13.00 Introduction to Ocean Science and Technology
Problem set 3

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1. Assuming that the dynamic viscosity μ is constant and the fluid is incompressible, perform the substitution of the constitutive relations (Equations 5.8a-f on page 35 of the notes) into Equations 5.7a-c (also on page 35) to yield the Navier-Stokes Equations (Equations 5.9a-c on page 35.)

Equations 5.7a-c are repeated here for your convenience:

$$\rho \frac{Du}{Dt} = \rho(\vec{i} \cdot \vec{g}) + \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right)$$

$$\rho \frac{Dv}{Dt} = \rho(\vec{j} \cdot \vec{g}) + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right)$$

$$\rho \frac{Dw}{Dt} = \rho(\vec{k} \cdot \vec{g}) + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right)$$

2. Figure 5-3 in the lecture notes shows what is often called *Couette flow.* It is assumed that the plates are very, very long in the x direction, so that there is no variation in any variable in the x direction. Also, there is no flow in the z direction and nothing varies in the z direction. Note that the solution given in the notes, $u(y) = Uy/h$, has the property that there is no slippage between the fluid and the plates, i.e., the fluid in contact with the plates has the same velocity as the plates (it "sticks" to the plates):

$$u = \frac{Uy}{h} \Longrightarrow \begin{cases} u = 0, \ y = 0 \\ u = U, \ y = h. \end{cases}$$

This is an essential *boundary condition* that must always be satisfied by a viscous fluid.

(a) Using the Navier-Stokes equations, show that the pressure in such a Couette flow must be a constant everywhere. [Ignore any effects of gravity.]

(b) Now suppose we impose a pressure gradient on the Couette flow such that $\frac{\partial p}{\partial x} = K$, a constant. What is then the velocity distribution? Sketch it for several values of K, both positive and negative. [Be sure your solution satisfies the boundary conditions on the plates. Continue to ignore the effects of gravity.]

(c) For the Couette flow with pressure gradient, what is the shear stress τ_{yx} on each plate? In a sketch, show its direction for postive and negative values of K.

Suggestion: Read the first three paragraphs at the heading **Surface Force** on pages 33-44 of the notes. Figure 5-1 may be especially helpful.

3. A baseball is travelling north for a distance of 60 feet at a velocity of 50 miles per hour at Fenway Park in Boston.

(a.) Calculate the direction and magnitude of the deflection of the baseball due to the Coriolis acceleration.

(b.) How does your answer to part (a) change if the ball is thrown east? south? west?

(c.) How does your answer to part (a) change if the ball was thrown in Wellington, New Zealand?

(d.) How does your answer to part (a) change if the ball was thrown in Quito, Ecuador?

(e.) How does your answer to part (a) change if instead the ball is thrown at 100 miles per hour?

Note: The approximate latitude of Boston is 42°N, the approximate latitude of Wellington, New Zealand is 42°S, and Quito lies approximately on the Equator.