Information Based Adaptive Robotic Exploration

Presented by Morten Rufus Blas

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- Motivation
- Introduction
- Related work
- Defining problem and model
- Solution:
  - Minimizing localization error
  - Maximize gain in explored map
  - Combined Information Utilities
  - Integrated Adaptive Information-based Exploration Algorithm
- Results
- Conclusion
  - Novelty
  - Problems
  - Extensions

Author: Morten Rufus Blas,
April 2004
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Motivation

SLAM:
- “There is little value in a robot exploring and mapping new areas when it has no idea of how accurately it knows its own location.”
- Come up with an algorithm to adapt controls to do better exploration.
Introduction

They attempt to maximize the accuracy and speed of their map building process.

- How well does the robot know its pose?
- How well have different areas been explored?

In this paper:
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Related work


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Defining problem and model

- Problem:
  - Optimize control step in order to:
    - Minimize localization error.
    - Maximize gain in explored map.

- Model:
  - Solve problem by maximizing information gain.
Defining problem and model

- We will be using:
  - EKF to model localization (Extended Kalman Filter).
  - OG to represent map (Occupation Grid).
  - Entropy map (more about this later).

Defining problem and model

A set of possible actions

- State estimate
  - Info. in state estimate
  - Info. gain in map
    - Composite Utility
    - Select most informative action
Solution: Minimizing localization error

- Localization is linked to two uncertainties:
  - Measurement,
  - And navigational uncertainty.
- Adaptively choose actions to maximize information about:
  - Robot position.
  - Feature positions (the map).
Solution: Minimizing localization error

- This can be modeled using a cost function $C(P)$:

$$C(P) = \pi \prod_j \sqrt{\lambda_j(P_{xx})} + \pi \sum_{i=1}^{n_v} \prod_j \sqrt{\lambda_j(P_{ii})}$$

$$= \pi \sqrt{\det(P_{xx})} + \pi \sum_{i=1}^{n_v} \sqrt{\det(P_{ii})} \quad (11)$$

- Maximizing information about a state estimate is equivalent to minimizing the determinant of the corresponding covariance matrix.

- $C(P)$ represents the sum of the uncertainty ellipses of both features and robot after the expected observation from the predicted state.
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Solution: Maximize gain in explored map

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<tr>
<th>Entropy map:</th>
<th>Occupation Grid:</th>
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<tr>
<td>0.0</td>
<td>1.0 (OCC)</td>
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Solution: Maximize gain in explored map

- The a priori entropy at time $t_k$ for grid cell $i$:

$$H_{k,i} = - E[\ln P_i(x_i)] = - \sum_{x_i \in X_i} P_i(x_i) \ln P_i(x_i)$$

- Given two possible states (OCC, EMP) for OG map this becomes:

$$H_{k,i} = - P_i(OCC) \ln P_i(OCC) - P_i(EMP) \ln P_i(EMP)$$

- So for unexplored cell at time $t_k$:

$$H_k = -0.5 \ln 0.5 - 0.5 \ln 0.5$$

$$= 0.693$$

- For occupied explored cell at $t_k$:

$$H_k = -1 \ln 1 - 0$$

$$= 0$$

- Analogous for empty explored cell.
Solution: Maximize gain in explored map

- Expected mutual information gain for cell i:

\[
\hat{I}_i(x_i) \equiv -E \left[ \ln \frac{P_i(x_i | z_k)}{P_i(x_i)} \right] = H_i - \overline{H}_i(x_i | z_k)
\]

Information gain = current entropy – new predicted entropy

Solution: Maximize gain in explored map

- Mean conditional entropy over all possible observations:

\[
\overline{H}_i = E[H_i(z_k)] = \int H_i(z_k) P_i(z_k) dz_k
\]

- It is the expectation of entropy left after an observation.
Solution: Maximize gain in explored map

- Conditional entropy for cell \( i \) after observation \( z_k \) at time \( t_k \):
  \[
  H_i(z_k) = - E[\ln P_i(x_i|z_k)] = - \sum_{x_i \in X_i} P_i(x_i|z_k) \ln P_i(x_i|z_k).
  \]

- Where Bayes rule says:
  \[
  P_i(x_i|z_k) = \frac{P_i(z_k|x_i)P_i(x_i)}{P_i(z_k)}.
  \]

- Using our two states (OCC, EMP) the conditional entropy can be rewritten as:
  \[
  H_{k,i}(z_k) = -P_i(\text{OCC} | z_k) \ln P_i(\text{OCC} | z_k) - P_i(\text{EMP} | z_k) \ln P_i(\text{EMP} | z_k)
  \]

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Information gain going up = 0.693 – 0.0
Going left = 0.693 – 0.0
Going down = 0.0 – 0.0

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Solution: Maximize gain in explored map

Entrophy map:  

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Solution: Maximize gain in explored map

- Total expected information gain from doing a specific action:

\[
\hat{I}_{S_j}(x_i|z_k) = \sum_{i \in S_j} I_i(x_i|z_k) = \text{sum of information gain for each explored cell}
\]

- Where S_j are the cells covered by scan.

- After you have done an action you update entropy map with measurements.
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Combined Information Utilities

- Constructed by linear combination:

\[
U_k = I_{\text{composite}}(x, x_c, u_j(k)) = w_1 I_{\text{SLAM}}(x, u_j(k)) + w_2 I_{\text{OG}}(x_c, u_j(k))
\]

\[
w_1(k) = \alpha I_{\text{SLAM}}_{\text{MAX}}(k) \quad w_2 = (1 - \alpha) I_{\text{OG}}_{\text{MAX}}
\]

- SLAM_{\text{MAX}} is an upper bound for the SLAM covariance matrix given a number of landmarks.
- OG_{\text{MAX}} is total information of a perfectly known OG map.
- Increasing alpha increases accuracy of OG map. Reducing it increases amount of exploration.
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Integrated Adaptive Information-based Exploration Algorithm

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Conclusion: Novelty

- Present an information based approach for exploration.
- Present a scheme for combining different types of information.
- Outline the Integrated Adaptive Information-based Exploration Algorithm
- Tests on an actual robot indicate the validity of these approaches.

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Conclusion: Problems

☐ Local Minima:
  ■ They claim it is robust, but is it?

☐ Global optimization:
  ■ Can be used in multi-step solutions such as path planning but:
    ☐ Computational costs grows very rapidly with amount of look-ahead.

☐ No notion of “closing the loop”.

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Conclusion: Extensions

- Can easily be extended with other types of information metrics.
- It is certainly interesting to extend this for multi-robot systems.
- Further reading/different approaches: