The computations and the discussion presented in this chapter have led to a set of testable conjectures concerning the vacuum of the bulk closed string tachyon of bosonic string theory. It seems likely that additional computations, using both string field theory, effective field theory, and conformal field theory will help test these ideas in the near future.

⁴In [74], a tachyon potential of the form $-T^2 e^{-T}$ is considered. Complications in fixing the kinetic terms made it unclear if $T = \infty$ was a point in the configuration space (see the discussion below eqn. (4.13) of [74]. For additional comments on the possible form of the tachyon potential, see Andreev [75].

Chapter 5

Rolling Closed String Tachyons and the Big Crunch

A tachyon of closed string theory is said to be a bulk tachyon if it lives throughout spacetime. In the presence of a tachyon one has an instability and two important and related questions arise:

1. Is there a ground state of the theory without the instability ?
2. What is the end-result of the physical decay process associated with the instability ?

The answers are presently known for the open string theory tachyons that live on the world-volume of unstable D-branes [81]. The ground state, or tachyon vacuum, is a state without the D-brane and without open strings – it is in fact the vacuum state of closed strings. In the associated physical decay process the D-brane disappears but the result is not quite the closed string vacuum but rather an excited state of closed strings that carries the original energy of the D-brane. The decay process is not simply a transition from the unstable to the stable vacuum.

The purpose of the present chapter is to study the physical decay induced by the bulk closed string tachyon of bosonic closed string theory – the second question above applied to bulk tachyons (for localized closed string tachyons, see [82]). Our work was prompted by new information about the first question: recently-found evidence that the closed string field theory “tachyon potential” has a critical point – a candidate for a closed string tachyon vacuum [83]. A set of considerations suggests that in such vacuum closed string states would not propagate and spacetime would cease to be dynamical. Our analysis of the physical decay aims to illuminate the nature of the tachyon vacuum. This may be possible because the physical decay turns out to be rather insensitive to the specific details of the tachyon potential, about which little is known.

We study here the low-energy field equations that couple the metric, the dilaton, and the tachyon. These equations are motivated by the conditions of conformal invariance of sigma models [71] and are expected to provide solutions that capture relevant features of exact string theory solutions. The low-energy field equations have been used in many papers to study all kinds of dynamical and cosmological issues (for a review and references see [84]). Few of these works, however, deal with key features of our present problem: a standard minimal coupling of the dilaton to other fields, an unstable

closed string vacuum with zero cosmological constant, a tachyon potential that is not positive, and a rolling process induced by the tachyon. The related problem of light (bulk) tachyons that arise from circle compactification has been studied by Dine *et.al.* [85] and Suyama [86]. These authors compute quadratic and quartic terms in the tachyon potential and consider a cosmological evolution that involves the metric, the dilaton, the radion, and the tachyon. The authors of [85] state that numerical studies show that rather general initial conditions lead to a radius that evolves to make the tachyon more tachyonic and a dilaton that evolves to make the system strongly coupled (the simplicity of a light tachyon seems to be illusory). The author of [86] freezes the radion and discusses explicitly the simplified system, showing in a numerical solution how it appears to be driven to strong coupling.

Our analysis assumes *arbitrary* tachyonic potentials $V(T) = -\frac{1}{2}m^2T^2 + \mathcal{O}(T^3)$ and reveals a few surprises. We have found that if the rolling process is triggered by the tachyon the string metric does *not* evolve. Moreover, the dilaton expectation value Φ will always increase as time goes by. If the tachyon history $T(t)$ is such that the potential is negative, $V(T(t)) \leq 0$, the evolution reaches a singular point in finite time: both \dot{T} and $\dot{\Phi}$ become infinite, and so do T and Φ . In the string frame this is a system with infinite string coupling, while in the Einstein frame the universe undergoes a big crunch. While negative potentials help accelerate its occurrence, a crunch occurs at finite time (both in the string and Einstein frames) for a wide class of potentials that grow arbitrarily large and positive for large T , $V(T) = -T^2 + T^4$, for example. The growing dilaton acts on the tachyon like anti-friction, a force proportional to the tachyon velocity in the direction of the velocity. This generally enables the tachyon to reach infinite value in finite time, even if it has to climb an infinite potential.

This chapter is organized as follows. In Section 2 we write the relevant coupled equations and examine them in the cosmological setting. We use the string metric and emphasize how the dilaton time derivative plays a role similar to that of *minus* the Hubble parameter $H(t)$. In Section 3 we define tachyon-induced rolling and show that it results in a constant string metric. The rolling problem simplifies considerably and becomes the coupled dynamics of a dilaton and a tachyon. Analytic solutions are possible if one can solve a certain first-order nonlinear differential equation. Up to numerical constants, the dilaton-tachyon equations can be mapped to those that describe a single scalar field rolling in Einstein's theory. In Section 4 we establish that the Einstein metric crunches in finite time if the tachyon potential is negative throughout the rolling solution. In Section 5 we consider potentials that can be positive and develop tools to decide if there is a big crunch and if it occurs in finite time. Conclusions are offered in Section 6.

5.1 The coupled system of rolling fields

Consider the action that describes the low-energy dynamics of the metric, the dilaton, and the tachyon:

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} e^{-2\Phi} \left(R + 4(\partial_\mu \Phi)^2 - (\partial_\mu T)^2 - 2V(T) \right). \quad (5.1)$$

Here $g_{\mu\nu}$ is the string metric, Φ is the dilaton, and T is the tachyon, with potential $V(T)$. The number of spatial dimensions is d . We are following the conventions of [88], with their dilaton ϕ replaced by

(-2Φ) . The metric–dilaton part of the action is that in [78]. The equations of motion are:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - (\partial_\mu T)(\partial_\nu T) &= 0, \\ \nabla^2 T - 2(\partial_\mu \Phi)(\partial^\mu T) - V'(T) &= 0, \\ \nabla^2 \Phi - 2(\partial_\mu \Phi)^2 - V(T) &= 0. \end{aligned} \tag{5.2}$$

To evaluate the action on-shell we multiply the first equation by $g^{\mu\nu}$, use the third equation to eliminate $\nabla^2 \Phi$, and find that $R + 4(\partial_\mu \Phi)^2 - (\partial_\mu T)^2 = -2V(T)$. Using this,

$$S_{on-shell} = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} e^{-2\Phi} (-4V(T)). \tag{5.3}$$

We look for solutions of (5.2) that represent a rolling tachyon field $T(t)$ accompanied by a time dependent dilaton $\Phi(t)$ and a time dependent string metric of the form

$$ds^2 = -(dt)^2 + a^2(t)(dx_1^2 + dx_2^2 + \dots dx_d^2), \quad H(t) \equiv \frac{\dot{a}(t)}{a(t)}. \tag{5.4}$$

With this metric, the gravitational equations of motion (first line in (5.2)) give two equations

$$d \frac{\ddot{a}}{a} + \dot{T}^2 - 2\ddot{\Phi} = 0, \quad \frac{\ddot{a}}{a} + (d-1) \left(\frac{\dot{a}}{a} \right)^2 - 2 \frac{\dot{a}}{a} \dot{\Phi} = 0. \tag{5.5}$$

The equations of motion for the dilaton and the tachyon are:

$$\ddot{\Phi} + (dH - 2\dot{\Phi})\dot{\Phi} + V(T) = 0, \tag{5.6}$$

$$\ddot{T} + (dH - 2\dot{\Phi})\dot{T} + V'(T) = 0. \tag{5.7}$$

We recognize the familiar Hubble “friction” term that couples H to the field velocity. Indeed, for $H > 0$ the force is opposite to the velocity and slows down the field. Similarly, the dilaton velocity $\dot{\Phi}$ is *anti*-friction: if $\dot{\Phi} > 0$ the force is in the direction of the velocity and accelerates the field. The dilaton is driven by $-V(T)$; it will tend to go to strong coupling while $V(T) < 0$.

The gravity equations (5.5) can be rearranged into two equivalent equations:

$$\frac{1}{2}(d-1)\dot{H} = -\frac{1}{2}\dot{T}^2 + \ddot{\Phi} - H\dot{\Phi}, \tag{5.8}$$

$$\frac{1}{2}d(d-1)H^2 = \frac{1}{2}\dot{T}^2 - \ddot{\Phi} + dH\dot{\Phi}. \tag{5.9}$$

It can be shown that if equation (5.9) holds at some time, equations (5.7), (5.6), and (5.8) guarantee that it holds for all times.

It is instructive to compare of the previous equations with those that govern the dynamics of a scalar field ϕ with potential $V(\phi)$ coupled to gravity *without* a dilaton:

$$\frac{1}{2}(d-1)\dot{H} = -\frac{1}{2}\dot{\phi}^2, \tag{5.10}$$

$$\frac{1}{2}d(d-1)H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{5.11}$$

$$\ddot{\phi} + dH\dot{\phi} + V'(\phi) = 0. \tag{5.12}$$

Note that $\dot{H} \sim -\dot{\phi}^2 < 0$, which means decelerating expansion or accelerating contraction. On the other hand, the analogous equation in the presence of a dilaton, (5.8), allows the possibility that \dot{H} vanishes. Equation (5.11) is analogous to (5.9). Comparison of (5.12) with (5.7) confirms that the rolling scalar is only affected by the addition of the dilaton-induced anti-friction ($\dot{\Phi} > 0$).

The Einstein metric $g_{\mu\nu}^E$ is determined by the string metric and the dilaton: $g_{\mu\nu}^E = \exp(-\frac{4}{d-1}\Phi) g_{\mu\nu}$. For a fixed string metric, the Einstein metric goes to zero if the dilaton expectation value goes to infinity. This corresponds to infinite string coupling.

5.2 Tachyon-driven rolling and the string metric

We now consider a general class of potentials $V(T)$ for a tachyon T that satisfy the condition $V(0) = 0$ and can be written as

$$V(T) = -\frac{1}{2}m^2 T^2 + \mathcal{O}(T^3). \quad (5.13)$$

We build a solution where $T \rightarrow 0$ for $t \rightarrow -\infty$, and the field rolls to positive values:

$$T(t) = e^{mt} + \sum_{n \geq 2} t_n e^{nmt}. \quad (5.14)$$

The first term in this ansatz is the solution to the linearized tachyon equation of motion. The arbitrary constant multiplying this term can be absorbed, as we did, by a redefinition of time. The exponentials in the sum are subleading to e^{mt} for large negative t . We say that the tachyon drives the rolling if the other fields, in this case $H(t)$ and $\Phi(t)$, have solutions with exponentials subleading to e^{mt} :

$$\Phi(t) = \sum_{n \geq 2} \phi_n e^{nmt}, \quad H(t) = \sum_{n \geq 2} h_n e^{nmt}. \quad (5.15)$$

Given (5.14), the dilaton equation (5.6) gives

$$\Phi(t) = \frac{1}{8} e^{2mt} + \mathcal{O}(e^{3mt}). \quad (5.16)$$

This leading behavior is valid for all potentials of the form (5.13). The dilaton begins to run towards *stronger* coupling. Evaluating the right-hand side of equation (5.8) we see that

$$-\dot{T}^2 + 2\ddot{\Phi} = -m^2 e^{2mt} + 2 \cdot \frac{1}{8} \cdot (4m^2) e^{2mt} + \mathcal{O}(e^{3mt}) = 0 \cdot e^{2mt} + \mathcal{O}(e^{3mt}). \quad (5.17)$$

Since the other term on the right-hand side, $H\dot{\Phi} \sim e^{4mt}$, we deduce that $\dot{H} \sim e^{3mt}$ and therefore the contribution of order e^{2mt} to H vanishes: $h_2 = 0$. The string metric is not affected to this order. This is actually the beginning of a pattern: we now prove that $H(t)$ vanishes identically for tachyon-induced rolling. Adding equations (5.8) and (5.9) we find

$$\dot{H} = -(dH - 2\dot{\Phi})H. \quad (5.18)$$

Now assume that $h_2 = h_3 = \dots = h_N = 0$ for some $N \geq 2$. Since $\dot{\Phi} \sim e^{2mt}$, the above equation gives $\dot{H} \sim e^{(N+3)mt}$, which implies that $h_{N+1} = 0$. By induction, $H(t)$ vanishes identically.

We now reconsider the equations of motion with $H = 0$. The gravitational equations (5.8) and (5.9) give a single equation, $\ddot{\Phi} = \frac{1}{2}\dot{T}^2$. Additionally, we have the equations of motion (5.7) and (5.6). With small rearrangements, the equations are:

$$\ddot{\Phi} = \frac{1}{2}\dot{T}^2, \quad (5.19)$$

$$2\dot{\Phi}^2 = \frac{1}{2}\dot{T}^2 + V(T), \quad (5.20)$$

$$\ddot{T} - 2\dot{\Phi}\dot{T} + V'(T) = 0. \quad (5.21)$$

Since $\ddot{\Phi} \geq 0$, the dilaton velocity $\dot{\Phi}(t)$ never decreases. Given that $\dot{\Phi}(t) > 0$ for sufficiently early times (see (5.16)), the dilaton $\Phi(t)$ increases without bound. If the evolution is regular, $\Phi \rightarrow \infty$ as $t \rightarrow \infty$ (the universe takes infinite time to crunch). More generally, the evolution produces a singular point at some finite time for which, as we shall see, both $\dot{\Phi}$ and Φ become infinite. Note also the complete correspondance between the above equations and equations (5.10), (5.11), and (5.12) for an ordinary scalar coupled to gravity. The sets of equations match, up to constants, when we set $H \sim -\dot{\Phi}$. Out of the three equations above, the last two suffice. Taking the time derivative of (5.20) and using (5.21), we find that (5.19) holds as long as $\dot{\Phi} \neq 0$. The rolling of ordinary scalars with negative potentials was studied by Felder *et.al.*[87], who noted that the final state is roughly independent of the shape of the potential. Given the correspondance with dilaton/tachyon rolling, this is also true in our problem.

We derived the final equations (5.21) and (5.20) using a class of initial conditions that implied $H = 0$. These equations, viewed as the original equations with the *ansatz* $H = 0$, allow more general initial conditions. For an initial time t_i we can take arbitrary $T(t_i)$ and $\dot{T}(t_i)$ as long as

$$(\dot{T}^2 + 2V(T))|_{t_i} \geq 0. \quad (5.22)$$

The evolution is fixed by choosing a square-root branch for $\dot{\Phi}$ in (5.20). Since $\dot{\Phi}$ is positive for tachyon-driven rolling, we take

$$2\dot{\Phi} = \sqrt{\dot{T}^2 + 2V(T)}. \quad (5.23)$$

This enables us to rewrite (5.21) as a second-order nonlinear differential equation for the tachyon alone, an equation that is quite convenient for numerical integration:

$$\ddot{T} - \sqrt{\dot{T}^2 + 2V(T)} \dot{T} + V'(T) = 0. \quad (5.24)$$

The general rolling problem can be reduced to the problem of solving a first-order nonlinear differential equation. For this we consider the “energy” E defined as

$$E \equiv \mathcal{E}^2 \equiv \frac{1}{2}\dot{T}^2 + V(T). \quad (5.25)$$

One readily checks that

$$\frac{dE}{dt} = \dot{T}\dot{T}(2\dot{\Phi}). \quad (5.26)$$

Since $\dot{\Phi} > 0$, E can only increase. The desired equation arises by rewriting (5.26) as

$$\frac{dE}{dt} = \pm\sqrt{E-V} \frac{dT}{dt} 2\sqrt{E} \quad \rightarrow \quad \frac{dE}{dT} = \pm 2\sqrt{E(E-V)}. \quad (5.27)$$

This is an equation for $E(T)$. The sign choice arises from solving for \dot{T} in terms of E and V . During evolution the sign must be changed each time \dot{T} goes through zero. We will use the above mostly when $\dot{T} > 0$, so we will take the plus sign. The equation becomes a little simpler in terms of $\mathcal{E} = \sqrt{E}$:

$$\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V}. \quad (5.28)$$

Equipped with $\mathcal{E}(T)$, one finds $T(t)$ by solving the first-order linear equation that follows from (5.25).

A reverse engineering problem can also be solved. Suppose we are given a tachyon rolling solution specified by a function $T(t)$ that has an inverse $t(T)$. It is then possible to find the associated dilaton $\Phi(t)$ and the potential $V(T)$. We use (5.19) to find $\dot{\Phi}(t)$ by integration, and (5.20) to find $V(t)$, which gives the potential $V(t(T))$. As a simple illustration we take the leading solution in (5.14) to be exact: $T(t) = e^{mt}$. Setting integration constants to zero we find

$$T(t) = e^{mt}, \quad \Phi(t) = \frac{1}{8}e^{2mt}, \quad V(T) = m^2 \left(-\frac{1}{2}T^2 + \frac{1}{8}T^4 \right). \quad (5.29)$$

In this solution the crunch happens at infinite string time (but finite Einstein time). Related rolling solutions have been considered using two-dimensional Liouville field theory to provide conformal invariant sigma model with spacetime background fields that typically include a linear dilaton and a constant string metric [67, 68, 89, 70]. In some of these solutions $T(t) = e^{mt}$ and the linear dilaton vanishes. This is unexpected given our analysis, which shows that the dilaton is sourced. It would be interesting to use this discrepancy to find constraints on the form of the effective action for the coupled system of fields.

5.3 Finite-time crunch with negative scalar potentials

We now show that for non-positive potentials $V(T) \leq 0$, if $\dot{\Phi}(t_0) > 0$ for some time t_0 then $\dot{\Phi}(t_*) = \infty$ for some finite time $t_* > t_0$. To do this we combine (5.19) and (5.20) to write

$$-\frac{\ddot{\Phi}}{\dot{\Phi}^2} = -2 + \frac{V(T)}{\dot{\Phi}^2}. \quad (5.30)$$

Integrating both sides of the equation from an initial time t_0 up to a time t we find

$$\frac{1}{\dot{\Phi}(t)} = \frac{1}{\dot{\Phi}(t_0)} - 2(t - t_0) + \int_{t_0}^t \frac{V(T(t'))}{\dot{\Phi}^2(t')} dt'. \quad (5.31)$$

To have a divergent $\dot{\Phi}$ we need the terms on the right-hand side to add up to zero. If we ignore the integral on the right-hand side, the first two terms cancel for $t = t_1$, with $t_1 - t_0 = 1/(2\dot{\Phi}(t_0))$. Since the integral vanishes at $t = t_0$ and can only decrease afterwards, the cancellation will actually occur for a time earlier than t_1 :

$$t_* \leq t_0 + \frac{1}{2\dot{\Phi}(t_0)}. \quad (5.32)$$

This is what we wanted to prove. It follows from (5.23) and $V \leq 0$ that as $\dot{\Phi} \rightarrow \infty$ we also have $\dot{T} \rightarrow \pm\infty$. The time evolution reaches a singular point at finite time.

To understand how the dilaton Φ itself diverges we can do an estimate that proves to be self-consistent. The integral term in (5.31) is assumed to be negligible. This is certainly the case if $V(T)$ is also bounded below since then, the integral is negligible for sufficiently large $\dot{\Phi}$. In fact, we will see that the integral is negligible under far more general circumstances. It then follows that

$$\dot{\Phi}(t) \simeq \frac{1}{2(t_* - t)} \quad \text{and} \quad \dot{T}(t) \simeq \frac{\pm 1}{(t_* - t)}. \quad (5.33)$$

For such solutions the tachyon and dilaton diverge logarithmically:

$$\Phi(t) = -\frac{1}{2} \ln(t_* - t) + \Phi_0 \quad T(t) = \mp \ln(t_* - t) + T_0. \quad (5.34)$$

For any polynomial potential $V(T)$ the integrand in (5.31) is of the form $(t_* - t)^2 V(\ln(t_* - t))$ and goes to zero as we approach collapse, thus justifying our approximations. Note that \dot{T}^2 is much larger than both $V(T)$ and $V'(T)$. This fact alone implies finite time collapse: the tachyon equation (5.24) becomes $\ddot{T} \mp \dot{T}^2 \simeq 0$, whose general solution describes a \dot{T} that diverges at an adjustable finite time. Note that the dilaton prefactor in the spacetime action, $e^{-2\Phi} \rightarrow e^{-2\Phi_0} \cdot (t_* - t)$, vanishes linearly with time as we approach the collapse. The string coupling becomes infinity, the Einstein metric crunches, and the value of the on-shell action (5.3) goes to zero. Since the Einstein metric becomes much smaller than the string metric as the dilaton diverges, the collapse also occurs in finite time in Einstein frame.

5.4 Crunching with arbitrary potentials

In this section we consider rolling solutions for rather general potentials $V(T)$, not necessarily tachyonic. As before, we take $H = 0$ and $\dot{\Phi} > 0$; these conditions ensure that we are dealing with a problem qualitatively related to tachyon-induced rolling. As a warmup we consider the case where the potential is positive and bounded and show that for a sufficiently large initial tachyon velocity the Einstein metric crunches in finite time. We then discuss a related question for more general potentials: is there an initial tachyon velocity $\dot{T} > 0$ for which $T = \infty$ (and crunching) is reached in finite time even if $V(T \rightarrow \infty) \rightarrow \infty$? We discuss a set of tools that enable one to approach this question systematically, at least in a case by case basis. We find that potentials of the form $V(T) \sim \exp(nT)$ with $n \geq 2$ are too steep, and no positive tachyon velocity allows the tachyon to reach $T = \infty$. We also analyze in detail the simple potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$.

Crunching with bounded potentials. We claim that for a bounded potential $0 \leq V(T) < \beta^2$ with bounded derivative $V'(T) < \gamma^2$, there is an initial tachyon velocity $\dot{T}(t = 0)$ for which crunching occurs in finite time. Here is a short proof. Take $\dot{T}(0) = \sqrt{\alpha^2 + 2\beta^2}$ with $\alpha^2 > \gamma^2$. Since the energy E (see (5.26)) cannot decrease in time, for $t > 0$:

$$2E(t) = \dot{T}^2(t) + 2V(T(t)) > 2E(0) = \alpha^2 + 2\beta^2 + 2V(T(0)) \geq \alpha^2 + 2\beta^2. \quad (5.35)$$

Therefore, $\dot{T}^2(t) > \alpha^2 + 2\beta^2 - 2V(T(t)) > \alpha^2$ and, as a result, $\dot{T}^2(t) > \alpha^2$ for all times. The tachyon equation of motion (5.24) then gives

$$\ddot{T}(t) > \sqrt{\alpha^2 + 2V(T)} \alpha - \gamma^2 \geq \alpha^2 - \gamma^2 > 0. \quad (5.36)$$

Since $\ddot{\Phi} = \frac{1}{2}\dot{T}^2$, one finds $\ddot{\Phi}(t) = \dot{T}(t)\ddot{T}(t) > 0$, so $\dot{\Phi}(t)$ is convex and grows without bound. On the other hand, with our bounds, equation (5.31) gives

$$\frac{1}{\dot{\Phi}(t)} < \frac{1}{\dot{\Phi}(t_0)} - \left(2 - \frac{\beta^2}{\dot{\Phi}^2(t_0)}\right)(t - t_0). \quad (5.37)$$

Since $\dot{\Phi}$ grows without bound, there is a time t_0 for which $2 - \frac{\beta^2}{\dot{\Phi}^2(t_0)} > 0$. Then at some time t_1 , the right hand side of equation (5.37) vanishes. Therefore, at some finite time $t_* < t_1$, $\dot{\Phi}(t_*) = \infty$.

General techniques. We now consider a general class of potentials $V(T)$, well-defined for all T , and unbounded above as T grows positive and large. We examine an initial configuration with some fixed value $T(t_0)$ and variable initial velocity $\dot{T}(t_0) > 0$. We wish to find out if the tachyon reaches $T = \infty$ and if it does so in finite time, causing the dilaton to diverge and the Einstein metric to crunch. We find that, typically, there is a critical tachyon velocity for which it takes infinite time to reach $T = \infty$. For velocities larger than critical, $T = \infty$ is reached in finite time. For velocities smaller than critical the tachyon evolution gives a turning point.

Three curves can be defined in the (T, \mathcal{E}) plane and help us understand the integral curves $\mathcal{E}(T)$ that solve our first-order differential equation $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V(T)} \equiv h(T, \mathcal{E})$:

- $h(T, \mathcal{E}) = 0$ is the *turning point* curve.
- $\frac{d}{dT}h(T, \mathcal{E}) = 0$ is the *inflection* curve. It separates a region where the integral curves are convex from a region where they are concave.
- $\mathcal{E}^2 - V(T) = f(T)$, with f specified below, is the *separating* curve. Any integral curve starting above the separating curve will remain above it.

Since $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V} = \frac{1}{\sqrt{2}}|\dot{T}(t)|$ the turning point curve is the locus of points where we get turning points for the tachyon time evolution. If an integral curve hits the turning point curve, the time evolution of the tachyon has a turning point. Moreover, since $\frac{d}{dT}\dot{T} = \frac{\ddot{T}}{T}$, the inflection curve also controls the convexity or concavity of $T(t)$.

Consider the curve $\mathcal{E}^2 - V(T) = f(T)$. On this curve the slope of the integral curve is $\sqrt{f(T)}$. Moreover, the slope of this curve itself is $\frac{f' + V'}{2\sqrt{f+V}}$. In order to be a separating curve we require the former to be larger than the latter:

$$\sqrt{f(T)} \geq \frac{f' + V'}{2\sqrt{f+V}}. \quad (5.38)$$

The equality gives an integral curve – integral curves separate because they cannot cross, but are hard to find. If $V(T) \geq 0$, a suitable $f(T)$ is obtained by setting:

$$\sqrt{f(T)} = \frac{f' + V'}{2\sqrt{f}} \quad \rightarrow \quad 2f - f' = V'. \quad (5.39)$$

For a polynomial $V(T)$, a convenient choice is $f(T) = \sum_{n=1} \frac{1}{2^n} \frac{d^n V}{dT^n}$.

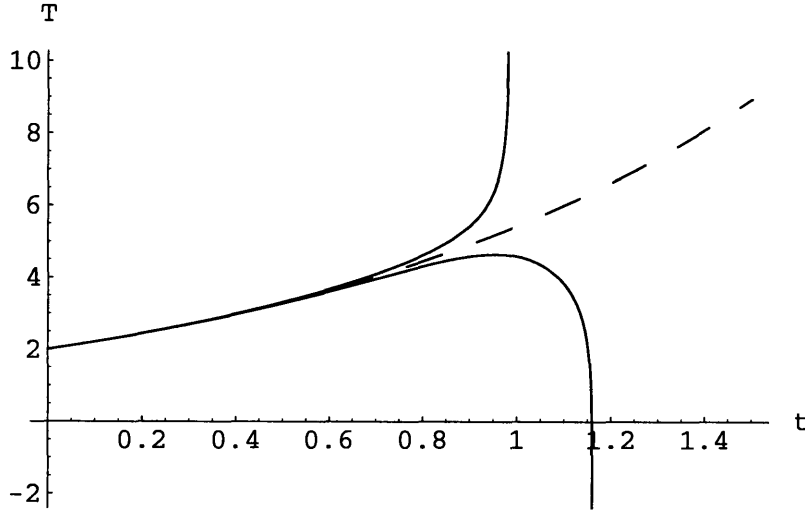


Figure 5.1: Tachyon histories $T(t)$ in the potential $V(T) = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, starting with $T(t=0) = 2$. The critical trajectory is dashed, begins with critical velocity $\dot{T}(0) = 2$, and crunches at infinite time. The solid line turning upwards starts with $\dot{T}(0) = 2.01$ and hits $T = \infty$ at $t \simeq 0.9808$. The solid line turning downward starts with initial velocity $\dot{T}(0) = 1.99$, encounters a turning point, and actually reaches minus infinity at $t \simeq 1.1609$.

A worked out example. We illustrate the above discussion with the potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, which is tachyonic near $T = 0$, vanishes for $T = \pm 2$, and grows arbitrarily large for large $|T|$. We consider arbitrary velocities for a tachyon for which $T = 2$ for $t = 0$. The potential was chosen so that it has an easily obtained solution with critical velocity: $T(t) = 2 \exp(t)$. This is, in fact, the tachyon-induced rolling solution that starts at $T = 0$ for $t = -\infty$. Here $T(0) = 2$, $\dot{T}(0) = 2$, $\mathcal{E}(T = 2) = \sqrt{2}$, and the solution reaches $T = \infty$ at $t = \infty$. We have verified numerically that solutions with larger initial velocity reach $T = \infty$ in finite time, while solutions with lesser initial velocity encounter a turning point. In Fig. 5.1, we show the critical trajectory $T(t)$ and two additional solutions corresponding to initial velocities slightly higher and slightly lower than critical.

For the potential in question, the turning point curve $\mathcal{E}(T) = \sqrt{V(T)}$ lies below the inflection curve $\mathcal{E}(T) = (\frac{V + \sqrt{V^2 + V'^2}}{2})^{1/2}$, which in turn lies below the separating curve ($T \geq 2$) defined with $f(T) = -\frac{1}{16}(1 + 2T - 6T^2 - 4T^3)$. In Fig. 5.2 we plot these three curves along with three solutions. The solution with lowest initial energy has velocity smaller than critical: it crosses the inflection curve and hits the turning point curve. We also show the critical solution and a solution with velocity larger than critical that lies above the separating curve. By construction any solution above the separating curve cannot have a turning point.

Potentials of the form $V(T) = \exp(nT)$. We now show that for $V(T) = \exp(nT)$, with $n \geq 2$, there is no initial (positive) tachyon velocity for which the tachyon can reach $T = \infty$. When $V(T) = \exp(nT)$

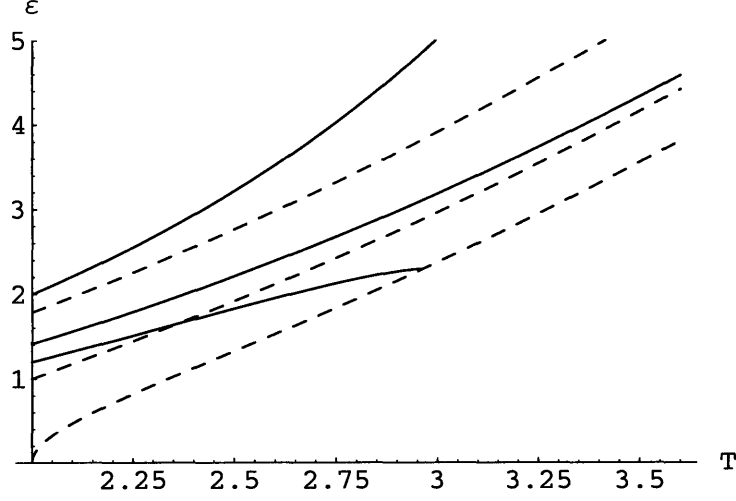


Figure 5.2: Curves and solutions for $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V}$ with potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, plotted in the (T, \mathcal{E}) plane with vertical axis at $T = 2$. From bottom to top, the three dashed curves are: the turning point curve ($\mathcal{E}(T = 2) = 0$), the inflection curve ($\mathcal{E}(2) = 1$), and the separating curve ($\mathcal{E}(2) = \sqrt{51}/4 \simeq 1.7854$). The three solid curves are solutions. From bottom to top they are: a solution with subcritical $\mathcal{E}(2) = 1.20$ that hits the turning point curve, the critical solution, with $\mathcal{E}(2) = \sqrt{2}$, and a solution above the separating curve, with $\mathcal{E}(2) = 2.0$.

the differential equation (5.28) is solvable. Setting $\mathcal{E} = \sqrt{V}g(T)$, we find

$$g' + \frac{V'}{2V}g = \sqrt{g^2 - 1} \quad \rightarrow \quad g' = \sqrt{g^2 - 1} - \frac{n}{2}g. \quad (5.40)$$

We have a turning point at T_* if $g(T_*) = 1$. Dividing both sides of the equation by g and integrating,

$$\ln g(T) = \ln g(T_0) - \frac{n}{2}(T - T_0) + \int_{T_0}^T dT' \left(1 - \frac{1}{g^2(T')}\right)^{1/2} < \ln g(T_0) - \left(\frac{n}{2} - 1\right)(T - T_0). \quad (5.41)$$

For $n > 2$, the right hand side vanishes at some $T_1 > T_0$. There is therefore some $T_* < T_1$ with $g(T_*) = 1$, and thus a turning point, as we wanted to show. The above argument in fact applies for any potential V such that $\frac{V'}{V} > 2$ for sufficiently large T . For $n = 2$, the solution of (5.40) is

$$T = T_0 + \frac{1}{2}(F(g) - F(g_0)), \quad F(g) \equiv \ln(g + \sqrt{g^2 - 1}) - g(g + \sqrt{g^2 - 1}). \quad (5.42)$$

It is readily checked that $F(1) = -1$ and $F(g)$ decreases monotonically for $g > 1$. Therefore, $g(T^*) = 1$ for $T^* = T_0 - \frac{1}{2}(1 + F(g_0)) > T_0$. This proves that all solutions have a turning point.

5.5 Conclusions

Our analysis of tachyon-induced rolling has revealed two general facts: 1) the string metric is constant and, 2) the dilaton rolls toward stronger coupling. These facts match precisely the properties of the

candidate tachyon vacuum identified in [83]. Consider fact one. In the tachyon vacuum both the tachyon and the dilaton take expectation values, and so do an infinite number of massive fields. The string metric, however, is not sourced and need not acquire an expectation value – this is guaranteed by rather general string field theory universality arguments [64, 83]. The cosmological constancy of the string metric appears to be the sigma model version of the universality result. Consider now fact two. It was shown in [83] that the dilaton expectation value in the candidate solution corresponds to stronger string coupling. The qualitative agreement makes it plausible that the rolling solutions discussed here represent rolling towards the tachyon vacuum conjectured in [83]. In the case of open strings, the end-product of the rolling solution is different but somewhat related to the tachyon vacuum. Our rolling solutions represent an Einstein metric big crunch or closed strings at infinite coupling. The tachyon vacuum may represent the disappearance of dynamical spacetime. We feel these two states could be related. The crunch certainly lies beyond the applicability of the action (5.1), which should be supplemented by terms of higher order in α' . The generality of the evolution and the almost complete independence on the details of the tachyon potential¹ suggest to us that the cosmological solutions presented here are relevant, modulo some stringy resolution of the big crunch singularity. It would be rather interesting if the stringy resolution would push the crunch to infinite time. A big crunch, followed by a big bang, is the key element in cyclic universe models [72]. The crunch is induced by a scalar field rolling down a *negative* potential with a steep region – the rest of the potential is largely undetermined. Negative, initially steep potentials, are the hallmark of bulk closed string tachyons. We found that generally a big crunch ensues, although in our case the gravitational part of the solution is carried by the dilaton, and thus the crunch has the alternative interpretation of a closed string theory at infinite coupling. It is tempting to speculate that closed string tachyons may play a role in cyclic universe scenarios – the central difficulty remaining the mysterious transition from a big crunch to a big bang. In such studies it would be useful to focus on tachyonic heterotic models and Type-0 strings.

If the vacuum of the bulk closed string tachyon truly represents the demise of fluctuating spacetime, understanding properly this state and how it fits into a consistent cosmology would give invaluable insight into the mechanisms by which a universe could come into existence. The tachyon vacuum would be roughly imagined to be the state of a universe before the Big Bang.

¹For comments on the specific form of the tachyon potential in closed string theory see Figure 1 and the conclusion section of [83].

Appendix A

Quartic Computations

A.1 The setup

We normalize correlators using $\langle 0|c_{-1}\bar{c}_{-1}c_0^+c_0^-c_1\bar{c}_1|0\rangle = 1$ with $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$. All states in this chapter have zero momentum. For convenience, all spacetime coordinates have been compactified and the volume of spacetime is equal to one. To use results from open string field theory, we note that

$$\langle c(z_1)c(z_2)c(z_3)\bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3)\rangle = -2\langle c(z_1)c(z_2)c(z_3)\rangle_o \cdot \langle \bar{c}(\bar{w}_1)\bar{c}(\bar{w}_2)\bar{c}(\bar{w}_3)\rangle_o, \quad (\text{A.1})$$

since open string field theory uses $\langle c(z_1)c(z_2)c(z_3)\rangle_o = (z_1 - z_2)(z_1 - z_3)(z_2 - z_3)$. Then:

$$\langle c_1\bar{c}_1, c_1\bar{c}_1, c_1\bar{c}_1\rangle = 2 \cdot \langle c_1, c_1, c_1\rangle_o \cdot \langle \bar{c}_1, \bar{c}_1, \bar{c}_1\rangle_o = 2 \cdot \mathcal{R}^3 \cdot \mathcal{R}^3 = 2\mathcal{R}^6, \quad (\text{A.2})$$

where $\mathcal{R} \equiv 1/\rho = 3\sqrt{3}/4 \simeq 1.2990$, and ρ is the mapping radius of the disks in the three-string vertex.

To construct four-string amplitudes we use antighost insertions [26, 30]

$$\mathcal{B} = \sum_{I=1}^4 \sum_{m=-1}^{\infty} (B_m^I b_m^J + \overline{C_m^I} \bar{b}_m^I), \quad \mathcal{B}^\star = \sum_{I=1}^4 \sum_{m=-1}^{\infty} (C_m^I b_m^I + \overline{B_m^I} \bar{b}_m^I), \quad (\text{A.3})$$

where \mathcal{B}^\star is the \star -conjugate of \mathcal{B} . The multilinear function in string field theory is

$$\{\Psi_1, \Psi_2, \Psi_3, \Psi_4\} \equiv \frac{1}{\pi} \int_{\mathcal{V}_{0,4}} dx \wedge dy \langle \Sigma | \mathcal{B} \mathcal{B}^\star | \Psi_1 \rangle | \Psi_2 \rangle | \Psi_3 \rangle | \Psi_4 \rangle. \quad (\text{A.4})$$

The first, second, third, and fourth states are inserted at $0, 1, \xi = x + iy$, and ∞ , respectively. Operationally, the fourth state is inserted at $t = 0$ with $z = 1/t$, where z is the global uniformizer. For further details and explanations the reader should consult [28]. We record that

$$\begin{aligned} B_{-1}^J &= \delta_{3J}/\rho_3, & C_{-1}^J &= 0, \\ B_1^I &= \rho_I \partial \beta_I + \frac{1}{2} \rho_3 \varepsilon_3 \delta_{I3}, & C_1^I &= \rho_I \bar{\partial} \beta_I, \\ B_2^I &= \frac{1}{6} \rho_I^2 \partial (2\beta_I^2 - \varepsilon_I) + \rho_I^2 (-4\delta_I - 2\varepsilon_I \beta_I + 8\beta_I^3) \delta_{3I}, & C_2^I &= \frac{1}{6} \rho_I^2 \bar{\partial} (2\beta_I^2 - \varepsilon_I). \end{aligned} \quad (\text{A.5})$$

Here $\bar{\partial} \equiv \frac{\partial}{\partial \bar{\xi}}$ and $\partial \equiv \frac{\partial}{\partial \xi}$. Since our string fields are annihilated both by b_0 and \bar{b}_0 , the coefficients B_0^I and C_0^I are not needed. Taking note of the vanishing coefficients, we see that for states in the Siegel gauge the antighost factor \mathcal{B} is given by

$$\mathcal{B} = B_{-1}^3 b_{-1}^{(3)} + \sum_{I=1}^4 (B_1^I b_1^I + \overline{C_1^I} \bar{b}_1^I) + \sum_{I=1}^4 (B_2^I b_2^I + \overline{C_2^I} \bar{b}_2^I) + \dots \quad (\text{A.6})$$

The Strebel quadratic differential on the surfaces determines:

$$\beta_1 = \frac{a}{2\xi} - \frac{1}{\xi} - 1, \quad \beta_2 = \frac{a-2\xi}{2(1-\xi)}, \quad \beta_3 = \frac{a-2}{2\xi(\xi-1)}, \quad \beta_4 = \frac{a}{2} - 1 - \xi. \quad (\text{A.7})$$

Here $a(\xi, \bar{\xi})$ is a function that determines the quadratic differential completely. We also have

$$\begin{aligned} \varepsilon_1 &= 2 + \frac{1}{\xi}(a-2) + \frac{1}{\xi^2} \left(2 + a - \frac{5}{8}a^2 \right), \\ \varepsilon_2 &= \frac{-5a^2 + 16\xi(\xi-3) + 8a(\xi+3)}{8(\xi-1)^2}, \\ \varepsilon_3 &= \frac{16 + 8a - 5a^2 + 24(a-2)\xi}{8\xi^2(\xi-1)^2}, \\ \varepsilon_4 &= 2 + a - \frac{5}{8}a^2 - 2\xi + a\xi + 2\xi^2. \end{aligned} \quad (\text{A.8})$$

The function $a(\xi)$ is known numerically to high accuracy for $\xi \in \mathcal{A}$, where \mathcal{A} is a specific subspace of $\mathcal{V}_{0,4}$ described in detail in Figures 3 and 6 of ref. [27]. The full space $\mathcal{V}_{0,4}$ is obtained by acting on \mathcal{A} with the transformations generated by $\xi \rightarrow 1-\xi$ and $\xi \rightarrow 1/\xi$, together with complex conjugation $\xi \rightarrow \bar{\xi}$. In fact $\mathcal{V}_{0,4}$ contains twelve copies of \mathcal{A} . Let $f(\mathcal{A})$ denote the region obtained by mapping each point $\xi \in \mathcal{A}$ to $f(\xi)$. Then $\mathcal{V}_{0,4}$ is composed of the six regions

$$\mathcal{A}, \quad \frac{1}{\mathcal{A}}, \quad 1-\mathcal{A}, \quad \frac{1}{1-\mathcal{A}}, \quad 1-\frac{1}{\mathcal{A}}, \quad \frac{\mathcal{A}}{1-\mathcal{A}}, \quad (\text{A.9})$$

together with their complex conjugates. The values of a in these regions follow from the values of a on \mathcal{A} via the relations

$$a(1-\xi) = 4 - a(\xi), \quad a\left(\frac{1}{\xi}\right) = \frac{a(\xi)}{\xi}, \quad a(\bar{\xi}) = \overline{a(\xi)}. \quad (\text{A.10})$$

For states of the form $|M_i\rangle = \mathcal{O}_i c_1 \bar{c}_1 |0\rangle$, where \mathcal{O}_i is built with matter oscillator, one finds

$$\{M_1, M_2, M_3, M_4\} = -\frac{2}{\pi} \int_{\mathcal{V}_{0,4}} \frac{dx \wedge dy}{(\rho_1 \rho_2 \rho_3 \rho_4)^2} \langle\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle\rangle_{\xi}. \quad (\text{A.11})$$

Here $\langle\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle\rangle_{\xi} \equiv \langle h_1 \circ \mathcal{O}_1 h_2 \circ \mathcal{O}_2 h_3 \circ \mathcal{O}_3 h_4 \circ \mathcal{O}_4 \rangle_{\Sigma_{\xi}}$, where the right-hand side is a matter correlator computed after the local operators \mathcal{O}_i have been mapped to the uniformizer.

A.2 Couplings of dilatons and tachyons

Elementary contribution to $t^3 d$. We insert the dilaton on the moving puncture to make the integration identical over each of the 12 regions of the moduli space. Since all the states inserted on the fixed punctures have ghost oscillators $c_1 \bar{c}_1$, the antighost factor $\mathcal{B}\mathcal{B}^*$ is only supported on the moving puncture:

$$\mathcal{B}\mathcal{B}^*(c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})^{(3)}|0\rangle = -(B_{-1}^3 C_1^3 + \overline{B_{-1}^3 C_1^3})|0\rangle = -(\bar{\partial}\beta_3 + \partial\bar{\beta}_3)|0\rangle. \quad (\text{A.12})$$

There are no matter operators, thus the correlator just involves the ghosts:

$$\begin{aligned} \langle \Sigma_P | \mathcal{B}\mathcal{B}^* | T \rangle | T \rangle | D \rangle | T \rangle &= -(\bar{\partial}\beta_3 + \partial\bar{\beta}_3) \langle (c_1 \bar{c}_1)^{(1)} (c_1 \bar{c}_1)^{(2)} (c_1 \bar{c}_1)^{(4)} \rangle \\ &= -(\bar{\partial}\beta_3 + \partial\bar{\beta}_3) \frac{2}{(\rho_1 \rho_2 \rho_4)^2}. \end{aligned} \quad (\text{A.13})$$

Using (A.4), the amplitude is:

$$\{T^3 D\} = -\frac{24}{\pi} \int_{\mathcal{A}} dx dy (\bar{\partial}\beta_3 + \partial\bar{\beta}_3) \frac{1}{(\rho_1 \rho_2 \rho_4)^2} = 23.2323. \quad (\text{A.14})$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!} \{T^3 D\} t^3 d = 3.8721 t^3 d$.

Elementary contribution to $t^2 d^2$. We insert the dilatons at $z_2 = 1$ and $z_3 = \xi$. The amplitude to be integrated is identical to the ghost part of the amplitude for the quartic interaction $a^2 d^2$, as given in [28], equation (4.9):

$$\langle \Sigma | \mathcal{B}\mathcal{B}^* | T \rangle | D \rangle | D \rangle | T \rangle = \frac{2}{(\rho_1 \rho_4)^2} \left(\bar{\partial}\beta_2 \partial(\bar{\xi}\bar{\beta}_3) - \partial\beta_2 \bar{\partial}(\bar{\xi}\bar{\beta}_3) + \text{*conj} \right). \quad (\text{A.15})$$

The four-point amplitude is then

$$\{T^2 D^2\} = \frac{4}{\pi} \int_{\mathcal{V}_{0,4}} \frac{dx dy}{(\rho_1 \rho_4)^2} \text{Re} \left(\bar{\partial}\beta_2 \partial(\bar{\xi}\bar{\beta}_3) - \partial\beta_2 \bar{\partial}(\bar{\xi}\bar{\beta}_3) \right). \quad (\text{A.16})$$

Since we have the same states on punctures one and four, and these punctures are exchanged by the transformation $z \rightarrow 1/z$, the integral over \mathcal{A} gives the same contribution as the integral over $1/\mathcal{A}$. The conjugation properties of the amplitude also imply that $\bar{\mathcal{A}}$ contributes the same as \mathcal{A} . Consequently, the four regions \mathcal{A} , $1/\mathcal{A}$, $\bar{\mathcal{A}}$, and $1/\bar{\mathcal{A}}$ all give the same contribution. To get the full amplitude we must multiply the contributions of \mathcal{A} , of $1 - \mathcal{A}$, and $1 - 1/\mathcal{A}$ by four:

$$\{T^2 D^2\} = 4 \cdot \frac{4}{\pi} \left[\int_{\mathcal{A}} + \int_{1-\mathcal{A}} + \int_{1-1/\mathcal{A}} \right] \frac{dx dy}{(\rho_1 \rho_4)^2} \text{Re} \left(\bar{\partial}\beta_2 \partial(\bar{\xi}\bar{\beta}_3) - \partial\beta_2 \bar{\partial}(\bar{\xi}\bar{\beta}_3) \right). \quad (\text{A.17})$$

The transformation laws given in Appendix B of [28] allow one to rewrite the second and third integrals as integrals over \mathcal{A} , where they can be easily evaluated. We find

$$\{T^2 D^2\} = 4 \cdot (-0.2410 + 0.4031 + 1.2065) = 5.4726. \quad (\text{A.18})$$

The contribution to potential is $\kappa^2 V = \frac{6}{4!} \{T^2 D^2\} t^2 d^2 = 1.3682 t^2 d^2$.

Elementary contribution to td^3 . The tachyon field is inserted at $z_3 = \xi$. We then have

$$\begin{aligned} & \mathcal{B} \mathcal{B}^* (c_1 \bar{c}_1)^{(3)} D^{(1)} D^{(2)} D^{(4)} |0\rangle \\ &= \left\{ B_{-1}^3 b_{-1}^{(3)} + \sum_{J \neq 3} (B_1^J b_1^{(J)} + \overline{C_1^J} \bar{b}_1^{(J)}) \right\} \left\{ \overline{B_{-1}^3} \bar{b}_{-1}^{(3)} + \sum_{J \neq 3} (\overline{B_1^J} \bar{b}_1^{(J)} + C_1^J b_1^{(J)}) \right\} (c_1 \bar{c}_1)^{(3)} D^{(1)} D^{(2)} D^{(4)} |0\rangle \\ &= \sum_{I \neq J \neq K \neq 3} \left(\frac{1}{2} B_{-1}^3 C_1^I D^{(J)} D^{(K)} c_1^{(I)} \bar{c}_1^{(3)} + B_1^I C_1^J (c_1 \bar{c}_1)^{(3)} c_1^{(I)} c_1^{(J)} (\bar{c}_{-1} \bar{c}_1)^{(K)} \right) |0\rangle - \star\text{-conj}. \end{aligned} \quad (\text{A.19})$$

Therefore, the correlator $\mathcal{C}_{td^3} = \langle \Sigma | \mathcal{B} \mathcal{B}^* T D^3 | 0 \rangle$ is:

$$\mathcal{C}_{td^3} = \sum_{I \neq J \neq K \neq 3} \left\langle -B_{-1}^3 C_1^I (\bar{c}_{-1} \bar{c}_1)^{(J)} (c_{-1} c_1)^{(K)} c_1^{(I)} \bar{c}_1^{(3)} + B_1^I C_1^J (\bar{c}_{-1} \bar{c}_1)^{(K)} c_1^{(I)} c_1^{(J)} (c_1 \bar{c}_1)^{(3)} \right\rangle + \star\text{-conj}.$$

Factorizing into holomorphic and antiholomorphic parts we get

$$\mathcal{C}_{td^3} = 2 \sum_{I \neq J \neq K \neq 3} \left(B_{-1}^3 C_1^I B_{KI} (B_{J3})^* - B_1^I C_1^J D_{IJ} (B_{K3})^* \right) + \star\text{-conj}, \quad (\text{A.20})$$

where $B_{IJ} \equiv \langle (c_{-1} c_1)^{(I)}, c_1^{(J)} \rangle$ was introduced and evaluated in [28], eqns. (4.18), (4.20), and (4.21). Additionally,

$$D_{IJ} \equiv \langle c_1^{(I)}, c_1^{(J)}, c_1^{(3)} \rangle = \frac{z_{IJ} z_{I3} z_{J3}}{\rho_I \rho_J \rho_3}, \quad D_{I4} = -D_{4I} = \frac{z_{I3}}{\rho_I \rho_3 \rho_4}, \quad I, J \neq 4. \quad (\text{A.21})$$

The full amplitude is

$$\{TD^3\} = \frac{12}{\pi} \int_{\mathcal{A}} dx dy \mathcal{C}_{td^3} = -5.7168. \quad (\text{A.22})$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!} \{TD^3\} t d^3 = -0.9528 t d^3$.

A.3 Couplings of tachyon to massive fields

In all cases the massive field will be inserted on the moving puncture $z_3 = \xi$.

Elementary contribution to $t^3 f_1$. With $F_1 \equiv c_{-1} \bar{c}_{-1}$ inserted at $z_3 = \xi$ we find:

$$\mathcal{B} \mathcal{B}^* (c_{-1} \bar{c}_{-1})^{(3)} |0\rangle = (C_1^3 \overline{C_1^3} - B_1^3 \overline{B_1^3}) |0\rangle. \quad (\text{A.23})$$

$$\{T^3 F_1\} = \frac{12}{\pi} \int_{\mathcal{A}} dx dy \frac{2}{(\rho_1 \rho_2 \rho_4)^2} (C_1^3 \overline{C_1^3} - B_1^3 \overline{B_1^3}) = -2.6261. \quad (\text{A.24})$$

The contribution to the potential is: $\kappa^2 V = \frac{4}{4!} \{T^3 F_1\} t^3 f_1 = -0.4377 t^3 f_1$.

Elementary contribution to $t^3 f_2$. With $F_2 \equiv c_1 \bar{c}_1 L_{-2} \bar{L}_{-2}$ at $z_3 = \xi$, the ghost part is that of the four-tachyon amplitude (eqn. (3.34) of [28]). With $w = 0$ corresponding to $z = z_3$, and $S(z, w)$ denoting the Schwarzian derivative, the holomorphic matter correlator is:

$$\langle L_{-2}^{(3)} \rangle = \langle T^{(3)}(w = 0) \rangle = \rho_3^2 \langle T(z_3) \rangle + \frac{26}{12} S(z, w) = \frac{13}{6} \rho_3^2 (2\beta_3^2 - \varepsilon_3). \quad (\text{A.25})$$

Therefore, the amplitude is

$$\{T^3 F_2\} = -\frac{24}{\pi} \int_{\mathcal{A}} \frac{dxdy}{(\rho_1 \rho_2 \rho_3 \rho_4)^2} \left| \frac{13}{6} \rho_3^2 (2\beta_3^2 - \varepsilon_3) \right|^2 = -337.571. \quad (\text{A.26})$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!} \{T^3 F_2\} t^3 f_2 = -56.262 t^3 f_2$.

Elementary contribution to $t^3 f_3$. With $L_{-2} c_1 \bar{c}_{-1}$ inserted at $z_3 = \xi$ we find

$$B B^* (c_1 \bar{c}_{-1})^{(3)} |0\rangle = -B_{-1}^3 \overline{B_1^3} |0\rangle. \quad (\text{A.27})$$

$$C_{t^3 f_3} \equiv \langle \Sigma | B B^* T T (c_1 \bar{c}_{-1})^{(3)} T |0\rangle \cdot \langle L_{-2}^{(3)} \rangle = -\frac{2B_{-1}^3 \overline{B_1^3}}{(\rho_1 \rho_2 \rho_4)^2} \cdot \frac{13}{6} \rho_3^2 (2\beta_3^2 - \varepsilon_3). \quad (\text{A.28})$$

With $F_3 \equiv L_{-2} c_1 \bar{c}_{-1} + c_{-1} \bar{L}_{-2} \bar{c}_1$, the string amplitude relevant to $t^3 f_3$ is:

$$\{T^3 F_3\} = \frac{12}{\pi} \int_{\mathcal{A}} dxdy (C_{t^3 f_3} + C_{t^3 f_3}^*) = 78.1432. \quad (\text{A.29})$$

The contribution to the potential is: $\kappa^2 V = \frac{4}{4!} \{T^3 F_3\} t^3 f_3 = 13.024 t^3 f_3$.

Elementary contribution to $t^3 g_1$. With $b_{-2} c_1 \bar{c}_{-2} \bar{c}_1$ at $z_3 = \xi$, one finds

$$B B^* (b_{-2} c_1 \bar{c}_{-2} \bar{c}_1)^{(3)} |0\rangle = \overline{C_2^3} \overline{B_{-1}^3} (c_1 b_{-2})^{(3)} |0\rangle. \quad (\text{A.30})$$

The state $c_1 b_{-2} |0\rangle$ is created by the non-primary ghost current $j(z) = cb(z)$ by acting on the vacuum. For the ghost current

$$j(w) = j(z) \frac{dz}{dw} - \frac{3}{2} \frac{z''}{z'} \quad \rightarrow \quad j(w=0) = \rho_3 (j(z_3) - 3\beta_3). \quad (\text{A.31})$$

We thus have the correlator:

$$\begin{aligned} C_{t^3 g_1} &\equiv \langle \Sigma | B B^* T T (b_{-2} c_1 \bar{c}_{-2} \bar{c}_1)^{(3)} T |0\rangle \\ &= \overline{C_2^3} \overline{B_{-1}^3} \frac{1}{(\rho_1 \rho_2 \rho_4)^2} \left\langle c \bar{c}(0) c \bar{c}(1) \rho_3 (j(z_3) - 3\beta_3) c \bar{c}(t=0) \right\rangle \\ &= \overline{C_2^3} \overline{B_{-1}^3} \frac{\rho_3}{(\rho_1 \rho_2 \rho_4)^2} \cdot 2 \left(\frac{1}{\xi} + \frac{1}{\xi-1} - 3\beta_3 \right). \end{aligned} \quad (\text{A.32})$$

With $G_1 \equiv b_{-2} c_1 \bar{c}_{-2} \bar{c}_1 - c_{-2} c_1 \bar{b}_{-2} \bar{c}_1$, the amplitude relevant for the $t^3 g_1$ coupling is

$$\{T^3 G_1\} = \frac{12}{\pi} \int_{\mathcal{A}} dxdy (C_{t^3 g_1} + C_{t^3 g_1}^*) = 1.6350. \quad (\text{A.33})$$

The contribution to the potential is $\kappa^2 V = \frac{4}{4!} \{T^3 G_1\} t^3 g_1 = 0.2725 t^3 g_1$.

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