

Lecture Notes 9 Supplement, 3/18/03

4.2.1, 4.3.1, Table 4.3.1 from Electromagnetic Fields and Energy, by Hermann A. Haus and James R. Melcher.

I. Air-Gap Magnetic Machines

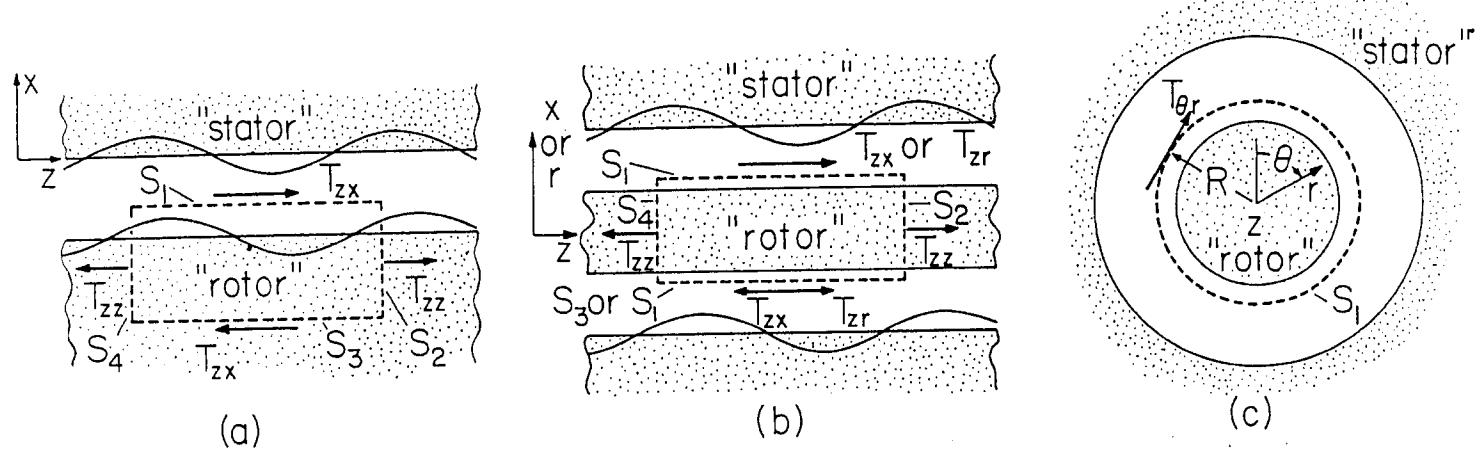
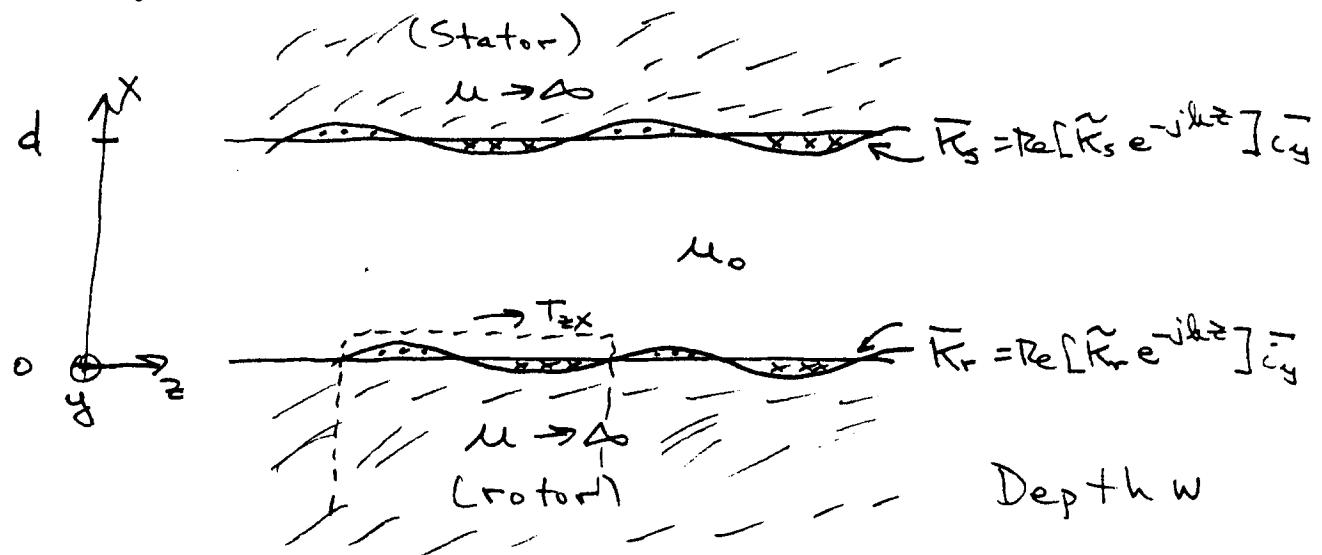


Fig. 4.2.1. Typical "air-gap" configurations in which a force or torque on a rigid "rotor" results from spatially periodic sources interacting with spatially periodic excitations on a rigid "stator." Because of the periodicity, the force or torque can be represented in terms of the electric or magnetic stress acting at the air-gap surfaces S_1 : (a) planar geometry or developed model; (b) planar or cylindrical beam; (c) cylindrical rotor.

A. Generalized Description



$$\begin{aligned} \text{B.C. : } \tilde{H}_z(x=\Delta) &= \tilde{K}_s \\ \tilde{H}_z(x=0) &= -\tilde{K}_r \end{aligned}$$

$$f_z = \oint_S T_{zx} n_x dz dy = w \int_0^{2\pi/l} u_0 H_z H_x \Big|_{x=0} dz = w \int_0^{2\pi/l} u_0 H_z^r H_x^r dz$$

↑ force over wavelength

$$\frac{l}{2\pi} \int_0^{2\pi/l} a(z, t) b(z, t) dz = \frac{1}{2} \operatorname{Re} [\tilde{A} \tilde{B}^*] = \frac{1}{2} \operatorname{Re} [\tilde{A}^* \tilde{B}]$$

$$f_z = \frac{2\pi w}{l} \frac{u_0}{2} \operatorname{Re} [\tilde{H}_z^r \tilde{H}_x^{r*}]$$

$$= \frac{\chi \pi w u_0}{l e \chi} \operatorname{Re} [-\tilde{k}_r \tilde{H}_x^{r*}]$$

$$\begin{bmatrix} \tilde{B}_x^s \\ \tilde{B}_x^r \end{bmatrix} = M_{0k} \begin{bmatrix} -\operatorname{coth}kd & \frac{1}{\sinh kd} \\ -\frac{1}{\sinh kd} & \operatorname{coth}kd \end{bmatrix} \begin{bmatrix} \tilde{\chi}^s \\ \tilde{\chi}^r \end{bmatrix}$$

$$\tilde{H}_z = +jlk \tilde{\chi} \Rightarrow \tilde{\chi}^s = \frac{1}{jk} \tilde{H}_z^s = \frac{\tilde{k}^s}{jk}$$

$$\tilde{\chi}^r = \frac{\tilde{H}_z^r}{jk} = -\frac{\tilde{k}^r}{jk}$$

$$u_0 \tilde{H}_x^r = u_0 l e \left[-\frac{\tilde{\chi}^s}{\sinh kd} + \tilde{\chi}^r \operatorname{coth}kd \right]$$

$$= u_0 l e \left[-\frac{\tilde{k}^s}{jksinhkd} - \frac{\tilde{k}^r}{jk} \operatorname{coth}kd \right]$$

$$\operatorname{Re} [-\tilde{k}^r \tilde{H}_x^r u_0] = -\operatorname{Re} \left[\frac{j u_0 k}{l} \left(\frac{\tilde{k}_r^* \tilde{k}_s^s}{\sinh kd} + \tilde{k}_r^* \tilde{k}_s^r \operatorname{coth}kd \right) \right]$$

$$= \operatorname{Re} [-u_0 j \frac{\tilde{k}_r^* \tilde{k}_s^s}{\sinh kd}]$$

$$f_z = -\frac{\pi w}{l} \frac{\mu_0}{\sin \theta d} \operatorname{Re} [j \tilde{k}_r^s + \tilde{k}_s] \quad (\text{Force on each wavelength})$$

B. Synchronous Interaction

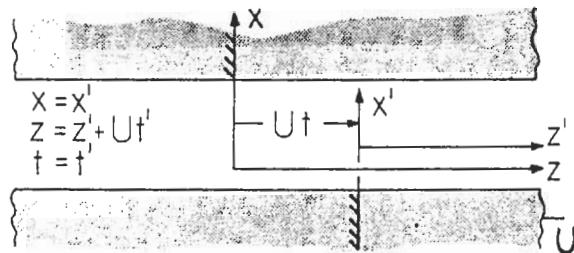


Fig. 4.3.1. Rotor and stator reference frames z' and z .

$$k^s = K_0^s \sin[\omega_s t - k_z z] = \operatorname{Re} -j K_0^s e^{j(\omega_s t - k_z z)}$$

$$\begin{aligned} k^r &= K_0^r \sin[\omega_r t - k_z(z' - s)] ; \quad z' = z - Ut \\ &= K_0^r \sin[(\omega_r + k_0 v)t - k_z(z - s)] \\ &= \operatorname{Re} -j K_0^r e^{j(\omega_r + k_0 v)t} e^{jk_z s} \end{aligned}$$

$$\begin{aligned} \tilde{k}^s &= -j K_0^s e^{j\omega_s t} \\ \tilde{k}^r &= -j K_0^r e^{jk_z s} e^{j(\omega_r + k_0 v)t} \end{aligned}$$

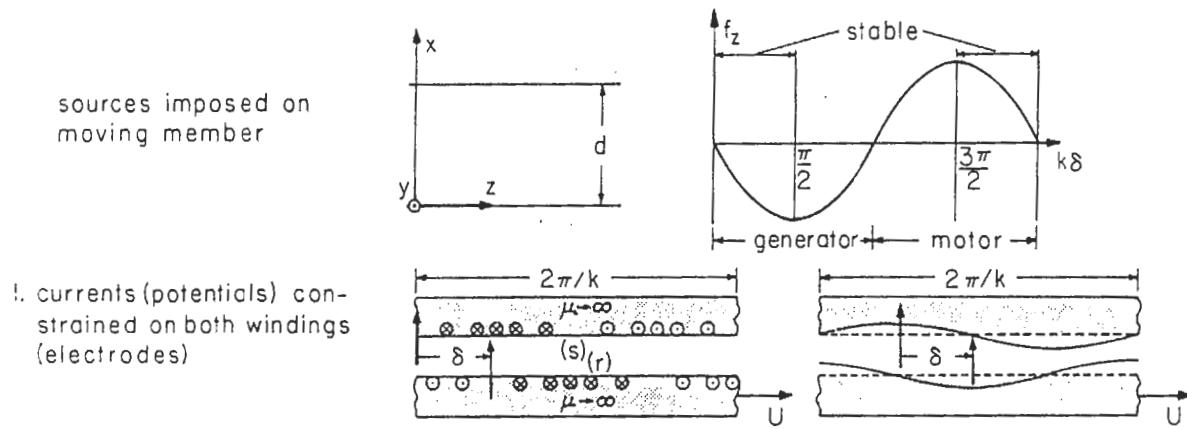
$$\begin{aligned} f_z &= -\frac{\pi w}{l} \frac{\mu_0}{\sin \theta d} \operatorname{Re} [j (-j K_0^s) e^{j\omega_s t} (j K_0^r e^{-jk_z s}) e^{-j(\omega_r + k_0 v)t}] \\ &= -\frac{\pi w}{l} \frac{\mu_0}{\sin \theta d} K_0^s K_0^r \operatorname{Re} [j e^{-jk_z s} e^{j(\omega_s - \omega_r - k_0 v)t}] \end{aligned}$$

For time average force $\Rightarrow \omega_s = \omega_r + kU$ (synchronous condition)

Usually $\omega_r = 0 \Rightarrow \omega_s = kU$

$$\langle f_z \rangle = -\frac{\pi \mu_0}{l} \frac{w_0}{\sinh \delta} K_0^S K_0^T \sin \delta S$$

Table 4.3.1. Basic configurations illustrating classes of electromechanical interactions and devices. MQS and EQS systems respectively in left and right columns.



II. Electrostatic Machine

Diagram of an electrostatic machine with a moving member of thickness d and width $2\pi/l$. The moving member has a surface charge density σ and a relative permittivity ϵ_r . The fixed electrode has a surface charge density σ_0 and a relative permittivity ϵ_0 .

The electric field distribution is given by:

$$\Phi^S = \sigma_0 [\tilde{V}_S e^{-jkz}]$$

$$\Phi^R = \sigma_0 [\tilde{V}_R e^{-jkz}]$$

The force per unit length is given by:

$$f_z = \frac{w_0^2 \pi}{l} \int_{x=0}^{2\pi/l} T_z \times dz = \frac{2\pi w}{l} \int_{x=0}^{2\pi/l} \epsilon_0 E_z E_x \Big|_{x=0}$$

$$\tilde{E}_z = jk \tilde{V}_r$$

$$f_z = \frac{1}{2} \frac{2\pi w}{k} \operatorname{Re} [\epsilon_0 \tilde{E}_z^r * \tilde{E}_x^r]$$

$$= \frac{\pi w}{k} \operatorname{Re} [\epsilon_0 (-jL \tilde{V}_r^*) \tilde{E}_x^r]$$

$$\begin{bmatrix} \tilde{D}_x^S \\ \tilde{D}_x^r \end{bmatrix} = \epsilon_0 L \begin{bmatrix} -\cot \theta d & \frac{1}{\sin \theta d} \\ -\frac{1}{\sin \theta d} & \cot \theta d \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_r \end{bmatrix}$$

$$\epsilon_0 \tilde{E}_x^r = \epsilon_0 L \left[\frac{-\tilde{V}_s}{\sin \theta d} + \tilde{V}_r \cot \theta d \right]$$

$$\begin{aligned} \operatorname{Re} [-jL \epsilon_0 \tilde{V}_r \tilde{E}_x^r] &= \operatorname{Re} [-jL \epsilon_0 \tilde{V}_r^* \left(-\frac{\tilde{V}_s}{\sin \theta d} + \tilde{V}_r \cot \theta d \right)] \\ &= \operatorname{Re} [+jL \epsilon_0 \tilde{V}_s \tilde{V}_r^* / \sin \theta d] \end{aligned}$$

$$f_z = \frac{\pi w}{k} \frac{L \epsilon_0}{\sin \theta d} \operatorname{Re} [\tilde{V}_s \tilde{V}_r^*]$$

$$V_s = V_o^s \cos(\omega_s t - k z)$$

$$V_r = V_o^r \cos(\omega_r t - k(z' - \delta)) ; z' = z - vt$$

$$\tilde{V}_r = -V_o^r e^{j(\omega_r + kv)t} e^{jk\delta}$$

$$\tilde{V}_s = V_o^s e^{j\omega_s t}$$

$$\langle f_z \rangle = \frac{\pi w L \epsilon_0}{\sin \theta d} \operatorname{Re} \left[j V_o^s V_o^r e^{-jk\delta} e^{j(\omega_s - \omega_r - kv)t} \right]$$

$$\omega_s = \omega_r + kv$$

$$\langle f_z \rangle = -\frac{\pi w L \epsilon_0}{\sin \theta d} V_o^s V_o^r \sin \delta$$