13.00 Introduction to Ocean Science and Technology
Problem Set 1 Solutions

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1. $\phi = xy^2z^3 + \cos(y^2+z^2)$

(a.) $\vec{\nabla}\phi = y^2z^3\hat{\mathbf{i}} + 2xyz^3 - 2y\sin(y^2+z^2)\hat{\mathbf{j}} + 3xy^2z^2 - 2z\sin(y^2+z^2)\hat{\mathbf{k}}$

(b.) $\vec{\nabla}\cdot\vec{\nabla}\phi = 2xy^3 + 6xy^2z - 4\sin(y^2+z^2) - 4(y^2+z^2)\cos(y^2+z^2)$

(c.) $\vec{\nabla}\phi\cdot\vec{\nabla}\phi = y^4z^6 + (x2yz^3 - 2y\sin(y^2+z^2))^2 + (3xy^2z^2 - 2z\sin(y^2+z^2))^2$

(d.) $\vec{\nabla}\times\vec{\nabla}\phi = 0$

2. From 3/21 to 9/20, the increase in the heat content of the water column between the surface and some depth is:

$$Q = \int_0^z c_p\rho[T_{9/20} - T_{3/21}]dz' = c_p\rho\int_0^z [T_{9/20} - T_{3/21}]dz'$$

Read the temperatures from the graphs at 10 m intervals and integrate the difference between the 9/20 values and the 3/21 values using Simpson's Rule:

Depth (m)	Simpson Multiplier	9/20	3/21	Δ T	SM$*\Delta T$
0	1	9.5	5.3	4.2	4.2
10	4	9.4	5.1	4.3	17.2
20	2	9.3	4.9	4.4	8.8
30	4	9.2	4.8	4.4	17.6
40	2	8.6	4.7	3.9	7.8
50	4	6.2	4.6	1.6	6.4
60	2	5.1	4.5	0.6	1.2
70	4	4.7	4.5	0.2	0.8
80	2	4.4	4.4	0.0	0.0
90	4	4.4	4.4	0.0	0.0
100	1	4.3	4.3	0.0	0.0
	Average:	6.83	4.68	Sum:	64.0

Then Q is given by:

$$\begin{aligned} Q &= \frac{\Delta z}{3} c_p\rho\Sigma(\text{Simpson Multiplier}) * \Delta T) \\ &= \frac{10}{3}(4.185\times 10^3)(10^3)(64.0) \end{aligned}$$

$$= \quad 893 \times 10^6 \text{ J/m}^2 \tag{1}$$

Note that we have taken the upper limit of the integral as $z = 100$ m. It does not matter what value we use, just so long as it is equal to or greater than 80 m, since the temperature does not change below this depth.

One could use another method for integrating the temperature difference, for example a simple trapezoidal rule, or even use the difference of the averages, which are given in the table above. Simpson's Rule is so easy to use, however, and it gives so much more accurate answers than, say, a trapezoidal rule that one is usually well-advised to use Simpson's Rule.

3. The density can be calculated either using matlab with the function `sw_dens` or by using the tables in the notes. The steps in the calculation of densities using the tables are as follows:

	3/21	9/20
Mean T in column	4.7	6.8
σ_t base (from table)	23.75	23.51
f (from table)	0.794	0.789
σ_t (calculated)	27.72	27.46
ρ kg/m^3	1,027.72	1,027.46

The value for σ_t is calculated using the instructions for Table A-2,

$$\sigma_t = \sigma_t - base + f \times (S - 30.0)$$

The water column will expand in inverse proportion to the density change. Thus the column that has height 100 m on 3/21 will have a height on 9/20 of:

$$100 \times \left(\frac{1027.72}{1027.46}\right) = 100.025 \text{ meters.}$$

The height of the column (and thus the ocean level) will increase by 2.5 cm.

4. The figure below shows the x and y axes superimposed on the figure previously given.

From the lecture notes (p. 140), we find that the average radius of the Earth is 6.367×10^6 meters. From the temperature data provided, we have T values at intervals of 0.4° in both the East/West and the North/South directions. In the North/South (y) direction, a 0.4° arc has length

$$\Delta y = 2\pi \times 6.367 \times 10^6 \times \frac{0.4}{360} = 44,500 \text{ meters}$$

In the East/West direction, the distance around the world is reduced by $\cos\phi$, where ϕ is the latitude. The mean value of the latitude for our data is 32.2° North. So we have:

$$\Delta x = 2\pi \times 6.367 \times 10^6 \times \frac{0.4}{360} \times \cos 32.2 = 37,600 \text{ meters}$$

In both the x and y directions, the change in T is -2°C over these distances. Thus

$$\frac{\partial T}{\partial x} \approx \frac{\Delta T}{\Delta x} = \frac{-2°\text{C}}{37,600 \text{ m}} = -5.32 \times 10^{-5} \text{ °C/meter};$$

$$\frac{\partial T}{\partial y} \approx \frac{\Delta T}{\Delta y} = \frac{-2°\text{C}}{44,500 \text{ m}} = -4.49 \times 10^{-5} \text{ °C/meter}.$$

The magnitude and direction of the gradient vector are

$$|\vec{\nabla} T| = \sqrt{(\frac{\partial T}{\partial x})^2 + (\frac{\partial T}{\partial y})^2} = 7.0 \times 10^{-5} \text{ °C/meter};$$

$$\tan^{-1}\left(\frac{\partial T/\partial y}{\partial T/\partial x}\right) = \tan^{-1}\left(\frac{-4.49 \times 10^{-5}}{-5.32 \times 10^{-5}}\right) = -140°.$$

The gradient vector must be perpendicular to lines of constant T. In the figure above, this is a vector pointing toward the lower right. If the length scales were identical, the vector would point downwards at 45°. Because the two scales are not identical, the direction is shifted by about 5°.