Fall 2002 13.00 Problem Set 5 — Answers

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1. (a) The Ekman transport is at right angles to the wind direction. Since we are considering the Southern Hemisphere, the water motion is to the left of the wind, i.e., north.

(b) The Antarctic Continent blocks the supply of surface water for the Ekman transport. Thus there is upwelling near the coast, and the water surface slopes upward toward the north (away from the coast). The pressure is higher in the region where the surface is elevated (the north), and so the pressure gradient is from south to north.

(c) The surface water is warmer than the water below it. Since the surface water is driven north and replaced near the continent with colder water from below, density near the surface decreases from south to north, i.e., the gradient is from north to south.

(d) In the Southern Hemisphere, a geostrophic flow moves such that the higher water is on the left. So the geostrophic current associated with the Ekman transport is from west to east, i.e., in the same direction as the wind.

2. Calculate the vorticity, $\vec{\omega}$

$$\vec{\omega} = \vec{k}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) = C\vec{k}\left[\frac{x^2 + y^2 - 2x^2 + x^2 + y^2 - 2y^2}{(x^2 + y^2)^2}\right] = 0$$

Hence, this is an irrotational flow.

Given $\phi(x, y) = C\tan^{-1}(\frac{y}{x})$,

$$\frac{\partial \phi}{\partial x} = \frac{-Cy}{x^2 + y^2} = u,$$
$$\frac{\partial \phi}{\partial y} = \frac{Cx}{x^2 + y^2} = v,$$

as required

3. Calculate the vorticity, $\vec{\omega}$

$$\vec{\omega} = \vec{k}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) = \vec{k}(2E) \neq 0$$

Hence this flow is not irrotational.

$$|\vec{u}| = \sqrt{u^2 + v^2} = Er.$$

In terms of θ,

$$\frac{D\theta}{Dt} = \frac{|u|}{r} = E,$$

which is independent of r. Hence the fluid is rotating as a solid block.

One way to create this would be to rotate a beaker of water for a long time.

4. A ship 140 meters long steams at a velocity V in deep water into a series of approaching waves. It is noticed that successive wave crests pass under the bow at intervals of 6 seconds and that the wavelength is just equal to the length of the ship. What is the velocity V in meters per second?

$$\lambda = 140 \text{ m}.$$

$$c_p = \sqrt{\frac{g\lambda}{2\pi}} = 14.77 \text{ m/sec}.$$

From the time when one wave crest hits the bow until time when the next wave crest hits the bow, ship moves the distance $D_S = V \cdot (6 \text{ seconds})$. The wave crest moves the distance $D_W = c_p \cdot (6 \text{ seconds})$. The sum of D_S and D_W is equal to the λ, length of the ship.

$$140 = D_S + D_W = V \cdot (6 \text{ seconds}) + c_p \cdot (6 \text{ seconds})$$

Solve for V $= 23.33 - 8.77 = 8.56$ meters/second.