6.045J/18.400J: Automata, Computability and Complexity Prof. Ron Rivest

Handout 10: Quiz 2

Write your full name on each page.

Name: ____

Problem 1: Short Answer: (3 points each)

1. Give a *new* undecidable language, and a short proof that it is, in fact, undecidable. (A "new" language is one neither presented nor discussed in lecture, your recitation section, or the text.)

2. True or False: Every subset of every regular language is recognizable. Briefly explain your answer.

3. True or False: The language $A = \left\{ \langle M \rangle \ : \ L(M) \text{ is recognizable or } \overline{L(M)} \text{ is recognizable} \right\}$ is undecidable. (Note that both $L(M)$ and $\overline{L(M)}$ can be recognizable (e.g. if $L(M) = \emptyset$).) Briefly explain your answer.

4. Consider the machine:

   $M =$ on input w
   (1) Obtain, by recursion theorem, $\langle M \rangle$
   (2) Simulate $\langle M \rangle$ on w.
   (3) If $\langle M \rangle$ accepts w, reject.
   (4) If $\langle M \rangle$ rejects w, accept.

   Can such a machine exist? If so, what is its language? If not, why not? (What contradiction results?)

5. True or False: If A and B are languages such that $A \leq_m (A \cup B)$ where B is decidable, then A is decidable. Briefly explain your answer.

Name:

Problem 2: **(15 points)** Prove that there exists a Turing Machine M that accepts exactly one string: $\langle M \rangle$, the encoding of M. For the purposes of this problem, assume that each Turing machine A has a unique representation $\langle A \rangle$. (Hint: Recursion Theorem)

Name:

Problem 3: **(15 points)** Let

$$L = \{\langle M_1, M_2\rangle \ : \ M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \subseteq L(M_2)\}.$$

Show that L is undecidable.

Name:

Problem 4: **(15 points)** Let A and B be two disjoint languages. We say that a language C *separates* A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that if A and B are disjoint languages and both are co-recognizable, then A and B are separated by some decidable language C. (Hint: consider running the enumerators for $\overline{A}$ and $\overline{B}$ in parallel.)

Name:

Problem 5: **(15 points)** A *right/reset machine* (RRM) is like an ordinary Turing machine (with one semi-infinite tape), except that at each step the head can either

- move right one square, or
- move back to the leftmost square (a "reset" of the head).

This problem has three parts. In any part, you may assume the result of previous parts:

1. Suppose that the tape alphabet of a RRM contains two types of characters: letters, and letters wearing a hat ˆ. Suppose that exactly one cell in the tape contains a letter with a hat. Show how to move the hat one cell right. (That is, keep the letters the same, moving only the hat.)
2. Suppose that exactly one cell in the tape contains a letter with a hat. Show how to move the hat one cell left. You may expand the tape alphabet to include a second kind of hat ˜ if you wish.
3. Prove that RRMs are as powerful as Turing Machines.