Relativistic Harmonic Gyrotron Traveling-Wave Tube Amplifier Experiments

by

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Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Abstract

The first multi-megawatt (4 MW, \( \eta = 8\% \)) harmonic (\( \omega = s\Omega_c, s = 2 \) or \( 3 \)) relativistic gyrotron traveling-wave tube amplifier (gyro-twt) experiment has been designed, built, and tested. Results from this experimental setup, including the first ever reported third harmonic gyro-twt results, are presented. The first detailed phase measurements of a gyro-twt are also reported. The electron beam source is SNOMAD-II, a solid-state nonlinear magnetic accelerator driver with nominal parameters of 400 kV and 350 A. The flat-top pulse width is 30 ns. The electron beam is focused using a Pierce geometry and then imparted with transverse momentum using a bifilar helical wiggler magnet. The imparted beam pitch is \( \alpha = \beta_\perp / \beta_\parallel \approx 1 \).

Experimental operation involving both a second harmonic interaction with the TE\(_{21}\) mode and a third harmonic interaction with the TE\(_{31}\) mode has been characterized. The third harmonic interaction resulted in 4 MW output power and 50 dB single-pass gain, with an efficiency of up to \( \sim 8\% \) (for 115 A beam current). The best measured phase stability of the TE\(_{31}\) amplified pulse was \( \pm 10^\circ \) over a 9 ns period. The phase stability was limited because the maximum rf power was attained when operating far from wiggler resonance. The second harmonic, TE\(_{21}\) had a peak amplified power of 2 MW corresponding to 40 dB single-pass gain and 4% efficiency. The second harmonic interaction showed stronger superradiant emission than the third harmonic interaction. Characterizations of the second and third harmonic gyro-twt experiments presented in this thesis include measurement of far-field radiation patterns, gain and phase versus interaction length, frequency spectrum, phase, and output power versus input power. The absolute power measurements are based both on angular radiation scans with a calibrated horn and diode, and on propagation of the TE\(_{31}\) mode through an efficient in-guide converter and measurement of the converted TE\(_{10}\) power in rectangular waveguide through a calibrated coupling port. Beam parameters of \( \alpha = 0.9, \sigma_\gamma / \langle \gamma \rangle \approx 3\%, \sigma_{pz} / \langle p_z \rangle \approx 12\%, \) and \( \sigma_\beta / \langle \beta_\perp \rangle \approx 20\% \) are consistent with nonlinear numerical simulations of the harmonic gyro-twt interaction based on the measured growth rate and rf power from the experiments.

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Chapter 1

Introduction

1.1 Background

The generation of high power centimeter and millimeter wavelength electromagnetic radiation (the traditional microwave range—see Fig. 1-1) is an important area of scientific research. Microwaves have many applications in modern society, ranging from military (radar, weapons systems) to industrial (communications, materials testing and heating, atmospheric sensing and conditioning) to domestic (microwave ovens, radar detectors). More specifically, in the high power (> 1 MW) regime, sources of microwave power are needed for applications such as driving rf accelerators and heating hydrogen plasmas to several million degrees for the purpose of eventually igniting a fusion reaction in a deuterium-tritium plasma. Because of these demands, several high power microwave devices are currently under investigation at frequencies ranging from 3 GHz (SLAC klystrons) to 170 GHz (gyrotrons for ITER) and beyond.

Masers can be divided into two groups: amplifiers and oscillators. Oscillators use a feedback mechanism (usually a cavity) to build up a strong microwave signal from very small (noise) levels. Amplifiers require an injected microwave signal which is then amplified, usually by several orders of magnitude in a single pass. Oscillators have the advantage of simplicity since they do not require an input rf device or coupler. They are used in applications where rf phase is not important, e.g. heating applications. Amplifiers, on the other hand, require no cavity, and they produce an output signal coherently phased to the input signal. Such phase coherence is required for applications such as accelerator drivers and phase critical radar and antenna systems.

Conventional microwave devices, such as the magnetron, the klystron, and the traveling wave tube rely on structures with characteristic dimensions that are comparable to the wavelength of the radiation. Cooling and electrical breakdown problems result in the output power from such devices dropping as the inverse square of the operating frequency[48]. Due to this fundamental limitation, overmoded and harmonic devices such as the free electron laser and the cyclotron resonance maser (CRM), which reduce the cooling requirements by spreading the microwaves over a larger cross-sectional area, have generated a substantial amount of interest for providing high power, high frequency microwave sources.

The theory behind the cyclotron resonance maser was developed in the late 1950’s independently by several scientists[81], including Richard Q. Twiss[83], Jurgen Schneider[78], Andrei
Figure 1-1: The electromagnetic wave spectrum [49].
Gapanov[35], and Richard Pantell[68]. Hirschfield[43] first experimentally demonstrated the CRM interaction in the 1960's, and Bers[9] developed the instability theory for anisotropic plasmas during the same time period.

In the 1970's, the first high power magnetron injection gun (MIG) CRM oscillator, known as a gyrotron, was demonstrated[32]. The first CRM amplifier, in the form of a gyrotron traveling-wave tube (gyro-twt), was also demonstrated[40], and Petelin presented initial theories on the cyclotron autoresonance maser (CARM)[74]. Vomvoridis and Sprangle developed linear and nonlinear analyses for the CRM interaction for arbitrary phase velocity in open resonators[82].

The 1980's saw rapid growth in both experimental gyro-twt progress and in gyro-twt theory. A list of gyro-twt experiments is shown in Sec. 1.3. Russian researchers provided a one-dimensional theory to calculate the nonlinear efficiency of CARM oscillators and amplifiers[14, 12, 36]. Kreischer and Temkin[51] also provided a comprehensive linear theory for the gyrotron for arbitrary waveguide modes. Fliflet presented a comprehensive theory for TE and TM mode CRM interactions[31]. Magnetic field tapering to enhance the CRM interaction efficiency was studied[18, 66].

Continuing into the 1990's, numerical simulations of CRM devices[73, 33, 58] have become commonplace tools as the processing power available to the average scientist has grown exponentially over the last decade. More variants of the gyro-twt are being tested today, with some devices employing electron beam prebunching (gyroklystron, gyrotwystron) and/or special waveguide structures.

At the time the experiments presented in this thesis were designed and initially operated, no CRM amplifier had achieved > 1 MW output power at frequencies above X-band (see Fig. 1-1). This is partly due to the difficulty of designing a stable, high power amplifier, and partly due to a lack of driving applications. CRM amplifiers have many promising features. They are relatively easy to build, have wide gain-bandwidth and high power capability, and are reasonably efficient. In addition, CARM and harmonic CRM amplifiers can operate at high frequencies with relatively low magnetic field requirements.

1.2 Introductory Theory

The cyclotron resonance maser interaction is driven by electrons traveling helically in a uniform magnetic field. A schematic of the basic CRM interaction is shown in Fig. 1-2. The randomly phased electrons first bunch together in velocity space both axially and azimuthally. The axial bunching is caused by non-relativistic effects and leads to the Weibel instability. In typical CRM devices, however, the azimuthal bunching, which is due to the (relativistic) negative mass instability, dominates the axial bunching[22] and leads to the CRM interaction, where the electrons emit radiation as they resonantly interact with a rotating TE wave. The CRM interaction is relativistic in nature, as the coupling is proportional to the square of the relativistic transverse velocity, $\beta_z^2$, and goes to zero as $c \rightarrow \infty$. The CRM resonance condition between the electrons and the wave is:

$$\omega = s\Omega_c + k_z v_z,$$

where $\omega$ and $k_z$ are the frequency and axial wave number respectively, $v_z$ is the axial electron velocity, $\Omega_c \equiv q_e B_0/(m_e \gamma)$ is the relativistic cyclotron frequency in the guiding magnetic field.
of amplitude $B_0$, and $s$ is the harmonic number. In the expression for $\Omega_c$, $\gamma$ is the normalized relativistic energy of the electrons, $\gamma = (1 - v^2/c^2)^{-1/2}$. In a gyrotron or a gyro-twt, the electrons travel with a high pitch ratio ($v_z$ small) and resonate with a wave near cut-off ($k_z$ small). In a CARM, the electrons travel with more axial velocity, the resonant frequency has a significant Doppler upshift (provided by the $k_zv_z$ term in Eq. 1.1), and the electromagnetic wave has a phase velocity close to the speed of light. For a harmonic CRM device, $s > 1$. Both the harmonic gyrotron/gyro-twt (due to the factor of $s$ in front of $\Omega_c$) and the CARM (due to the $k_zv_z$ term) have a higher resonant operating frequency, $\omega$, than the fundamental ($s = 1$) gyrotron for the same magnetic field strength.

Solving for $\omega$ simultaneously in Eq. 1.1 and the dispersion equation for the relevant electromagnetic waveguide mode,

$$\omega^2 = c^2(k^2 + k_z^2), \quad (1.2)$$

yields the resonant radiation frequency. Here, $k_\perp$ is the transverse wave number of a given waveguide structure, $ck_\perp$ being the cut-off frequency of the structure. The detailed theory for the CRM interaction will be presented in Chapter 2.

### 1.2.1 Autoresonance

The CARM earns its name from the fact that for the case of luminous waves ($\beta_\phi = \omega/(ck_z) \approx 1$), an electron which is injected in resonance with the wave will remain in resonance even as it loses energy to the wave. The autoresonant property of the CARM, simply explained, occurs because the decrease in the $k_zv_z$ term in Eq. 1.1 balances the increase in the $s\Omega_c$ term (due to the decrease
in $\gamma$) as the electrons lose parallel energy. This can be proven by rewriting Eq. 1.1 as

$$\omega = \frac{s\Omega_0}{\gamma(1 - \beta_z/\beta_\phi)}, \quad (1.3)$$

where $\gamma$ is the relativistic energy factor of the electron, $\Omega_0 \equiv \Omega_\epsilon \gamma$, and $\beta_z = v_z/c$. Note that $\beta_\phi$ has a dependence on $\omega$, so Eq. 1.3 is not explicitly solved for $\omega$. For the case where $\beta_\phi \approx 1$, Eq. 1.3 becomes

$$\omega \approx \frac{s\Omega_0}{\gamma(1 - \beta_z/\beta_\phi)}. \quad (1.4)$$

The denominator of Eq. 1.4 is a constant of motion for an electron moving in a constant amplitude TE wave. This is seen by writing the energy equation for an electron:

$$\frac{d\gamma}{dt} = -\frac{q_e}{m_0c^2}v \cdot E, \quad (1.5)$$

and the $z$-component of the Lorentz force equation:

$$\frac{dp_z}{dt} = -q_e(v \times B) \cdot \hat{z} = q_e v \cdot (\hat{z} \times B), \quad (1.6)$$

where $E$ and $B$ are the electric and magnetic fields of the TE wave, and $p_z = m_0c^2\gamma\beta_z$ is the axial momentum of the electron. For a TE wave, the relation between the $E$ and $B$ rf field vectors is

$$E = -c\beta_\phi(\hat{z} \times B). \quad (1.7)$$

Substituting Eq. 1.7 into Eq. 1.6 and also making use of Eq. 1.5, it is easily seen that

$$\frac{d}{dt} \left( \gamma - \frac{p_z\beta_\phi}{m_0c} \right) = 0. \quad (1.8)$$

Thus $\gamma(1 - \beta_z\beta_\phi)$ remains constant, and the electron stays in resonance even as it loses energy. Again, Eq. 1.8 is based on Eq. 1.4, which was derived assuming $\beta_\phi \approx 1$. In actuality, however, a CARM interaction occurring inside of a waveguide must operate at $\beta_\phi \gtrsim 1$ and also must operate slightly detuned from the wave frequency for maximum efficiency. Even so, the autoresonant feature increases the resonant interaction length and potential efficiency of the CARM.

### 1.3 Previous Experiments

Singling out the gyro-twt and the CARM amplifier, experimental history began in the late 1970's. Since then only a handful of experiments have been carried out on these devices. Two CARM amplifier experiments have been reported, as seen in Table 1.1. These amplifier experiments were both moderately successful, demonstrating the feasibility of the CARM interaction at modest efficiencies. Both experiments attribute the low efficiencies to poor electron beam quality. By way of comparison to the CARM amplifier experiments, Table 1.2 highlights the more numerous
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<td>Frequency (GHz)</td>
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<td>Mode</td>
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<td>TE_{11}</td>
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<tr>
<td>Voltage (kV)</td>
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<td>500</td>
</tr>
<tr>
<td>Current (A)</td>
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<td>500</td>
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<tr>
<td>P_{max} (MW)</td>
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<td>10</td>
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<tr>
<td>η (%)</td>
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<td>4</td>
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<tr>
<td>Total gain (dB)</td>
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</tr>
<tr>
<td>Growth rate (dB/cm)</td>
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<td>1.4</td>
</tr>
<tr>
<td>Doppler Upshift</td>
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</tr>
<tr>
<td>τ_{PULSE} (ns)</td>
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<td>20</td>
</tr>
<tr>
<td>α</td>
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<td>0.4</td>
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<td>–</td>
</tr>
<tr>
<td>β_{φ}</td>
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<td>1.27</td>
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Table 1.1: CARM amplifier experiments.
CARM oscillator experiments that have been done up to the present day. Note in Tables 1.1 and 1.2 that the upshift number, which gives the frequency ratio between the radiation frequency and the relativistic cyclotron frequency, is always significantly larger than unity. As well, the phase velocity, $\beta_\phi$, is always close to unity. This is not the case for Table 1.3, which lists gyro-twt experiments beginning with the pioneering effort by Granatstein in 1975. As opposed to the CARM amplifier experiments, the majority of the gyro-twt experiments have been performed using “long-pulse” systems, meaning $\tau > 1 \mu s$, so that competition between the amplifying mode and the oscillating modes becomes a chief factor limiting beam current and therefore gain and output power. All but the first gyro-twt experiment are relatively low in power (until recently—see caption in Table 1.3). At the time the gyro-twt experiments which will be reported in this thesis were designed and initially operated, Furuno[34] had completed the only other harmonic gyro-twt experiment, and no gyro-twt experiment had demonstrated $> 1$ MW of output power except for the pioneering effort, which had a negligible efficiency. Furuno’s experiment was done at very low power levels and achieved only a modest efficiency, partly due to the harmonic number being so large. Issues involving the phase of the rf output in gyro-twt and CARM amplifier experiments have rarely been addressed. To date, no one has published phase measurements of any gyro-twt or CARM amplifier.

<table>
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<td>0.1</td>
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<td>&lt;0.1</td>
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</tr>
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<td>$\alpha$</td>
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<td>.35</td>
<td>.4</td>
<td>&lt;.6</td>
<td>&lt;.32</td>
<td>&lt;.32</td>
<td>.7</td>
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<td>$\sigma_{pz}/p_z$</td>
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<td>.04</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>.045</td>
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</tbody>
</table>

Table 1.2: CARM oscillator experiments. IAP is the Institute of Applied Physics of the Russian Academy of Sciences in Nizhny Novgorod, Russia. JINR is the Joint Institute of Nuclear Research. Several other research groups have designed and built CARM oscillators[62, 20, 52, 65]. Many of these experiments (as did many in the above table) experienced severe mode competition, and in some cases the CARM mode was not observed.
Table 1.3: Gyro-twt experiments. NTHU is the National Tsing Hua University in Hsinchu, Taiwan. NRL is the Naval Research Laboratory in Washington, DC. NRL[6] also conducted lossy wall and wide-band gyro-twt experiments in 1980. Both were conducted in the small signal regime, with the wide-band TWT achieving a 12% bandwidth[6] and the lossy wall experiment achieving 56 dB single stage gain[4]. Recently (1993), Maryland operated a 1 μs second harmonic gyrokystron at 20 GHz with a peak power exceeding 21 MW and an efficiency near 21%[54]. UCLA also very recently (1994) operated a 1 μs second harmonic gyro-twt at 16 GHz with efficiencies of ~ 15% and power levels of ~ 200 kW.
experiment. Very little has been published about the spectral “quality” of the amplified pulses, as well.

1.4 Thesis Overview

This thesis discusses the design and implementation of experiments that were able to address many of the aforementioned issues. The experimental design includes operation both as a CARM amplifier and as a second and third harmonic gyro-twt experiment at 17 GHz with minimal changes. The thesis is divided up in the following manner: Chapter 2 discusses the theory of gyro-twt and CARM amplifiers. Chapter 3 is a detailed design study for the gyro-twt and CARM experiments. Chapter 4 presents the design for the pulsed power source (SNOMAD II) and the electron beam focusing system. Chapter 5 analyzes the design and theory for the beam spin-up mechanism, a bifilar helical wiggler. Chapter 6 discusses the interaction section of the experiment, including focusing coils, beam diagnostics, input coupler, and output window. Chapter 7 presents all significant experimental data. Chapter 8 is a discussion of the experimental results with a summary section.

All units, equations, and formulae in this thesis are in S.I. units. Often, more appropriate units are used for numerical values given in tables and figures, for example mm and cm for small dimensions and GHz for frequency.
Chapter 2
CRM Amplifier Theory

2.1 Introduction

In order to effectively design a CRM amplifier, its performance must be predicted, or simulated, by an adequate theoretical model. A theory is needed that describes how the rf electric field being amplified interacts with the electron beam, first bunching the beam, and then being amplified by it. There are two common approaches to such theory: kinetic and single particle. Both theories must be linearized to give meaningful analytical results, but the non-linear single particle theory of a CRM amplifier lends itself well to numerical simulations. The linearized kinetic theory, on the other hand, provides an easily calculated, accurate analytical estimate of the rf growth rate in a gyro-twt before the wave reaches saturation power levels (i.e. the electron beam can give no more of its energy to the wave). This treatment of both theories assumes that the electron beam is tenuous—that self-field effects are not important. This approximation is discussed further in Sections 2.6 and 3.8. In the following sections, both the kinetic model and the single particle model results are presented.

2.2 Guided Wave Propagation

The equations for the electric and magnetic field for a TE wave traveling through a circular waveguide with perfectly conducting walls are shown below. The radius of the waveguide is \( r_w \). The TE wave has azimuthal index \( m \) and radial index \( n \), and it is a right-handed rotating wave. The term “right-handed” means that if the thumb of the right hand is pointed along the direction of propagation—the positive \( z \) direction in this case—then at a fixed point in space the entire transverse magnetic and electric field pattern rotates in the direction of the fingers in time. This is not in general true, however, for the electric field or magnetic field vectors at each point in space. The equations for the E and H fields in the guide, written in terms of the independent variables \((r, \theta, z, t)\), are

\[
\tilde{E}_r = \tilde{E}(z)C_{mn}\frac{m}{k_r r} J_m(k_{\perp} r) e^{ik_z z - i\omega t + im\theta},
\]

(2.1)

\[
\tilde{E}_\theta = i\tilde{E}(z)C_{mn} J'_m(k_{\perp} r) e^{ik_z z - i\omega t + im\theta},
\]

(2.2)
\[ \tilde{H}_r = -i \tilde{H}(z) C_{mn} J_m'(k_{\perp} r) e^{i{k_{\perp} z - \omega t + im \theta}}, \quad (2.3) \]
\[ \tilde{H}_\theta = \tilde{H}(z) C_{mn} \frac{k_{\perp}}{k_z} J_m(k_{\perp} r) e^{i{k_{\perp} z - \omega t + im \theta}}, \quad (2.4) \]
and
\[ \tilde{H}_z = -\tilde{H}(z) C_{mn} \frac{k_{\perp}}{k_z} J_m(k_{\perp} r) e^{i{k_{\perp} z - \omega t + im \theta}}, \quad (2.5) \]

where
\[ \tilde{H}(z) = \frac{\tilde{E}(z)}{\eta_0 \beta_{\phi}} - \frac{i}{\mu_0 \omega} \frac{d \tilde{E}(z)}{dz}, \quad (2.6) \]
\[ C_{mn} = \frac{1}{J_m(\nu_{mn}) \sqrt{\pi (\nu_{mn}^2 - m^2)}}, \quad (2.7) \]
\[ \beta_{\phi} = \frac{\omega}{ck_z}, \quad (2.8) \]

and \( J_m \) and \( J_m' \) are the Bessel function of order \( m \) and its derivative (Fig. 2-1).

Here, the fields are written in complex notation. The real field values are recovered by taking the real part of the complex value for each vector component. From Eqs. 2.1–2.5, the actual vector fields are recovered as
\[ E(r, \theta, z, t) = \text{Re} \left\{ \tilde{E} \right\} = \text{Re} \left\{ \tilde{E}_r \right\} \hat{r} + \text{Re} \left\{ \tilde{E}_\theta \right\} \hat{\theta} \quad (2.9) \]
\[ H(r, \theta, z, t) = \text{Re} \{ \mathbf{H} \} = \text{Re} \{ \mathbf{H}_r \} \hat{r} + \text{Re} \{ \mathbf{H}_\theta \} \hat{\theta} + \text{Re} \{ \mathbf{H}_z \} \hat{z}. \] (2.10)

In Eqs. 2.1, 2.2, and 2.6, \( \mathbf{E}(z) \) is the wave amplitude. For a non-interacting wave in a perfectly conducting guide, \( \mathbf{E}(z) \) is constant over \( z \). In this case, Eq. 2.6 simplifies due to the \( d\mathbf{E}(z)/dz \) term vanishing. Eq. 2.6 is derived by satisfying Maxwell's equations using the forms given by Eqs. 2.1–2.5. The time-averaged power flowing through a full cross section of the waveguide at any \( z \) is expressed as

\[ \mathcal{P}(z) = \frac{1}{2} \int_{S} \text{Re} \{ \mathbf{E} \times \mathbf{H}^* \} \cdot \hat{z} \, ds, \] (2.11)

where the integral above is over the cross-section of the waveguide. The mode-dependent \( C_{mn} \) factor (Eq. 2.7) is multiplied into Eqs. 2.1–2.5 so that the result of Eq. 2.11 is simply

\[ \mathcal{P}(z) = \frac{1}{2k^{2}} \text{Re} \{ \mathbf{\tilde{E}}(z)\mathbf{\tilde{H}}^*(z) \}. \] (2.12)

Other parameters used in the above equations are as follows: \( k = \nu_{mn}/r_{w} \) is the transverse wave number, where \( \nu_{mn} \) is the \( n \)th non-zero root of \( J_{m}(x) \); \( k_z = \sqrt{\omega^2 \mu_0 - k^2} \) is the axial wave number, \( \omega \) is the angular frequency of the rf wave; \( \epsilon_0 = 8.85419 \times 10^{-12} \) F/m is the permittivity of free space; \( \mu_0 = 4\pi \times 10^{-7} \) H/m is the permeability of free space; \( \eta_0 = \sqrt{\mu_0/\epsilon_0} = 376.73 \) \( \Omega \) is the impedance of free space, and \( c = 1/\sqrt{\mu_0\epsilon_0} = 2.997925 \times 10^8 \) m/s is the speed of light in free space.

### 2.3 Single Particle Theory

When the rf wave described by Eqs. 2.1–2.5 interacts with an electron beam, \( \mathbf{E}(z) \) is no longer constant over \( z \) and instead can slowly change. By “slowly change,” it is meant that the change in the wave amplitude is small over one rf wavelength \( 2\pi/k_z \). The single particle equations of motion for a gyro-twt then predict how \( \mathbf{E}(z) \) behaves over the length of the interaction.

The electrons with which the wave interacts are described by an axial velocity, \( v_z \equiv c\beta_z \), a transverse velocity, \( v_\perp \equiv c\beta_\perp \), and a transverse velocity angle of \( \phi \), such that \( v_x = v_\perp \cos(\phi) \) and \( v_y = v_\perp \sin(\phi) \). The electrons are also described by the relativistic energy factor, \( \gamma = (1 - \beta^2)^{-1/2} \), and momentum variables: \( p_x = \gamma m_0 v_z \), \( p_\perp = \gamma m_0 v_\perp \). Denoting an individual electron’s parameters with a \( j \) subscript, the cyclotron resonance maser single particle equations of motion for the interaction of a right-hand rotating TE_{mn} wave with the \( s \)th harmonic of the beam motion are[16, 31, 14]

\[
\frac{d\gamma_j}{dz} = -\frac{q_j e C_{mn}}{m_0 c \omega} J_{m-s}(k_\perp R_{gj}) J'_s(k_\perp R_{Lj}) \hat{p}_j \cdot \mathbf{p}_j \mathbf{H}_z, \quad \text{Re} \{ \mathbf{E} e^{-i\Lambda_j} \}, \tag{2.13}
\]

\[
\frac{d\mathbf{p}_j}{dz} = -\frac{q_j e C_{mn}}{m_0 c \omega} J_{m-s}(k_\perp R_{gj}) J'_s(k_\perp R_{Lj}) \hat{p}_j \cdot \mathbf{p}_j \mathbf{H}_z \left[ \frac{1}{\beta \phi} \text{Re} \{ \mathbf{E} e^{-i\Lambda_j} \} + \text{Im} \left\{ \frac{d\mathbf{E}}{dz} e^{-i\Lambda_j} \right\} \right], \tag{2.14}
\]
\[
\begin{align*}
\frac{d\Lambda_j}{d\bar{z}} &= \frac{\gamma_j}{\hat{p}_{jz}} \frac{1}{\beta_\phi} \frac{s\Omega_{c0}}{\omega \hat{p}_{jz}} - \frac{s^2 q_e C_m n J_{m-s} (k_\perp R_{gj}) J_s (k_\perp r_{Lj})}{m_0 \omega k_\perp r_{Lj} \hat{p}_{jz} \hat{p}_{j1}} \\
&\quad \times \left[ \left( \gamma_j - \frac{\hat{p}_{jz}}{\beta_\phi} \right) \text{Im} \{ \bar{E} e^{-i\Lambda_j} \} + \hat{p}_{jz} \text{Re} \left\{ \frac{d\bar{E}}{d\bar{z}} e^{-i\Lambda_j} \right\} \right], \quad (2.15)
\end{align*}
\]

and
\[
\begin{align*}
\left[ \frac{d^2}{d\bar{z}^2} + 2i \frac{d}{\bar{z}} - \frac{(1 - i)(1 - \beta_\phi^{-2})}{\delta_r} \right] \bar{E} &= \frac{2i k_\perp^2 C_m n I_0}{\omega e_0} \sum_{j=1}^N \frac{1}{N} \sum_{j=1}^N J_{m-s} (k_\perp R_{gj}) J'_s (k_\perp r_{Lj}) \hat{p}_{j1} e^{i\Lambda_j}. \quad (2.16)
\end{align*}
\]

In the above equations, \( \gamma_j \) is the relativistic energy factor of the electron; \( \beta_\phi \) is the normalized velocity of the electron; \( \hat{p}_{jz} \equiv \gamma_j \beta_{jz} \) is the normalized transverse momentum of the electron; \( \hat{p}_{jz} \equiv \gamma_j \beta_{jz} \) is the normalized axial momentum of the electron; \( \bar{z} \equiv \omega \bar{z} / c \) is the normalized variable for \( z \); \( q_e = 1.60219 \times 10^{-19} \text{ C} \) is the unsigned charge of an electron; \( m_0 = 9.1095 \times 10^{-31} \text{ kg} \) is the rest mass of an electron; \( B_0 \) is the externally supplied axial magnetic field seen by the electrons; \( \Omega_{c0} = q_e B_0 / m_0 \) is the non-relativistic cyclotron frequency; \( s \) is the cyclotron harmonic number; \( R_{gj} \) is the guiding center of the electron’s orbit; \( r_{Lj} \) is the Larmor radius of the electron’s orbit,

\[
\frac{r_{Lj}}{\Omega_{c0}} = \frac{\gamma_j v_{j1}}{\Omega_{c0}} = \frac{\hat{p}_{j1} c}{\Omega_{c0}} ; \quad (2.17)
\]

\( \Lambda_j \) is the relative phase between the electron and the rf-wave,

\[
\Lambda_j = \omega t_j - \bar{z} / \beta_\phi - s \phi_j + \pi / 2; \quad (2.18)
\]

and \( \delta \) is the skin depth of the waveguide walls, which vanishes for a perfectly conducting wall. In these equations of motion, \( \bar{E}, \gamma_j, \hat{p}_{jz}, \hat{p}_{j1}, \Lambda_j, \omega, \) and \( B_0 \) are all implicitly functions of (and only of) \( \bar{z} \). The \( t_j \) variable used in Eq. 2.18 is the propagation time of a given particle,

\[
t_j(z) = t_j(z = 0) + \int_{z' = 0}^{z = z} \frac{dz'}{v_{jz}(z')}, \quad (2.19)
\]

where \( t_j(z = 0) \) differs for each particle and depends on the initial beam distribution.

Note that Eq. 2.16 is necessary for a self-consistent set of equations for a CRM amplifier. In the case of a CRM oscillator, Eq. 2.16 is not always used. In some cases, electric field amplitude and phase profiles in \( z \) are assumed, and Eqs. 2.13–2.15 integrate the particles through the assumed field profile. This is allowable when the electric field amplitude is high enough that perturbations in the field profile due to the beam are small (e.g. in high Q cavities). For a CRM amplifier, because the wave amplitude starts small and is changed dramatically by the beam, Eq. 2.16 must be used. For self-consistent oscillator simulation, Eq. 2.16 can be used with added terms to describe the coupling between the forward and backward traveling waves in the oscillator cavity. In all oscillator simulations, calculating the efficiency is an iterative process since boundary conditions must be matched at both the input and output of the device.
In order to suit the CRM amplifier equations to numerical analysis, it is useful to write them in terms of \( d\gamma_j /d\hat{z} \) and \( d\hat{L}_j /d\hat{z} \) rather than \( dy_j /d\hat{z} \) and \( d\hat{L}_j /d\hat{z} \). This is due to \( \gamma_j \) being physically restricted to be larger than unity. Small numerical round-off errors can result in violations of this bound, which can cause unpredictable results. The normalized momentum variables, on the other hand, have no such restrictions. Using the simply derived relation

\[
\frac{d\gamma_j}{d\hat{z}} = \frac{1}{\gamma_j} \left[ \hat{L}_j \frac{d\hat{L}_j}{d\hat{z}} + \frac{d\hat{L}_j}{d\hat{z}} \right],
\]

the full equation of motion for \( \hat{L}_j \) is derived from Eqs. 2.13 and 2.14:

\[
\frac{d\hat{L}_j}{d\hat{z}} = \left( \frac{\beta_\phi - \hat{L}_j}{\hat{L}_j} - \gamma_j \right) \frac{q_0 C_{m,n}}{m_0 c \omega} J_{m-s}(k_r R_{L_j}) J_{s}(k_r R_{L_j}) \frac{\hat{L}_j}{\hat{L}_j} \text{Re} \left\{ \frac{\hat{E} e^{-i\Lambda_j}}{d\hat{z}} \right\}
\]

\[
+ \frac{q_0 C_{m,n}}{m_0 c \omega} J_{m-s}(k_r R_{L_j}) J_{s}(k_r R_{L_j}) \text{Im} \left\{ \frac{d\hat{E}}{d\hat{z}} e^{-i\Lambda_j} \right\}
\]

\[
+ \hat{L}_j \frac{1}{2B_0} \frac{dB_0}{d\hat{z}}.
\]

Equations 2.13–2.16 and 2.21 assume that \( B_0(z) \) varies adiabatically:

\[
\frac{2\pi v_z}{\Omega_c \omega} \frac{1}{B_0} \frac{dB_0}{dz} \ll 1.
\]

In other words, the magnetic field is assumed to change by a small fraction over the distance an electron travels in a single cyclotron orbit. When this is the case, the magnetic moment of the electron, \( \hat{L}_j / B_0 \), remains constant, and differentiating yields

\[
\frac{d\hat{L}_j}{d\hat{z}} = \hat{L}_j \frac{1}{2B_0} \frac{dB_0}{d\hat{z}},
\]

which explains the last term in Eqs. 2.14 and 2.21.

A frequent assumption with the cyclotron resonance maser equations of motion (Eqs. 2.13–2.16) is that \( r_{L_j} \) is small, which typically leads to the replacement of \( J_{s}(k_r R_{L_j}) \) and \( J_{s}(k_r R_{L_j}) \) with their small argument approximations:

\[
\lim_{k_r R_{L_j} \to 0} J_{s}(k_r R_{L_j}) = \lim_{k_r R_{L_j} \to 0} \frac{s J_{s}(k_r R_{L_j})}{k_r R_{L_j}} = \frac{1}{2} \left( \frac{k_r R_{L_j}}{2} \right)^{s-1}.
\]

These small argument approximations are valid for CARM operation at the fundamental \((s = 1)\), but for gyro-twt operation at second and third harmonics, both \( k_r \) and \( r_{L_j} \) are larger than in the CARM case, and the approximations are not as good (see Fig. 2-4). A typical gyro-twt case may have \( k_r r_L \sim 2 \), in which case, for third harmonic, \( J_3(2) = 0.16 \), whereas the small argument approximation is 0.25. Another interesting point about the \( J_{s}(k_r R_{L_j}) \) term is that it starts at 0.5 and decreases for increasing \( r_L \) for the fundamental, but it starts at 0 and increases for all harmonics (see Fig. 2-1). A higher beam pitch is therefore detrimental to fundamental operation (only in the
sense that it decreases this coupling term—in general, higher beam pitch is necessary for good
efficiency in gyro-twts), but beneficial to harmonic operation. More design issues will be discussed
in Chapter 3.

2.4 Linearized Kinetic Theory for a “Cold” Beam

Linear kinetic theory analyzes perturbations of a system at equilibrium. The electron beam is
described by a distribution function. This distribution function is analogous to a density function,
except that as well as depending on space and time, the distribution function also depends on mo-
momentum phase space. Maxwell’s equations and the Vlasov equation are used to relate perturbations
of this distribution function to electro-magnetic perturbations. These perturbations are usually
linearized to obtain meaningful analytical results in the form of rf wave growth rates.

Consider the CRM interaction in infinite space with no waveguide boundary restrictions. A
circularly polarized plane wave travels in the positive z-direction:

\[
E(z, t) = \frac{\tilde{E}_0}{\sqrt{2}}(\hat{x} - i\hat{y})e^{-i(\omega t - k_z z)} \tag{2.25}
\]

\[
H(z, t) = \frac{k_z \tilde{E}_0}{\sqrt{2}\omega \mu_0}(\hat{y} + i\hat{x})e^{-i(\omega t - k_z z)}. \tag{2.26}
\]

\(\tilde{E}_0\) is the amplitude of the input wave and is a complex constant with no temporal or spatial
dependence. The components of \(E(z, t)\), written out explicitly, are:

\[
E_x(z, t) = \text{Re}\left\{\frac{\tilde{E}_0}{\sqrt{2}}e^{-i(\omega t - k_z z)}\right\} \tag{2.27}
\]

\[
E_y(z, t) = \text{Im}\left\{\frac{\tilde{E}_0}{\sqrt{2}}e^{-i(\omega t - k_z z)}\right\} \tag{2.28}
\]

\[
\phi(z, t) = \tan^{-1}\left(\frac{E_y(z, t)}{E_x(z, t)}\right) \tag{2.29}
\]

where \(\phi\) is the phase of the wave.

This plane wave interacts with an electron distribution that is uniform over all space with elec-
trons traveling in the z-direction in a helical fashion, following the lines of a uniform, z-directed
magnetic guiding field. The electrons all start with the same values for axial and transverse mo-
menta: \(\hat{p}_z(z = 0) = \hat{p}_{z0}\) and \(\hat{p}_\perp(z = 0) = \hat{p}_{\perp0}\). Thus the equilibrium (no rf field perturbations
present) electron distribution function is singular and anisotropic. A singular anisotropic distribu-
tion is commonly referred to as a “cold” electron distribution when discussing the CRM interaction
(though a true plasma physicist will tell you that such a distribution is, in fact, very hot). The
expression for the distribution function is

\[
f_0(\hat{p}_\perp, \hat{p}_z) = \frac{n_0}{2\pi \hat{p}_\perp}\delta(\hat{p}_\perp - \hat{p}_{\perp0})\delta(\hat{p}_z - \hat{p}_{z0}) \tag{2.30}
\]

27
where the distribution function, $f_0$, has the property that
\[ -q_e \int f_0(\hat{p}_\perp, \hat{p}_z) d^3\hat{p} = \rho_0, \]  
(2.31)
and
\[ -q_e \int f_0(\hat{p}_\perp, \hat{p}_z) v_z d^3\hat{p} = J_{0z}. \]  
(2.32)

Normally, $f_0$ can also depend on the spatial variables $x$, $y$, and $z$, but in order to keep the theory here tractable, $f_0$ is assumed uniform over all space. Here, $\hat{p}_\perp$ and $\hat{p}_z$ are the transverse and axial momenta of the electrons, $\delta$ is the Dirac-delta distribution, $n_0$ is the equilibrium number of electrons per unit volume, which is assumed to be constant over all space, $\rho_0$ is the equilibrium space-charge density of the electrons, and $J_{0z}$ is the equilibrium current in the axial direction. A subscript of zero on a parameter, such as $\hat{p}_{\perp0}$ and $\hat{p}_{z0}$, denotes initial conditions just before the gyro-twt interaction. While the distribution discussed here is a cold distribution, in reality, electrons in a beam have a whole range of momenta centered about some average. This range is often referred to as the momentum “spread,” and the distribution is frequently approximated by a gaussian. The quality of an electron beam is often associated with low momentum spread. The following kinetic theory treatment assumes zero spread.

If the electrons are allowed to have a vanishingly small space charge, the following simple dispersion relation results:
\[ (\omega - k_z v_{\perp0} - \Omega_c)^2 (\omega^2 - c^2 k_z^2) = 0. \]  
(2.33)

Here, $\Omega_c \equiv \omega_0 / \gamma_0$ is the relativistic cyclotron frequency, and $\gamma_0$ is the initial energy of the electrons. Eq. 2.33 is referred to as the uncoupled CRM dispersion relation. The first term in Eq. 2.33 is the CRM resonance term from Eq. 1.1, and the second term is the plane wave dispersion term. The electrons are naturally coupled to the plane wave, however. The derivation of this coupling is based on combining the linearized Vlasov equation,
\[ \frac{df_1(\hat{p}_\perp, \hat{p}_z, \phi, t)}{dt} = -q_e (E + v \times B) \cdot \nabla_p f_0(\hat{p}_\perp, \hat{p}_z), \]  
(2.34)
with the self-consistent wave equation for a TE mode (in cylindrical coordinates),
\[ \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) H_z = \frac{1}{r} \frac{\partial}{\partial r} (r J_{1\theta}) - \frac{1}{r} \frac{\partial J_{1r}}{\partial \theta} \]  
(2.35)

In this case, $J_{1\theta}$ and $J_{1r}$ are the perturbed current densities of the electron distribution, and they are related to $f_1$ through the following integrals:
\[ \int f_1(\hat{p}_\perp, \hat{p}_z, \phi, t) v_{\theta0} d^3\hat{p} = J_{1\theta}. \]  
(2.36)
\[ \int f_1(\hat{p}_\perp, \hat{p}_z, \phi, t) v_{r0} d^3\hat{p} = J_{1r}. \]  
(2.37)
The self-consistent coupled dispersion relation, as derived by Davidson in this manner[26], is
\[
(\omega - k_z v_{z0} - \Omega_c)^2 \left( \omega^2 - c^2 k_z^2 \right) \\
= - \frac{\omega_p^2}{\gamma_0} \left[ \frac{\beta_{10}^2}{2} \left( \omega^2 - c^2 k_z^2 \right) - (\omega - k_z v_{z0} - \Omega_c) (\omega - k_z v_{z0}) \right] .
\] (2.38)

Here, \( \omega_p \) is the plasma frequency of the electron distribution, defined as
\[
\omega_p \equiv q_e \sqrt{\frac{n_e}{\epsilon_0 m_0}} = q_e \sqrt{\frac{I_b}{\pi e^2 q_e \epsilon_0 m_0 r_b^2 \beta_{z0}}}
\] (2.39)
where \( \epsilon_0 \) is the permittivity of free space, and \( I_b \) and \( r_b \) are the total current and beam radius when the electrons are considered to form an electron beam.

Reconstruction of the electric field phase and amplitude as a function of \( z \) by an inverse Fourier transform involves careful consideration of the dispersion equation in the complex \( \omega \) and \( k_z \) planes. If the particle-wave interaction does not have a significant axial component, i.e. if \( k_z v_z \) is small, then the dispersion equation will likely have positive imaginary “pinch” singularities in the complex \( \omega \) plane, and the time-asymptotic electric field will grow in time at all points in space, resulting in an absolute instability[8]. This condition is further discussed in Section 3.3.1. Here, it is assumed that the electrons have sufficient axial momentum such that no such singularities exist, so that the \( \omega \) integration can be done along the real \( \omega \) axis and be guaranteed to have an analytic integrand. In such a case, at certain values of \( \omega \) (\( \omega \) now purely real), some roots of \( k_z \) may be imaginary, representing a convective instability. It is this instability that drives the CARM or gyro-twt amplifier. The coupling term responsible for the CRM instability in Eq. 2.38 is the relativistic \( \beta_{10}^2 \omega^2 \) term. In Eq. 2.38, the CRM instability causes the growth of fast waves (\( \beta_\phi > 1 \)). In the limit of a non-relativistic interaction (\( c \to \infty \)), the CRM instability vanishes, and the Weibel instability, which causes the growth of slow waves (\( \beta_\phi < 1 \)) dominates[22].

To accurately model a physical CRM amplifier, the dispersion relation must take waveguide effects into account. The uncoupled waveguide dispersion relation (\( \omega_p \to 0 \)) is
\[
(\omega - k_z v_{z0} - s \Omega_c)^2 \left( \omega^2 - c^2 k_{\perp}^2 - c^2 k_z^2 \right) = 0
\] (2.40)
where, as in Eq. 2.33, the first term is the CRM resonance term, also called the “beam line” term, and the second term is the waveguide dispersion term. The coupled waveguide dispersion relation has been derived by several authors[23, 31]. From Fliflet, the coupled waveguide dispersion relation for a CRM amplifier operating in the TE_{mn} mode and interacting with cyclotron harmonic number \( s \) is
\[
(\omega - k_z v_{z0} - s \Omega_c)^2 \left( \omega^2 - c^2 k_{\perp}^2 - c^2 k_z^2 \right) \\
= - \frac{\omega_p^2}{\gamma_0} \left[ \frac{H'}{\beta_{10}^2} \left( \omega^2 - c^2 k_z^2 \right) - Q' (\omega - k_z v_{z0} - s \Omega_c) (\omega - k_z v_{z0}) \right] .
\] (2.41)
where
\[ H' = 2 \frac{r_b^2}{r_w^2} \left[ \frac{J_{s-m}(k_{\perp}R_g)J_s^2(k_{\perp}r_L)}{J_m^2(\nu_{mn})(1 - \frac{m^2}{\nu_{mn}^2})} \right] \]  
(2.42)

and
\[ Q' = 2 \frac{r_b^2}{r_w^2} \left[ \frac{J_{s-m}(k_{\perp}R_g)J_s^2(k_{\perp}r_L)J_s(k_{\perp}r_L)J_s''(k_{\perp}r_L)}{J_m^2(\nu_{mn})(1 - \frac{m^2}{\nu_{mn}^2})} \right] \]  
(2.43)

Here, \( r_b \) is the radius of the solid, axis-encircling electron beam. Examining Eq. 2.41, it is useful to point out that the \( H' \) term drives the CRM instability, while the \( Q' \) term acts to stabilize the interaction. Because the \( Q' \) term in Eq. 2.41 involves the factor \( (\omega - k_z v_0 - s\Omega_c) \), which is small near CRM resonance, it is often neglected.

By finding the complex forward traveling \( k_z \) roots of Eq. 2.38 for the given operating frequency \( (\omega = 2\pi \times 17.136 \times 10^9) \), carefully considering initial conditions, and applying an inverse Fourier transform, \( E(z, t) \) is found for all time and space for the given operating frequency. Again, this is done after an instability analysis determines which \( k_z \) roots are growing in the forward direction (Sec. 3.3.1). Let \( \kappa \equiv k_z c/\omega \). Also let Eq. 2.41 be recast as a polynomial in \( \kappa \) with the \( Q' \) term neglected (for simplicity). Call this dispersion polynomial \( D(\kappa) \). Then an equivalent equation to Eq. 2.41 with the \( Q' \) term neglected is
\[ D(\kappa) = 0. \]  
(2.44)

The residue of each root of Eq. 2.44, assuming no double roots, is written as
\[ R_j = \frac{\kappa - \kappa_j}{D(\kappa)} \bigg|_{\kappa = \kappa_j}. \]  
(2.45)

Here, the \( \kappa_j \) are the roots of Eq. 2.44. \( D(\kappa) \) is a quartic polynomial, so there are four roots. The complete electric field is reconstructed by summing over each root that corresponds to a forward traveling wave. Each root is multiplied by its residue, and the following initial conditions are assumed:
\[ |E(z, t)|_{z=0} = |E_0| \]  
(2.46)
\[ \frac{\partial |E(z, t)|}{\partial z} \bigg|_{z=0} = 0. \]  
(2.47)

The final result is given by Chen and Wurtele [19]:
\[ E(z, t) = \frac{C_E}{\sqrt{2}} e^{-i\omega t} \sum_{\kappa_j, \text{ forward}} \left[ \left( 1 - \frac{s\Omega_c}{\omega} \right) c - \beta_{s0}\kappa_j \right]^2 \kappa_j + \omega_{p1}^2\beta_{s0}^2\beta_{s0} H' \] \[ R_j e^{in_j\omega z/c} \]  
(2.48)

where \( C_E \) is a complex constant satisfying the initial condition in Eq. 2.46:
\[ C_E = \frac{E_0}{\sum_{\kappa_j > 0} R_j \left[ \left( 1 - \frac{s\Omega_c}{\omega} \right) c - \beta_{s0}\kappa_j \right]^2 \kappa_j + \omega_{p1}^2\beta_{s0}^2\beta_{s0} H'/(2\gamma_0\omega^2) $.  
(2.49)
In Eqs. 2.48 and 2.49, $\kappa_{kj} = \text{Re}\{\kappa_j\}$. The large factor inside the summation arises from initial conditions, and the summation is over all roots of $D(\kappa)$ that have a positive real part. For a typical CRM amplifier design, three of the four roots have a positive real part and correspond to a forward traveling wave. The amplitude of the input wave is $E_0$. Because the dispersion equation is fourth order, the expression for the roots is lengthy and not presented here, but it is straightforward and presented in several mathematical handbooks. When the phase or amplitude of the rf wave is needed for a specific set of design parameters, it is calculated by evaluating the roots of the dispersion equation and substituting them into Eq. 2.48. The gain and total phase at any point $z$ in the CRM amplifier interaction region are:

$$\text{Gain (dB)} = 20 \log_{10} \left( \frac{|E(z, t)|}{|E_0|} \right)$$

$$\phi_{\text{total}} = \tan^{-1} \left( \frac{\text{Im}\{E(z, t)\}}{\text{Re}\{E(z, t)\}} \right).$$

The slow phase of the rf wave is defined as the total phase with the fast phase term, $k z_{\omega} z$, removed:

$$\phi_{\text{slow}} = \phi_{\text{total}} - k_{\omega} z,$$

where $k_{\omega} = \sqrt{\omega^2/c^2 - k_{\perp}^2}$. The phase variability of a CRM amplifier is described by the correlation between $\phi_{\text{slow}}$ at the output of the CRM amplifier and the CRM input parameters. Some obvious CRM input parameters are the initial beam voltage, $V_0$, which is directly related to $\gamma_0$: $\gamma_0 = 1 + q_e V_0/(m_0 c^2)$; the beam current, $I$; the beam transverse velocity, $\beta_{\perp} z_{\omega}$, or beam pitch, $\alpha_0 \equiv \beta_{\perp}/\beta_{\omega}$; and the initial power of the injected rf wave, $P_{\text{IN}}$. There are many other input parameters (e.g. wave phase velocity, velocity spread) that affect the performance of the amplifier, as well.

### 2.5 Normalized Parameters

It has been shown by both Bratman[14] and Fliflet[31] that the number of input parameters describing a unique CRM amplifier case can be distilled down to a minimum set of normalized parameters which give a better sense of how the amplifier will perform than the straightforward parameters of voltage, current, beam pitch, etc. Two important parameters that will be referred to later in this thesis are the normalized beam-wave detuning, $\Delta$, and the recoil parameter, $b$, which characterizes how strongly the axial momentum changes with a change in electron energy. The formulas for these two parameters are

$$\Delta \equiv \frac{2 \left( 1 - \beta_{\omega} / \beta_{\phi} \right)}{\beta_{\perp}^2 \left( 1 - \beta_{\phi}^{-2} \right)} \left( 1 - \frac{\beta_{\omega} z_{\omega} - s \Omega_e}{\omega} \right)$$

$$b \equiv \frac{\beta_{\perp}^2}{2 \beta_{\omega} \beta_{\phi} (1 - \beta_{\omega} / \beta_{\phi})},$$
In Eqs. 2.53–2.54, the zero subscripts denote initial values (before the interaction begins). The detuning, $\Delta$, is most closely associated with the strength of the axial guide field, $B_0$, since they are linearly related (though negatively correlated). For the CRM design cases that will be considered for this thesis, the optimum detuning falls between 0.2 and 0.8 (see Section 3.8). The recoil parameter combines the beam pitch, energy, and wave phase velocity into one parameter which gives one measure of how "CARM-like" a device is. The regime $b \approx 0$ corresponds to a pure gyrotron interaction, which gives the highest normalized efficiency[31]. The regime $b \approx 0.5$ corresponds to a CARM[14]. Again, for the design cases, $b$ will generally be in the range 0.2–0.3.

Other parameters that indicate CARM versus gyro-twt operation are the Doppler frequency upshift, $\omega/(s\Omega_e) \gtrsim 2$ for CARMs, and the wave phase velocity, $\beta_\phi$, which is close to unity for CARMs. The essence of the CARM, however, is in the name, autoresonance. For highly relativistic beams, the CARM interaction is much more efficient than the gyrotron interaction. For the gyrotron, a large $\gamma$ value means the interaction will quickly shift out of resonance as the beam energy decreases. For the CARM, however, autoresonance maintains the interaction. Bratman gives the following conditions as determining when a device is a CARM[14]:

$$\beta_\perp \sim \gamma^{-1}$$

$$\left| 1 - \beta_\phi^{-2} \right| \ll \gamma^{-2}.$$  \hspace{1cm} (2.55) \hspace{1cm} (2.56)

These conditions result in high single particle efficiencies (See Eq. 3.4 in Sec. 3.3) which result from CARM operation in relativistic gyro-devices. The gyro-twt and CARM cases are easiest to distinguish when viewed on a dispersion diagram, as in Fig. 2-2.

The derivation of the normalized parameters as done by Bratman and Fliflet depends upon the approximation made in Eq. 2.24. If this approximation loses validity, as discussed earlier, the saturated efficiency will not strictly depend on $\Delta$, $b$, and other parameters as derived by Fliflet and Bratman, and these parameters lose some of their usefulness.

### 2.6 Numerical Simulations

The ready availability of computer power on a desktop PC that was unheard of only a few years ago makes numerical computing more feasible than ever. Two programs to simulate CRM amplifiers have been developed specifically for this thesis. The first to be developed, LCRM32, is based on the linear kinetic theory and uses Eq. 2.48 to determine the growth of the electric field amplitude and phase. LCRM32 employs the analytical formula for the solution to a quartic equation to solve the roots of the polynomial from Eq. 2.41. Because of the theory it is based on, LCRM32 assumes no spread and cannot predict nonlinear properties such as saturated power level or efficiency. Its greatest usefulness is accurately predicting initial growth rates and launching losses. Launching loss is an initial decrease in the wave amplitude due to electron phase bunching.

Because saturated power and efficiency are critical quantities associated with an amplifier, a program based on the non-linear equations, Eqs. 2.13–2.16, was also developed. This program is CRM32 (The "32" in LCRM32 and CRM32 was originally adopted due to the programs being compiled with a 32-bit compiler. It remains now more to distinguish these programs from other CRM simulators). CRM32 loads an arbitrary number of macro-particles with arbitrary (usually
Figure 2-2: Dispersion diagrams for a gyro-twt interaction and a CARM interaction. The dashed lines are the beam lines (Eq. 1.1), and the solid curves are the interacting waveguide modes (Eq. 1.2). Two different wall radii are used. The CARM mode is characterized by a phase velocity close to unity and a large upshift in interaction frequency as compared to the cyclotron frequency.
gaussian) distributions in energy, momentum, guiding-center radius, and phase. For CRM simulations, the initial phase distribution is always uniform. (A non-uniform phase distribution results in gyro-twistron simulation.) CRM32 has been thoroughly benchmarked against other existing CRM simulations. Improvements to CRM32 over other (available) CRM simulations include using no approximations for Bessel functions (Eq. 2.24), removal of particles that satisfy the condition $R_{gj} + r_{Lj} \geq r_w$ (waveguide wall interception), and ability to use arbitrary magnetic field profiles and wall radius profiles. CRM32 uses the Bulirsch-Stoer integration algorithm[76].

Both LCRM32 and CRM32, by the nature of the equations they are based on, consider interactions with only one TE waveguide mode, and they both neglect the effects of beam space charge. For most optimized run parameters, the single TE mode approximation has been shown to be a reasonable approximation[16]. The neglect of space charge is valid if the beam is tenuous, that is, if the plasma frequency of the beam is significantly less than the relativistic cyclotron frequency. Davidson, in Sec. 7.2 of his book[26], defines the following parameters related to space charge in a beam:

$$s_e \equiv \frac{\gamma \omega_p^2}{\Omega_{ce}^2}$$

(2.57)

$$s_e^0 \equiv \frac{2 \beta_y^2}{(1 - \beta_y^2)^2}$$

(2.58)

Davidson's exact condition for a tenuous beam, then, is

$$s_e \ll s_e^0.$$  

(2.59)

As $s_e$ approaches $s_e^0$, Davidson shows that the temporal CRM growth rate first increases above the tenuous beam growth rate, but then drops to zero as $s_e \to s_e^0$. The tenuous beam condition is discussed further in Section 3.8. Chen[17] discusses longitudinal space charge effects on the CRM instability, but does not provide a universal condition for when longitudinal space charge affects the growth rate. For the examples shown in his paper involving a 500 A, 500 kV beam, the effect is quite small in regions where the growth rate is highest.

The most significant disadvantage to using CRM32 is the CPU time it requires in comparison to LCRM32. Whereas LCRM32 uses much less than one second of CPU time for a single simulation on a 386 based PC, CRM32 uses nearly one minute of CPU time on a Cray-2S/4128 for a simulation involving 4096 macro-particles ($N = 4096$), or, equivalently, nearly 5 hours on a 386 based PC. $N = 4096$ macro-particles was determined to be optimal when running simulations with significantly non-uniform energy and momentum distributions.

Results from two cases simulated by both LCRM32 and CRM32 are shown in Fig. 2-3. For both cases, the energy, momentum, and guiding-center radius distributions loaded into CRM32 are single-valued, with $R_g = 0$. The number of macro-particles used is 32. For both cases, $V = 400$ kV, $f = 17.136$ GHz, $r_w = 1.27$ cm, and the input power, $P_{IN} = 10$ W. For the $TE_{11}$ case $I = 350$ A, $\alpha = 0.4$, and $\Delta = 0.5$ ($B_0 = 0.31$ T). For the $TE_{31}$ case, $I = 180$ A, $\alpha = 0.7$, and $\Delta = 0.12$ ($B_0 = 0.262$ T). Initially, the two theories agree reasonably well, but as the wave amplitude increases and the linear theory breaks down, the non-linear theory begins to differ substantially. Even in the linear regime, however, Fig. 2-3 shows some discrepancy between
Figure 2-3: Linear kinetic theory and single particle (non-linear) theory results for two cases: (a) a first harmonic and (b) a third harmonic interaction. Note the good agreement between initial launching loss and growth rate. Parameters: $V = 400$ kV, $f = 17.136$ GHz, $r_w = 1.27$ cm, $P_{IN} = 10$ W. For the $TE_{11}$: $I = 350$ A, $\alpha = 0.4$, and $\Delta = 0.5$ ($B_0 = 0.31$ T). For the $TE_{31}$: $I = 180$ A, $\alpha = 0.7$, and $\Delta = 0.5$ ($B_0 = 0.262$ T).
the single particle theory and the linear kinetic theory. This is likely explained by approximations that are made in the derivation of Eq. 2.41. All terms involving transverse spatial gradients in the equilibrium beam distribution function, $f_0$, have been neglected. Chu[23] has derived a more thorough dispersion relation that includes stabilization terms in addition to the $Q'$ term in Eq. 2.41. These terms come from considering the spatial gradient in $f_0$ and generally have the effect of lowering the CRM growth rate, which would improve the agreement in Fig. 2-3. For example, the effective $Q'$ value as derived by Chu is 30% higher than the value from Eq. 2.43 for the TE_{31} case in Fig. 2-3, and 10% higher for the TE_{11} case.

As discussed at the end of Section 2.3, the Bessel function approximation used in Eq. 2.24 loses validity most noticeably when the electrons have a high pitch value ($\alpha \gtrsim 1$). The results from a simulation of such a case by CRM32 are shown in Fig. 2-4 for a third harmonic TE_{31} interaction. As expected, the case where the actual Bessel function coupling term is used has a slower growth rate. The parameters for the simulation are $V = 400$ kV, $I = 200$ A, $f = 17.136$ GHz, $r_w = 1.27$ cm, $P_N = 10$ W, $\alpha = 1$, $B_0 = 0.265$ T, and $N = 4096$. The particles are loaded uniformly in guiding-center radius out to a radius of 0.6 cm. Energy and momentum distributions are single-valued.

With the theory for the CRM interaction established and LCRM32 and CRM32 to be used as the essential design tools, a practical gyro-twt design must now be selected. Chapter 3 addresses this issue.
Figure 2-4: Gain curves for a TE$_{31}$ third harmonic interaction where the approximation from Eq. 2.24 is (dashed curve) and is not (solid curve) used. Parameters: $V = 400$ kV, $I = 200$ A, $f = 17.136$ GHz, $r_w = 1.27$ cm, $P_{IN} = 10$ W, $\alpha = 1$, $B_0 = 0.265$ T. The simulation uses 4096 macro-particles and a beam width (diameter) of 1.2 cm. Energy and momentum distributions are single-valued.
Chapter 3

Gyro-TWT Amplifier Design

3.1 Introduction

In the design of any high power microwave device, there are several issues to consider. What is the desired operating frequency? What is the required magnetic field? What is the best operating mode? More generally, what is the goal of the experiment? These and other design issues are addressed in this section.

3.2 Experimental Objective

The purpose of this thesis is to experimentally investigate relativistic harmonic gyro-twt amplifiers. One motivation for such a study is to examine the practicality of gyro-twt amplifiers for use as drive sources for electron beam accelerating structures. The frequency expected to be used for the next generation of high energy colliders is presently a hotly debated topic, though authorities agree that it will be somewhere between 2.8 GHz (the frequency used at SLAC) and $\sim 30 \text{ GHz}[85, 67]$. Many people believe that six times the SLAC frequency, or 17.136 GHz, is the best choice[47], and this is the primary motivation for the design choice of 17.136 GHz as the operating frequency for the gyro-twt experiments that will be investigated by this thesis. The energy and current of the electron beam are determined by the chosen beam source for the experiments, which is presented in Section 4.2. Based on this source, the beam energy used in the design is 400 kV, and the beam current is 350 A. Though SNOMAD-II can run at lower voltages and currents, two key experimental objectives are to investigate relativistic amplifiers and to achieve high power, and these objectives are better met at high voltages and currents.

Because an accelerating structure needs high power rf with a very pure frequency spectrum, the emphasis in the design of these gyro-twt experiments is primarily to maximize efficiency and, secondarily, to minimize phase variability. The gain-bandwidth of the experiments is not considered to be critical, though large gain-bandwidth is commonly desired.
3.3 Interaction Efficiency

To choose design parameters which yield high efficiency, it is useful to begin by considering the ideal single particle efficiency for a CRM device. This formula has been derived in the literature[14], and the derivation begins by combining the general relativistic energy-momentum relation,

\[ E = q_e V + m_0 c^2 = \sqrt{m_0^2 c^4 + \frac{c^2 p^2}{k_z} + \frac{c^2 p^2}{k_z}} \]  

with the result from Eq. 1.8,

\[ \frac{dE}{dp_z} = \frac{\omega}{k_z} = \text{constant}, \]  

yielding

\[ \Delta E \approx \frac{\beta^2 \Delta p_z}{2E_0 (1 - \beta \omega / \beta_\phi)}. \]  

Substituting in the general formula for efficiency, \( \eta = (E_0 - E)/(E_0 - m_0 c^2) \), and a total loss of perpendicular momentum, \( \Delta p_\perp = -p_{10} \), the final result for the ideal single particle efficiency is

\[ \eta_{sp} \approx \frac{\beta_{10}}{2(1 - \beta \omega / \beta_\phi)(1 - \gamma_0^{-1})}. \]  

Shown in Fig. 3-1 is a contour map of the single particle efficiency for the chosen case of \( V = 400 \text{kV} \). Clearly, the efficiency is a strong function of the beam pitch, \( \alpha \equiv \beta_{10} / \beta_\omega \). This strong dependence is a well known feature of the CRM interaction. In choosing the design case beam pitch, then, \( \alpha \) should be chosen as high as possible. There are four factors that limit the \( \alpha \) value for a CRM design:

- **Wall radius.** Because \( r_L \) is linearly proportional to \( \beta_{10} \), a high enough value of \( \alpha \) will cause the beam to be intercepted by the beam tube wall.

- **Beam spin-up mechanism.** The way in which perpendicular momentum is imparted to the electron beam generally limits the available \( \alpha \) range. This is discussed in detail in Chapter 5.

- **Beam quality.** Often a higher \( \alpha \) value comes at the expense of beam quality. This is discussed in detail in Sections 5.7.3 and 5.7.4.

- **Absolute instability.** For any CRM design, a threshold value of \( \alpha \) exists above which the interaction generates an absolute (stationary in \( z \)) instability. The convective (moving in \( z \)) instability may also still be excited, but the presence of an absolute instability generally has deleterious effects on the convective interaction.

3.3.1 Absolute Instability

For now, the first three limits on \( \alpha \) listed above are not a concern. The absolute instability condition is expected to be the most limiting factor. It is important to minimize the possible occurrence of oscillations which may spoil the beam quality and greatly reduce the efficiency of the desired
Figure 3-1: Lines of constant single particle efficiency plotted for $V = 400$ kV. Calculations are based on Eq. 3.4. The efficiency depends most strongly on the beam pitch, $\alpha$. 
convective instability. An example of the absolute instability threshold is shown in Fig. 3-2 as a dispersion diagram. It is not at all obvious from a dispersion diagram, however, how to determine the threshold for absolute instability. The exact conditions for CRM absolute instability in an infinite system have been determined by Davies[27]. A brief synopsis of that work is presented here. The coupling term in a cyclotron resonance maser, $\epsilon$, is given by Davies as

$$\epsilon = \frac{4\beta_z^2 J_{s-m}^2(k_R) J_{s}^2(k_L r_L) I}{\gamma_0 \beta_z (\nu_{mn}^2 - m^2) J_{m}^2(\nu_{mn}) I_A},$$

(3.5)

where $I_A \equiv q_e/(4\pi \epsilon_0 m_0 c^3) \approx 17$ kA. The coupling term, $\epsilon$, is simply the first term on the right-hand side of Eq. 2.41 with $(\omega^2 - c^2 k^2)$ replaced by $c^2 k_1^2$. This replacement is valid near resonance. A critical coupling is now determined. If the coupling constant from Eq. 3.5 exceeds this critical coupling, the interaction becomes absolutely unstable. This critical coupling, $\epsilon_c$ has the form

$$\epsilon_c = 27 \beta_z^2 k_1^4,$$

(3.6)
where

$$\hat{k}_s = \frac{-4\beta_s s b + \sqrt{16\beta_s^2 s^2 b^2 + 2(1 + 8\beta_s^2)(1 - s^2 b^2)}}{2(1 + 8\beta_s^2)},$$  \hspace{1cm} (3.7)$$

and $b \equiv \Omega_c/(ck_{\perp}) = q_e B_0/(m_0 \gamma_0 c k_{\perp})$. The frequency of the absolute instability is $\omega_s$, where

$$\omega_s = 4\beta_s \hat{k}_s + s b.$$  \hspace{1cm} (3.8)$$

There is one caveat, however, that being that Eq. 3.6 is only valid when $\hat{k}_s$ and $\omega_s$, as determined by Eqs. 3.7 and 3.8, satisfy the following condition:

$$\omega_s - \hat{k}_s/\beta_s > 0.$$  \hspace{1cm} (3.9)$$

If Eq. 3.9 is not satisfied, the critical coupling value is given by

$$\epsilon_c = \frac{\hat{k}_s'}{\beta_s'} (b_0^2 \hat{k}_s' - \beta_s s b)^3,$$$$

where

$$\hat{k}_s' = \frac{\beta_s}{4b_0^2} \left[ s b + \sqrt{s^2 b^2 + 8b_0^2} \right],$$  \hspace{1cm} (3.11)$$

and $b_0 \equiv \sqrt{1 - \beta_s^2}$. In this case, the frequency of the instability is $\omega_s'$, where

$$\omega_s' = \hat{k}_s'/\beta_s.$$  \hspace{1cm} (3.12)$$

The instability threshold from Eqs. 3.6 and 3.10 can place restrictions on experimental parameters in a number of ways. Most commonly, it is used to derive a threshold beam current, above which the interaction is unstable. However, since our beam current has been chosen at 350 A, for these designs the absolute instability threshold will be used to limit the value of beam pitch, $\alpha$. The above theory does not take into account reflectivity in the system. Detailed analysis of backward wave oscillations in finite length CRM systems with reflectivity is in the literature[53, 50, 59]. Due to the short voltage pulse for these experiments, however, it is expected that keeping the beam parameters below the instability threshold for an infinite system will suffice to stabilize oscillations.

### 3.4 Investigation of First Four Harmonics

In evaluating the CRM interaction at the first four harmonics, a proper waveguide mode must be selected for interaction with each harmonic. Because the average guiding center radius, $\langle R_g \rangle$, of the source electron beam will be zero (due to the beam being generated by a Pierce-wiggler configuration as discussed in Chapter 5), and the CRM coupling scales as $J_{s-m}(k_{\perp} R_g)$ (Eq. 2.41), the only selection for the azimuthal index that results in non-zero coupling is $m = s$. The natural selection for the radial index, $n$, is 1, since a higher value leads to mode competition with all lower values of $n$. Thus, for each harmonic interaction $s = 1, 2, 3, \text{ and } 4$, the chosen interaction mode is the $\text{TE}_{11}$, $\text{TE}_{21}$, $\text{TE}_{31}$, and $\text{TE}_{41}$, respectively.
The results for threshold beam pitch values as calculated from Eqs. 3.6 and 3.10 are shown in Fig. 3-3 for each of the harmonics. Curves of constant $\alpha_{\text{CRT}}$ are plotted against waveguide radius, $r_w$, and interaction detuning, $\Delta$, where $\alpha > \alpha_{\text{CRT}}$ leads to absolute instability. The cut-off radii at 17.136 GHz are 5.13 mm, 8.5 mm, 11.7 mm, and 14.8 mm for the TE$_{11}$, TE$_{21}$, TE$_{31}$, and TE$_{41}$ modes, respectively. The salient result from Fig. 3-3 is that the higher harmonics compensate for their lower coupling by having a higher threshold beam pitch before absolute instability occurs.

The next logical step in the design process is to generate the same sort of plots as in Fig. 3-3, but instead of plotting $\alpha_{\text{CRT}}$ contours, plotting contours of the saturated efficiency of the CRM interaction for $\alpha = \alpha_{\text{CRT}}$. Fig. 3-4 shows such plots, with the range of the axes changed from Fig. 3-3 in order to highlight the region of interest. For the parameters chosen so far: $V = 400$ kV, $I = 350$ A, $f = 17.136$ GHz, and the chosen interaction modes, Fig. 3-4 demonstrates the regions of maximum attainable efficiency without generating an absolute instability. The efficiency values in Fig. 3-4 are calculated from CRM32 using $N = 32$ and single-valued energy, momentum, and guiding-center radius distributions with $R_g = 0$. The simulations were done with “wall-hit-checking” on, which means that particles satisfying $R_g + r_L > r_w$ are assumed to have been intercepted by the waveguide wall and are removed from the simulation at the point when they first satisfy $R_g + r_L > r_w$. The most interesting result from Fig. 3-4 is that the best efficiency results are for the second harmonic case rather than the fundamental. This is due to the beam current being too high for optimal efficiency in the TE$_{11}$ interaction, as will be shown. For the third and fourth harmonics, the efficiency steadily drops off, a result of the weaker coupling in higher harmonics.

An additional consideration for the design of the experiments is the saturation length of the amplifier. A short saturation length is desired since it reduces the overall size and cost of the experiment. A short length can also reduce the growth rate of absolute instabilities. Fig. 3-5 shows the saturation length contours corresponding to the efficiency contours in Fig. 3-4. The highest efficiency regions in Fig. 3-4 also have the longest saturation lengths. There are, however, regions of efficiency quite close to the maximum where the saturation lengths are much shorter. Due to limitations in available magnetic coils, a saturation length of under 1 m is desired for these experiments. Such a condition is easily satisfied without sacrificing a large amount of efficiency. Based on Figs. 3-3-3-5, four specific cases (one for each harmonic) are selected for further study. The parameters for these cases are listed in Table 3.1. The selection procedure used was to choose a wall radius as small as possible without sacrificing efficiency and without having a large saturation length. A smaller wall radius results in a higher phase velocity and a shorter saturation length, both of which reduce the amplifier’s sensitivity to poor beam quality. This will be shown in Section 3.6.

The efficiency of each design case reflects the general trend from Fig. 3-4, with the TE$_{21}$ having the highest efficiency at $\eta = 35\%$. The TE$_{31}$ also has good efficiency, nearing 30%. A useful plot for a design study is that of efficiency contours plotted against normalized detuning and current. Fig. 3-6 shows such efficiency contours for the four cases listed in Table 3.1. For all cases but the TE$_{11}$ case, the peak efficiency occurs near the designed beam current (350 A). This is not necessarily a serendipitous result, however. After all, the method used to choose these cases was based on finding the optimum parameters for a beam current of 350 A. Even so, in the TE$_{11}$ case, the optimum current is substantially less than 350 A. At 60 A, the peak efficiency for the TE$_{11}$ case is 38%, nearly 50% higher than the efficiency in Table 3.1 (26%). This optimum current is better thought of as an optimum rf power limit (since current and rf power are linearly related.
Figure 3-3: Curves of constant $\alpha_{\text{CRIT}}$ plotted against waveguide radius, $r_w$, and normalized detuning, $\Delta$ (from Eq. 2.53), for the first four CRM harmonics. The CRM interaction for the listed mode becomes unstable when $\alpha > \alpha_{\text{CRIT}}$. The cut-off radii at 17.136 GHz are 5.13 mm, 8.5 mm, 11.7 mm, and 14.8 mm for figures (a), (b), (c), and (d) respectively. Fixed parameters are $V = 400$ kV, $I = 350$ A, and $f = 17.136$ GHz.
Figure 3-4: Curves of constant efficiency plotted against waveguide radius, $r_w$, and normalized detuning, $\Delta$ (from Eq. 2.53), for the first four CRM harmonics. The $\alpha$ value used in the calculation of the efficiency is the threshold $\alpha$ for absolute instability (see Fig. 3-3). Fixed parameters are $V = 400$ kV, $I = 350$ A, and $f = 17.136$ GHz. CRM32 is used to calculate the efficiencies with $N = 32$, $R_g = 0$, and single-valued energy and momentum distributions.
Figure 3-5: Curves of constant saturation length plotted against waveguide radius, $r_w$, and normalized detuning, $\Delta$ (from Eq. 2.53), for the first four CRM harmonics. These contours correspond to the efficiency contours plotted in Fig. 3-4. Fixed parameters are $V = 400$ kV, $I = 350$ A, and $f = 17.136$ GHz.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>s = 1</th>
<th>s = 2</th>
<th>s = 3</th>
<th>s = 4</th>
</tr>
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<tbody>
<tr>
<td>Mode</td>
<td>TE$_{11}$</td>
<td>TE$_{21}$</td>
<td>TE$_{31}$</td>
<td>TE$_{41}$</td>
</tr>
<tr>
<td>Wall radius, $r_w$</td>
<td>6.6 mm</td>
<td>9.5 mm</td>
<td>12.8 mm</td>
<td>16.0 mm</td>
</tr>
<tr>
<td>Beam pitch, $\alpha$</td>
<td>0.867</td>
<td>1.04</td>
<td>1.18</td>
<td>1.27</td>
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<tr>
<td>Detuning, $\Delta$</td>
<td>0.75</td>
<td>0.48</td>
<td>0.33</td>
<td>0.23</td>
</tr>
<tr>
<td>Axial Field, $B_0$</td>
<td>0.542 T</td>
<td>0.356 T</td>
<td>0.259 T</td>
<td>0.206 T</td>
</tr>
<tr>
<td>Phase velocity, $\beta_\phi$</td>
<td>1.59</td>
<td>2.24</td>
<td>2.46</td>
<td>2.64</td>
</tr>
<tr>
<td>Normalized $b$ parameter</td>
<td>0.244</td>
<td>0.186</td>
<td>0.193</td>
<td>0.194</td>
</tr>
<tr>
<td>Upshift, $\omega/(\Omega_c)$</td>
<td>2.02</td>
<td>1.53</td>
<td>1.40</td>
<td>1.33</td>
</tr>
<tr>
<td>Single particle efficiency, $\eta_{sp}$</td>
<td>55.3%</td>
<td>54.5%</td>
<td>58.0%</td>
<td>59.8%</td>
</tr>
<tr>
<td>Overall efficiency, $\eta$</td>
<td>25.7%</td>
<td>35.2%</td>
<td>29.2%</td>
<td>21.5%</td>
</tr>
<tr>
<td>Saturation length, $z_{SAT}$</td>
<td>0.36 m</td>
<td>0.44 m</td>
<td>0.56 m</td>
<td>0.55 m</td>
</tr>
</tbody>
</table>

Table 3.1: Four selected cases (one for each harmonic) for further study. Parameters fixed across all cases are $V = 400$ kV, $I = 350$ A, and $f = 17.136$ GHz. Each case corresponds to parameters near the optimal efficiency for each harmonic. The single particle efficiency, $\eta_{sp}$, is given by Eq. 3.4. The parameters $\Delta$ and $\delta$ are given by Eqs. 2.53 and 2.54, respectively. The overall efficiency is from Fig. 3-4.
Figure 3-6: Curves of constant efficiency plotted against beam current, $I$, and normalized detuning, $\Delta$, for the four cases listed in Table 3.1. The dashed curves show the absolute instability threshold for each case. The region above and to the left of the dashed curve (lower current, higher $\Delta$) is stable. Note that the detuning ranges differ from graph to graph.
assuming a constant efficiency). With high enough beam current, the rf power eventually reaches a level such that the local electric fields become too strong to interact efficiently with the electron beam. For higher order modes, the electric field is more widely distributed across the waveguide cross-section, which reduces the local field strength for a given rf power level, thus increasing the current threshold at which the interaction begins to lose efficiency. In addition, the larger radius waveguide associated with higher order modes gives a high current beam more room to propagate at reasonable current densities.

3.5 Investigation of CARM Interaction

By analyzing the TE_{11} case for s = 1 at higher values of the wall radius, r_w, the CARM performance at V = 400 kV, I = 350 A, and f = 17.136 GHz can be gauged. As the wall radius increases, the phase velocity approaches unity and the Doppler upshift in frequency increases. This combination of qualities (phase velocity near unity, high upshift) is the trademark of a CARM amplifier. Fig. 3-7 contains all of the data from a CARM analysis done for s = 1 with the TE_{11} mode. Figures 3-7(a), 3-7(b), and 3-7(c) are identical to Figs. 3-3(a), 3-4(a), and 3-5(a) respectively, but they are plotted over a wider range in waveguide radius, r_w. The maximum allowable beam pitch for the CARM interaction is much more limited by absolute instability than for the gyro-twt interactions, which reduces the achievable efficiency for these CARM cases. In choosing a specific CARM amplifier case based on Figs. 3-7(b) and 3-7(c), there is a clear trade-off between efficiency and wall radius. A wall radius too small results in a gyro-twt instead of a CARM, and a radius too small does not allow for propagation of a finite thickness beam. The effect of a finite thickness beam is not included in the simulations since they are run with a single valued guiding center radius of R_g = 0 for all particles (the effect of the Larmor radius, however, is fully included in the simulations). A compromise radius of r_w = 12.7 mm is chosen for the sake of CARM analysis. The best predicted efficiency at r_w = 12.7 mm is η = 9% with α = 0.38 and Δ = 0.53. Higher values for α result in absolute instability. Because 350 A is clearly not the optimum beam current for good efficiency in this case, α = 0.45 is used for the efficiency contour plot in Fig. 3-7(d) even though it results in absolute instability at 350 A. The optimum current from Fig. 3-7(d) is 40 A, with an efficiency of 24% and a saturation length of 1.6 m. These parameter settings, due to the low current, do not result in absolute instability. The parameters for the two CARM designs just discussed (one at 40 A and one at 350 A) are presented in Table 3.2. Note that the output power in the 350 A case, despite having a lower efficiency than the 40 A case, is significantly higher (13 MW versus 4 MW). The cases presented in Table 3.2 do not do justice to a CARM amplifier because of the 400 kV beam energy limitation. For more relativistic beams, the CARM interaction efficiency improves substantially.

3.6 Sensitivity to Beam Quality

An important component of any high power microwave device, particularly CRM devices, is the electron beam source. The beam formation is critical in determining the uniformity of the individual electron velocities within the beam, which is, in turn, critical to the efficiency of CRM masers.
Figure 3-7: Contours for a fundamental CARM interaction with the TE\textsubscript{11} mode. Figure (a) shows beam pitch threshold values, above which absolute instability occurs. Figure (b) shows interaction efficiency for the $\alpha$ values in Figure (a). Figure (c) shows saturation length contours. Figure (d) shows efficiency versus current and detuning for $\alpha = 0.45$ and $r_w = 12.7$ mm. The dashed curve in Figure (d) shows the absolute instability threshold. Above and to the left of this curve, the interaction is stable. Fixed parameters are $V = 400$ kV, $I = 350$ A, and $f = 17.136$ GHz.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$I = 40 \text{ A}$</th>
<th>$I = 350 \text{ A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic, $s$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mode</td>
<td>TE$_{11}$</td>
<td>TE$_{11}$</td>
</tr>
<tr>
<td>Wall radius, $r_w$</td>
<td>12.7 mm</td>
<td>12.7 mm</td>
</tr>
<tr>
<td>Beam pitch, $\alpha$</td>
<td>0.45</td>
<td>0.38</td>
</tr>
<tr>
<td>Detuning, $\Delta$</td>
<td>0.4</td>
<td>0.53</td>
</tr>
<tr>
<td>Axial Field, $B_0$</td>
<td>0.324 T</td>
<td>0.305 T</td>
</tr>
<tr>
<td>Phase velocity, $\beta_\phi$</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>Normalized $b$ parameter</td>
<td>0.226</td>
<td>0.175</td>
</tr>
<tr>
<td>Upshift, $\omega/(s\Omega_c)$</td>
<td>3.37</td>
<td>3.58</td>
</tr>
<tr>
<td>Single particle efficiency, $\eta_{sp}$</td>
<td>42.5%</td>
<td>33.7%</td>
</tr>
<tr>
<td>Overall efficiency, $\eta$</td>
<td>24%</td>
<td>9%</td>
</tr>
<tr>
<td>Saturation length, $z_{SAT}$</td>
<td>1.6 m</td>
<td>1.2 m</td>
</tr>
</tbody>
</table>

Table 3.2: Two selected CARM cases. Parameters fixed across both cases are $V = 400 \text{ kV}$ and $f = 17.136 \text{ GHz}$. The two designs are close to optimal in efficiency for low and high current. The single particle efficiency, $\eta_{sp}$, is given by Eq. 3.4. The parameters $\Delta$ and $b$ are given by Eqs. 2.53 and 2.54, respectively. The overall efficiency values are from the plots in Fig. 3-7.
As an example of allowable non-uniformity in electron velocity (velocity spread), consider the resonance equation, Eq. 1.1. The velocity of each electron directly affects its relative phase with the rf wave. If $\Lambda_j$ is this relative phase, then

$$\frac{d\Lambda_j}{dz} = \frac{1}{v_{xj}} \left( \omega - k_z v_{xj} - s\Omega_c \right)$$

(3.13)

Assuming that the CRM interaction extracts most of its energy from the perpendicular component of the velocity and leaves $v_{xj}$ relatively unchanged,

$$\Lambda_j(z) = \frac{L}{v_{xj}} \left( \omega - k_z v_{xj} - s\Omega_c \right),$$

(3.14)

where $L$ is the length of the interaction. If two electrons have different velocities, their phases will eventually differ by 180 degrees, at which point they will not be able to impart further energy to the rf wave. If this criterion is used to establish a velocity spread limit on the beam, the limit can be written as

$$\pi \geq \Lambda_1(z) - \Lambda_2(z) \geq \frac{L}{v_{x1}} (\omega - k_z v_{x1} - s\Omega_{c1}) - \frac{L}{v_{x2}} (\omega - k_z v_{x2} - s\Omega_{c2})$$

(3.15)

Assuming that $\Delta v_z = v_{z2} - v_{z1}$ is small, using $v_z = (v_{z1} + v_{z2})/2$, keeping $\gamma$ fixed, and dropping terms of order $(\Delta v_z)^2$, the spread limit becomes

$$\frac{\Delta v_z}{v_z} \leq \frac{\pi v_z}{L(\omega - s\Omega_c)}.$$  

(3.16)

If the axial velocity is kept fixed and instead $\gamma$ varies, the same procedure yields an energy spread limit of

$$\frac{\Delta \gamma}{\gamma} \leq \frac{\pi v_z}{Ls\Omega_c}.$$  

(3.17)

Of course, these spread limits make significant approximations, neglecting any coupling between the beam and the wave, and only considering the point at which two particles become 180 degrees out of phase. Nonetheless, it is instructive to find out what kind of velocity and energy spread numbers Eqs. 3.16 and 3.17 yield for specific cases. The value of $L$ will be set to the growth length (the length in which $E$ increases by a factor of $e$) of the amplifier, which is typically $\sim 5$ cm for these cases. After one or two growth lengths, the rf field amplitude becomes strong enough that the phase of the particles is less dominated by Eq. 3.13 and more dominated by the local rf field.

For the high current CARM case from Table 3.2: $f = 17.136$ GHz, $V = 400$ kV, $\alpha = 0.38$, $B_0 = 0.305$ T, and $L = 5$ cm, the spread limits for this case are:

$$\frac{\Delta v_z}{v_z} \leq 19\%$$
\[ \frac{\Delta \gamma}{\gamma} \leq 48\%. \quad (3.18) \]

For the third harmonic gyro-twt case from Table 3.1: \( f = 17.136 \text{ GHz}, \ V = 400 \text{ kV}, \ \alpha = 1.15, \ B_0 = 0.262 \text{ T}, \) and \( L = 5 \text{ cm}, \) the spread limits are:

\[ \frac{\Delta v_z}{v_z} \leq 34\% \quad \frac{\Delta \gamma}{\gamma} \leq 13\%. \quad (3.19) \]

While the velocity spread limit for the harmonic gyro-twt is higher, the energy spread limit, due to the harmonic number, \( s, \) in the denominator, is lower. Eq. 3.16 makes it appear that the velocity spread limit is circumvented altogether in the gyrotron limit, \( \omega = s\Omega_c. \) This is partially true. Gyrotrons are much less sensitive to beam spreads than CARMs, but they do have finite velocity spread limits beyond which an efficient interaction does not occur. The gyro-twt because the \( k_z v_z \) term is substantially smaller than for a CARM, is also less sensitive to spreads in \( v_z \) than the CARM.

The spread limits in Eqs. 3.18 and 3.19 are only approximations, and they are typically upper bounds. For a better prediction of the effects of beam spreads, full scale simulations were run on the four harmonic design cases and the 40 A CARM design case from Tables 3.1 and 3.2, respectively. The results are shown in Fig. 3-8, where interaction efficiency for each case is plotted against both axial momentum spread and energy spread. The interaction efficiency is normalized to unity at spreads of zero. The curves in Fig. 3-8 were calculated from CRM32 with \( N = 4096 \) and a beam width of 1.2 cm. The energy and momentum distributions are gaussians centered about the design values with \( \sigma, \) and \( \sigma_{rz} \) representing the usual definition of a standard deviation of a gaussian. From \(-\sigma \) to \(+\sigma\) covers 68\% of the area underneath the gaussian. Both plots in Fig. 3-8 show the gyro-twt cases to be significantly less sensitive to poor beam quality than the CARM case. For only 1\% energy spread or 2\% axial momentum spread, the CARM interaction efficiency is reduced to near zero, whereas the gyro-twt interaction efficiency remains significant even at very large spreads. Based on Fig. 3-8, a CARM design operating at the fundamental for a 400 kV, 350 A electron beam does not look promising unless the beam quality is exceptionally good. The beam transport system is discussed in detail in Chapters 4 and 5.

Although Fig. 3-8 suggests that the sensitivity to axial momentum spread decreases with increasing harmonic, this trend more likely results from the corresponding increase in phase velocity, \( \beta_p, \) beam pitch, \( \alpha, \) and upshift, \( \omega/(s\Omega_c), \) as \( s \) goes from 1 to 4 in the design cases. Note also that the results from Fig. 3-8 contradict the results from the simple theory. The harmonic cases do not show a significant increase in sensitivity to energy spread for rising harmonic number as Eq. 3.17 suggests, and the CARM design is more sensitive to both energy and momentum spread than the gyro-twt designs. The plots in Fig. 3-8 were calculated using single-valued \( \gamma \) (top graph) and \( p_z \) (bottom graph) distributions, however, and Eqs. 3.16 and 3.17 assume single-valued \( \gamma \) and \( v_z \) distributions, respectively.
Figure 3-8: Normalized efficiency curves plotted against axial momentum and energy spread. The efficiencies are normalized to 1 at energy and momentum spreads of zero. The cases are from Tables 3.1 and 3.2. The CARM case is at $I = 40$ A. The curves were calculated with CRM32 using $N = 4096$ and a beam width of 1.2 cm. The top graph assumes zero energy spread, and the bottom graph assumes zero axial momentum spread.
3.7 Efficiency Enhancement by Magnetic Field Tapering

All simulations to this point have been done using a uniform (in \( z \)) axial magnetic guide field. Several authors have shown[73, 18, 66] that a taper in the magnetic field just before the rf wave saturates can lead to significantly increased efficiency. As the electrons lose energy to the wave in a gyro-twt, \( \gamma \) will decrease, causing the relativistic cyclotron frequency, \( \Omega_c \), to increase. For a gyro-twt this is the dominant change in the resonance condition as the electrons lose energy (see Eq. 1.1). For a CARM, the \( k_z v_z \) term also decreases, tending to offset the increase in \( \Omega_c \), hence autoresonance. Since the gyro-twt is not autoresonant, down-tapering the magnetic field, thereby keeping the relativistic cyclotron frequency constant as the particles lose energy, can be used to maintain resonance, which results in higher efficiency. Under certain conditions, an uptaper also can improve efficiency because it pumps more transverse velocity into the beam, which increases the beam-wave coupling. The optimal taper typically begins just before saturation and has a down-slope of \( 0.04 \Gamma B_0 - 0.08 \Gamma B_0 \), where \( \Gamma \) is the electric field growth rate of the gyro-twt interaction, \( E \propto e^{\Gamma z} \).[18]

A specific design for a magnetic field taper is not critical for the gyro-twt design process. Rather, the enhanced efficiency that results from tapering is a motivation to design the interaction field magnet system so that the field can be easily tapered. This is most easily done by using several consecutive, identical coils. The interaction magnet system is discussed in detail in Section 6.2. Fig. 3-9 shows the optimum taper for the third harmonic TE_{31} case from Table 3.1. Again, CRM32 was used to simulate the interaction with \( N = 32 \) macro-particles and single-valued distributions in energy and momentum, and guiding-center radius (\( R_g = 0 \)). The peak efficiency increases from 29% to 40% as a result of the taper. The taper begins at \( z = 48.75 \text{ cm} \), which is 7.25 cm before the original efficiency saturation point of \( z_{SAT} = 56 \text{ cm} \). The taper value is \(-0.24 \text{ T/m}\), and the growth rate of the amplifier is \( \Gamma = 0.083 \text{ m}^{-1} \), so the absolute value of the taper slope is \( 0.083 \Gamma B_0 (B_0 = 0.2592 \text{ T}) \), just as Chen predicts[18].

3.8 Final Design

Often, limitations from available equipment, funding, and materials place restraints on the design of an experiment. For example, because copper pipe is readily available with inner diameters in steps of 1/8 inch in the United States, it is cost-effective to use wall radii that convert nicely to inch units. Therefore, the design radii for the TE_{21} and TE_{31} cases will be shifted slightly from 9.5 mm and 12.8 mm to 9.525 mm and 12.7 mm, respectively. In order to keep this investigation focused, the \( s = 2 \) and \( s = 3 \) cases from Table 3.1 have been selected for experimental testing. The \( s = 1 \) and \( s = 4 \) cases were presented primarily for comparison purposes, and they clearly show that the optimum harmonics for the beam parameters of \( V = 400 \text{ kV} \) and \( I = 350 \text{ A} \) are \( s = 2 \) and \( s = 3 \). The \( s = 1 \) case, in particular, was not especially realistic since the simulations assumed a guiding-center radius of zero for all particles. In reality, the beam has a radius of \( \sim 6 \text{ mm} \) (see Sec. 4.6.2)—just barely small enough to propagate through a 6.6 mm tube, in which case the beam would have no room to spin up to a pitch of \( \alpha = 0.87 \). Because the wall radius used in the 350 A CARM case in Table 3.2 is identical to the \( s = 3 \) gyro-twt case (12.7 mm), this CARM case can be investigated using the identical TE_{31} design, with the necessary changes in \( \alpha \) and \( \Delta \) effected...
Figure 3-9: A demonstration of the efficiency enhancement that results from magnetic field tapering in a gyro-twt interaction. The long-dashed curve shows the efficiency resulting from the tapered magnetic field profile (solid curve). The short-dashed curve shows the efficiency resulting from a uniform field of the same strength as the initial value of the shown tapered field. The parameters of the simulations are from the $s = 3$ case in Table 3.1, with CRM32 performing the calculations.
simply by tuning magnetic coil currents.

The advantage of a lower interaction field for harmonic gyro-twt interactions is demonstrated by the parameters in Table 3.1. The required field falls from 0.54 T for the first harmonic to 0.2 T for the fourth harmonic. The CARM interaction has this same advantage, with the magnetic field required for the two cases in Table 3.2 being ~ 0.31 T. The axial magnetic field for the interaction region will be provided by several identical magnetic coils built by Livermore National Laboratory. These coils are discussed in detail in Section 6.2. The maximum field they can achieve is ~ 0.375 T. The only design which is infeasible due to this constraint is the \( s = 1 \) gyro-twt design in Table 3.1, which was not intended for experimental investigation.

The parameters for the final designs and predicted efficiencies, with spread included, are shown in Table 3.3. The dispersion diagrams are shown in Fig. 3-10 (CARM) and Fig. 3-11 (harmonic gyro-twt). The spreads in Table 3.3 were chosen arbitrarily. A more complete analysis of the expected beam spread is presented in Chapters 4 and 5. Note that the efficiencies from Table 3.3, even for zero spread, are lower than those presented in Tables 3.1 and 3.2. This is not due to the slight shift in parameters from those tables. It is due to the guiding-center radius now being distributed over a full beam width of 1.2 cm (in a top-hat distribution). As with the efficiency contours (Figs. 3-6, 3-7), CRM32 was run with particles satisfying \( R_g + r_L > r_w \) removed from the problem at each \( z \) step.

The gain-bandwidth numbers listed in Table 3.3 for each design case were calculated using CRM32 with \( N = 32 \) rather than \( N = 4096 \) as in the spread cases. All parameters of the simulation except the rf drive frequency were held fixed for the gain-bandwidth calculation. Because a harmonic interaction more quickly shifts out of resonance for a shift in cyclotron frequency due to the \( s \Omega_e \) term, it has a naturally narrower bandwidth. The gain-bandwidth for the CARM is typical of its autoresonant property. As seen in Fig. 3-10, the beam line has a wide area of interaction with the waveguide mode. Note that the CARM bandwidth is centered around lower frequencies, 10 GHz–18 GHz, while the gyro-twt bandwhits are in a higher range: 16.5 GHz–22.2 GHz for the TE_{21} and 16.9 GHz–20.9 GHz for the TE_{31}.

The dispersion diagrams show other potential interactions that may compete with the intended operating modes. As discussed at the beginning of Section 3.4, viable interaction modes must have a harmonic number, \( s \), equal to the azimuthal index number, \( m \). Such interactions are marked in Figs. 3-10 and 3-11 with a 'C' if they are convectively unstable and an 'A' if they are absolutely unstable. The absolutely unstable modes in the CARM case (Fig. 3-10) only occur for the fourth and fifth harmonics, which have very small coupling due to the low beam pitch. Considering the pulse width of the experiment, these modes are not likely to attain significant power levels. The only other unstable mode is the fourth harmonic interaction for the bottom figure in Fig. 3-11. This mode is just unstable, with \( \epsilon/\epsilon_{\text{CRIT}} \sim 1 \). The stronger coupling of the third harmonic interaction should result in a dominant interaction. Again, the short pulse length of the experiment will help the dominant (higher coupling) mode. In Fig. 3-11, the significant detuning of the gyro-twt designs necessary for good efficiency is manifested by the beam lines not even intersecting the waveguide dispersion curves for the intended interactions.

In Section 2.6, the tenuous beam condition was discussed as a way to validate the neglect of space-charge effects in the simulations. For the design cases in Table 3.3, the tenuous beam parameters given by Davidson (Eqs. 2.57–2.59) are shown for each case in Table 3.3 assuming a
<table>
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<tr>
<th>Interaction type</th>
<th>CARM</th>
<th>Gyro-twt</th>
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</tr>
</thead>
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<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
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<td>TE₂₁</td>
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<td>TE₂₁</td>
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<td>Voltage, V</td>
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<td>Current, I</td>
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<td>350 A</td>
<td>350 A</td>
</tr>
<tr>
<td>Frequency, f</td>
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<td>17.136 GHz</td>
<td>17.136 GHz</td>
</tr>
<tr>
<td>Wall radius, r₀</td>
<td>12.7 mm</td>
<td>9.525 mm</td>
<td>12.7 mm</td>
</tr>
<tr>
<td>Beam pitch, α</td>
<td>0.38</td>
<td>0.95</td>
<td>1.15</td>
</tr>
<tr>
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<td>0.53</td>
<td>0.44</td>
<td>0.33</td>
</tr>
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<tr>
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<td>2.22</td>
<td>2.57</td>
</tr>
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<td>3.58</td>
<td>1.53</td>
<td>1.39</td>
</tr>
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<td>50.7%</td>
<td>56.3%</td>
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<td>100 W</td>
<td>100 W</td>
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<tr>
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<td>4.0 GHz</td>
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<tr>
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<td>0.16</td>
<td>0.33</td>
</tr>
<tr>
<td>sᵯₑ (Eq. 2.58)</td>
<td>1.07</td>
<td>1.59</td>
<td>1.57</td>
</tr>
</tbody>
</table>

| σᵣ/⟨γ⟩ = 0%, σₚz/⟨pₑ⟩ = 0% | 9.2% | 19.4% | 17.7% |
| Efficiency, η            | 12.9 MW | 27.2 MW | 24.7 MW |
| Power                      | 51.1 dB | 54.4 dB | 53.9 dB |
| Saturation length, zₑSAT | 1.21 m | 0.53 m | 0.88 m |

| σᵣ/⟨γ⟩ = 1%, σₚz/⟨pₑ⟩ = 5% | 0.0% | 13.6% | 12.1% |
| Efficiency, η            | – | 19.0 MW | 17.0 MW |
| Power                      | – | 52.8 dB | 52.3 dB |
| Saturation length, zₑSAT | – | 0.54 m | 0.72 m |

| σᵣ/⟨γ⟩ = 2%, σₚz/⟨pₑ⟩ = 10% | 0.0% | 6.4% | 6.0% |
| Efficiency, η            | – | 9.0 MW | 8.4 MW |
| Power                      | – | 49.5 dB | 49.3 dB |
| Saturation length, zₑSAT | – | 0.63 m | 0.77 m |

Table 3.3: Final design parameters for the CARM and gyro-twt experiments. Each case corresponds to parameters near the optimal efficiency for each harmonic. The single particle efficiency, ηₛₚ, is given by Eq. 3.4. The parameter Δ is given by Eq. 2.53. The overall efficiencies were calculated by CRM32 using N = 4096 and a 1.2 cm wide top-hat distribution in guiding-center radius.
Figure 3-10: Dispersion diagram for the designed CARM interaction. The solid curves represent the fundamental beam dispersion equation and the TE_{11} waveguide mode. The dashed curves represent harmonic beam lines and higher order modes. A solid flat line marks the operating frequency of 17.136 GHz. Convectively unstable interactions are marked with a 'C,' and absolutely unstable interactions are marked with an 'A' (See Sec. 3.3.1). The parameters for this mode are listed in Table 3.3 (350 Å).
Figure 3-11: Dispersion diagrams for the designed gyro-twt interactions. The solid curves represent beam lines and waveguide modes involved in the intended operating mode interaction. The dashed curves represent other harmonic beam lines and waveguide modes. A solid flat line marks the operating frequency of 17.136 GHz. Convectively unstable interactions are marked with a 'C,' and absolutely unstable interactions are marked with an 'A' (See Sec. 3.3.1). The parameters for these diagrams are listed in Table 3.3.
beam radius of 6 mm. The most questionable case is the third harmonic, where \( s_\varepsilon = 0.33 \). This is still well below \( s_\varepsilon^0 \), however \( s_\varepsilon/s_\varepsilon^0 = 0.21 \). In the example shown on p. 359 of Davidson's book[26], the CRM growth rate is not substantially changed until \( s_\varepsilon/s_\varepsilon^0 > 0.8 \). The CRM32 and LCRM32 codes, then, should give reasonable estimates for these cases.

3.9 Phase Variability

The \( rf \) wave injected into a gyro-twt or CARM amplifier experiences a shift due to the interaction. This shift depends on the exact parameters of the interaction, so a change in system parameters results in a change in this phase shift, which will henceforth be referred to simply as the output \( rf \) phase of the amplifier. Applications like Doppler-shifted radar and accelerator drivers require phase stability and reproducibility. There are two types of phase fluctuations—shot-to-shot and during a single shot, and there are two ways to reduce both types of fluctuations: reduce the sensitivity of the amplifier phase to fluctuations in system parameters, and reduce the fluctuation in system parameters. A commonly used benchmark for phase stability is that of the klystrons driving the Stanford Linear Accelerator (SLAC). These 2.8 GHz klystrons have a phase stability of 8°/% change in voltage[64]. The actual equation for klystron phase stability depends on the interaction length, \( L \), and on \( \gamma \), and is given by[41]

\[
\frac{\partial \phi_k}{\partial V_k} = \left( \frac{\partial}{\partial V_k} \right) \left( \frac{2\pi L\gamma}{\lambda_0\sqrt{\gamma^2 - 1}} \right), \tag{3.20}
\]

where \( \lambda_0 \) is the free space wavelength, \( \phi_k \) is the output phase of the klystron, and \( V_k \) is the beam voltage. The modulators driving the klystrons at SLAC provide voltages accurate to 0.25%, so the total phase fluctuation is \( \sim 2° \).

At the beginning of this section, phase stability was mentioned as a goal of the experimental design, yet it was not used as a criterion for selecting any of the final design parameters in Table 3.3. This is because phase stability in a gyro-twt is largely determined by how various beam parameters are correlated, which is a function of how the beam is formed rather than the CRM interaction itself. The method of using a bifilar helical wiggler to spin-up an axis-encircling beam from a Pierce gun can be used to correlate beam parameters so that the phase variability of the amplifier is minimized[63]. The two parameters that most directly affect the output phase of a gyro-twt are the initial values of beam energy, \( \gamma_0 \), and beam pitch, here represented as in terms of the beam transverse velocity, \( \beta_{\perp 0} \). The equations for the effect of these parameters on the real part of \( k_z \), which directly affects the phase of the amplifier, are[63]

\[
\frac{\partial k_z}{\partial \gamma_0} = \frac{1}{v_\perp^2 \gamma_0^2} \left[ s\Omega_\perp \gamma_0 - \frac{\omega}{\gamma_0 \beta_0 \beta_{\perp 0}} \right], \tag{3.21}
\]

\[
\frac{\partial k_z}{\partial \beta_{\perp 0}} = \frac{\beta_{\perp 0} \sqrt{(\omega/c)^2 - k_{\parallel}^2}}{\beta_{\perp 0}^2}. \tag{3.22}
\]

These equations, as they are derived solely from the CRM beam resonance equation, involve
significant approximations which make them invalid for the design cases presented here. Instead, CRM32 has been used to calculate constant phase contours for the design cases from Table 3.3. These contours are shown in Fig. 3-12 plotted against changes in the beam transverse velocity and the beam energy. The design values from Table 3.3 are denoted by a D subscript, as in $\beta_{1D}$ and $V_D$ for the designed transverse velocity and beam voltage, respectively. The initial values are allowed to fluctuate around the design values by $\pm 2\%$ for the plots. Close to the design value, the phase variation with beam energy is $3.4^\circ/\%$, $9.3^\circ/\%$, and $14.5^\circ/\%$ for the TE_{11}, TE_{21}, and TE_{31} cases, respectively. The variation with transverse velocity is $1.8^\circ/\%$, $10.5^\circ/\%$, and $-29^\circ/\%$, respectively. In each case, the zero degree contour describes a correlation between beam voltage and transverse velocity that, if realized, would result in perfect phase stability against fluctuations in beam energy and transverse velocity. In Section 5.6.1, the correlation induced between beam energy and transverse velocity by a bifilar helical wiggler will be discussed in detail.

With the designs complete, the next step in the experimental process is to construct a system of components that can achieve the desired parameters for each design. This process begins in the next section.
Figure 3-12: Curves of constant amplifier phase plotted against fluctuations in beam energy and transverse velocity. The three cases (a), (b), and (c) represent the designs in Table 3.3 for the TE_{11}, TE_{21}, and TE_{31} cases, respectively. The values V_D and \( \beta_{1D} \) represent the design values from Table 3.3. The zero degree contours indicate the correlation between fluctuations in beam energy and transverse velocity that would be most phase stable.
Chapter 4

Relativistic Electron Beam Transport

4.1 Introduction

This section discusses the proposed method of generating the electron beam for the gyro-twt experiments. Several components work together to form a high quality electron beam, including a high-voltage pulse generator, a carefully designed cathode-anode geometry, a magnetic focusing system, and a good vacuum system.

Armed with the velocity spread criteria from Section 3.6, a beam transport system is needed that can meet these requirements. There are several common beam transport geometries used in maser design, including magnetron injection guns (MIG), Pierce guns, and field emission cathodes. All cathode anode systems involve pulling electrons from a surface (the cathode) by means of a temporary high electric field at the surface created by applying a very high voltage pulse to the cathode. The voltage pulse, depending on the maser design and application, must typically satisfy some nominal conditions involving duration, flatness, and repeatability, and efficiency.

4.2 SNOMAD II: High Voltage Pulse Compression

One means of generating a short, high voltage pulse is through pulse compression. SNOMAD-II, the second incarnation of a solid-state, nonlinear magnetic accelerator driver, was designed and built by Dan Birx of Science Research Laboratory and installed at the MIT Plasma Fusion Center in 1989. SNOMAD-II uses a sophisticated pulse compression technique that generates a 30 ns wide pulse at voltage levels up to 500 kV. Because SNOMAD-II uses all solid-state circuitry and magnetic switches, it can pulse at repetition rates up to 1 kHz. A picture of SNOMAD-II is shown in Fig. 4-1.

The pulse compression mechanism used in SNOMAD-II can be explained with a few simple circuit diagrams. In Fig. 4-2, the capacitor \( C_1 \) represents the main charging capacitor which holds a DC voltage \( V_0 \) until the beginning of the voltage pulse, at which time, \( t = 0 \), the switch in Fig. 4-2 is latched shut. The time evolution of the voltages in the circuit is easily derivable as

\[
V_1(t) = V_0 \left[ 1 - \frac{C_2}{C_1 + C_2} (1 - \cos \omega t) \right] \quad (4.1)
\]
Figure 4-1: SNOMAD-II, built by Science Research Laboratory. For scale, the cement blocks surrounding SNOMAD-II are 1.22 m (4 feet) in height.
\[ V_2(t) = V_0 \frac{C_1}{C_1 + C_2} \left( 1 - \cos \omega t \right), \]  

(4.2)

where

\[ \omega = \sqrt{\frac{C_1 + C_2}{LC_1 C_2}}. \]  

(4.3)

In the case where \( C_1 = C_2 \), all of the energy from \( C_1 \) is transferred to \( C_2 \), with \( V_1(t) \) and \( V_2(t) \) having amplitudes of \( V_0 \). This is an efficient circuit for pulse compression. In the case where \( C_2 \ll C_1 \), the voltage \( V_2(t) \) rises to \( 2V_0 \) in time \( \Delta t = \pi / \omega \). The energy transfer is inefficient (much of the energy stays in capacitor \( C_0 \)) but the voltage amplitude is doubled.

If another switch, inductor, and capacitor are added to the peaking capacitor circuit in Fig. 4-2 and the new switch is thrown when the voltage \( V_2 \) peaks, then with a properly designed value for \( L_2 \), the voltage pulse compresses further. To simplify the circuit, the switches, which must be latched with precise timing, are replaced by saturable inductors, as in Fig. 4-3, which, if designed properly, play the role of switches which automatically latch at the correct time. A saturable inductor is an inductor loaded with ferromagnetic material that is designed to saturate at a specified number of volt-seconds. Before saturation, the saturable inductor \( L_2 \) looks like an open circuit to an AC signal because of its very high inductance. After saturation, the inductance is lowered by orders
of magnitude to a designed value, and energy can quickly pass through the inductor into the next capacitor in the pulse compression chain. A hysteresis curve for a saturable inductor is shown in Fig. 4-4. \( L_2 \) initially must be reversed biased with either a pulsed negative current or a DC negative current so that it initially has an internal magnetic field of \( B \leq -B_r \). In Fig. 4-3, as current flows into \( C_2 \) and the voltage on \( C_2 \), \( V_2(t) \) rises, the magnetic field in \( L_2 \), \( B_2(t) \), will be related to \( V_2(t) \) in the following manner:

\[
B_2(t) = -B_r + \frac{1}{NA_c} \int_0^t V_2(t')dt',
\]  

(4.4)

where \( N \) is the number of turns in \( L_2 \) and \( A_c \) is the area of the core of \( L_2 \). This relation will hold until \( L_2 \) saturates. At this point, the effective inductance of \( L_2 \) will be greatly reduced. The \( H \) axis in Fig. 4-4 can be thought of as representing how much current is flowing through \( L_2 \), and the slope of the hysteresis curve can be thought of as the effective inductance of \( L_2 \) at an instantaneous point in time. When \( L_2 \) is “forward” saturated, current can then flow quickly into \( C_3 \). The time at which \( L_2 \) saturates is given by \( t_{sat} \), where

\[
\int_{t_{sat}}^t V_2(t')dt' = NA_c(B_s + B_r).
\]  

(4.5)

The design for \( L_2 \) must be such that \( L_2 \) saturates when \( V_2(t) \) is just peaking. From Eq. 4.2, \( V_2 \) saturates when \( t = \pi/\omega \). A simple formula for designing the \( L_2 \) inductor can then be derived.
directly from Eq. 4.5:

\[ NA_c(B_s + B_r) = \frac{V_0 \pi}{2\omega} \quad (4.6) \]

Eq. 4.6 makes the assumption that \( V_2(t) \) peaks at \( V_0 \). For multiple compression stages, more saturable inductors and capacitors are added, as in Fig. 4-5. Letting the subscript \( n \) denote the \( n^{th} \)

![Figure 4-5: Pulse compression chain.](image)

stage of compression, the following criteria establish the guideline for designing the capacitances and inductances of each stage:

- **Efficiency.** If all of the capacitors have the same capacitance, the energy transfer will be the most efficient. \( C_1 = C_2 = \cdots = C_{n-1} = C_n = \cdots = C \).

- **Switching Time.** From Eqs. 4.3 and 4.6, design the inductors so that

\[
\pi V_0 \sqrt{\frac{L_{n-1,\text{sat}} C}{2}} = 2 N_n A_{c,n} (B_{s,n} + B_{r,n})
\]

- **Small Prepulse.** For a small prepulse, \( L_{n-1,\text{sat}} \ll L_{n,\text{unsat}} \).

- **Power Moves Forward.** When a capacitor has just been charged to its peak voltage, the power can either move forward through the next saturable inductor, which is still unsaturated, or backwards through the last saturable inductor, which just saturated. Either way, it must reverse the polarity of a saturable inductor, so the unsaturated inductor must be the easier inductor to "reverse." This condition is expressed as

\[
N_{n-1} A_{c,n-1} (B_{s,n-1} + B_{r,n-1}) \gg N_n A_{c,n} (B_{s,n} + B_{r,n})
\]

or, assuming that the hysteresis curve is symmetric, another way of stating the above condition is \( L_{n-1,\text{unsat}} \gg L_{n,\text{unsat}} \).

An example sequence of voltage pulses on consecutive capacitors in a properly designed pulse compression circuit is shown in Fig. 4-6.

The actual circuit used in SNOMAD-II is shown in Fig. 4-7. The DC voltage supply provides
Figure 4-6: Voltage pulses on consecutive capacitors in a pulse compression chain.
Figure 4-7: SNOMAD-II schematic.
up to 500 V to the first stage of charging capacitors. The pulse after the first set of saturable inductors is 10 $\mu$s wide. After the second stage, it is 2 $\mu$s wide. The pulse is then stepped up in voltage by a factor of 50 using a transformer, where it can now be as high as 25 kV. After two more pulse compression stages, the pulse is 120 ns wide. It is stepped up by another factor of 5 so that it reaches up to 125 kV and is sent through a pulse shaping line. Another saturable inductor stage compresses the pulse to 40 ns, and the pulse is sent to four accelerating cells in series via four drive lines in parallel. The four accelerating gaps add together to give a peak voltage of up to 500 kV on the cathode. An accelerating cell is shown in Fig. 4-8. The ferrite core surrounding the accelerating cell effectively acts as one last saturable inductor. It is initialized to a reverse biased state. When the voltage pulse arrives, it propagates through the ferrite core with velocity $v_{\text{unsat}} = 1/\sqrt{\epsilon\mu}$, where $\epsilon$ is the permittivity of the core, typically $\epsilon = 10\epsilon_0$, and $\mu$ is the permeability of the core, typically $\mu = 1000\mu_0$. The propagation speed of the pulse and the length of the core, $h$, determine how long the accelerating gap will be maintained. An example of the normalized equipotential lines for the ferrite core in the middle of the voltage pulse is shown in Fig. 4-9. Looking back at Fig. 4-1, the pulse compression circuitry up to the drive lines is all housed in the top of SNOMAD-II, in the large, vertical, mushroom shaped part. The pulse begins compression at the top of SNOMAD-II and moves down through the enclosed circuitry until finally the drive lines bring it down to the ferrite core accelerating cells which are aligned horizontally along the cement pad. The entire SNOMAD-II housing is filled with transformer oil to prevent breakdown. Finally, Fig. 4-10 shows a typical voltage and current trace from a cathode installed in SNOMAD-II. As a practical note, though SNOMAD-II nominally operates at 500 kV, the latest experiments involving SNOMAD-II have had problems running above $\sim$420 kV reliably due to voltage breakdown on the supports that hold the cathode in place.

4.3 Cathode Types

The voltage pulse from SNOMAD-II is used to generate a high electric field on a cathode surface. A cathode, in general, emits electrons. For most maser applications, the cathode consists of a material surface which is enclosed in a vacuum of less than $10^{-6}$ Torr (atmospheric pressure is 760 Torr). The emitted electrons travel through free space, forming a non-neutral plasma, or electron beam. The vacuum helps the electrons move more easily through space and prevents certain types of cathode surfaces from oxidizing. The cathode emits electrons when the electrons are given enough energy to overcome the work function of the metal comprising the cathode surface. The electric field generated by the voltage pulse helps the electrons overcome this barrier, but an electric field alone is usually insufficient to pull off electrons. There are various physical methods of electron emission from a material surface.[57]

4.3.1 Thermionic Emission

For thermionic emission, the cathode surface is heated to a high temperature, commonly $\sim$1000°C, in order to increase the number of electrons with high enough energy to overcome the work function with the addition of an electric field. The applied electric field then releases a high number of electrons and accelerates them away from the cathode. The current density emitted from a
Figure 4-8: SNOMAD-II ferrite loaded accelerating cell.
Figure 4-9: Normalized equipotential lines in the middle of a voltage pulse on a SNOMAD-II ferrite core.

Figure 4-10: Typical voltage and current pulse from SNOMAD-II.
heated material is determined by the Richardson-Dushman equation, here modified to include the Schottky-effect[25]:

\[
J = \frac{4\pi m_0 q_e k^2}{h^3} T^2 e^{-(\phi_m - \phi_E + \alpha T)/(kT)},
\]

(4.9)

where \( T \) is the absolute surface temperature of the material in K, \( \phi_m \) is the material’s work function, \( \phi_E \) is the electric field work function, which is proportional to the square-root of the amplitude of the electric field normal to the surface of the cathode, \( \alpha \) is the temperature coefficient of the work function, \( k = 1.38 \times 10^{-23} \) J/K is Boltzmann’s constant, and \( A_0 \) is the thermionic emission constant. The theoretical value for \( A_0 \) is \( A_0 \equiv 4\pi m_0 q_e k^2 / h^3 = 120 \) A/cm²/K², where \( h = 6.626 \times 10^{-34} \) J·s is Planck’s constant.

A table of thermionic emission data for common materials[57, 25, 45] is presented in Table 4.1. Note that the actual value for \( A_0 \) differs from its theoretical value by a significant amount for most materials. This is due to crystal surface imperfections and poor emission efficiency.

Tungsten is frequently used as the primary cathode material since it has a high melting point. The Tungsten may then be coated with an oxide such as Barium oxide since oxides have very low electron work functions. A single coating of Barium evaporates very quickly, so most modern thermionic cathodes are dispenser cathodes, which consist of porous tungsten impregnated with chemical compounds that continuously generate barium when heated. Thermionic cathodes are widely used in masers, and almost entirely in commercial masers, as they have long lifetimes, produce beams of good quality, and can be used at a high duty rate.

### 4.3.2 Photoelectric Emission

For photoelectric emission, a light source is shined onto the cathode during the voltage pulse. The light source, much like the heat for the thermionic case, plays the role of lowering the energy needed to emit the electrons. Photoelectric cathodes have the capability to produce very high current, high quality electron beams, but are not well suited to long pulses. Because of expense and complication, photoelectric emitters are not widely used in masers.

### 4.3.3 Secondary Emission

Secondary emission of electrons results when the cathode surface is bombarded with electrons and/or ions. This process is similar to photoelectric emission, with the bombarding electrons/ions taking the place of the light source.

### 4.3.4 High-Field Emission

For high-field emission, the electric field from the voltage pulse is simply made strong enough to overcome the work function of a cold (room temperature) cathode. No other process is used, as in the previous cases. High-field emission occurs in every day life when one experiences a shock due to static electricity. High-field emitters are used mostly in experimental masers where duty rate, lifetime, and overall “wall-plug efficiency” are not important. Beams from high-field emitters are
<table>
<thead>
<tr>
<th></th>
<th>Emission constant $A_0$ (A/cm²/K²)</th>
<th>Work function $\phi_m$ (eV)</th>
<th>Temperature coefficient $\alpha$ (×10⁻³eV/K)</th>
<th>Melting Temperature Temperature (°C)</th>
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</thead>
<tbody>
<tr>
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<td>60</td>
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<td></td>
<td>3370</td>
</tr>
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<td></td>
<td>3000</td>
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<tr>
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<td>32</td>
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<tr>
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<td>30</td>
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<td></td>
<td></td>
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<td>48</td>
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<tr>
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</tr>
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<td>55</td>
<td>4.20</td>
<td></td>
<td></td>
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<tr>
<td>Cesium on W</td>
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<tr>
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<td>.573</td>
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<tr>
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<td>.317</td>
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<tr>
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<td>1.43</td>
<td>.435</td>
<td></td>
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<tr>
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<td>.436</td>
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</tr>
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<td>1.67</td>
<td>.307</td>
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<tr>
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<td></td>
<td>1.43</td>
<td>.425</td>
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<td>Scandate 3:1:1</td>
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<td>.425</td>
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<td>.399</td>
<td></td>
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<tr>
<td>Scandate 4:1:1</td>
<td>352</td>
<td>1.43</td>
<td>.401</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Thermionic emission data collected from several sources[25, 57, 45]. The B-type cathodes are made up of Tungsten impregnated with BaO, CaO, and Al₂O₃ in the ratios given (e.g. 5:3:2 is 5BaO:3CaO:2Al₂O₃). An M-type cathode is a B-type coated with a thin layer of osmium-ruthenium or osmium-iridium to enhance the emission properties. The scandate cathodes have a 2%-7% mix of scandate (Sc₂O₃) in the B-type to provide high current at low temperatures with low evaporation rate. The cathode type used in the gyro-twt experiments for this thesis is the M-type, 3:1:1.
usually run through an emittance selector (beam scraper) so that only the highest quality (central) part of the beam is kept.

4.4 Cathode Anode Geometries

Aside from the type of emission, cathode-anode geometry also plays a key role in the formation of a beam. Two common geometries are:

4.4.1 MIG Geometry

The common feature of all MIG geometries is that they emit electrons perpendicular to the local magnetic field lines. The electrons coming from the cathode are then immediately trapped by the magnetic field lines and spiral along them. Many MIG geometries use two anodes, or a triode system. The voltage of the middle, or “mod” anode determines how much transverse momentum the beam will have. Other MIG systems have a single anode and use the geometry of the gap to determine the transverse momentum properties of the beam. A triode MIG is shown in Fig. 4-11.

4.4.2 Pierce Geometry

The Pierce gun geometry is named after its original designer, John R. Pierce. The idea of Pierce geometry is to produce a laminar beam with uniform current density.[45] Laminar beam movement is quite similar to laminar water flow. Particle orbits do not cross each other, and they flow in layers, or laminae. A spherical Pierce geometry is shown in Fig. 4-12. The cathode and the anode are spherical in shape, centered about the same point. The design causes the electrons to compress together towards the central point until the field perturbations at the anode and self-forces within the beam prevent the beam from compressing any further. The compression of the beam allows an emitter of limited current density to generate a high current beam.

4.5 Electron Beam Propagation

The cathode-anode geometry proposed for the gyro-twt experiments is a Pierce geometry with a thermionic dispenser cathode. The design was done by J. Haimson of Haimson Research Corporation and modeled after a SLAC X-band klystron gun design[30]. This cathode design has already been successfully employed to propagate a beam for a relativistic klystron amplifier at 11.4 GHz using SNOMAD-II. The beam quality was good enough for the klystron to achieve over 40% efficiency[38]. This geometry is shown in Fig. 4-13 and in the simulation from Fig. 4-12 (and Fig. 4-19 in Section 4.6.4). The details of the geometry used in the EGUN simulations are shown in Fig. 4-14. The cathode-anode gap, shown as \( D \) in Fig. 4-14, is adjustable by changing the length of the cathode supports. Increasing \( D \) lowers the perveance of the geometry (Sec. 4.5.1). The design value for \( D \) is 36.32 mm. The cathode used is manufactured by Spectramat. It is an M-type cathode with a \( 3\text{BaO}:\text{CaO}:\text{Al}_2\text{O}_3 \) molar ratio (M-type 3:1:1—See Table 4.1). The coating is an
Figure 4-11: A Magnetron Injection Gun simulation done with the EGUN program[42]. The electrons are accelerated through increasing equipotential lines as they follow the path of and spiral about B-field lines (not shown). A cross cut of the beam at a fixed axial distance is annular in shape.
Figure 4-12: A Pierce Gun simulation done with the EGUN program.[42] The electrons are accelerated through increasing equipotential lines as the beam compresses together. A cross cut of the beam at a fixed axial distance is circular in shape.
Figure 4-13: The cathode-anode design done by Haimson Research Corporation, plotted at half scale. The design was taken from the SLAC 5045 klystron cathode[30]. The equipotential shaping surface is inclined at an angle of 65.5 degrees, as shown in the figure, which is very close to the optimum value of 67.5 degrees as calculated by Pierce[75]. The beam converges from the emitting surface and travels down the beam tube, as shown in Fig. 4-12.
Figure 4-14: The cathode-anode measurements used in the EGUN simulations shown at 13:8 scale. These measurements are valid only when the cathode is hot (~1000°C). When the cathode is at room temperature, it contracts, and the measurements change slightly. The design value for $D$ is 36.32 mm.
osmium-ruthenium alloy. The cathode is spherical in shape, with a radius of curvature of 6.528 cm and an edge to edge distance of 8.962 cm. The total area of the cathode surface is 73.0 cm².

If the cathode is to generate a maximum of 500 A of current, this means it must deliver up to 6.8 A/cm² in current density, which is reasonable for a dispenser cathode. The lower the current density, the longer the cathode will last. Some modern dispenser cathodes deliver 80 A/cm² but have short lifetimes of only a few thousand hours. Space tubes, on the other hand, which must have long lifetimes (hundreds of thousands of hours), typically draw 2–3 A/cm². The Spectramat cathode is heated indirectly by a filament coil placed beneath the emitting surface. It is heated to a nominal value of ~1000°C.

4.5.1 Space Charge Limited Flow

In Sec. 4.3.1, the Richardson–Dushman equation with Schottky effect (Eq. 4.9) gives the relation between emitted cathode current, surface temperature, and electric field. When the temperature of the cathode is low enough, the \( kT \) term in Eq. 4.9 equation dominates the exponent, and the current density is not sensitive to the cathode voltage. The "cathode voltage" is the voltage potential on the cathode surface (usually negative) with respect to the anode (usually ground). As the cathode temperature increases, assuming that the current density is not limited, the magnitude of the exponent in Eq. 4.9 decreases, and the current drawn from the cathode both increases and depends more on the cathode voltage. Eventually, Eq. 4.9 loses validity. This happens because the electric field due to the cloud of emitted electrons next to the cathode surface begins to cancel the electric field due to the cathode voltage. When these fields totally cancel each other, the current drawn from the cathode depends entirely upon the cathode voltage and is governed by the Child-Langmuir law (below, Eq. 4.10) for nonrelativistic beams. Emission in the low temperature regime, where cathode current depends entirely on temperature, is referred to as "temperature limited." Emission in the high temperature regime, where cathode current depends entirely on voltage, is referred to as "space charge limited." If the current density of the cathode limits the flow, the emission is "source limited." The transition from temperature limited emission to space charge limited emission is shown in Fig. 4-15. For nonrelativistic and mildly relativistic anode-cathode gaps, space charge limited emission is governed by the well known Child-Langmuir law[45]:

\[
I = \alpha V^{\frac{3}{2}},
\]

where \( I \) is the cathode current, \( V \) is the cathode voltage, and \( \alpha \) is the perveance. The SI unit of perveance is the perv, equal to 1 amp/(volt)³/2, with perveance commonly specified in micro-pervs (\( \mu \)P). The perveance of a cathode-anode system depends entirely upon the geometry for nonrelativistic and mildly relativistic anode-cathode gaps. For a spherical cathode-anode system such as ours, the perveance has a theoretical value of[45]

\[
\alpha = \frac{4\varepsilon_0}{9} \left( \frac{2e}{m_0} \right)^{1/2} \frac{4\pi V^{3/2}}{a^2},
\]
Figure 4-15: Experimentally measured emitted cathode current versus both cathode temperature and cathode voltage. The current is emitted from a Spectramat 6.5 cm radius thermionic dispenser cathode. Both figures demonstrate the changing of current dependence from the temperature limited regime ($T < 900^\circ C$) to the space charge limited regime ($T > 1000^\circ C$). A curve fit of Eq. 4.10 to the 1053$^\circ C$ case in the top figure resulted in a perveance of 1.3 $\mu$P.
where \( a \) is a function of \( R_A \), the anode radius, and \( R_C \), the cathode radius. This function is known as the Langmuir function, which is the solution to the differential equation

\[
3a \frac{d^2a}{dx^2} + \left( \frac{da}{dx} \right)^2 + 3a \frac{da}{dx} = 1. \tag{4.12}
\]

In Eq. 4.12, \( x \equiv \ln(R_A/R_C) \). An approximate solution to \( a \) valid near \( x = 0 \), is

\[
a = x - 0.3x^2 + 0.075x^3 - 0.0143182x^4 + 0.0021609x^5 - 0.00026791x^6 + \cdots. \tag{4.13}
\]

Because the entrance for the anode piece in the gyro-twt cathode geometry (Fig. 4-13) is so large, the anode is not well approximated by a sphere, and Eq. 4.11 does not accurately predict the perveance of this geometry. Instead, the perveance can be computed through the use of the program EGUN, developed by W.B. Herrmannsfeldt at the Stanford Linear Accelerator Center (SLAC)[42]. EGUN simulates electron trajectories from an emitting surface in electrostatic and magnetostatic fields, including space charge and self-magnetic field forces. In a cylindrical geometry, EGUN assumes azimuthal symmetry and calculates the electron trajectories in the \( r-z \) plane, as shown in Figs. 4-11 and 4-12. EGUN predicts a perveance of \( \sim 1.5 \mu \text{P} \) for our geometry, which is not far different from the measured value of \( 1.3 \mu \text{P} \) (Fig. 4-15).

The gyro-twt experiments will operate in the space charge limited regime. This both eliminates the effects of non-uniform heating across the cathode and reduces the sensitivity of current to temporal temperature fluctuations. From Eq. 4.9, a 0.5% change in temperature results in a 35% change in current density (at 1000 degrees C), whereas from Eq. 4.10, a 1% change in voltage results in only a 1.5% change in current. Note the shaping electrode surrounding the cathode in Fig. 4-13. This electrode helps to shape the equipotential lines so that the electron beam will converge. It compensates for the missing part of the complete sphere in an ideal spherical cathode. Pierce calculated the optimum angle of this electrode to be 67.5 degrees with respect to vertical. This electrode ensures that the beam will properly converge into the beam tube; however, without magnetic focusing, the beam will eventually diverge due to repulsive space charge forces. Magnetic focusing is discussed in the next section.

### 4.6 Magnetic Focusing

In order to propagate a high density electron beam over a long distance, magnetic focusing is needed to balance the radially outward space charge force. It is not within the scope of this thesis to fully examine magnetic focusing and “beam matching” (propagating a beam between regions with differing magnetic fluxes). Such discussions can be found in a number of references[77]. An introductory discussion of the topic follows, largely taken from Humphries[45].

#### 4.6.1 Brillouin Flow

Under the context of magnetically focusing a flow of electrons, Brillouin flow results when, starting with zero magnetic field at the cathode surface, the magnetic field is established such that the force
it produces on the electrons in the beam at every point exactly balances the space charge force. Such a magnetic field is called a Brillouin field. At first, a nonrelativistic treatment of Brillouin flow is covered, and then the treatment of relativistic beams is discussed. For the planned gyro-twt experimental voltage of 400 kV, a nonrelativistic treatment is not a very good approximation, but it at least provides a first order estimate for the magnetic field requirements.

The beam is assumed to leave the cathode with zero canonical angular momentum and uniform energy. Canonical angular momentum, $P_\phi$, is defined as

$$P_\phi \equiv \gamma m_0 r v_\phi - q_e r A_\phi,$$  \hspace{1cm} (4.14)

where $\phi$ is the azimuthal angle, $r$ is the radius of the electron, and $A_\phi$ is the magnetostatic vector potential in the $\phi$ direction. For an axial magnetic field $B_0$, $A_\phi = r B_0/2$. The energy distribution of the beam will remain relatively uniform since the fields are electrostatic, and the canonical momentum will remain zero because the magnetostatic forces have azimuthal symmetry[45].

Because the Brillouin field must increase from zero at the cathode to an equilibrium value, the beam passes through a transition region of increasing axial field strength. This increasing axial field necessarily also results in a radial field which imparts azimuthal velocity to the beam. Although the beam orbits in this transition region are complex, the energy and canonical angular momentum distributions retain their form. Once the beam is propagating in the uniform field region, the condition $P_\phi = 0$ determines the azimuthal velocity of each electron:

$$v_\phi = \frac{q_e B_0 r}{2 m_0}.$$  \hspace{1cm} (4.15)

In Eq. 4.15, $\gamma$ is assumed to be close to unity for a non-relativistic beam. Because the azimuthal velocity is exactly proportional to $r$, the azimuthal angular velocity, $\omega$, is constant at $\omega = q_e B_0/(2m_0)$. This frequency is half the normal non-relativistic cyclotron frequency, $\Omega_0$. The factor of 2 comes from the balance of space charge forces with magnetic forces. The uniform angular velocity of all of the beam electrons is referred to as rigid rotor equilibrium.

The matched beam condition for Brillouin flow is derived by balancing the space charge force, the centrifugal force, and the magnetic focusing force. The magnetic forces generated by the beam itself are generally neglected because they are relatively small. The resulting force balance equation is[45]

$$m_0 \frac{d^2 r}{dt^2} = \frac{q_e^2 n_0 r}{2 \varepsilon_0} + \frac{m_0}{r} \left[ \frac{q_e r B_0}{2 m_0} \right]^2 - \frac{(q_e B_0)^2 r}{2 m_0} = 0,$$  \hspace{1cm} (4.16)

where $n_0$ is the beam density. Converting $n_0$ to current density, $n_0 = J/(q_e v_z)$, and setting the right side of Eq. 4.16 to zero results in the Brillouin matched condition:

$$B_{BRILLOUIN} = \sqrt{\frac{2 m_0 I}{\pi r_b^3 q_e \varepsilon_0 c \beta_z}},$$  \hspace{1cm} (4.17)

where $r_b$ is the total radius of the beam and $m_0 c^2 \beta_z^2/2 = q_e V$ for a nonrelativistic beam with mostly axial velocity.

The relativistic calculation for Brillouin flow is complicated by the inclusion of beam generated
magnetic fields and the variation of $\gamma$, $v_z$, and density with radius. In the paraxial limit, however, the calculation is greatly simplified. The paraxial limit involves several assumptions, all of which are quite valid for the beam parameters for the gyro-twt experiments. The beam velocity is assumed to be almost entirely axial, $v_z \gg v_\phi$, and all particles are assumed to have nearly the same kinetic energy and axial velocity, which are then represented by average values $\gamma$ and $\beta$.

The paraxial ray equation, from Humphries[45], is

$$R'' = -\frac{\gamma' R'}{\beta^2 \gamma} - \left[\frac{\gamma''}{2 \beta^2 \gamma}\right] R - \left[\frac{q B_z(0, z)}{2 \beta \gamma m_0 c}\right]^2 R + \frac{e^2}{R^3} + \left[\frac{q \psi_0}{2 \pi \beta \gamma m_0 c}\right]^2 \frac{1}{R^3} + \frac{K}{R},$$

(4.18)

where the quantity $R$ is the envelope radius of the beam, the prime (') symbol denotes a derivative with respect to $z$, $B_z(0, z)$ is the on-axis axial magnetic field, $\epsilon$ is the beam emittance, $K$ is the perveance of the cathode-anode geometry, and $\psi_0$ is the total magnetic flux enclosed within the beam envelope at the cathode:

$$\psi_0 = \int_0^{R_z} 2\pi r dr B_z(r, Z_z).$$

(4.19)

The first three terms on the right-hand side of Eq. 4.18 represent focusing due to acceleration, electrostatic forces, and magnetostatic forces, respectively. The last three terms represent defocusing due to emittance, immersed flow, and beam generated electric and magnetic fields, respectively. Immersed flow is the term used to describe beams that are generated in a finite magnetic field (immersed in a magnetic field). The relativistic beam matching condition now results from setting $R'' = 0$ in Eq. 4.18:

$$B_{\text{BRILLOUIN}} = \sqrt{\frac{2 I m_0}{\pi R^2 q e_0 c \beta \gamma}}.$$ 

(4.20)

Eq. 4.20 assumes that the beam emittance is negligible. Note that Eq. 4.20 is identical to Eq. 4.17 except for the factor $\gamma^{-1/2}$.

### 4.6.2 Confined Flow

In practice, the Brillouin match condition is not often used because of a high sensitivity to operating parameters[57]. Instead, confined flow is used, where the operating magnetic field is 2–5 times larger than the matched Brillouin field. The $m$ number of the field is the ratio of the operating field to the Brillouin field:

$$m \equiv \frac{B_{\text{OPERATING}}}{B_{\text{BRILLOUIN}}}.$$ 

(4.21)

Factors that are discussed in Chapter 5 will end up determining the final choice for the focusing field strength provided by the focusing magnet system. The axial field strength at the cathode that minimizes beam scalloping (oscillations of the beam envelope in $z$) for a given desired beam radius and operating guide field can be calculated using two simple equations. Assuming that the electrons adiabatically compress as the magnetic field increases, the magnetic moment is conserved and

$$\frac{r(z_2)}{r(z_1)} = \sqrt{\frac{B_z(z_1)}{B_z(z_2)}}.$$ 

(4.22)
The equation that minimizes beam scalloping is [2]

\[ B_{\text{OPERATING}} = B_{\text{BRILLOUN}} + B_{\text{DESIRED RADIUS}}, \quad (4.23) \]

where \( B_{\text{DESIRED RADIUS}} \) is the magnetic field value at the point where the beam first reaches the desired radius. Using Eqs. 4.21 and 4.23, the field at the cathode is determined:

\[ B_{\text{CATHODE}} = \frac{r_{\text{DESIRED}}^2}{r_{\text{CATHODE}}^2} (B_{\text{OPERATING}} - B_{\text{BRILLOUN}}). \quad (4.24) \]

For a design field of 1600 G, Table 4.2 lists the optimum cathode field for several different final beam radii. The numbers from Table 4.2 are rough figures at best. The best confirmation of a magnetic field match is to use EGUN to simulate the propagation of the beam through the magnetic field, as in Figs. 4-11 and 4-12. An EGUN simulation more specific to the gyro-twt experiments is presented in the following section.

### Table 4.2

<table>
<thead>
<tr>
<th>(r_{\text{DESIRED}}) (mm)</th>
<th>(B_{\text{BRILLOUN}}) (G)</th>
<th>(B_{\text{CATHODE}}) (G)</th>
<th>(m) number</th>
</tr>
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<tr>
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<td>1.82</td>
</tr>
<tr>
<td>7</td>
<td>752</td>
<td>21</td>
<td>2.13</td>
</tr>
</tbody>
</table>

Table 4.2: Optimum magnetic field values at the cathode for given desired beam radii. The minimum obtainable radius \((m = 1)\) is just above 3 mm. The operating field is 1600 G. The beam voltage and current are 400 kV and 300 A, respectively. The cathode radius is 4.48 cm (from the Spectramat cathode). The results are from Eqs. 4.24 and 4.20.

### 4.6.3 Beam Spreads Resulting from Brillouin Flow

Relatively simple analysis can give a first approximation to the beam quality that results from Brillouin flow. Starting with Eq. 4.14, setting \( P_0 \) to zero for a matched beam, and solving for \( p_\perp \),

\[ p_\perp (r) = \frac{q_e B_0 r}{2}. \quad (4.25) \]

Assuming that the beam has uniform density and almost entirely axial velocity, the charge density of the beam is \( \rho = I/(\pi r^2 v_z) \approx I/(\pi r^2 v) \). Gauss’s law applied to an infinitely long cylinder of
uniform negative charge then results in

\[ E_v(r) = \begin{cases} \frac{-I}{2\pi \varepsilon_0 v}, & r \leq r_b \\ \frac{-I}{2\pi \varepsilon_0 v}, & r \geq r_b, \end{cases} \quad (4.26) \]

where \( I \) and \( v \) are defined as positive quantities. Integrating to find the voltage potential of the beam at different \( r \),

\[ \Phi(r) = \begin{cases} \frac{I}{4\pi \varepsilon_0 v} \left[ r^2 - r_b^2 \left( 1 - 2 \ln \frac{r}{r_b} \right) \right], & r \leq r_b \\ \frac{I}{2\pi \varepsilon_0 v} \ln \frac{r}{r_w}, & r \geq r_b, \end{cases} \quad (4.27) \]

where \( \Phi_{\text{CATHODE}} = 0 \) is the voltage potential at the beam tube wall. The voltage potential steadily decreases as \( r \) decreases from \( r_b \) to 0. The kinetic energy of the electron beam is directly correlated to \( \Phi(r) \):

\[ E_K(r) = -e [\Phi_{\text{CATHODE}} - \Phi(r)], \quad (4.28) \]

where \( \Phi_{\text{CATHODE}} \), the voltage potential at the cathode, is a large negative value, typically \(-400 \text{ kV}\) for the gyro-twt experiments. Because \( \Phi(r) < 0 \) for all particles in the beam, the beam is said to have a depressed voltage, meaning that the energy of the beam particles does not exactly correspond to the anode-cathode potential. The particles with the highest kinetic energy, at the outer edge of the beam, have a depressed potential of

\[ \Phi(r_b) = \frac{I}{2\pi \varepsilon_0 v} \ln \frac{r_b}{r_w}. \quad (4.29) \]

The particles with the lowest kinetic energy, at the center of the beam, have a depressed potential of

\[ \Phi(0) = \frac{I}{2\pi \varepsilon_0 v} \ln \frac{r_b}{r_w} - \frac{I}{4\pi \varepsilon_0 v}. \quad (4.30) \]

For typical gyro-twt run parameters of 400 kV and 350 A, with a beam radius of 6 mm, \( \Phi(r_b) = -29 \text{ kV} \) and \( \Phi(0) = -42 \text{ kV} \) in the focusing tube region (\( r_w = 1.905 \text{ cm} \)). The voltage depression is \( \sim 10\% \). As the beam spins up and enters the interaction region (with smaller wall radius), this depression becomes less significant.

The beam kinetic energy distribution, \( E_K(r) \) (Eq. 4.28), and transverse momentum distribution, \( p(r) \) (Eq. 4.25), can now be used to derive any other pertinent distribution for Brillouin flow. As examples, the spreads most frequently referred to in this thesis, \( \sigma_\gamma/\langle \gamma \rangle \) and \( \sigma_{pz}/\langle p_z \rangle \), are derived here. The definition used here for “average” spread (in this case for \( \sigma_\gamma \)) is

\[ \sigma_\gamma = \sqrt{\langle \gamma^2(r) \rangle - \langle \gamma(r) \rangle^2}. \quad (4.31) \]

Here, because the beam density is assumed uniform in radius, arguments in angle brackets take the form

\[ \langle x(r) \rangle = \frac{1}{\pi r_b^2} \int_0^{r_b} x(r) 2\pi r dr. \]
\[
= \frac{2}{r^2} \int_{r_0}^{r_b} x(r) r \, dr. \quad (4.32)
\]

Relating \( \gamma \) to the \( E_K(r) \) distribution,

\[
\gamma(r) = 1 + \frac{q_e E_K(r)}{m_0 c^2}, \quad (4.33)
\]

and substituting \( \gamma_0 \equiv 1 + \frac{q_e E_K(0)}{(m_0 c^2)} \) results in

\[
\begin{align*}
\gamma_{\text{MAX}} &= \gamma_0 + \frac{1}{\beta} \left( \frac{I}{I_A} \right) \\
\gamma_{\text{MIN}} &= \gamma_0 \\
\langle \gamma(r) \rangle &= \gamma_0 + \left( \frac{I}{2\beta I_A} \right) \\
\langle \gamma^2(r) \rangle &= \gamma_0^2 + 2\gamma_0 \left( \frac{I}{2\beta I_A} \right) + \frac{4}{3} \left( \frac{I}{2\beta I_A} \right)^2,
\end{align*} \quad (4.34-4.37)
\]

where \( I_A \equiv 4\pi \varepsilon_0 m_0 c^3 / q_e = 17045 \) A is the Alfvén current and \( \gamma_0 \) is the energy of the on-axis \((r = 0)\) particles. The final result for the maximum \( \gamma \) spread, \( \Delta \gamma \equiv \gamma_{\text{MAX}} - \gamma_{\text{MIN}} \), is:

\[
\frac{\Delta \gamma}{\langle \gamma(r) \rangle} = \frac{\left( \frac{I}{\beta I_A} \right)}{\gamma_0 + \left( \frac{I}{2\beta I_A} \right)}. \quad (4.38)
\]

The average \( \gamma \) spread is then

\[
\frac{\sigma_\gamma}{\langle \gamma(r) \rangle} = \frac{1}{2\sqrt{3}} \frac{\Delta \gamma}{\langle \gamma(r) \rangle}. \quad (4.39)
\]

Following a similar (though more involved) procedure for \( p_z \) results in the following relations for maximum and average axial momentum spread:

\[
\begin{align*}
\frac{\Delta p_z}{\langle p_z(r) \rangle} &= \frac{\Delta \gamma}{\langle \gamma(r) \rangle} \\
\frac{\sigma_{p_z}}{\langle p_z(r) \rangle} &\approx \frac{\left( \frac{I}{I_A} \right)}{\sqrt{12(\gamma_0 - 1)}}. \quad (4.40-4.41)
\end{align*}
\]

For 400 kV and 350 A, Eqs. 4.39 and 4.41 predict \( \sigma_\gamma / \langle \gamma(r) \rangle = 0.40\% \) and \( \sigma_{p_z} / \langle p_z(r) \rangle = 0.67\% \), respectively.

**4.6.4 Experimental Focusing System**

The easiest way to design and build a system that attains a desired azimuthally symmetric magnetic field profile is to use azimuthally symmetric current carrying coils and (if necessary) iron pole pieces. Coils with square cross sections are particularly easy to manufacture and to model. Coils
usually require some form of cooling, particularly for high fields. For fields above 1 T (depending on the bore size), superconducting magnets are frequently used. In the field range of the gyro-twt experiments, 0.1–0.4 T, hollow-core copper wound coils are a good choice. Because the water flows through the center of the conductor, hollow-core coils can maintain high currents with a moderate amount of water pressure.

The magnetic system built for the 11.4 GHz relativistic klystron amplifier experiment easily produces the magnetic field strengths necessary for the gyro-twt experiment. This system consists of twelve hollow-core, copper-wound, square cross-section pancake coils, all identical in size, spaced evenly along a central axis with a 4 inch I.D. bore. The twelve coils, which are referred to as H₁–H₁₂, are surrounded by iron pieces. A “bucking,” or “lens” coil with its own iron pieces completes the system. The lens coil is wired to produce an opposing field from the rest of the coils, thereby enabling the reduction of field at the cathode surface. A scaled drawing of these focusing coils is shown in Fig. 4-16, with detailed positional measurements shown in Fig. 4-17. The focusing coils, including the lens coil, are mounted on sliding rails that allow the magnet to be positioned axially in a precise manner. Table 4.3 lists the coil parameters, and Table 4.4 lists the positions of the pole pieces with respect to the \((z, r) = (0, 0)\) coordinate shown in Fig. 4-17. Note that the left pole face of the focusing coil geometry does not have an inner diameter equal to

Figure 4-16: A 1:6 scale drawing of the magnetic focusing coils designed by Haimson Research Corporation. The lens coil reduces the field strength at the cathode. The iron pieces cause the field to drop off more rapidly outside of the magnet system and to be more uniform inside the magnet system. The twelve main coils, numbered H₁ through H₁₂, focus the beam into the beam tube, where a peak field of > 1 T on axis can be achieved. The gyro-twt experimental requirement is much more modest at only .15–.25 T.
Figure 4-17: A 1:2.5 scale drawing showing the position of the focusing coils with respect to the cathode and the beam tube. The \((z, r)\) coordinate shown as \((0, 0)\) in the figure is used as the reference point for Table 4.4.
### Table 4.3: Parameters describing the focusing coils. The end-to-end length of the focusing magnet system, including the pole pieces but excluding the lens, is 80.5 cm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Main coils</th>
<th>Lens coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Turns</td>
<td>128</td>
<td>20</td>
</tr>
<tr>
<td>Total resistance</td>
<td>0.049 Ω</td>
<td>0.014 Ω</td>
</tr>
<tr>
<td>Weight</td>
<td>25 kG</td>
<td>8 kG</td>
</tr>
<tr>
<td>Dissipation @200 A</td>
<td>2 kW</td>
<td>0.56 kW</td>
</tr>
<tr>
<td>Inner diameter</td>
<td>11.6 cm</td>
<td>38.2 cm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>33.8 cm</td>
<td>43.2 cm</td>
</tr>
<tr>
<td>Width</td>
<td>5.56 cm</td>
<td>2.8 cm</td>
</tr>
<tr>
<td>Inter-gap spacing</td>
<td>0.54 cm</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 4.4: Pole piece positions and dimensions for the focusing coils. The $z_{\text{MIN}}$ point is with respect to the $z = 0$ coordinate shown in Fig. 4-17. The $z_{\text{MIN}}$ point is the left most position of the piece, where “left” mean towards the cathode, or negative $z$. Note that the main left pole piece actually has two parts and is not identical to the main right pole piece (not indicated in any figures).

<table>
<thead>
<tr>
<th>Pole piece description</th>
<th>$z_{\text{MIN}}$ (cm)</th>
<th>inner diameter (cm)</th>
<th>outer diameter (cm)</th>
<th>width (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lens far left</td>
<td>-15.3</td>
<td>39.8</td>
<td>41.0</td>
<td>6.4</td>
</tr>
<tr>
<td>lens left</td>
<td>-8.9</td>
<td>39.8</td>
<td>51.2</td>
<td>0.6</td>
</tr>
<tr>
<td>lens top</td>
<td>-8.3</td>
<td>43.2</td>
<td>44.4</td>
<td>2.8</td>
</tr>
<tr>
<td>lens right</td>
<td>-5.5</td>
<td>35.8</td>
<td>51.2</td>
<td>1.0</td>
</tr>
<tr>
<td>main left lower</td>
<td>1.3</td>
<td>17.8</td>
<td>21.6</td>
<td>1.9</td>
</tr>
<tr>
<td>main left upper</td>
<td>0.0</td>
<td>21.6</td>
<td>43.2</td>
<td>3.2</td>
</tr>
<tr>
<td>main top</td>
<td>3.2</td>
<td>35.1</td>
<td>40.2</td>
<td>74.1</td>
</tr>
<tr>
<td>main right</td>
<td>77.3</td>
<td>10.2</td>
<td>43.2</td>
<td>3.2</td>
</tr>
</tbody>
</table>
the bore of the focusing coils. A cut-out around the bore is actually made of non-ferromagnetic material. This cut-out is not shown in any figures, but it is accurately described in Table 4.4.

Modeling the magnetic field produced by the focusing coils is a relatively straightforward process when using the POISSON/SUPERFISH group of magnetics programs from the Los Alamos Accelerator Code Group at Los Alamos National Laboratory. POISSON solves magnetostatic problems with iron by a method of iteration and relaxation. A typical POISSON run to simulate the magnetic field from the focusing coils takes 1–2 minutes of CPU time on the Livermore Cray-2S/4128. Results from two POISSON simulations of the focusing coils are shown in Fig. 4-18 along with actual magnetic field data for the focusing coils measured with a Hall probe. Supplying EGUN with the correct cathode-anode geometry and with magnetic field input from POISSON, beam focusing simulations are done. Fig. 4-19 shows one of the best matches from several EGUN simulations for the gyro-twt experiments. This particular simulation used 2.5 minutes of CPU on a Cray-2S/4128. The field at the cathode for this case is 20 G, and the final beam radius is 6–7 mm at an on-axis field value of 1620 G. These parameters are quite close to the predictions in Table 4.2. The spread numbers are also very close to those predicted at the end of Section 4.6.3. Of the beam simulations done with EGUN for the gyro-twt case, it should be noted that the beam match was most sensitive to changes in the field close to the cathode, as would be expected. Keeping the field profile uniform, the beam match was not sensitive to beam voltages ranging from 350 kV–450 kV.

4.7 Vacuum System

Experiments utilizing thermionic cathodes such as the Spectramat cathode that the gyro-twt experiments will use require vacuums < 10^{-6} Torr, and perform best at vacuums < 10^{-7} Torr. All of the pumping for the gyro-twt experiments will be provided by two Balzers 500 l/s water-cooled turbopumps. The turbopumps attach to the chamber surrounding the cathode. The cathode cannot be sealed from the beam tube due to the bore size of the focusing coils. There is not enough room for a valve. No pump is planned for the window end of the experiment. Flanges are joined by copper gaskets where practical, although the window design, the waveguide flanges, and the seals in the linac (behind the cathode) incorporate several o-rings made of viton rubber. A very similar system was used for the 11.4 GHz relativistic klystron amplifier, and the vacuum easily met the requirements of the amplifier.

The design and simulation of beam transport up to the bifilar helical wiggler entrance has now been established. The beam quality to this point is predicted to be very good, with < 1% spread for both energy and axial momentum (Fig. 4-19). The beam lacks only one critical property for a CRM interaction. It has very little perpendicular momentum. In the next section, a method for imparting perpendicular momentum to the beam is discussed and implemented into the design of the gyro-twt experiments.
Figure 4-18: Measured on-axis field from the Haimson Research focusing coils and the theoretical predictions from POISSON. In the top figure, note how the field decreases faster on the pole-side of $H_1$ than on the non-pole-side. For all of the coils at the same current, 26 G/A is the peak on-axis field conversion factor.
Figure 4-19: Results of an EGUN beam trajectory simulation for the cathode at 390 kV. Note the distorted aspect ratio of the figure. EGUN predicts a current of 360 A (1.48 $\mu$P), initial beam spreads of $\sigma_\gamma/\langle \gamma \rangle = 0.4\%$ and $\sigma_{p_z}/\langle p_z \rangle = 0.6\%$, and an initial beam pitch of $\alpha = 0.06$. The focusing coils are set as follows: Lens = $-35$ A, $H_1 = 80$ A, $H_2-H_{11} = 71$ A, $H_{12} = 90$ A. The field at the cathode is 20 G.
Chapter 5

Bifilar Helical Wiggler Theory and Design

5.1 Introduction

In Section 3.3, it is made clear that the CRM resonance extracts primarily perpendicular energy from the electron beam, yet, to this point in the study and design of beam transport, a beam emitted from a Pierce gun with momentum primarily in the axial direction has been discussed. A means of imparting perpendicular momentum to this beam is required. This impartation must meet several requirements. It must achieve beam pitch values within a desired range (\( \alpha = 0.3-1.2 \) for the gyro-twt and CARM experiments). It must not degrade the beam quality to the point where CRM interactions do not efficiently occur. It must not change the overall energy of the beam. Finally, it must be practically feasible. One solution to these requirements is a bifilar helical wiggler magnet. Such magnets are commonly used for exactly this application—to “spin up” small radius, axis-encircling, solid electron beams emitted from a Pierce gun geometry. The wiggler converts axial momentum to transverse momentum as the beam travels in \( z \), and the beam begins to travel in a helical path, corkscrewing about the \( z \)-axis. A Pierce gun in combination with a bifilar helical wiggler is often referred to as a “Pierce wiggler” system.

The bifilar helical wiggler magnet is constructed relatively simply by interleaving two helical conductors (hence “bifilar”), offset by a half period from each other, around a fixed radius core. The two conductors carry current in opposite directions. This causes the axial component of the on-axis field generated by the wiggler to approximately cancel, leaving a transverse field component in a direction that rotates with the same periodicity as each helix. A simple schematic of a bifilar helical wiggler is shown in Fig. 5-1.

5.2 Particle Motion in a Wiggler Field

Assuming that the wiggler is infinitely long, symmetry dictates that the transverse magnetic field pattern rotates uniformly in \( z \). The initial approximation used for the wiggler field will be the “idealized” form:

\[
\mathbf{B}_w = B_w [\cos(k_w z)\mathbf{x} + \sin(k_w z)\mathbf{y}],
\]

(5.1)
Figure 5-1: Simple diagram of a bifilar helical wiggler with four wiggler periods. Two helical coils of wire are wound around a core in the same direction, but they are offset by half a period and also carry current in opposite directions. In an actual wiggler, the wires terminate at each end in a loop to prevent a deflecting field. The wiggler period is $\lambda_w$. The radius of the wiggler core is $r_0$.

where $B_w$ is constant, $k_w \equiv 2\pi/\lambda_w$, $\lambda_w$ is the wiggler period or wavelength, and $z$ is axial position. The axially directed magnetic field produced by the wiggler is assumed to cancel. A uniform axial guide field, $B_z$, is assumed to be provided by a separate magnetic system (e.g. the focusing coils) in order to keep the beam focused. Although the wiggler field represented by Eq. 5.1 is not physically realizable (see Section 5.4), it serves to illustrate particle motion in a wiggler very nicely, and it turns out to be a very good approximation in the case of the gyro-twt wiggler design (Section 5.5).

The equations of motion for a particle in such a field are derived from the Lorentz force equation,

$$\frac{dp}{dt} = -\frac{q_p}{m_0\gamma} \times (B_w + B_z \hat{z}),$$

where $p$ is the relativistic momentum vector of an electron. Here, electric fields are assumed to be absent. If the beam has a significant amount of space charge, this approximation loses validity. This is discussed in Section 5.7.1. Because a magnetostatic field cannot do work on a charged particle, the particle energy, $\gamma$, and overall momentum, $p \equiv |p|$, will remain constant throughout the wiggler. Since it is most useful to know $p_\perp$ as a function of $z$, the independent variable in Eq. 5.2 is changed from $t$ to $z$ using the relation

$$\frac{d}{dt} = \frac{dz}{d\gamma} \frac{d}{dz}.$$

As in Section 2.3, the transverse momentum of the electron is written in terms of a magnitude and a phase:

$$p = p_\perp (\cos \phi \hat{x} + \sin \phi \hat{y}) + p_\parallel \hat{z}.$$

Then

$$\frac{dp_x}{dz} = \frac{dp_\perp}{dz} \cos \phi - p_\perp \frac{d\phi}{dz} \sin \phi$$

96
\[
\frac{dp_y}{dz} = \frac{dp_\perp}{dz} \sin \phi + p_\perp \frac{d\phi}{dz} \cos \phi. \tag{5.6}
\]

Substituting Eqs. 5.5, 5.6, 5.4, and 5.3 into Eq. 5.2 yields

\[
\frac{dp_\perp}{dz} \cos \phi - p_\perp \frac{d\phi}{dz} \sin \phi = -q_e \left[ \frac{p_\perp}{p_z} B_z \sin \phi - B_w \sin k_w z \right] \tag{5.7}
\]

\[
\frac{dp_\perp}{dz} \sin \phi + p_\perp \frac{d\phi}{dz} \cos \phi = q_e \left[ \frac{p_\perp}{p_z} B_z \cos \phi - B_w \cos k_w z \right]. \tag{5.8}
\]

Solving Eqs. 5.7 and 5.8 explicitly,

\[
\frac{dp_\perp}{dz} = q_e B_w \sin (k_w z - \phi) \tag{5.9}
\]

\[
\frac{d\phi}{dz} = q_e \left[ \frac{B_z}{p_z} - \frac{B_w}{p_\perp} \cos (k_w z - \phi) \right]. \tag{5.10}
\]

Using the following normalizations:

\[
\theta \equiv k_w z - \phi \tag{5.11}
\]

\[
Z \equiv k_w z \tag{5.12}
\]

\[
a_w \equiv \frac{q_e B_w}{m_0 c k_w} \tag{5.13}
\]

\[
\Delta_w \equiv 1 - \frac{q_e B_z}{k_w p_z}, \tag{5.14}
\]

Eqs. 5.9 and 5.10 are now written

\[
\frac{dp_\perp}{dZ} = m_0 c a_w \sin \theta \tag{5.15}
\]

\[
\frac{d\theta}{dZ} = \Delta_w + \frac{m_0 c a_w}{p_\perp} \cos \theta. \tag{5.16}
\]

The axial momentum of the electron can be calculated easily from the perpendicular momentum since the overall momentum and energy of each electron remain constant: \( p_z = \sqrt{p^2 - p_\perp^2} \).

The perpendicular momentum of a single electron in the wiggler magnetic field is now described by two coupled, nonlinear differential equations of motion. The parameter \( \Delta_w \) is the wiggler detuning. When \( \Delta_w = 0 \), the electrons will gain perpendicular momentum the most rapidly. This condition is equivalent to the relativistic cyclotron frequency of the electron being equal to the frequency at which the electron passes through one wiggler period. Note that as the electron gains perpendicular momentum, \( \Delta_w \) does not remain constant, so the electron does not remain in resonance throughout the interaction. If, however, \( a_w \) is small, the change in \( \Delta_w \) will be gradual.
5.3 First Order Analytical Solution

Because $a_w$ is small for typical wiggler designs ($a_w < 0.06$ for the gyro-twt designed wiggler), it is a useful parameter to use for expanding Eqs. 5.15 and 5.16 in order to get a first order analytical solution for $p_\perp(Z)$. First, $p_\perp$ and $\theta$ are expanded in powers of $a_w$:

$$p_\perp = p_\perp(0) + a_w p_\perp(1) + a_w^2 p_\perp(2) + \cdots$$  \hspace{1cm} (5.17)

$$\theta = \theta(0) + a_w \theta(1) + a_w^2 \theta(2) + \cdots$$  \hspace{1cm} (5.18)

Using these expansions, terms involving $p_\perp$ and $\theta$ are rewritten to first order:

$$\sin \theta = \sin \theta(0) + a_w \theta(1) \cos \theta(0) + \mathcal{O}(a_w^2)$$  \hspace{1cm} (5.19)

$$\cos \theta = \cos \theta(0) - a_w \theta(1) \sin \theta(0) + \mathcal{O}(a_w^2)$$  \hspace{1cm} (5.20)

$$p_\perp^{-1} = p_\perp^{-1}(0) - a_w p_\perp(1) p_\perp(0) + \mathcal{O}(a_w^2)$$  \hspace{1cm} (5.21)

$$\Delta_w = \Delta_w(0) - a_w \frac{q_e B_z p_\perp(0)}{k_w p_z(0)} + \mathcal{O}(a_w^2)$$  \hspace{1cm} (5.22)

where $\Delta_w(0)$ and $p_z(0)$ are defined as

$$\Delta_w(0) \equiv 1 - \frac{q_e B_z}{k_w p_z(0)}$$  \hspace{1cm} (5.23)

$$p_z(0) \equiv \sqrt{p^2 - p_\perp(0)^2}.$$  \hspace{1cm} (5.24)

Following standard perturbation analysis, Eqs. 5.17–5.22 are substituted into Eqs. 5.15 and 5.16, and zeroth order terms are collected to yield the zeroth order result:

$$\frac{dp_\perp(0)}{dZ} = 0$$  \hspace{1cm} (5.25)

$$\frac{d\theta(0)}{dZ} = \Delta_w(0).$$  \hspace{1cm} (5.26)

Eqs. 5.25 and 5.26 are then solved subject to the initial conditions

$$p_\perp(Z = 0) = 0$$  \hspace{1cm} (5.27)

$$\theta(Z = 0) = \frac{\pi}{2},$$  \hspace{1cm} (5.28)

yielding

$$p_\perp(0) = \frac{q_e B_z}{k_w p_z(0)}$$  \hspace{1cm} (5.29)

$$\theta(0) = -\Delta_w(0) Z + \frac{\pi}{2}.$$  \hspace{1cm} (5.30)

Note that the initial condition for $\theta$ is arbitrarily chosen since $\theta$ is not defined if $p_\perp = 0$. To get the first order solution to $p_\perp$, all first order terms are collected after substituting Eqs. 5.17–5.22...
into Eqs. 5.15 and 5.16. Once this is done, the zeroth order solutions, Eqs. 5.29 and 5.30, are substituted into the result. Following these steps yields

\[
\frac{dp_{\perp}(1)}{dZ} = m_0 c \sin \theta(0) = m_0 c \cos(\Delta_w(0) Z),
\]

which is directly solved:

\[
p_{\perp}(1) = \frac{m_0 c \sin(\Delta_w(0) Z)}{\Delta_w(0)}.
\]

Therefore, the solution for \(p_{\perp}\) in the wiggler, to first order, is

\[
p_{\perp}(Z) = \frac{a_w m_0 c \sin(\Delta_{w0} Z)}{\Delta_{w0}} + \mathcal{O}(a_w^2),
\]

where \(\Delta_{w0}\) has been replaced by the value of the wiggler detuning at \(Z = 0, \Delta_{w0}\). This substitution comes from Eqs. 5.23 and 5.29. For \(\Delta_{w0} \ll 1\), Eq. 5.33 simplifies even further:

\[
p_{\perp}(Z) \approx a_w m_0 c Z.
\]

Eqs. 5.33 and 5.34 clearly show how \(p_{\perp}(Z)\) depends on \(\Delta_{w0}\) and \(a_w\). As one might expect, \(p_{\perp}\) initially increases linearly with \(Z\) before saturating at the predicted value of \(a_w m_0 c / \Delta_{w0}\).

The first order solution to \(p_{\perp}\) has serious limitations. When \(\Delta_{w0} = 0\), Eq. 5.33 predicts that \(p_{\perp}\) increases indefinitely with \(Z\), which is obviously not physically possible. Eq. 5.33 is best used to get the initial rate of increase of \(p_{\perp}\) in the wiggler. A much more accurate way to analyze electron motion in the wiggler region is to integrate Eqs. 5.15 and 5.16 numerically. Such a program, henceforth referred to as WIGGLE32, has been written and is used to present much of the data in Section 5.5. WIGGLE32 uses a Runge-Kutta integration method and is typically run with 100 steps per wiggler period. Each integration through the wiggler takes only 0.2 s of CPU time on a 386-based PC. Fig. 5-2 shows an example of the first order solution from Eq. 5.33, plotting \(\beta_{\perp}\) against \(z\) for a given set of wiggler parameters. Also plotted is the exact solution from Eqs. 5.15 and 5.16 as calculated by WIGGLE32. The parameters are listed in the figure caption.

### 5.4 A Realizable Wiggler Magnetic Field

The magnetic field used from Eq. 5.1 is unphysical. That is to say, it violates Maxwell's equations. The actual field profile from an infinitely long bifilar helical wiggler with an infinitesimally thin winding is[29, 46]

\[
B_r = \frac{2}{\pi} \mu_0 I r_0 k_w^2 \sum_{n=1,3,5...}^{\infty} n K'_n(nk_w r_0) I'_n(nk_w r) \sin \left[ n(\phi - k_w z) \right]
\]

\[
B_\phi = \frac{2}{\pi} \mu_0 I r_0 k_w^2 \sum_{n=1,3,5...}^{\infty} \frac{n}{k_w r} K'_n(nk_w r_0) I_n(nk_w r) \cos \left[ n(\phi - k_w z) \right]
\]
Figure 5-2: The first order solution (Eq. 5.33) to $\beta_\perp$ in an ideal wiggler field and the exact solution (Eq. 5.15) plotted against wiggler interaction length. Parameters are $\lambda_w = 9.21$ cm, $B_w = 60$ G, $B_z = 1750$ G, $V = 400$ kV, $\Delta_{v0} = -0.0197$, and $a_w = 0.0516$. 
\[ B_z = -\frac{2}{\pi} \mu_0 I r_0 k_w^2 \sum_{n=1,3,5,\ldots}^{\infty} n K'_n(n k_w r_0) I_n(n k_w r) \cos [n(\phi - k_w z)] , \quad (5.37) \]

where \( K_n \) and \( I_n \) are the modified Bessel functions, \( r_0 \) is the radius of the coil windings, and \( I \) is the total current through the winding. The summations are a result of harmonics of sinusoidally varying current sheets that add up to an infinitesimally thin winding. The first three odd-index modified Bessel functions for \( I_n, I'_n, \) and \( K'_n \) are shown in Fig. 5-3.

![Figure 5-3: \( I_n, I'_n, \) and \( K'_n \) for \( n = 1, 3, \) and 5. See Eqs. 5.35–5.37.](image)

As will be thoroughly discussed in Section 5.5, the numbers for the gyro-twt wiggler are \( k_w = 68.2 \) m\(^{-1}\) and \( r_0 = 2.86 \) cm. From the beam transport design in Section 4.6.4, \( r_b \approx 6-7 \) mm. Due to wall radius constraints in the gyro-twt interaction region, the maximum extent of the beam is limited to approximately 12 mm (neglecting adiabatic compression). Thus, for the design, \( k_w r < 0.8 \). Given this restriction, the ratio of the \( n = 3 \) terms to the \( n = 1 \) terms in Eqs. 5.35–5.37 is \( \sim 0.05 \). For the gyro-twt wiggler simulations, then, all terms but the \( n = 1 \) terms will be neglected. Eqs. 5.35–5.37 become

\[ B_r \approx -\frac{2}{\pi} \mu_0 I r_0 k_w^2 K'_1(k_w r_0) I'_1(k_w r) \sin(\phi - k_w z) \quad (5.38) \]

\[ B_\phi \approx -\frac{2}{\pi} \mu_0 I r_0 k_w^2 K'_1(k_w r_0) \frac{I_1(k_w r)}{k_w r} \cos(\phi - k_w z) \quad (5.39) \]
\[ B_z \approx -\frac{2}{\pi} \mu_0 I r_0 k_w^2 K'_1(k_w r_0) I_1(k_w r) \cos(\phi - k_w z) \] (5.40)

The ideal wiggler field is now recovered in the limit \( k_w r \ll 1 \): 

\[ \lim_{k_w r \to 0} I'_1(k_w r) = \lim_{k_w r \to 0} \frac{I_1(k_w r)}{k_w r} = 0.5, \] (5.41)

which when substituted into Eqs. 5.38–5.40 yields

\[ B_r \approx B_w \sin(\phi - k_w z) \] (5.42)
\[ B_\phi \approx B_w \cos(\phi - k_w z) \] (5.43)
\[ B_z \approx 0, \] (5.44)

where

\[ B_w = \frac{\mu_0}{\pi} K'_1(k_w r_0) I r_0 k_w^2. \] (5.45)

With the arbitrary starting phase of the wiggler set to the appropriate value, Eqs. 5.42–5.44 are equivalent to Eq. 5.1. For the maximum value of \( k_w r \) mentioned above, \( k_w r = 0.8 \), the Bessel function \( I'_1(k_w r) \) differs from 0.5 by over 20%. For more typical values of \( k_w r \), however, the difference is much less, and the WIGGLE32 program, which assumes an ideal wiggler field as in Eq. 5.1, is expected to give a good estimate of the transverse velocity imparted to the beam by the bifilar wiggler magnet. For a more thorough analysis, and as a check on the results of WIGGLE32, a multiple-particle code, TRAJ, will be used to simulate beam propagation through the wiggler. TRAJ is discussed in Section 5.7.

### 5.5 Final Wiggler Design

The first order of business in the wiggler design is to decide on basic dimensions. The most straightforward dimension to choose first is the radius of the windings, \( r_0 \). Because the Bessel function \( K'_1(k_w r_0) \) drops off rapidly as \( k_w r_0 \) increases, it is best to make the radius of the wiggler form as small as possible. The wiggler form is the “housing” for the wiggler windings—a machined piece with grooves that hold the wiggler windings in place. The wiggler form must be designed to fit inside the bore of the Haimson Research focusing coils and slide along the outside of the beam tube so that it can be positioned easily along the axis. The outer radius of the beam tube, 2.54 cm (1 inch), is the constraint on the minimum radius for the wiggler form. Due to tolerances and machining, this radius is set at 2.86 cm. Using this radius, the wiggler wavelength, \( \lambda_w \), and overall length must be chosen. Fig. 5-4 shows how \( B_w \) depends on \( \lambda_w \) for a fixed current through the wiggler winding. The desire is to run minimal current through the wiggler, therefore higher values of \( B_w \) for the same amount of current are better. For \( \lambda_w < 10 \) cm, there is a substantial drop off in \( B_w \). With this drop-off in \( B_w \) in mind, and considering that the designed axial field strength for the CRM interaction in the TE31 mode (Section 3.8) is \( \sim 2600 \) G, \( \lambda_w \) and the overall length of the wiggler need to be selected in order to realize beam pitch values, \( \alpha \), up to the maximum design value of 1.15. For the purposes here, the \( \alpha \) value for a particle entering the CRM interaction region
Figure 5-4: The amplitude of the transverse, on-axis wiggler field, $B_w$ (Eq. 5.1) plotted against wiggler wavelength, $\lambda_w$ (Fig. 5-1) for 160 A through the wiggler winding and $r_0 = 2.86$ cm. The calculations are from Eq. 5.45.
is calculated by integrating it through the wiggler with the WIGGLE32 code and then assuming that the particle undergoes adiabatic compression as the axial guide field increases from its value at the exit of the wiggler to the beginning of the CRM interaction region. The transverse velocity of a particle that undergoes adiabatic compression is discussed in Section 2.3. If $B_{z,\text{WIGGLER}}$ is the axial field in the wiggler region and $\beta_{\perp,\text{WIGGLER}}$ is the transverse particle velocity at the wiggler exit, then the transverse velocity in the interaction region, assuming adiabatic compression, is

$$\beta_{\perp,\text{INTERACTION REGION}} = \beta_{\perp,\text{WIGGLER}} \sqrt{\frac{B_{z,\text{INTERACTION REGION}}}{B_{z,\text{WIGGLER}}}},$$

(5.46)

where $B_{z,\text{INTERACTION REGION}}$ is the axial field in the interaction region. The value of $\beta_{\perp}$ in the interaction region is then directly related to $\alpha$ by assuming that the beam energy remains constant. The wiggler form must be short enough that the focusing coils can achieve a flat field over its entire length. Given this constraint and the space needed to initially focus the beam and also to adiabatically compress it, a length of $\sim 30$ cm is chosen. For this length and a transverse field of $B_w = 20$ G, Fig. 5-5 shows the beam pitch, $\alpha$, at the beginning of the CRM interaction region plotted as a series of contours against both wiggler wavelength, $\lambda_w$, and axial guide field in the wiggler region, $B_z$. The interaction field is 2600 G. As Fig. 5-5 demonstrates, lower values of $\lambda_w$ are necessary to impart significant $\alpha$ to particles at higher values of $B_z$. Moreover, the maximum $\alpha$ value decreases for both increasing $B_z$ and decreasing $\lambda_w$. For $\lambda_w$ too small, the wiggler is more difficult and expensive to machine, and the $\alpha$ values are lower. For $\lambda_w$ too large, $B_z$ starts to become too low to achieve a good beam match, as discussed in Section 4.6.2. With these considerations in mind, the design value for $\lambda_w$ was chosen to be 9.21 cm (3 5/8 inches), with a total length of three periods (27.63 cm). A scale drawing of the final machined wiggler form is shown in Fig. 5-6.

The wiggler form was wound with a single 14 gauge ($\sim 1.6$ mm diameter), enamel insulated, solid copper wire. A total of 16 passes were run through each wiggler channel, and the impedance of the entire length of the coil is 0.23 $\Omega$. For an impedance of 0.23 $\Omega$, the heat generated from the windings with the wiggler current at 50 A is 575 W, which results in a water temperature rise of 7.8$^\circ$ C for a flow rate of 5 l/s through the wiggler. The measured flow rate through the wiggler is well over 5 l/s at reasonable water pressure, so operation of the wiggler at currents up to 50 A is easily achievable. The wiggler transverse field was measured with an accurate Hall probe at angles of 0 and 90 degrees. The results of these measurements and the component sum is shown in Fig. 5-7. With the component sum fitted to 45.4 G and the current through the wiggler at 33 A, the conversion ratio from current to $B_w$ is 1.37 G/A. The variations in the flat region of the component sum are most likely attributable to errors in the measurement of the Hall probe axial position. The value of $r_0$ predicted by Eq. 5.45 that results in a field of 1.37 G/A (with 16 windings) is $r_0 = 3.18$ cm. Given that the winding wire was wound in two layers (a top and bottom layer), and that there was a small amount of slack in the winding, this is a good prediction. The more significant value is 1.37 G/A, which will be used to convert wiggler current to transverse guide field for experimental analysis.
Figure 5-5: Curves of constant $\alpha$ plotted against wiggler wavelength, $\lambda_w$, and guide field in the wiggler region, $B_z$. The $\alpha$ values are calculated at the CRM interaction guide field of 2600 G assuming adiabatic compression from $B_z$ to 2600 G. WIGGLE32 was used for the computations with a fixed wiggler length of 30 cm and a transverse wiggler field amplitude of 20 G.

Figure 5-6: A 1:3 scale drawing of the wiggler form. The wiggler wavelength, $\lambda_w$, was chosen at 9.21 cm. The wiggler form radius is 2.86 cm. A cover slips over the form (the O-ring grooves are shown on each end) to allow cooling water to flow through the channels where the wire coil is laid. Wire is wrapped around the form a total of 16 times.
Figure 5-7: Measured wiggler field plotted against axial position in the wiggler. A Hall probe was used to measure the transverse on-axis field. The wiggler was scanned with the probe at 0 (open triangles) and 90 (filled circles) degrees. The component sum of the two measured scans is shown by the open boxes. The flat part of the component sum averages to 45.4 G. The wiggler current is 33 A ($\times$ 16 windings).
5.6 Single Particle Analysis

With the wiggler design established, simulations can be done using the designed wiggler dimensions and the single particle code, WIGGLE32, introduced at the end of Section 5.3. With the beam voltage fixed at 400 kV, the analysis is straightforward. The only parameters to vary are $B_w$ and $B_z$. By analyzing particles with different initial energies and $\alpha$ values, the spread at the beginning of the gyro-twt interaction region can be predicted. In particular, the EGUN simulations from Section 4.6.4 predict that the outer-most rays of the beam for a good match are $\sim 10$ kV more energetic than the inner-most rays. In addition, the outer-most rays have a starting pitch of $\alpha_0 \approx 0.05$, while the inner rays have a starting pitch of $\alpha_0 = 0$. For simulations, then, two different particles are integrated through the wiggler—one representing an outer beam ray at $V = 405$ kV and $\alpha_0 = 0.05$, and the other representing an inner beam ray at $V = 395$ kV and $\alpha_0 = 0$. The final $\alpha$ value for each of these particles at the beginning of the CRM interaction region is expected to demonstrate the range for the entire beam, and thus give a sense of the spreads that the wiggler will add to the beam. Fig. 5-8 shows the results of three such simulations, each graph representing a different value of $B_w$. The plots show the mean $\alpha$ for each of the two particles, and the percentage difference in both $\beta_\perp$ and $p_z$ for the two particles, all evaluated at the beginning of the CRM interaction region where the axial field is 2600 G. The values are plotted against $B_z$, the axial guide field at the wiggler. For $B_z$ near the wiggler resonance (peak values of $\alpha$), the percentage difference, or spread, in both $\alpha$ and $p_z$ is large, but on either side of the wiggler resonance, the spread decreases. For $B_z \sim 1450$ G, just below wiggler resonance, all three plots show an area where both spreads are near zero. This would seem to be an optimal value of $B_z$ for running the experiment; however, the low value of $B_z$ lowers the $m$ number for the beam confinement and makes the match more unstable. As $B_z$ increases from 1450 G to 1600 G and then higher, the spreads first increase but then decrease again. In fact, the axial momentum spread falls to near zero for $B_z \sim 1900$ G in each case. At 1900 G, however, $\alpha$ has dropped off substantially. The maximum value for $\alpha$ at $B_z = 1900$ G is 0.6 for $B_w = 65$ G. A trade-off point certainly exists that will optimize the gyro-twt interaction, most likely in the range $B_z = 1600$–1900 G. This point will be easiest to determine empirically. The significant insights gained from Fig. 5-8 are that spread increases with beam pitch, that there are better and worse axial guide fields to use based on spread values, and that the spreads appear to be within the bounds necessary ($\Delta p_z/\langle p_z \rangle \sim 5\%$ for $\alpha \sim 1$) for attaining high power output from the gyro-twt interactions (see Fig. 3-8 in Section 3.6).

5.6.1 Phase Variability

In Section 3.9, critical correlations between $\beta_\perp$ and $\gamma$ was discussed that would result in optimal phase stable operation of the gyro-twt amplifier. The remaining issue is how to achieve such a correlation. Because of the nature of wiggler resonance, the wiggler interaction can induce a wide range of $\beta_\perp-\gamma$ correlations depending on the setting of the axial guide field. This is demonstrated in Fig. 5-9, where the ratio of percentage change in $\beta_\perp$, $\Delta \beta_\perp/\beta_\perp$, to the percentage change in beam energy, $\Delta V/V$, is plotted against wiggler axial guide field, $B_z$. From Fig. 3-12 in Section 3.9, the optimal phase stable correlation between $\beta_\perp$ and $\gamma$ may be either positive (TE$_{31}$ case) or negative (TE$_{21}$ and TE$_{11}$ cases). In either case, Fig. 5-9 shows that the appropriate setting for wiggler axial guide field attains the desired correlation. The value of $\beta_\perp$, in theory, can then be adjusted by
Figure 5-8: Predicted beam pitch and spread results from the WIGGLE32 code. The transverse field is $B_z =$ 45 G, 55 G, and 65 G for graphs (a), (b), and (c), respectively. For each graph, two particles with different initial $V$ (395 kV and 405 kV) and $\alpha$ (0 and 0.05, respectively), are integrated through the wiggler region at the guide field value $B_z$, and adiabatically compressed to the CRM interaction field of 2600 G. The curves show average beam pitch, $\langle \alpha \rangle$, and the percentage difference in both transverse velocity, $\Delta \beta_1 / \langle \beta_1 \rangle$, and axial momentum, $\Delta p_z / \langle p_z \rangle$, all evaluated at the beginning of the CRM interaction region.
Figure 5-9: Due to the wiggler's resonant nature, the ratio of change in $\beta_\perp$ to change in $V$ changes steadily over a range of axial guide field strengths. Depending on run parameters, a negative or a positive correlation may be desired for optimal phase stability in a gyro-twt. See Fig. 3-12 in Section 3.9. WIGGLE32 was used to calculate these curves using the designed wiggler parameters, $V = 400$ kV, and $B_w = 65$ G.
changing the value of the transverse wiggler field, $B_w$. Of course, the beam quality will also be of concern, as just discussed, and some run parameters may optimize gyro-twt power while others may optimize phase stability.

5.7 Multiple Particle Analysis—The TRAJ Code

To include the effects of space charge and self magnetic fields when finding particle orbits through the wiggler, it is necessary to expand the single-particle model discussed in the previous section to include multiple particles. All particles are then integrated in $z$ with the $x$ and $y$ positions being allowed to change with $z$. Unlike in the CRM32 code, particles in the wiggler have to be traced in full 3D space. The problem is not axisymmetric since the beam corkscrews about the axis. The MIT TRAJ code [71, 84, 80] was written to perform such a multiple particle 3D integration through a wiggler and also through a region of adiabatic compression as the guide field increases to the proper level for the gyro-twt interaction. TRAJ includes several options for calculating the wiggler field and initializing the particle parameters. By solving Poisson’s equation in the transverse plane at each $z$-step, TRAJ includes the effects of the transverse electric field due to space-charge and the transverse magnetic field due to the axial beam current. Because TRAJ steps in $z$, it does not include fields that require a $z$ history—longitudinal space charge forces and self-$B_z$ fields. This limitation will be discussed.

5.7.1 Transverse Space Charge Forces

TRAJ extends Eq. 5.2 to integrate the particles through the wiggler. One such extension is to add a force term corresponding to the space charge force seen by each particle at each $z$-step. For the purposes of calculating this force, consider a solid, cylindrical, uniform density beam. Gauss’s law is used to evaluate the radial electric field at points inside the beam, just as was done for Eq. 4.26:

$$E_r = \frac{I_r}{2\pi\varepsilon_0 r_b^2 v_z},$$ (5.47)

where $r$ is the radius of a particle inside the beam, $I$ is the total beam current, $r_b$ is the outer radius of the beam, and $v_z$ is the beam velocity. The force due to this field will affect $\hat{p}_x$ and $\hat{p}_y$:

$$\frac{d\hat{p}_x}{dz} = -\frac{q_e}{v_z} E_x = \frac{-q_e I}{2\pi\varepsilon_0 r_b v_z^2} \cos(\phi)$$ (5.48)

$$\frac{d\hat{p}_y}{dz} = -\frac{q_e}{v_z} E_y = \frac{-q_e I}{2\pi\varepsilon_0 r_b v_z^2} \sin(\phi),$$ (5.49)

where $\hat{p} \equiv p/(m_0 c)$. As different particles see different forces, the beam cross-section loses symmetry. To evaluate the transverse E-field for an asymmetric cross-section of many charged particles, the particles are distributed on a polar grid, and the electric potential, $\Phi$, can be calculated numerically as outlined in Birdsall and Langdon[10].

It is instructive to calculate the peak value of $d\hat{p}_1/dz$ for beam parameters typical of the experiment. Substituting $I = 350$ A, $r = r_b = 6$ mm, and $v_z = 0.8c$ into Eq. 5.48, the amplitude
of $d\dot{p}_x/dz$ from Eq. 5.48 is 0.11 cm$^{-1}$. For every centimeter of movement, the space charge force is enough to change the transverse momentum by nearly 10%. In Brillouin flow and confined flow, as discussed in Sections 4.6.1 and 4.6.2, this force is counteracted by a slight rotation of the beam in a strong axial magnetic field.

If the beam is corkscrewing about the axis rather than traveling entirely axially, space charge will not only result in significant radial forces, it will result in significant longitudinal forces. The magnitude of the longitudinal forces for pitch values near unity is expected to be the same order of magnitude as the radial forces just discussed. The simple argument here is that with a pitch of $\alpha = 1$, the angle of the beam cylinder is such that the radial force from Eqs. 5.48 and 5.49 now has equal components pointing transversely and longitudinally. Unfortunately, TRAJ is not equipped to account for longitudinal forces, because they can only be calculated by knowing the full shape of the beam in $z$. TRAJ only makes calculations based on beam cross-sections at single values of $z$. Due to the potentially large values for longitudinal space charge forces ($d\dot{p}_z/dz \sim 0.1$ cm$^{-1}$ as discussed above), this limitation of TRAJ must be considered when analyzing data from TRAJ simulations, especially for the high current, high pitch values in these gyro-twt designs.

### 5.7.2 Self Magnetic Field Forces

In addition to particles in the beam being affected by the electric field from other particles, they are also affected by the magnetic field from other particles—the self magnetic field of the beam. The magnetic field due to the beam is calculated from Ampere’s law:

$$\nabla \times \mathbf{B}_{\text{self}} = \mu_0 \mathbf{J},$$

(5.50)

where $\mathbf{B}_{\text{self}}$ is the self magnetic field vector and $\mathbf{J}$ is the beam current density vector. Using the Coulomb gauge for the vector potential, $\nabla \cdot \mathbf{A} = 0$, and the definition of the vector potential, $\nabla \times \mathbf{A} = \mathbf{B}$, Eq. 5.50 becomes

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}.$$  

(5.51)

For the transverse self magnetic field, $A_z$, can be solved on a polar grid in the same manner as the electric potential, as discussed in the previous section.

Though, in principle, the solution for the axial self magnetic field can be found from Eq. 5.51, such a solution would not be accurate because it would only consider the current components of the beam at a single $z$ cross-section. The axial self magnetic field will depend strongly on the entire $z$ history of the beam path. As the wiggler causes the beam to begin corkscrewing about the axis with increasing pitch and Larmor radius, the beam begins to act as a large helical current conductor. If the beam is approximated by an infinitely long helical conductor with a fixed radius and pitch, an axial self magnetic field estimate can be made. For beam parameters $V = 400$ kV, $I = 350$ A, $r_b = 6$ mm, and $\alpha = 1$, the self field is readily approximated using the Biot-Savart equation. The result is a reverse axial self-field of $\sim 50$ G, maximum. The beam particles see a range of self-field from 0 to this maximum. From Fig. 5-8, a shift of $-50$ G in the guide field has a significant affect on $\alpha$ for certain values of $B_z$. This limitation of TRAJ will have to be considered when examining TRAJ simulations.
5.7.3 TRAJ Results Without Self-Fields

Because TRAJ includes a number of algorithms for calculating the wiggler field, the first simulations were done to compare the various models of wiggler field that have been presented in this section. There is (a) the ideal wiggler field from Eq. 5.1, (b) the first term in the wiggler harmonic series from Eqs. 5.38–5.40, (c) the complete harmonic sum from Eqs. 5.35–5.37, and (d) the field calculated by using the Biot-Savart law to integrate the actual current loops around the wiggler form. The pitch and spreads predicted by TRAJ for these four cases are shown in Fig. 5-10 along with the results from WIGGLE32. Particles for the TRAJ simulations were initialized with velocities and positions predicted by an EGUN simulation. The EGUN simulation used 416 kV at the cathode and predicted the outer rays to be at $V = 406$ kV and $\beta_\perp = 0.05$ and the inner rays to be at $V = 394$ kV and $\beta_\perp = 0$. The transverse field strength for Fig. 5-10 is $B_w = 60$ G. The WIGGLE32 curves in Fig. 5-10 were calculated exactly as in Fig. 5-8, except that the represented parameters are evaluated at the wiggler exit rather than at the beginning of the CRM interaction region, so adiabatic compression is not taken into account. Because Fig. 5-10 is only meant to compare different ways of calculating the wiggler field, self-field effects are not included in the TRAJ simulations. Because the computation time for the TRAJ (c) and (d) points is substantial, fewer points were calculated. For beam pitch, the TRAJ result with an ideal field is almost identical to WIGGLE32, while the more accurate field modeling has the expected result of slightly increasing the beam pitch due to the increase in $B_w$ as the beam Larmor radius increases. The most surprising result from Fig. 5-10 is that TRAJ does not predict the same decrease in spread at low guide field that WIGGLE32 does. The middle beam rays, which are not represented by WIGGLE32, must play a critical role in the spread when $B_z$ is below resonance. Though the spreads disagree substantially between TRAJ and WIGGLE32, both do predict a fall off in spread when the axial guide field is well above resonance.

5.7.4 TRAJ Results With Self-Fields

As a first test of the transverse self-field modeling in TRAJ, simulations were done with input rays from EGUN and no transverse wiggler field. The beam was simply allowed to continue to propagate in a uniform guide field for 20 cm. Fig. 5-11 shows such results. The TRAJ prediction of beam scalloping period and amplitude is identical to the EGUN prediction. Because axial self-fields are small when the beam is traveling with such small rotation, the good results are not surprising.

The next step was to run TRAJ through the wiggler with all self-fields included. Because such simulations are CPU intensive, it is only practical to have TRAJ approximate the transverse wiggler field using method (b) in the previous section, i.e., by calculating the Bessel functions from Eqs. 5.38–5.40 to third order in $k_{wr}$. With this method of approximation, the simulations take $\sim 10$ minutes of CPU time on a CRAY-2S/4128. The results from Fig. 5-10 show that this is a reasonable approximation. Each full simulation of the beam for a unique set of parameters also requires a corresponding POISSON run to calculate the magnetic field and an EGUN run to simulate the initial propagation of the beam. Three sets of parameters were chosen for full beam simulations with transverse self-fields included. These parameters and the corresponding results are shown in Table 5.1. An obvious result from the self-field TRAJ simulations is that the spread numbers are dramatically higher than both the WIGGLE32 predictions and the TRAJ predictions done without self-fields included. The most alarming numbers from Table 5.1 are the energy
Figure 5-10: Predicted beam pitch and spread results from WIGGLE32 and TRAJ for $B_w = 60$ G and $(V) = 400$ kV based on input rays from EGUN. Self-field effects are not taken into account. The figures from top to bottom show average beam pitch, transverse velocity spread, and axial velocity spread, respectively. For the four TRAJ cases, the transverse wiggler field is calculated by (a) Eq. 5.1 (ideal), (b) Eqs. 5.38-5.40 (ideal plus third-order radial term in Bessel function), (c) Eqs. 5.35-5.37 (first two terms), and (d) integration of the actual current conductors using the Biot-Savart law (most exact). Note: The WIGGLE32 spread curves actually represent maximum rather than standard deviation, just as in Fig. 5-8. All values are at the wiggler exit, not at the CRM interaction region.
Figure 5-11: Beam trajectories predicted by EGUN and TRAJ. The TRAJ simulation takes into account transverse self-fields, but not longitudinal self-fields. For the beam shown in the figure, this is not a bad approximation. Parameters: $V = 415$ kV, $B_z = 1860$ G, $I = 400$ A. Note the greatly distorted aspect ratio of the figure.
<table>
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Table 5.1: Results from TRAJ simulations with self-fields included. The three cases were done based on Fig. 5-8, and they represent three different settings that achieve the same values of $\alpha$ in the interaction region. The values shown for TRAJ and WIGGLE32 are evaluated at the beginning of the CRM interaction region ($B_z = 2600$ G).
spreads that TRAJ predicts for the beam. Energy spread increases due to space charge forces on the corkscrewing beam. Because particles on the outside of the corkscrewing beam rotate about the surface of the helical cylinder described by the beam, the space-charge force on them is constantly changing direction, resulting in an averaging, or phase-“mixing” process. The natural end result of mixing is a beam that loses spatial coherence since this results in all particles experiencing the same space-charge forces. Values such as 10% for energy spread in an electron beam meant for a CRM interaction are completely intolerable, as evidenced by Fig. 3-8 in Section 3.6. Several tolerance checks were made on TRAJ to verify that the spreads in Table 5.1 are not due to numerical inaccuracies. The WIGGLE32 results show no increase in energy spread since all particle energies are conserved.

To get an idea of how the spread develops over the length of the beam propagation, three figures are presented from Case C ($B_w = 65$ G, $B_z = 1860$ G) in Table 5.1. Fig. 5-12 shows an $x$-$z$ plot of the trajectories of all simulated particles that start at $y = 0$. The beam is shown to do exactly what is expected—gain transverse velocity and begin corkscrewing about the axis. Fig. 5-13 shows $x$-$y$ cross-sections of the beam at three different $z$ points. The beam is quite uniform upon exit from the wiggler, but after adiabatic compression into the interaction region, mixing has taken place, and the beam cross-section is not as coherent. Fig. 5-14 shows the axial history of the parameters listed in Table 5.1 for Case C. The beam pitch has the expected history, increasing throughout the wiggler,
Figure 5-13: Beam cross sections in $x$-$y$ predicted by TRAJ at three different $z$ points. The particles are initially grouped into $\sim 100$ beamlets, and each beamlet is traced through the interaction region. The cross sections show the $x$-$y$ position of the centroid of each beamlet. Self-field effects cause the cross-section to lose uniformity.
remaining flat during a short propagation, and then increasing again during adiabatic compression. The spreads clearly demonstrate the beam “mixing” discussed earlier. All three spreads show a steady increase throughout the beam propagation. The increase for the energy spread becomes more severe during compression. The $B_z$ curve in Fig. 5-14 has a slight bump in the adiabatic compression region due to the end iron pole-piece on the focusing coils. This is discussed further in Section 6.2.1.

In summary, the designed wiggler has met all of the requirements listed in the introduction except one, that being a tolerable energy spread, according to the TRAJ simulations. While this is a serious concern, it is also suspect due to TRAJ’s neglect of longitudinal self-fields. It may be that an optimized parameter space and field profile will be empirically found that will deliver improved beam quality compared to the cases in Table 5.1. Reducing the beam current, and thereby reducing the self-field effects that cause the energy spread, may also be considered. This could be done by either running the cathode temperature limited or by using a beam scraper. TRAJ also predicts a strong correlation between $\alpha$ and energy spread, so that a trade-off between $\alpha$ and energy spread may be empirically determined. This would reduce the optimum $\alpha$ predicted by Table 3.3. Shortening the beam propagation distance would also clearly be beneficial to the beam quality; however, the experiment is as short as possible given the existing mechanical and financial constraints.

If the 7%-12% energy spread predictions from TRAJ are accurate, measures will have to be taken to reduce the energy spread. Fig. 3-8 shows how significantly the gyro-twt efficiency drops for even 5% energy spread. The values predicted by TRAJ for axial momentum spread and pitch are more promising. They are in the range necessary to generate high power. The discussion shifts now to the interaction circuit for the gyro-twt experiments.
Figure 5-14: TRAJ predictions of $\langle \alpha \rangle$, $\langle \beta_\perp \rangle$, and various beam spreads throughout the wiggler and adiabatic compression regions. The axial magnetic field, $B_z$, is plotted on the bottom figure. Beam mixing is evidenced by the steady increase of spreads after the exit from the wiggler region. The adiabatic compression hastens the increase in spread.
Chapter 6

Interaction Circuit

6.1 Introduction

The final stage of the CRM amplifier design involves details of the interaction circuit. The resonant field in the wiggler region is in the range 1400–2000 G, and the field required for the gyro-twt interactions is 2600–3500 G, so a region of adiabatic compression from the wiggler field region to the CRM interaction region is necessary, followed by a region of at least 1 m where the axial field is kept constant (or tapered) at the optimum value for the gyro-twt interactions. Beam diagnostics are needed to measure current, and an rf input coupler is needed to inject the low power 17 GHz signal which is amplified in the gyro-twt interaction region. A source is needed which provides the rf input. Finally, a window is needed which will divide the vacuum system from free space and allow the rf power to propagate out of the beam tube with minimal reflected power. All of these issues are discussed in this section.

6.2 High Field Region—The X-Coils

The field in the gyro-twt interaction region will be provided by several identical, 22 cm long, 14 cm bore diameter coils. By running several identical coils together, the field can be precisely tapered. The coils were designed and built at Lawrence Livermore National Laboratory, and are referred to as the “X-coils.” Though the X-coils have known winding errors[70] which cause slight transverse field errors, these errors can be minimized by reversing the orientation of the leads on every other coil in the axial series of coils. A schematic of a single X-coil is shown in Fig. 6-1. The X-coils have 90 turns of hollow-core copper tubing and are cooled with up to 80 psi of water pressure. The electrical impedance of one X-coil ranges from 0.33 Ω at room temperature to 0.4 Ω when run at 750 A and hot to the touch. With five coils at 750 A, a pressure of 80 psi keeps the water temperature rise below 35° C.

The on-axis field of a single X-coil has been carefully measured at 480 A using a Hall probe. Fig. 6-2 (top) shows this measurement compared to the analytical solution for the on-axis field due to a square cross-section coil. The analytical formula is derived easily from the Biot-Savart law:
Figure 6-1: A 1:2 scale drawing of an X-coil cross section. The cross-hatched area shows the X-coil potting. The X-coil has 90 turns, but the number used by POISSON simulations is 91.3 turns in order to exactly match the measured field of the X-coil on-axis (see Fig. 6-2).

\[ B_z(z) = \mathcal{J} \left\{ \frac{[l/2 - (z - z_0)] \ln \left[ \frac{r_o + \sqrt{r_o^2 + [l/2 - (z - z_0)]^2}}{r_i + \sqrt{r_i^2 + [l/2 - (z - z_0)]^2}} \right]}{[l/2 + (z - z_0)] \ln \left[ \frac{r_o + \sqrt{r_o^2 + [l/2 + (z - z_0)]^2}}{r_i + \sqrt{r_i^2 + [l/2 + (z - z_0)]^2}} \right]} \right\}, \tag{6.1} \]

where \( r_o \) is the outer radius of the windings, \( r_i \) is the inner radius of the windings, \( l \) is the axial length of the windings, \( z_0 \) is the axial center of the windings, and \( \mathcal{J} \) is the current density through the windings. POISSON is not necessary to predict the field from a single X-coil since there is no iron involved in the problem. The current through the X-coil, 480 A, is accurate to 0.2%, and the field value to 0.1%. The factor of 1.015 difference between the theory and the measured field value (taken into account by using 91.3 rather than 90 windings) is most likely explained by the coil not having an exactly square cross-section and the geometry being difficult to measure precisely. Because it is important to simulate the measured field value as accurately as possible, the value of 91.3 windings will be used in all simulations of the X-coil fields to predict field values from X-coil currents. Fig. 6-2 (bottom) also shows the predicted field for five consecutively positioned X-coils all at \( I = 500 \) A. This prediction was made by summing the fields of individual coils and using the potting spacing in Fig. 6-1. Because of the known winding errors in the X-coils, the axial field has a \( \sim 3\% \) ripple. Though not shown, the 3% ripple has been confirmed by measurements with a Hall probe. The ripples are not expected to affect the gyro-twt performance.
Figure 6-2: Measured and simulated on-axis X-coil magnetic fields for a single X-coil at $I = 480$ A and for five consecutive X-coils at $I = 500$ A. The coil geometry is given in Fig. 6-1, and the theory here assumes 91.3 windings, as discussed in Fig. 6-1. In the top figure, $z = 0$ represents the center of the potting. In the bottom figure, the ripples on the field profile of the consecutive coils are due to the potting gaps between the X-coils.
6.2.1 Transition from Wiggler Field to High Field

The X-coils butt up against the end-pole piece of the Haimson Research focusing coils as close as possible allowing for 1/4 inch poly-flow water tubing to exit between the first X-coil and the end pole. The water tubing is for cooling the wiggler magnet. Allowing some tolerance, there will be a 3/8 inch gap between the first X-coil and the end pole of the focusing coils. Due to this gap and to the end-pole piece, a significant “dip” occurs in the axial magnetic field profile between the focusing coils and the X-coils. Although removing the end pole of the focusing coils would improve the field transition between the focusing coils and the X-coils and eliminate this field dip, this possibility is not mechanically feasible without destroying the delicate alignment of the focusing coils. Instead, a small matching coil was designed to be inserted directly inside the focusing coils just inside the end pole-piece in order to counter-act the field dip.

A scale drawing of the matching coil, the first X-coil, and other components to be discussed in later sections is shown in Fig. 6-3. The matching coil winding cross-section is 6.73 cm long (axially) with an inner diameter of 6.60 cm and an outer diameter of 9.22 cm. It has 475 turns, an impedance of 1.58 \( \Omega \), and is cooled by water tubing wound around the outside of the windings. The current limit is expected to be 8–10 A. It will be centered directly underneath the end-pole piece of the focusing coils. The effect of the matching coil on the transition field region is shown in Fig. 6-4. The axial field profile is shown for the matching coil at 0 A and 10 A. The transition is considerably more gradual (the dip is removed) when the matching coil is used. The field profiles in Fig. 6-4 are calculated using POISSON.

6.3 Beam Diagnostics

Two beam diagnostics have been integrated into one flange for the purpose of measuring total beam current, beam position, and beam pitch for the gyro-twt experiments. The flange consists of four “b-dot” wire loops (a single turn for each loop), each loop in a plane of constant angle, \( \phi \), at \( \phi = 0^\circ, 90^\circ, 180^\circ, \) and \( 270^\circ \). One other loop, a diamagnetic loop, is placed in the flange in a plane of constant \( z \). The beam will pass directly through the diamagnetic loop, which will measure the rotation of the beam. The inner and outer radii of the beam diagnostics flange are 5.1 cm and 8.6 cm, respectively. The diameter of the diamagnetic loop is 3.8 cm (1.5 in), and the b-dot loops extend to a minimum radius of 3.8 cm. The flange is shown in the schematic in Fig. 6-3, and a photograph of the completed flange is shown in Fig. 6-5. Each loop in the flange has one side attached to the flange itself, which will be electrically grounded, and one side attached to an APC 3.5 mm feedthrough connector which holds vacuum. The voltages on the loops can then be measured on the APC connectors.

6.3.1 B-dot Loops

The b-dot loops generate signals based on Faraday’s law: the voltage on a loop is proportional to the time derivative of the magnetic flux through the loop:

\[
V_{\text{loop}}(t) = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a},
\]  

(6.2)
Figure 6-3: A 1:2 scale drawing of the transition to and beginning of the gyro-twt interaction section. A matching coil is used to maintain a smooth increase in magnetic field from the lower field in the wiggler region to the higher field in the CRM interaction region provided by the X-coils. The b-dot loops (there are four) measure beam current and position, and the diamagnetic loop measures beam pitch. A 17 GHz rf input signal travels in WR62 rectangular waveguide in the TE_{10} mode and reflects off of a wire mesh in a quasi-optical manner to generate a mix of modes in the 1.27 cm radius circular copper waveguide. The selected mode will immediately begin CRM amplification in the interaction region.
Figure 6-4: The axial magnetic profile with and without the use of the matching coil between the focusing coils and the X-coils. The field profiles are calculated using POISSON.
Figure 6-5: A photograph of the beam diagnostics flange, slightly larger than actual size. The large loop concentric with the flange is the diamagnetic loop. The four smaller loops around the inside edge of the flange are the b-dot loops. The inner diameter of the flange is 5.08 cm (2 in), and the diameter of the diamagnetic loop is 3.81 cm (1.5 in).
where \( V_{\text{loop}}(t) \) is the voltage signal on the APC connector and the integral is over the area of the b-dot loop. The name "b-dot" comes from the time derivative of the b-field commonly being written as \( B \). By time-integrating the b-dot voltage signals, \( \int B \cdot da \) is recovered. Since the loop is in a plane of constant \( \phi \), the dot product is simply \( B_\phi da \), where \( B_\phi \) is the azimuthally directed B-field.

For an axially traveling beam, \( B_\phi \) is in turn proportional to the beam current. For an infinitely long cylinder of current,

\[
B_\phi = \frac{\mu_0 I}{2\pi r},
\]

where \( r \) is the distance from the center of the beam to the point where \( B_\phi \) is evaluated. So long as the beam position (and therefore \( r \)) is fixed, a strict proportionality will exist between the integrated b-dot loop signal and the beam current. This proportionality constant can be computed based on the dimensions of the b-dot loop, but it is more accurate to measure it by running a known current through the flange. When the beam is not centered, the \( 1/r \) factor in Eq. 6.3 must be considered. The beam will be closer to some loops, and thus generate a larger signal on those loops, and the beam will be farther from the other loops, and thus generate a smaller signal on them. By analyzing the signals on all four loops (after calibration), the beam position and total current can be uniquely determined.

To simulate the beam from SNOMAD-II, a long, straight, copper rod of \( \sim 1 \) cm diameter was placed straight through the b-dot loop flange (the rod perpendicular to the plane of the flange) and pulsed with 1 A of current for 50 ns durations. The rod was placed so as to intersect the plane of the flange at different \( x \) and \( y \) positions, and all four b-dot signals, labeled 'A' through 'D,' were stored at 1 billion samples per second (1 GS/s) on a 400 MHz bandwidth Lecroy digital storage oscilloscope. Each set of b-dot traces was then digitally time-integrated. A sample integrated b-dot trace with the corresponding current pulse (as measured on a 50 \( \Omega \) resistor) is shown in Fig. 6-6. The amplitude of every time-integrated b-dot signal was then determined and multiplied by \( \cos \theta \), where \( \theta \) for each trace is the angle between the plane of the corresponding b-dot loop and the line between the center of the copper rod and the center of the b-dot loop. The factor of \( \cos \theta \) is necessary to correctly calculate the magnetic flux through the loop. If the rod is centered in the flange, \( \cos \theta = 1 \).

After multiplication by \( \cos \theta \), the amplitudes of the b-dot signals were plotted as a function of the distance between the relevant loop and the straight copper rod in Fig. 6-7 (top). These points were then fitted to the relation \( a/(r - r_0) + b \), and the best fit results are \( a = 1.65 \) V-ns/(A-cm), \( r_0 = 0.36 \) mm, and \( b = -0.38 \) V-ns/A. The fit is shown in Fig. 6-7 (top). The \( a \) parameter is the proportionality constant, \( b \) accounts for d.c. offsets in the signal, and \( r_0 \) is a distance offset. The rms deviation between the fitted curve and the measured points is 0.028 V-ns/A. Using this fit and other calibration data for the different loops, a computer program was written that analyzes the four signals together and computes total beam current and beam position. The results of the position calibration are shown in Fig. 6-7 (bottom). The open circles mark where the calibration rod actually crossed through the flange. The crosses mark where the computer predicted the rod to be based on the b-dot signals. The rms error between the predicted and actual positions is 0.4 mm in both \( x \) and \( y \).

While the b-dot flange appears to be an excellent diagnostic for determining beam current and position, some caveats should be mentioned. The b-dot loops work well in this case because of the short pulse and fast rise time of SNOMAD-II. For longer pulse experiments, the \( d/dt \) factor
Figure 6-6: A time-integrated b-dot signal for a calibrated current pulse through a copper rod positioned at the center of the beam diagnostics flange. The signal is from b-dot loop 'A.'
Figure 6-7: The top figure shows the amplitude of time-integrated b-dot pulses as a function of the distance between the b-dot loop and the current carrying copper rod. The bottom figure shows an x-y cross-section of the beam diagnostics flange with open circles marking the actual position of the copper rod and crosses marking the prediction of the rod position based on the relative strengths of the four b-dot signals. The predictions are made by a computer program using the fit from the top figure. The rms error in each direction is 0.4 mm.
would decrease the overall b-dot signal strength. Also, the position of the calibration return current path was seen to make a difference of \(~10\%\) in b-dot signal strengths. This was accounted for in a crude manner (by subtracting offsets from the b-dot signals closest to the return current path).

Finally, when the actual beam is corkscrewing about the axis, in which case Eq. 6.3 is no longer accurate, the b-dot signals are expected to be proportional to the axially directed current—not the total current. This will be easily tested by measuring the effect of the wiggler on the b-dot signals. Even so, the b-dots will be used primarily to measure current and beam position when the wiggler is off, the desire being to have a perfectly centered beam in this situation.

### 6.3.2 Diamagnetic Loop

The diamagnetic loop is based on the exact same physics as the b-dot loops, except that it is positioned in the \(x-y\) plane, so instead of measuring axially directed current, it measures azimuthal current, or beam pitch. This can be seen if the beam is modeled as an infinitesimally thin winding shaped in a helix with radius \(r_L\). The diamagnetic loop is placed perpendicular to the axis of the helix, and the helix passes through the center of the loop, as shown in Fig. 6-8. Assuming the helix is infinitely long and wound tightly enough, the magnetic field on the inside of the helix will be

\[
B_x = \mu_0 n I = \mu_0 \frac{I}{\lambda_c},
\]

Figure 6-8: Schematic for discussion of diamagnetic loop theory. A pencil-thin electron beam with Larmor radius \(r_L\) and axial period \(\lambda_c\) passes through the center of the diamagnetic loop.
where \( n \) is the number of windings per unit length and \( \lambda_c \) is the axial period of the helical winding. Outside the helix, the field is assumed to be negligibly small. The flux through the diamagnetic loop is then \( B_x A \), where \( A = \pi r_L^2 \). Substituting this into Faraday's law,

\[
\int V(t)dt = \frac{\mu_0 I}{\lambda_c} \pi r_L^2
\]  

(6.5)

The beam pitch parameter, \( \alpha \), is related to \( \lambda_c \) and \( r_L \) by the formula \( \alpha = 2\pi r_L / \lambda_c \). Substituting this into Eq. 6.5 yields the pitch proportionality constant:

\[
\int V(t)dt = \frac{\mu_0 I}{2} \alpha r_L.
\]  

(6.6)

To verify that the diamagnetic loop signal is proportional to beam pitch, four different wire helices were constructed at four different pitches. Each helix was then placed through the diamagnetic loop (centered about the axis) and driven with a 50 ns, 1 A current pulse. The diamagnetic loop signal was stored, integrated in time, and the amplitude of the resultant pulse was measured. That amplitude is plotted against the pitch of the helix in Fig. 6-9. The measured pitch proportionality constant, factoring in the value \( r_L = 0.74 \text{ cm} \), is \( 0.15 \mu_0 / 2 \), or nearly six times smaller than the

![Graph](image)

Figure 6-9: The solid squares represent the amplitude of the time-integrated diamagnetic loop pulse measured for four different helical pitches. The slope of the fitted line is 0.69 V·ns/A. The radius of the test helix in each case is 0.74 cm.
simple theory predicts. This is likely attributable to the boundary conditions around the loop. The surrounding metal flanges generate opposing currents that reduce the proportionality constant.

To account for boundary conditions, an electrodynamic formalism was developed to describe the response of the diamagnetic loop inside a waveguide of radius \( r_w \) by L. Lin of MIT[61]. The theory assumes a centered beam and derives the loop response as a sum of \( \text{TE}_{op} \) modes. If the frequency response of the diamagnetic loop signal is much lower than the cut-off frequency of the \( \text{TE}_{01} \) mode in the waveguide (a safe assumption in our case), then the inductance of the loop can be written as

\[
\int V(t)\,dt = \kappa \frac{\mu_0 I}{2} r_L,
\]

where

\[
\kappa \equiv 4 \frac{r_{\text{loop}}}{r_L} \sum_{p=1}^{\infty} \frac{J_0'(\nu_{0p} r_{\text{loop}}/r_w)}{\nu_{0p} J_0' (\nu_{0p})} \frac{J_0'(\nu_{0p} r_L/r_w)}{J_0' (\nu_{0p})}.
\]

In Eq. 6.8, \( r_{\text{loop}} \) is the diamagnetic loop radius, \( J_0(x) \) is the zeroth order Bessel function, \( J_0'(x) = dJ_0(x)/dx \), and \( \nu_{0p} \) is the \( p^{th} \) non-zero root of \( J_0'(x) \). Unless \( r_L/r_{\text{loop}} \) is very close to unity, \( \kappa \) is relatively constant over \( r_L \) and depends most strongly on \( r_w/r_{\text{loop}} \), i.e. how close the diamagnetic loop is to the surrounding waveguide wall. As \( r_w/r_{\text{loop}} \to 1 \), \( \kappa \to 0 \). As \( r_w/r_{\text{loop}} \to \infty \), \( \kappa \to 1 \), and the magnetostatic, free-space result of Eq. 6.6 is recovered. A plot of \( \kappa \) versus \( r_w/r_{\text{loop}} \) is shown in Fig. 6-10. The calibrated value of \( \kappa \) from Fig. 6-9 is marked on Fig. 6-10, and it implies \( r_w/r_{\text{loop}} = 1.08 \). In actuality, the boundary around the diamagnetic loop is complex. The metal boundary of the flange to the right of the diamagnetic loop in Fig. 6-3 is certainly quite close to the loop and likely explains the calibrated measurement.

Unfortunately, simple theory based on the Biot-Savart law predicts that the diamagnetic loop signal is quite sensitive to a number of parameters: the phase of the helix passing through the loop (for low pitch values), the position of the center of the helix in \( r \) and \( \phi \), and the exact distribution of current that would be in an actual beam. This dependence on such a large number of parameters results in the diamagnetic loop being difficult to calibrate for a realistic electron beam. Thus the diamagnetic loop is used mostly as a qualitative diagnostic. The amplitude of the diamagnetic loop signal for any one beam pulse will be approximately proportional to beam pitch, so the pitch profile of the beam can be viewed in real time to determine when wiggler resonance is reached, but there are too many unknowns for the diamagnetic loop to predict the exact value of the beam pitch with useful precision.

### 6.4 RF Input

Because the design is simple and inexpensive, the initial rf input coupler for the gyro-twt experiments will consist of WR62 waveguide aimed radially into a 2.54 cm inner diameter circular tube with the plane of the narrow WR62 walls parallel to the beam axis. Inside the beam tube and directly beneath the rectangular aperture corresponding to the WR62 waveguide cross-section, a very thin copper wire mesh will be placed at 45° so that the rf electric field, which comes in parallel to the beam axis from the \( \text{TE}_{10} \) mode in the WR62 waveguide, reflects off of the mesh and travels vertically, launching primarily TE waves in the circular waveguide. A scale drawing of the coupler has already been presented in Fig. 6-3. The WR62 waveguide propagates frequencies
Figure 6-10: This plot of $\kappa$ (Eq. 6.8) versus $r_w/r_{\text{loop}}$ shows how the diamagnetic loop signal becomes less sensitive to beam pitch as the diamagnetic loop radius, $r_{\text{loop}}$, approaches the waveguide wall radius, $r_w$. In the other limit, $r_w/r_{\text{loop}} \to \infty$, $\kappa \to 1$, and the magnetostatic free-space response of the loop (Eq. 6.6) is recovered. The calibrated value of $\kappa$ (from Fig. 6-9) is shown by the dashed line.
from 9.52 GHz to 19.04 GHz in the fundamental mode (TE_{10}) only. The 2.54 cm diameter circular
guide will be overmoded. At 17 GHz, six modes can propagate: TE_{11} (f_c = 6.92 GHz), TE_{21}
(f_c = 11.47 GHz), TE_{01} (f_c = 14.4 GHz), TE_{31} (f_c = 15.78 GHz), TM_{01} (f_c = 9.03 GHz), and
TM_{11} (f_c = 14.4 GHz). The hope is that a combination of all of these modes will be launched due
to the quasi-optical nature of the coupler. The CRM interaction will then selectively amplify the
resonant mode (TE_{11}, TE_{21}, or TE_{31}). Because simple ray optics predict that the \( E \)-field will be
strongest in the center of the guide, TE_{11} is expected to be the dominant launched mode.

The interaction between the electron beam and the wire mesh is of limited concern. The mesh
wires will be extremely thin, and the beam is expected to vaporize the center of the mesh after
several shots. Though this will gradually change the rf coupling properties of the input coupler, it
is expected that enough input power will be provided that amplification will still start for whatever
mode is desired. The rf power will be provided in the form of a \( \sim 0.5 \, \mu s \) rf pulse from a magnetron
built by Varian Associates. The magnetron requires a 30 kV pulse which will be generated by
a power supply and pulse-forming network on loan from North Star Research. The magnetron
generates up to 50 kW of rf power and is tunable from 15.9 GHz to 17.3 GHz. It can pulse at
repetition rates up to 2 Hz. A sample magnetron rf pulse is shown in Fig. 6-11 at 24 kW power and
17.1 GHz. The signal was measured from a \(-66 \, \text{dB} \) broadwall coupler into a calibrated low-barrier
Schottky rf-detection diode. The magnetron was chosen primarily for its ability to generate high

![Detector signal (mV)](image)

**Figure 6-11:** An example rf pulse from the Varian magnetron at 17.1 GHz.

power rf since the input coupler is expected to inefficiently couple and to divide the power between
several modes. For an accelerator application, a master oscillator would drive several preamplifiers
(e.g. conventional TWT's) that would then deliver power in the hundreds-of-watts or kilowatt range to the gyro-twt.

6.5 Window

The interaction tube for the gyro-twt experiments, where the 17 GHz rf signal will amplify by several orders of magnitude, is ironically the simplest part of the experiment to manufacture. It is a straight copper pipe with a 2.54 cm inner diameter for the TE_{31} gyro-twt experiment and TE_{11} CARM experiment, or a 1.905 cm inner diameter for the TE_{21} experiment. For the 1.905 cm diameter tube, a gentle taper on each end will increase the inner diameter from 1.905 cm to 2.54 cm over a 10 cm length so that the TE_{21} tube can mate to the same parts (coupler on one end, window on the other) as the TE_{31}/TE_{11} tube. While each interaction pipe will be almost 2 m long, the actual interaction length will be determined by the number of X-coils placed around the interaction tube. Up to eight X-coils (each one 22 cm long axially) are available, but the saturation lengths predicted for the gyro-twt designs are on the order of 1 m maximum, so only five or six X-coils are expected to be required to reach saturation. The only length requirement for the interaction tube is that it be longer than the maximum predicted saturation length.

At the end of the interaction tube, a 2.54 cm to 5.08 cm uptaper will bring the high power rf to a 5.08 cm diameter alumina ceramic (99.5% Al_{2}O_{3}) window. A scale drawing of the uptaper and window is shown in Fig. 6-12. The uptaper reduces the power density in the waveguide, improving the match to free space and reducing the likelihood of breakdown. The larger diameter of the window also reduces the angular spread in the rf far-field pattern, making it easier to measure. The permittivity of the window is 9.6\varepsilon_{0}, and the permeability is the same as for free space, \mu_{0}. The most important property for the window is that it have a very low reflectivity at the operating frequency. In general, broad-bandwidth low reflectivity is desired. Other considerations in choosing the window

![Figure 6-12: A 1:2 scale drawing of the uptaper and the window at the end of the interaction region. The uptaper and interaction tube are made of OFHC copper. The window is made of alumina ceramic and has an index of refraction of 3.1 (\varepsilon = 9.6\varepsilon_{0}, \mu = \mu_{0}). Alumina ceramic is white and opaque.](image-url)
material are mechanical stresses due to possible electrical breakdown and vacuum pressure. The index of refraction of Al₂O₃ is \( n = 3.1 \), which is too high for the window to have broadband low reflectivity, but alumina ceramic is exceptionally strong and unlikely to be damaged by the kinds of powers that may be generated by the gyro-twt experiments.

Calculating the exact power reflected by a boundary layer inside of a waveguide is a complicated problem. Detailed theory must take into account that the signal may experience some mode conversion in reflection. A simple estimate for the window reflectivity is easily calculated, however, by substituting the in-guide value for \( k_z \) into the equations for the reflectivity of a TE plane-wave off of a layered medium. Kong presents a detailed solution for this problem[49]. The complex electric field amplitude reflectivity is

\[
R = \frac{R_{01} + R_{12}e^{i2k_{z1}\Delta z}}{1 + R_{01}R_{12}e^{i2k_{z1}\Delta z}}e^{i2k_0z_0},
\]

where \( R_{01} \) is the reflectivity between region 0 (the interaction tube) and region 1 (the alumina ceramic disk), \( R_{12} \) is the reflectivity between region 1 and 2 (beyond the window), \( k_{z1} \) is \( k_z \) in region 1, \( \Delta z \) is the width of region 1 (the window), \( k_0 \) is \( k_z \) in regions 0 and 2, and \( z_0 \) marks the boundary between regions 0 and 1. The boundary reflectivity between regions for a TE plane-wave is

\[
R_{n(n+1)} = \frac{1 - \mu_n k_{(n+1)z}/(\mu_{(n+1)}k_{nz})}{1 + \mu_n k_{(n+1)z}/(\mu_{(n+1)}k_{nz})},
\]

where \( R_{n(n+1)} \) is the electric field amplitude reflectivity of a TE plane-wave traveling from region \( n \) to region \( (n + 1) \), \( \mu_n \) is the permeability of region \( n \), and \( k_{nz} \) is the axial wavenumber in region \( n \). Inside a circular waveguide, for a TE mode, \( k_{nz} = \sqrt{\nu^2 \epsilon_n - \mu_n - \nu^2}/r_w^2 \), where \( \epsilon_n \) is the permittivity of region \( n \), \( r_w \) is the waveguide radius, and \( \nu_{mp} \) is the \( p^{th} \) non-zero root of \( J'_n(\nu) \). This model does not take into account the reflectivity between the end of the waveguide and free space, but for the gyro-twt window parameters of \( D/\lambda_0 = 5.08 \text{ cm}/1.75 \text{ cm} = 2.9 \), the effect of the transmission to free space has been shown to change the overall reflectivity by only 2%[44].

Using Eqs. 6.9 and 6.10, the total power reflectivity of the designed window is plotted in Fig. 6-13 for the three TE modes of interest. The predicted power reflectivity for the gyro-twt window at 17.136 GHz is 6%, 3%, and 0.3% for the TE₁₁, TE₂₁, and TE₃₁ modes, respectively. A cold test measurement was made using a network analyzer and a rectangular to circular transition piece to launch a linearly polarized TE₁₁ wave into the uptaper. The reflected power was directly measured. This measurement is shown by the filled squares in Fig. 6-13.

To calculate the maximum field gradient in the window, the equations from Section 2.2 are used. Substituting Eq. 2.6 into Eq. 2.12 yields

\[
P = \frac{1}{2\eta \beta_\phi k_1^2} \tilde{E}\tilde{E}^*.
\]

The TE₁₁ mode, because it is the lowest order mode, will have the highest peak field value of all modes at a given power, so it is sufficient to evaluate the peak field of the TE₁₁ mode. The TE₁₁ peak electric field occurs exactly on the axis \( (r = 0) \), where, for the TE₁₁ mode, \( |\tilde{E}_r| = 0.314|\tilde{E}| \) (from Eq. 2.1). Using this relation along with Eq. 6.11, the maximum field inside the waveguide

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Figure 6-13: Power reflectivity of the gyro-twt window for the three TE modes of interest. The curves show the prediction of Eq. 6.9. The solid squares show low power cold test measurements of the $\text{TE}_{11}$ reflectivity.
for a TE_{11} mode is

\[ E_{\text{MAX}} = 0.815 \sqrt[4]{\frac{\beta \eta P}{r_w}}, \]  

(6.12)

where \( \eta = \sqrt{\frac{\mu}{\varepsilon}} \) is the impedance of the material in the waveguide. For the alumina ceramic window, the peak field at \( P = 20 \text{ MW} \) of power is 15.8 kV/cm. This is well within the breakdown strength of 99.5\% Al_{2}O_{3}, which is \( \sim 300 \text{ kV/cm} \). Note that the maximum field scales as the inverse root of the permittivity of the material, so that alumina ceramic, with its high permittivity, is less susceptible to breakdown.

### 6.6 Summary

The gyro-twt and CARM experimental design is complete. The entire experimental schematic is shown in Fig. 6-14. Results from the three designed experiments are reported in Chapter 7.
Figure 6-14: A 1:15 scale drawing of the entire gyro-twt experiment.
Chapter 7

Experimental Results

7.1 Introduction

Installation of the gyro-twt experiment began in April, 1991, and all measurements were completed by April, 1994. An experimental time-line is presented in Table 7.1. This chapter presents all significant experimental measurements. These measurements will be compared to theoretical predictions, and a discussion of the experimental results takes place in Chapter 8. The data presented includes measurement of amplified rf gain and power, super-radiant rf power, far-field patterns of the amplified modes, gain and phase versus \( z \) in the interaction region, frequency purity, phase stability, and output power versus input power.

7.2 Initial Measurements

The final assembly of the experiment involved mounting the entire magnet system on sliding rails so that various parts of the experimental circuit could be accessed easily, in particular the rf input coupler area. The cathode and magnet system were aligned geometrically using a helium-neon laser, and the vacuum tube was centered inside the magnet coils with careful measurement. The geometric alignment method is considered accurate to \( \pm 1 \) mm.

All experimental diagnostic signals were measured using the same LeCroy oscilloscope mentioned in Sec. 6.3.1. This oscilloscope originally had two channels, each capable of storing signals at 1 GS/s. In March, 1992, a 500 MHz plug-in with two more 1 GS/s channels was added to the LeCroy 7200 for a total of four channels. After the addition of this plug-in, for example, all four b-dot loop signals from the same linac voltage pulse could be (and usually were) stored simultaneously.

7.2.1 SNOMAD-II Voltage Measurements

The voltage pulse of SNOMAD-II is measured from a capacitive probe (C-probe) next to the bus bar. A schematic of the capacitive probe is shown in Fig. 7-1. The feedthrough is in series with a 10 k\( \Omega \) resistor and then insulated from the high voltage bus bar by an oil-filled gap. The bus bar supplies the high voltage to each of the four linac accelerating gaps. The insulation around the edge
<table>
<thead>
<tr>
<th>Date</th>
<th>Activity</th>
<th>Tube I.D. (cm)</th>
<th>Cathode</th>
<th>Cathode Gap, $D$ (mm)</th>
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<tr>
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<td>First run</td>
<td>–</td>
<td>A</td>
<td>~31</td>
</tr>
<tr>
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<td>–</td>
<td>A</td>
<td>~31</td>
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<td>B</td>
<td>~31</td>
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<tr>
<td>Jan 1992–Feb 1992</td>
<td>RF measurements</td>
<td>2.54</td>
<td>B</td>
<td>~31</td>
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<tr>
<td>Mar 5, 1992</td>
<td>Refurbished cathode installation</td>
<td>2.54</td>
<td>A&lt;sub&gt;R&lt;/sub&gt;</td>
<td>29.64</td>
</tr>
<tr>
<td>Mar 1992</td>
<td>RF measurements</td>
<td>2.54</td>
<td>A&lt;sub&gt;R&lt;/sub&gt;</td>
<td>29.64</td>
</tr>
<tr>
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<td>29.64</td>
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<tr>
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<td>A&lt;sub&gt;R&lt;/sub&gt;</td>
<td>36.32</td>
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<td>A&lt;sub&gt;R&lt;/sub&gt;</td>
<td>36.32</td>
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<td>A&lt;sub&gt;R&lt;/sub&gt;</td>
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<td>36.32</td>
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<tr>
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<td>~40</td>
</tr>
<tr>
<td>Jan 13, 1994</td>
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<td>C</td>
<td>36.45</td>
</tr>
<tr>
<td>Jan 1994–Feb 1994</td>
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Table 7.1: Gyro-twt experimental time table. Results before April 1993 are not detailed in this thesis, as they were not significant compared to results after April 1993. Each of three different Spectramat cathodes (all identical) is designated by the letters A, B, and C, with an R subscript denoting that the cathode has been refurbished after having been poisoned. The refurbishing, done by Spectramat, involves scraping off the top ~0.25 mm of the cathode surface and recoating it. “Tube I.D.” refers to the interaction waveguide inner diameter. Refer to Fig. 4-14 for the meaning of the cathode-anode gap, $D$.  

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Figure 7-1: Capacitive probe schematic for SNOMAD-II, not to scale. An external feedthrough is capacitively coupled to the high voltage on the bus bar through an oil filled gap. The bus bar transports the high voltage to the accelerating gaps. The feedthrough is also capacitively coupled to ground (the outer casing) through the insulation.

of the capacitive probe couples it to the outer casing, which is at ground potential. The capacitance coupling the C-probe to the bus bar is denoted as $C_g$. The capacitance coupling the C-probe to ground is denoted as $C_p$. An equivalent circuit for the C-probe is shown in Fig. 7-2, where $V_{\text{GAP}}$ is the linac gap voltage and $V_{\text{CP}}$ is the C-probe signal. The formula relating $V_{\text{GAP}}(t)$ to $V_{\text{CP}}(t)$ is

$$ \frac{dV_{\text{GAP}}(t)}{dt} = \frac{R_f + R_t}{R_t} \frac{1}{C_g} \left[ \frac{V_{\text{CP}}(t)}{R_f + R_t} + \frac{(C_p + C_g)}{C_g} \frac{dV_{\text{CP}}}{dt} \right], \quad (7.1) $$

which, after integrating in time, can be expressed as

$$ V_{\text{GAP}}(t) = A \int V_{\text{CP}}(t')dt' + BV_{\text{CP}}(t). \quad (7.2) $$

In Eq. 7.2, $A$ and $B$ are constants that depend on the circuit components in Fig. 7-2. On November 26, 1991, the values of $A$ and $B$ were determined by using a high voltage probe to directly measure the linac voltage at voltages up to 30 kV. In this case, $V_{\text{GAP}}$ is actually replaced by the sum of the four gaps, but Eq. 7.2 still applies. The values of $A$ and $B$ simply multiply by four. Three different voltage pulses and their corresponding C-probe signals were measured and fitted to Eq. 7.2. The best fit results of the calibration were $A = 2.6 \times 10^{11}$ rad/s and $B = 2.5 \times 10^5$. Throughout this thesis, the C-probe signal measured from the linac is converted to a cathode voltage by use of Eq. 7.2 with the values $A = 2.6 \times 10^{11}$ rad/s and $B = 2.5 \times 10^5$. For typical voltage pulses ($> 300$ kV), the result is a cathode voltage-to-C-probe voltage ratio of $\sim 260$ kV/V.
7.2.2 SNOMAD-II Beam Current Measurements

In October 1991 and again in April 1993, the rf input coupler and interaction tube were replaced by a short tube with a ceramic break in the middle. At the end of the beam tube, the beam is collected by a blank-off flange with a copper collecting plate mounted on the vacuum side of the flange. The ceramic break is surrounded by a set of current viewing resistors (a CVR) which electrically connects the opposite sides of the tube around the ceramic break. As the beam strikes the copper collecting plate at the end of the tube, the return current is forced to travel through the CVR. The voltage across the CVR is measured, and the beam current is determined by Ohm’s Law. The resistance of the CVR used for this experiment is 0.373 \( \Omega \). Several beam measurements were made with the CVR in place. The perveance of the cathode was measured, current versus cathode temperature was measured, and the b-dot loop signals (Sec. 6.3) were calibrated.

The current delivered on any given day from SNOMAD-II is dependent on several critical factors: the cathode-anode gap, the condition of the cathode in use, the vacuum pressure, the cathode temperature, and, most importantly, the voltage applied to the cathode. During the time the gyro-twt experiments were in progress, all of these factors changed over time. Some on a shot-to-shot basis, others on a daily basis, and still others on more of a monthly basis. For these reasons, the current measured from the cathode varied substantially for different rf measurements made on different dates. The plotted curves of current versus voltage and temperature shown in Fig. 4-15 were measured on April 8, 1993, and the vacuum pressure that day was quite good at \( 9 \times 10^{-8} \) Torr. More typically, the initial pressure (before pulsing SNOMAD-II) was \( \sim 1-2 \times 10^{-7} \) Torr. With a repetition rate of 1 Hz, the vacuum pressure of SNOMAD-II slowly increases during experimental operation as the beam strikes the far end of the interaction tube and causes out-gassing. If the initial pressure is not good enough, the beam current steadily drops during the run due to pressure build-up.
On-line B-dot Calibration

Two sets of data were taken to calibrate the b-dot loop signal strengths. These calibrations were done with the CVR in place. For each linac pulse, the four b-dot signals were digitally captured, integrated in time, and combined into a weighted statistical average using the calibration curve from Fig. 6-7 (top). A typical time-integrated b-dot signal resulting from the linac current pulse is shown in Fig. 7-3 with the CVR signal and linac voltage pulse overlaid on top of the b-dot signal. The figure shows how well the shape of the b-dot signal matches the CVR signal, even for the second short spike of current resulting from voltage "ringing" after the initial pulse.

![B-dot signal (time-integrated) with CVR signal and voltage pulse overlaid.](image)

**Figure 7-3**: B-dot signal (time-integrated) with CVR signal and voltage pulse overlaid. The b-dot signal and the CVR signal are both proportional to beam current. Calibration of the signal strengths is shown in Fig. 7-4.

The average signal strength (in V·ns) of the b-dot loops for linac pulses of several different voltages is plotted against the beam current measured from the CVR in Fig. 7-4. Two sets of measurements were taken, one in October 1991, and one in February and April of 1993. The distinction between the two sets of measurements is made because the slope of the line of best fit differs by 20% from one case to the other. This discrepancy is not entirely understood. The most plausible explanation is that for the 1991 measurements, we now suspect that the beam was poorly matched and had significant scalloping and rotation. The bad match was primarily due to the small cathode-anode gap (see Table 7.1). The matching coil (Sec. 6.2.1) also was not installed for the
Figure 7-4: Calibration of the b-dot loop using a 0.37 Ω CVR. The current value (horizontal axis) is derived from the CVR signal. The b-dot signal value is an average of the signals from the four loops, each loop signal first being time-integrated.

\[
S = \frac{I}{2.0} \text{ A/(V-ns)}
\]

\[
S = \frac{I}{2.4} \text{ A/(V-ns)}
\]
1991 measurements. The 1991 b-dot signals (before time-integration) show significant noise that may attest to the beam scalloping enough so that electrons struck the b-dot loops. The noise may also result from microwaves generated by significant beam scalloping. Compare the b-dot signals in Fig. 7-5, both typical of the 1991 and 1993 calibrations. Rotation on the beam and electrons striking the loops would be expected to reduce the b-dot signals relative to the CVR signal. Because of the cleaner signals in the 1993 calibration, the calibration of $2.0 \text{ A/}(\text{V} \cdot \text{ns})$ is used to determine the beam current in Sec. 7.3.

**Wiggler Resonance Measurements**

Wiggler resonance was first seen in October 1991 by examining the CVR signal. With the wiggler transverse field set to $\sim 50 \text{ G}$, the wiggler guide field was tuned through wiggler resonance. The wiggler guide field value at resonance was determined to be the value at which the CVR signal showed the most significant current loss. Fig. 7-6 shows two beam current traces—one with the wiggler turned off, and the other with the wiggler on and at wiggler resonance. The guide field at which wiggler resonance occurred was measured at several different pulse voltages, and the results are plotted in Fig. 7-7 along with the theoretical resonance curve predicted by single particle theory.
Figure 7-6: Beam current traces from the CVR diagnostic in October 1991. The solid trace is a normal beam current pulse with the wiggler off. The dotted trace shows a current pulse with the wiggler on and at resonance. The beam spin-up is large enough that some of the beam is intercepted by the vacuum tube walls, and this reduces the detected beam current during the resonant part of the pulse.
Figure 7-7: Theoretical and measured wiggler resonance points from October, 1991. The solid curve shows the theoretical peak in beam pitch, \( \alpha \). This same line corresponds to peaks in the Larmor radius of the beam, \( r_L \). The theory was calculated by WIGGLE32. The measured values (filled squares) were obtained by tuning the guide field at a fixed voltage in each case until the CVR signal showed maximum current loss (Fig. 7-6). The measured voltage values are adjusted by \(-5\%\) from the capacitive probe reading to account for beam voltage depression (see Sec. 4.6.3).
from the WIGGLE32 code (Sec. 5.3). The resonance curve shows where $\alpha$ peaks in the beam voltage versus guide field coordinate space. The predicted voltage depression for the beam based on the run parameters from Fig. 7-7 is $\sim 5\%$ (see Sec. 4.6.3). This has been accounted for in Fig. 7-7.

More recent wiggler resonance measurements are based on using the shape of the diamagnetic loop pulse to qualitatively determine when wiggler resonance occurs. These measurements are from February, 1994. Because the diamagnetic loop signal scales approximately like $\alpha$ (Sec. 6.3.2), the shape of the time-integrated signal (which can be viewed real-time on the LeCroy 7200) shows the characteristic double-hump, then a flattening, and then a rounding-off, exactly as one would expect when tuning the wiggler guide field through wiggler resonance. These three traces, along with a baseline trace, are shown in Fig. 7-8. The double-humped trace shows a peak in $\alpha$ during the rise and the fall of the voltage pulse, thus the resonant voltage is lower than the peak pulse voltage. The flat trace shows the condition where the resonant voltage exactly equals the peak pulse voltage. The rounded trace shows the condition where the resonant voltage is higher than the peak pulse voltage.

Using the amplitude of the diamagnetic loop trace to indicate local maxima and minima in beam pitch, the wiggler guide field was tuned at a fixed beam voltage to find minima and maxima in the diamagnetic loop pulse. The results, at two different settings for the wiggler transverse field (14 G and 41 G), are shown in Fig. 7-9. The beam pitch maxima and minima were measured at an
Figure 7-9: Theoretical and measured minima and maxima in beam pitch plotted against beam energy and wiggler axial guide field. The measurements are from February, 1994. The solid lines show theoretical local maxima in $\alpha$. The dashed lines show theoretical local minima in $\alpha$. Filled-in circles mark measured maxima. Open triangles mark measured minima. The top figure is for $B_w = 14$ G, the bottom figure for $B_w = 41$ G. The theory lines were calculated by WIGGLE32, with the inset in the bottom figure showing the actual profile of beam pitch, $\alpha$, versus wiggler guide field for $V = 325$ kV. The guide field for the inset ranges from 500 to 2000 G on the horizontal axis, and $\alpha$ ranges from 0 to 1 on the vertical axis.
anode-cathode voltage of 360 kV, which, after a 10% reduction for voltage depression, corresponds
to a beam energy of 325 kV. The voltage depression is higher in this case than in the October 1991
case because the beam current is significantly larger. The plots in Fig. 7-9 are done over a range of
voltages to show how insensitive the maxima and minima positions are to beam voltage. The inset
on the bottom figure shows the actual $\alpha$ profile versus wiggler guide field at 325 kV. The measured
points are marked on the inset as well as on the top and bottom graphs, with open triangles marking
measured minima and filled circles marking measured maxima. Most of the measured points agree
with the theory, but some extraneous diamagnetic pulse minima and maxima were measured for
the low wiggler setting (14 G) near the theoretical local minima in beam pitch. These points are
not fully understood. They may be attributable to the equilibrium rotation already present on the
beam before it enters the wiggler. This may also explain the slight shifts in the measured minima
for the 41 G case. Or perhaps the non-ideal adiabatic transition region (Fig. 6-4) played some
role in the extra minima and maxima. For the theory, the particles were given an initial $\alpha$ of 0.05
before entering the wiggler and adiabatically compressed to an interaction field of 2200 G. The
initial pitch and adiabatic compression shifted the theoretical resonances only slightly, by 1–2% in
wiggler guide field.

Despite some unexplained discrepancies, the voltage calibration of the linac and the theory
predicted by WIGGLE32 are largely supported by the wiggler resonance measurements. With the
beam voltage and current measurement from the linac calibrated and the correct operation of the
wiggler verified, the rf input coupler was the next device to be tested.

7.2.3 Input Coupler Measurements

The performance of the rf input coupler described in Section 6.4 was measured primarily by
analysis of the far-field radiation pattern emitted from the gyro-twt window. In the absence of an
electron beam, the magnetron described in Section 6.4 was pulsed at 1 Hz. The magnetron signal,
propagating at 17.1 GHz in WR62 waveguide, is single-moded in the TE$_{10}$ mode. The signal
strength was measured through a nominal $-60$ dB broadwall coupler with a calibrated coupling of
$-59.0$ dB at 17.1 GHz. The signal then was directed through an isolator and into the gyro-twt input
coupler as in Fig. 6-3. The mix of modes propagating down the gyro-twt interaction tube then exits
through the rf window (Sec. 6.5). Just beyond the window, a short piece of circular waveguide
extends the gyro-twt tube so that the exiting waves are not disturbed by the bolts extending from
the gyro-twt window retainer flange. The microwaves exit from the waveguide extension into free
space where a detecting horn, attenuator, and diode measure the signal in the far field region. The
horn is constrained to pivot about the radiating aperture in the horizontal plane, moved by a 2D
scanning table. The entire setup is shown in Fig. 7-10.

**Far Field Radiation**

The free space wave equation for the electric field vector at frequency $\omega$, as derived from Maxwell's
equations, is

$$\nabla^2 E + k_f^2 E = 0,$$

(7.3)
Figure 7-10: A 10:1 scale schematic (side-view and overhead view) of the far field measurement system. The detector horn is moved in the horizontal (z-z) plane by a 2D scanning table which is computer controlled. The movement is accurate to < 1 mm. Anechoic material is placed around the aperture, the detector horn, and over the scanning table in order to minimize reflections. The coordinate system origin is at the gyro-twt radiating aperture, with $r = \sqrt{x^2 + y^2 + z^2}$, $\phi = \tan^{-1}(y/x)$, and $\theta = \cos^{-1}(z/r)$. 
where \( k_f \equiv \omega \sqrt{\varepsilon_0 \mu_0} \) is the free-space wave number. The Stratton-Chu solution of this equation applied to waves exiting a waveguide aperture is

\[
E = \frac{1}{4\pi} \int_S \left[ (\hat{n} \cdot E') \nabla' \psi + (\hat{n} \times E' \times \nabla' \psi) - i\omega \psi (\hat{n} \times B') \right] dS,
\]

(7.4)

where the integral is over the cross-sectional surface of the aperture, \( S \), the primed coordinate frame is on the aperture surface, the unprimed coordinate frame is at the observation point, \( \hat{n} \) is the unit vector normal to area element \( dS \), and \( \psi \) is the point source Green’s function, defined by

\[
\psi \equiv e^{-i k_f |r - r'|}.
\]

(7.5)

It is straightforward to evaluate the Stratton-Chu formula numerically to predict the radiation pattern from a mix of waveguide modes. The rf field amplitude can be evaluated at any point in space. The only approximation made in the Stratton-Chu formula is that the fields at the waveguide aperture are unperturbed by the aperture itself. Such an approximation is commonly made in predicting antenna radiation patterns. In the far field limit, the Green’s function can be approximated by

\[
\psi \approx e^{-i k_f (r - \hat{r} \cdot r')/r},
\]

(7.6)

and the gradient operator, \( \nabla \), can be approximated by \( i k_f \hat{r} \). The far field limit is the limit in which the distances from all points on the aperture to the observation point vary by less than \( 0.1 \lambda \), where \( \lambda \) is the free-space wavelength. Written in terms of \( D \), the diameter of the aperture, this condition is approximately

\[
r > \frac{2D^2}{\lambda}.
\]

(7.7)

For the gyro-twt rf window diameter of 5.08 cm and a free space wavelength of 1.75 cm for 17.136 GHz, this condition becomes \( r > 29.5 \) cm. To evaluate the Stratton-Chu formula in the far field limit, a coordinate system is first chosen. The \( z \)-axis is defined (as before) as the axis of the gyro-twt interaction tube, \( z = 0 \) being the aperture (rf window) position, and the \( x-z \) plane is defined as the horizontal plane. The variables \( r, \phi, \) and \( \theta \) represent the normal spherical coordinates, \( \theta = \cos^{-1}(z/r) \) being the polar angle, \( \phi = \tan^{-1}(y/x) \) being the azimuthal angle, and \( r = \sqrt{x^2 + y^2 + z^2} \) being the distance from the origin (see Fig. 7-10). Making use of this coordinate system, the far field limit solution of the Stratton-Chu equation is

\[
E_\theta = \frac{i k_f e^{-i k_f r}}{4\pi r} \left[ (f_x \cos \phi + f_y \sin \phi) + \eta_0 \cos \theta (g_y \cos \phi - g_z \sin \phi) \right],
\]

\[
E_\phi = \frac{i k_f e^{-i k_f r}}{4\pi r} \left[ \cos \theta (f_x \cos \phi - f_z \sin \phi) - \eta_0 (g_y \sin \phi + g_z \cos \phi) \right],
\]

(7.8)

where \( \eta_0 = \sqrt{\mu_0 / \varepsilon_0} \) and

\[
f(k_x, k_y) = \int_S E' e^{i(k_x x + k_y y)} dS
\]
\[ g(k_x, k_y) = \int_S H^i e^{i(k_x x + k_y y)} dS. \]  
(7.9)

In Eq. 7.9, \( k_x = k f \sin \theta \cos \phi \) and \( k_y = k f \sin \theta \sin \phi \). Eq. 7.8 can be independently derived using Schelkunoff's equivalence principle with equivalent magnetic and electric current sources (E-H formulation)[56]. For rotating \( \text{TE}_{mn} \) modes, Eq. 7.8 becomes

\[
E_\theta = (-i)^m \sqrt{\frac{P}{2\eta_0 \beta_\phi}} \frac{C_{mn} \omega \mu_0}{k_z} r e^{-ik_f r} \frac{e^{-im\phi}}{k_f} \left( 1 + \frac{k_z \cos \theta}{k_f} \right) \frac{J_m(\nu_{mn}) J_m(k f r \sin \theta)}{\sin \theta},
\]

\[
E_\phi = (-i)^{m+1} \sqrt{\frac{P}{2\eta_0 \beta_\phi}} \frac{C_{mn} k_z^2 r \omega \mu_0}{k_z} r e^{-im\phi} \frac{J_m(\nu_{mn}) J'_m(k f r \sin \theta)}{k_z - k_f \cos \theta},
\]

(7.10)

where the same notation as in Sec. 2.2 is used for the waveguide modes except that here \( r \) and \( \phi \) are observation point coordinates and not waveguide coordinates, and \( r_w \) is the aperture radius.

The difference between the Stratton-Chu equation (computed numerically from Eq. 7.4) and the far field approximation (Eq. 7.10) is shown in Fig. 7-11 for three different waveguide modes at 17.136 GHz: The \( \text{TE}_{11} \) mode, the \( \text{TE}_{21} \) mode, and the \( \text{TE}_{31} \) mode. In these plots the aperture radius is 2.54 cm, the observation point is at a constant radius of \( r = 70 \text{ cm} \), \( \phi \) is fixed at 0 degrees, and \( \theta \) varies from -40 to +40 degrees, just as would be the case for the gyro-twt detecting horn shown in Fig. 7-10. Except at low power levels, the match between Stratton-Chu and the far field approximation is very good for the parameters for Fig. 7-11, which represent typical measurement parameters for the gyro-twt experiments.

**Input coupler radiation patterns**

Because Eq. 7.10 is appropriate for our measurements, a computer program was written that matches an arbitrary measured far field radiation pattern to the theoretical pattern (from Eq. 7.10) resulting from a mix of a finite number of waveguide modes. The user selects the desired number of modes, and each mode selected for inclusion in the pattern matching is allowed to have an arbitrary phase and an arbitrary amplitude. The program searches through the parameter space of different phases and amplitudes for each mode using Powell’s search algorithm[76] to quickly find a mode mix giving a best fit to the measured data. The program that finds this best fit is called FFMATCH, and it was written specifically for this thesis. The first example of a match from FFMATCH is shown in Fig. 7-12, where the radiation patterns measured from the gyro-twt input coupler are shown for the two different interaction tube sizes used in the experiments. The setup is as was described earlier and as is shown in Fig. 7-10, with the distance from the gyro-twt window to the detector horn being 83 cm. The detecting horn (Waveline model #899) feeds the rf to a remotely controlled attenuator (Millitech MWA42), then to a WR42-to-SMA adapter, and finally to a low-barrier Schottky diode detector (HP #8473B). The attenuator is adjustable from 0 to 60 dB in steps of 0.1 dB. The entire horn-attenuator-diode assembly rests on a stand which can be moved by remote computer control of a 2D scanning table. The horn can be moved to any position within approximately a 4 x 4 square foot area. Physical constraints keep the horn pointed toward the aperture at all times. Anechoic material is placed around the aperture, around the detecting horn, and on the floor of the scanning table to minimize reflections (see Fig. 7-10). Each measured data
Figure 7-11: Exact and far field solutions to the Stratton-Chu equation for the TE$_{11}$ (top), TE$_{21}$ (middle), and TE$_{31}$ (bottom) modes. The solid curves represent the exact solution to the Stratton-Chu equation (Eq. 7.4), and the dashed curves represent the far field approximation (Eq. 7.10). The observation point is 70 cm from the radiating aperture, which is 2.54 cm in radius.
Figure 7-12: Cold test radiation patterns emitted from the gyro-twt rf window with only the input drive signal applied. Patterns are shown for the 2.54 cm I.D. tube (top figure) and the 1.905 cm I.D. tube (bottom figure). The radiation frequency is 17.1 GHz and the distance from the aperture to the detector horn is 81 cm. The measured points are averaged over several pulses, with the filled circles representing $E_\phi$ measurements and the open triangles representing $E_\theta$. The points are fitted to patterns resulting from mode mixes calculated by the FFMATCH program. The best fit patterns are shown as continuous curves for $E_\phi$ and dashed curves for $E_\theta$. FFMATCH also predicts the total rf power based on the mode mix. The best fit phase of each mode was calculated by FFMATCH, but the phases are not listed in the figure.
point in Fig. 7-12 represents an average over several magnetron pulses at 1 Hz repetition rate. The $E_\phi$ and $E_\theta$ data were measured on two separate passes, inserting a waveguide twist between passes so as to rotate the detecting horn by 90°, as shown in Fig. 7-13.

![Diagram](image)

Figure 7-13: For measurement of $E_\phi$, the detector horn is oriented so that the longer sides are parallel to the horizontal (x-z) plane. For $E_\theta$ measurement, the longer sides are perpendicular to the horizontal plane.

The radiation mix in Fig. 7-12 was allowed to contain only the TE$_{11}$, TE$_{21}$, and TE$_{31}$ modes. This resulted in excellent matches. Unfortunately, for radiation from a 5.08 cm I.D. aperture at 17.136 GHz, the other modes that may propagate: TE$_{01}$, TM$_{01}$, and TM$_{11}$, have radiation patterns that are difficult to distinguish from TE$_{21}$, TE$_{21}$, and TE$_{31}$ patterns, respectively. That is, knowing the radiation pattern in the horizontal plane, where it was always measured, cannot give any information about TE$_{31}$ content versus TM$_{11}$ content, for example. In this sense, the assumption to only match to TE modes is a large one for these cold tests, but it should give an approximate upper bound on the content of the TE modes in the mix. If there is TM$_{11}$ mode content in Fig. 7-12, then it will likely reduce the percentage of TE$_{31}$ predicted, since the two patterns are almost the same. The mode-mix percentages in Fig. 7-12 are quite approximate, then, but do give an estimate on the mode mix launched by the gyro-twt input coupler in the two different interaction tubes.

Calculating Radiated Power

The power estimate in Fig. 7-12 is the total cross-sectional power in the waveguide necessary to generate the best-fit mode mix radiation pattern. This estimate is dependent on the mix of modes chosen and is also critically dependent on accurate measurement of the rf intensity by the detecting horn. The calibrated horn gain at 17.1 GHz for the WR42 horn is 11.8 dB. This is converted to an effective area by use of the formula[3, p. 63]

$$A_{em} = \frac{\lambda^2 D_0}{4\pi},$$  \hspace{1cm} (7.11)

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where $A_{em}$ is the effective area of the horn, $D_0$ is the directivity (or gain) of the horn, and $\lambda$ is the wavelength of the radiation in free space. In our case, $D_0 = 10^{11.8/10} = 15.1$ and $A_{em} = 3.68 \text{ cm}$ for a frequency of 17.1 GHz. The detected intensity is then calculated as the power measured by the calibrated diode (adjusting for the attenuator setting) divided by the effective area of the horn. The series of components (shown in Fig. 7-10) that the radiation travels through also affects power measurement. Each component—the rf window, the detecting horn, the attenuator, the WR42-to-SMA adapter, and the Schottky diode—all have a measured voltage standing-wave ratio (SWR) that determines the accuracy of the power measurement. The SWR of a component is related to the complex reflection coefficient, $\Gamma$, by

$$\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|^2}. \quad (7.12)$$

The mismatch between two components determines the measurement error that may occur when connecting them. If the components have reflection coefficients $\Gamma_1$ and $\Gamma_2$, then the mismatch, in dB, is

$$\text{MISMATCH (dB)} = 20 \log_{10}(1 \pm |\Gamma_1 \Gamma_2|). \quad (7.13)$$

If several components are connected in series, all of the mismatch errors must be added together to give the total system mismatch. For the SWR's shown in Fig. 7-10, the total mismatch of the detecting system is $\sim 0.7$ dB. This gives one gauge on the expected error range of the far field scans. Such an error bar is in rough agreement with the quoted error bar on the gyrokystron anechoic chamber measurements done by Lawson[54], though some sources believe that calibrated diode detection used to measure peak power is no more accurate than $\pm 1.5$ dB[15, p. 169]. A better way to measure power is to measure the thermal effects of radiation incident on a calibrated, efficient absorber. Unfortunately, for low average power this is not practical.

The power estimated by the FFMATCH program is adjusted in one critical way. The following calculation is performed numerically on the far field radiation pattern for each different mode:

$$P_R = \frac{1}{2\eta_0} \int_0^{\pi/2} \int_0^{2\pi} \left( |E_\phi|^2 + |E_\theta|^2 \right) r \sin \theta d\phi d\theta, \quad (7.14)$$

where $E_\phi$ and $E_\theta$ are from Eq. 7.10, $P_R$ represents the total radiated power from the mode, and the integral is over the entire forward hemisphere ($z > 0$) of the radiating aperture. For the radiation patterns predicted by Eq. 7.10 for the gyro-twt parameters, the value of $P_R$ from Eq. 7.14 for modes other than the TE$_{11}$ mode is significantly less than the power assumed to be in the waveguide. That is, the radiated patterns to not conserve power for the gyro-twt operating parameters. With 1 W in waveguide for the TE$_{21}$ mode, $P_R = 0.85$ W. With 1 W in waveguide for the TE$_{31}$ mode, $P_R = 0.81$ W. This error is inherent in Eq. 7.10 for the gyro-twt operating parameters. The FFMATCH program forces power to be conserved by artificially increasing the intensity of the far field radiation patterns for the TE$_{21}$ and TE$_{31}$ modes by 1/0.85 and 1/0.81, respectively. In all far field power predictions made by FFMATCH, this adjustment is taken into account. This includes Fig. 7-12, already presented.

To test the accuracy of the power measurements predicted by FFMATCH, a calibration was done feeding the magnetron signal directly into the gyro-twt window as shown in Fig. 7-14. The signal
Figure 7-14: A 5:1 scale schematic (side-view) of the far field power measurement calibration system. The far field detection system is the same as in Fig. 7-10. Here, instead of going through the gyro-twt input coupler, the magnetron signal (after being detected with a −60 dB broadwall WR62 coupler) is injected directly into a rectangular-to-circular converter. This converts the TE$_{11}$ rectangular WR62 signal into a linearly polarized (in $\hat{y}$-direction) TE$_{11}$ circular mode. This mode is then launched into free space as shown. Optionally, for TE$_{31}$ calibration, an elliptic fixed-to-rotating mode converter and a rippled wall TE$_{11}$ to TE$_{31}$ converter are inserted between the rectangular-to-circular converter and the uptaper. Both converters together are > 90% efficient.
was first detected by a \(-60\) dB broadwall coupler in WR62 waveguide, then fed to a rectangular-to-circular (2.54 cm I.D.) converter that launches pure \(TE_{11}\) radiation. The rectangular-to-circular converter then went to the gyro-twt uptaper and window, where the radiation was launched and detected in the far field. The signal at the \(-60\) dB port on the WR62 coupler was detected by a calibrated diode, and this determined the total power launched. The measured radiation pattern, shown in Fig. 7-15, was then matched with FFMATCH (again assuming only TE modes), and a power was predicted by FFMATCH. Because the launched \(TE_{11}\) is linear in the \(\hat{y}\)-direction, only

![Graph](image-url)

Figure 7-15: Calibration of far field power prediction for \(TE_{11}\) mode. A \(TE_{11}\) wave of known power was launched from the gyro-twt window and measured in the far field. The FFMATCH program was then used to predict the total launched power based on the measured radiation pattern. The actual power launched was 2.9 kW, while FFMATCH predicts 3.9 kW. This is a difference of 1.25 dB. As in Fig. 7-12, filled circles and the continuous curve are the data and theory, respectively, for \(E_\phi\), and open triangles and the dashed curve are the data and theory, respectively, for \(E_\theta\).

The \(E_\phi\) pattern is significant. The high ratio of \(E_\phi\) to \(E_\theta\) at the zero angle in Fig. 7-15 demonstrates the quality of the polarization alignment. The FFMATCH program matches this measured pattern very well and predicts a power of 3.9 kW, while the detector at the \(-60\) dB port on the WR62 coupler predicts a power of 2.9 kW. The prediction by FFMATCH is 35% (1.25 dB) higher than the predicted power from the coupling port diode. The same sort of calibration was done for the \(TE_{31}\) mode by adding two special in-guide converters to the calibration assembly (see Fig. 7-14). J. Gonichon[37] designed the two converters. The first converts a purely linear \(TE_{11}\) mode to a purely rotating \(TE_{11}\) mode by using a varying, elliptically shaped waveguide wall. The second
converts a purely rotating TE$_{11}$ mode to a purely rotating TE$_{31}$ mode through the use of sinusoidal, helical waveguide wall ripples. Both converters together are > 90% efficient. The radiation pattern from the combination of converters is shown in Fig. 7-16. In this case, the power prediction by FFMATCH is 50% (1.8 dB) higher than the power prediction from the coupling port diode.

As a last check on the power measurement from the far field matching, radial scans were done at a constant detector (θ) angle. An example radial scan is shown in Fig. 7-17. The deviation from a best fit $1/r^2$ curve is < 1 dB for all measured values. Based on all of these results, and particularly based on the 1.8 dB discrepancy in Fig. 7-16, the error bar for the far field power predictions is determined to be ±2 dB.
Figure 7-17: Radial scan of the gyro-twt in the far field. The detector horn was kept at a fixed angle ($\theta = 20^\circ$), and the distance from the radiating aperture was varied. The results show $< 1$ dB variation from a best fit to a $1/r^2$ curve. In this case, the measured signal is a TE$_{31}$ amplified mode from the gyro-twt. Each data point represents an average over several shots. The points were checked at the end of the scan to ensure reproducibility.
7.3 Gyro-twt Power and Gain Measurements

7.3.1 Introduction

With the 2.54 cm I.D. tube installed, the gyro-twt operated reliably as a third harmonic TE\textsubscript{31} mode amplifier. The first amplification was seen in late March, 1992 (see Table 7.1). The pulse repetition rate of the gyro-twt was limited by the magnetron, which could not pulse faster than 2 Hz without a significant drop in rf output. On most occasions, the gyro-twt was pulsed at 1 Hz. Several exhaustive parameter space searches were done to search for amplified modes of the system, with emphasis primarily focused on the TE\textsubscript{31} amplified mode. Measurements characterizing the TE\textsubscript{31} amplified mode was taken over a two year span on approximately 70 different dates, with an accumulation of literally thousands of stored digital traces. During all of this time, no TE\textsubscript{11} CARM mode amplification was ever verified (from design in Table 3.3). With the 1.905 cm I.D. tube installed, the gyro-twt operated reliably as a second harmonic TE\textsubscript{21} mode amplifier, although it was more prone to spontaneous emission than in the TE\textsubscript{31} case. The TE\textsubscript{21} amplification was characterized over approximately 20 different run dates, again accumulating hundreds of stored traces. As with any complicated experiment over a two and a half year lifetime, there were numerous adjustments and down times. There were flaky components, blown power supplies, and failing turbo pumps. Most of the data presented in this thesis is from May 1993 and September 1993 for the third harmonic amplifier case, when we had the most reliable system operation and consequently our most fruitful data taking. The TE\textsubscript{21} data is from July and August of 1993, which were the only two months that the 1.905 cm I.D. interaction tube was installed.

Four specific cases have been selected for detailed data presentation in the following paragraphs. In each of these cases, a complete far field scan of the amplified mode was measured, and a complete axial scan of the power versus the interaction length was also measured. Done together, these measurements give a complete characterization of mode content, power, growth rate, gain, and efficiency. The data presented for these four cases represents typical operation of the gyro-twt around the time period when the presented data was measured. Data from other dates and run parameters is presented where necessary to discuss different aspects of the gyro-twt operation. A summary chart of the power and gain measured for the four selected cases (three third harmonic and one second harmonic) is shown in Table 7.2. The table includes all significant run parameters for each case, as well as theoretical predictions of beam pitch, single particle efficiency, and Davidson's space charge parameter, all based on the given run parameters. Each of the four cases has at least one aspect that separates it from the other cases, for example, high gain, TE\textsubscript{21} operation, or measured phase.

7.3.2 First High Power Results—April 30, 1993

The first high power results from the third harmonic gyro-twt experiment were seen in late April of 1993, soon after a thorough system realignment. Fig. 7-18 shows a voltage pulse, a time-integrated diamagnetic loop pulse, and the amplified rf pulse for the April 30 run. Because the gain of the amplifier is so high in this case, the magnetron signal level (the base line of the rf pulse) is undetectable in Fig. 7-18. Both the rf pulse and the diamagnetic loop pulse are substantially narrower than the voltage pulse, primarily due to the axial wiggler guide field (1830 G) being so
<table>
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<td>2.57</td>
<td>2.22</td>
<td>2.57</td>
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<td>1.4</td>
<td>1.3</td>
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<td>27%</td>
<td>29%</td>
</tr>
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</tr>
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</tr>
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</table>

Table 7.2: A list of run parameters and measured gain and power for four of the runs being discussed. The magnetic field values are predicted by POISSON for the recorded magnetic coil settings of the run. The interaction field was tapered in each case, so the $B_0$ value is an approximate value. The beam pitch value is predicted by single particle theory (WIGGLE32) for the given run parameters, and $\alpha_{CRT}$, the beam pitch at which absolute instability theoretically occurs, is calculated as in Sec. 3.3.1. The $s_\epsilon$ parameter is for a 5 mm radius beam. The input power, output power, gain, and efficiency are all based on far field measurements and have significant error bars ($\pm 2$ dB—See Sec. 7.2.3). The single particle efficiency, $\eta_{sp}$, is given by Eq. 3.4. The parameter $\Delta$ is given by Eq. 2.53.
Figure 7-18: A voltage pulse, a time-integrated diamagnetic loop pulse, and an amplified TE_{31} rf pulse from April 30, 1993. The rf pulse, proportional to the diode signal, and the diamagnetic loop signal, proportional to V·s, are in arbitrary units. The pulses are from consecutive shots.
far above the resonance value (1560 G). This is discussed further in Sec. 7.4.3.

For the April 30 run, the injected magnetron power was set quite high, with a far field scan predicting ~ 2400 W total injected power in the TE$_{31}$ mode. With the input coupler assumed to launch fixed modes, half of the ~ 2400 W predicted is in the correctly rotating mode, resulting in ~ 1200 W input power. On a shot-to-shot basis, the amplified power was very stable, the diode signal at times being so steady that the change between shots was difficult to detect by the naked eye. The amplified signal was only present when both the wiggler (at a transverse field of 65 G) and the magnetron were turned on. With the injected rf signal not present, the gyro-twt rf output level was > 15 dB lower than with the injected signal present (at $\theta = 25$ degrees). The frequency of the amplified pulse was verified to be 17.1 GHz by passing it through a YIG tuned filter (after attenuating the amplified power so that the YIG filter was used in the linear regime). A comparison of the rf pulse with its YIG-filtered counterpart is shown in Fig. 7-19. The cut-off frequency of

![Image of Figure 7-19]

Figure 7-19: A comparison of a TE$_{31}$ pulse with and without filtering through a 350 MHz bandwidth YIG filter (Ferrettec #FD1095) with the center frequency tuned to 17.1 GHz. The YIG-filtered pulse is attenuated 3 dB less than the unfiltered signal, which corresponds to the insertion loss of the filter. As an interesting side-note, the noise on the YIG-filtered pulse is not present on the non-YIG filtered pulse because the cabling from the diode detector to the oscilloscope was switched from RG-58 to Andrews cable during this particular run. Also, as a testament to the reproducibility of the experiment, these pulses were measured six hours apart from each other.

WR42 waveguide (the detector horn and attenuator) is 14 GHz. When lower frequency detector horns were not in use (a WR62 and WR90 horns were both occasionally used), only rf signals above 14 GHz were detected. The HP detector diodes used for rf detection in these experiments
have a frequency range 0.01–18 GHz.

The far field scan of the amplified pulse predicts 4 MW of power. This power is plotted against interaction length in Fig. 7-20. The rf power was measured against interaction length by sliding a pair of “kicker” magnets along the length of the interaction tube. The strong transverse field from the kicker magnets deflects the electron beam into the wall of the interaction tube, causing the gyro-twt interaction to cease. For this particular run, both \( E_0 \) and \( E_\phi \) were measured on separate passes. For all of the gain versus interaction length curves shown in this thesis, the detector horn was kept at a fixed angle (\( \theta = 270^\circ \)) and distance in the far field, and the measured signal amplitude was averaged over several gyro-twt pulses for each data point. After completing each axial scan, certain points were rechecked to verify reproducibility. All measured values for Fig. 7-20 are normalized to 4 MW based on the far field power prediction. With each axial scan taking approximately 30 minutes, comparison of the \( E_\phi \) and \( E_0 \) data from Fig. 7-20 demonstrates the reproducibility of the measurements in Fig. 7-20.

Though the single particle theory predicts a beam pitch of 0.65 for the run parameters of April 30, the theoretical matches were done at a variety of beam pitch and beam energy and velocity spreads to demonstrate the wide range of parameter space that yields a reasonable fit to the measured data. The gain curves are calculated using CRM32 (nonlinear single particle equations of motion) with \( N = 4096 \) particles. The exact field profile, as predicted by POISSON based on the recorded magnet coil settings, is used by CRM32 in the simulation. In general, as the initial value of \( \alpha \) increases, the initial spreads necessary to match the data also increase, offsetting the increased coupling resulting from a higher initial value of \( \alpha \). The theoretical gain curves are quite sensitive to initial energy spread. For example, for the \( \alpha = 0.9 \) curve in Fig. 7-20, the only initial energy spread resulting in a reasonable match to the data is close to 3%. An energy spread of 2% or 4% resulted in saturated power values that were too high or too low regardless of the axial momentum spread. Discussion of the initial \( \alpha \) and initial beam spread values predicted by TRAJ for a TE\(_{31}\) case similar in parameters to this case will be presented in Chapter 8 (see also Sec. 5.7.4).

### 7.3.3 High Gain Results—May 12, 1993

On May 12, 1993, the input rf level for the gyro-twt was reduced to a minimum level necessary to sustain saturated amplification, and the axial gain history and total power of the amplifier were again measured in an identical fashion to the April 30 case. The measured far field scan with matching theory for the May 12 case is shown in Fig. 7-21. Only TE modes were allowed to be used in the mode-mix calculation since the CRM interaction couples to TE modes much more strongly than to TM modes\([31]\). Not surprisingly, the mode-mix prediction is almost entirely TE\(_{31}\). Again, the predicted power is 4 MW. Unlike the April 30 case, however, the input power is estimated at only 30 W, implying a gain of > 50 dB. The beam current was also significantly lower on May 12 than on April 30 (partly due to cathode heater supply problems), and the corresponding efficiency of the amplifier is 6.5% for the May 12 run. The measured axial gain history for this run is shown in Fig. 7-22. For the May 12 case, the August 3 case, and the September 30 case, only \( E_\phi \) was measured for the axial gain history scan. Again, like the April 30 case, a range of initial parameters was chosen as input to CRM32 to demonstrate plausible theoretical matches to the data in Fig. 7-22. The dispersion curve for the May 12 case, which is typical of most of the TE\(_{31}\) cases,
Figure 7-20: Measured and theoretical $\text{TE}_{31}$ amplified rf power plotted against interaction length for the April 30, 1993 run (see Table 7.2). The rf power is plotted in dB-Watts, where, for example, 0 dBW = 1 W and 60 dBW = 1 MW. The peak measured power is 4 MW. The theory curves are generated by CRM32 with $N = 4096$ particles, a beam radius of 5 mm, and initial average values of $\alpha, \sigma_{\gamma}/\langle\sigma_{\gamma}\rangle$, and $\sigma_{p_{z}}/\langle p_{z}\rangle$ as specified for each curve. These are the average values at the beginning of the interaction ($z = 0$ cm). The magnetic profile shown is predicted by POISSON based on the recorded magnetic coil settings. This exact field profile is used by CRM32 in the simulations. The absolute error range of the measured data is $\pm 2$ dB, as discussed in Sec. 7.2.3. The relative error between points, however, is substantially smaller, $\sim \pm 0.5$ dB.
Figure 7-21: Measured and best-fit $TE_{31}$ radiation pattern from the gyro-twt operation on May 12, 1993. The filled circles show measurement of $E_\phi$, and the open triangles show measurement of $E_\theta$. Each measured value is an average over several gyro-twt pulses. The mode mix and total power corresponding to the theory curves are calculated by the FFMATCH program.
Figure 7-22: Measured and theoretical TE_{31} amplified rf power plotted against interaction length for the May 12, 1993 run (see Table 7.2). The peak measured power is 4 MW. The theory curves are generated as in Fig. 7-20. The absolute error range of the measured data is ±2 dB, and the relative error is ±0.5 dB.
7.3.4 TE$_{21}$ Power and Gain—August 3, 1993

After installation of the 1.905 cm I.D. interaction tube and an extensive mode search, it was found that a relatively low beam energy, $\sim$ 320 keV, yielded high TE$_{21}$ second harmonic amplification. Optimum settings at higher beam energies were also found. The 1.905 cm I.D. tube experienced more noticeable oscillations than the 2.54 cm I.D. tube. For cases of the highest amplified TE$_{21}$ mode power, the gyro-twt output rf level in the absence of an injected signal was often only 10 dB lower than the amplified output level. For lower beam energies and lower $\alpha$ values, this could be improved to 20 dB or even 30 dB, but usually at the expense of amplified power. The oscillations, as measured with both the YIG tuned filter and with a frequency mixing system, typically had a frequency slightly lower than 17.1 GHz, e.g. 16.6–16.9 GHz. The oscillations always radiated in a characteristic TE$_{21}$ pattern, though they were not stable enough to accurately measure their far field pattern.

The TE$_{21}$ pulses tended to be wider than their TE$_{31}$ counterparts, but, as will be discussed in Sec. 7.4.1, the wider pulses also had significant frequency variation, or chirping. An example TE$_{21}$
Figure 7-24: A voltage pulse and an amplified TE$_{21}$ rf pulse from August 3, 1993. The rf pulse, proportional to the diode signal, is plotted in arbitrary units. The pulses are from consecutive shots. The parameter settings for this pulse were slightly different from those listed in Table 7.2.
pulse from August 3 is shown in Fig. 7-24 with the corresponding linac voltage pulse. The far field scan used to measure the gyro-twt output power for the August 3 run is shown in Fig. 7-25. The prediction is a mode which is almost purely TE$_{21}$ with 2 MW of power. As with the

![Figure 7-25: Measured and best-fit TE$_{21}$ radiation pattern from the gyro-twt operation on August 3, 1993. The filled circles show measurement of $E_\theta$, and the open triangles show measurement of $E_\phi$. Each measured value is an average over several gyro-twt pulses. The mode mix and total power corresponding to the theory curves are calculated by the FFMATCH program.](image)

two TE$_{31}$ axial gain history curves already shown, the power level of 2 MW acts as an absolute calibration for Fig. 7-26, which shows the amplified power plotted against interaction length for the TE$_{21}$ mode from August 3. In Fig. 7-26, the gyro-twt power in the absence of an injected signal (the superradiant gyro-twt signal) is plotted against the interaction length along with the amplified signal. The superradiant signal has the same gain history profile as the amplified signal, but is lower in power by $\sim 10$ dB. The superradiant TE$_{31}$ signal was usually not measured in the 2.54 cm I.D. tube because it was substantially weaker than the amplified signal, in some cases being virtually non-existent ($<-40$ dB). The dispersion curve for the August 3 case is shown in Fig. 7-27.

7.3.5 Measured Phase versus Interaction Length—September 30, 1993

With the 2.54 cm I.D. tube reinstalled in late August, 1993, TE$_{31}$ measurements continued with more emphasis on the spectral analysis of the pulses. A phase discriminator was added to the experimental
Figure 7-26: Measured and theoretical TE$_{21}$ amplified rf power plotted against interaction length for the August 3, 1993 run (see Table 7.2). The peak measured power is 2 MW. The filled circles represent the measured amplified signal level. The open circles represent the measured gyro-twt output level with no injected rf signal (magnetron turned off). The theory curves are generated as in Fig. 7-20. The absolute error range of the measured data is ±2 dB, and the relative error is ±0.5 dB.
Figure 7-27: Dispersion curve for the $TE_{21}$ mode based on the run parameters of August 3, 1993 (see Table 7.2).
setup. This is further discussed in Sec. 7.4. Two measured far field scans representative of the September operation of the gyro-twt are shown in Fig. 7-28. The scans were taken at settings similar to the September 30 case listed in Table 7.2. The top scan, for September 13 has a predicted power of 3 MW. The bottom scan in Fig. 7-28 is significant because it corresponds to the best measured efficiency of the gyro-twt at 8%. The current for this case was particularly low at only 115 A. A ~ 3 MW power level was typical of the September runs, and particularly of the run on September 30, where the gain and phase of the gyro-twt amplified signal were measured against interaction length. A full far field scan of the September 30 mode was not measured, but the peak points on the TE31 radiation pattern (θ = ±27°) were at the same level as in Fig. 7-28 (top), so the saturated output power is assumed to be ~ 3 MW in Fig. 7-29, which presents measured gain and phase versus interaction length for September 30. The initial beam pitch for the theory curves in Fig. 7-29 was chosen at α = 0.9 because this was clearly the best theoretical match to the data. Lower initial α values of 0.6 or 0.7 did not match the growth rate of the measured data, even at low beam energy and velocity spreads.

7.3.6 Output Power versus Input Power

To verify that the gyro-twt is in fact an amplifier and not simply a mode-locked oscillator, it is important to establish that the gain of the system remains relatively constant over some region of input power, particularly when the gyro-twt is operated in the small-signal (unsaturated) regime. This is clearly evident in Fig. 7-30, which shows the amplified pulse power plotted against the rf drive power. Measurements are shown for both a short, unsaturated interaction and for a longer interaction which demonstrates saturation for high rf drive power levels. The measurements were made by increasing the magnetron cathode voltage and measuring both the input and the amplified power with the detector horn placed at a fixed angle (27°) and distance in the far field. The absolute power is based on a far field angular scan.

7.4 Gyro-twt Frequency and Phase Measurements

Though the gyro-twt operates as an amplifier and should theoretically have one uniform output frequency, for real-life pulses this is not the case. Any variation in beam voltage, beam current, or beam pitch during the pulse modifies the phase of the gyro-twt output, and real-time variations in phase directly correspond to real-time variations in frequency. In particular, if the gyro-twt input signal is at a frequency of ω and the gyro-twt interaction adds a corresponding phase shift φ(t) to this injected frequency, the output signal will have the form

\[ E_{\text{out}}(t) = E_0 \cos(\omega t + \phi(t)) \]  \hspace{1cm} (7.15)

If \( \phi(t) = \omega_0 t \) is proportional to t, a frequency shift of \( d\phi(t)/dt = \omega_0 \) results. If \( \phi(t) = (\omega_0/\tau)t^2 \) has a significant \( t^2 \) dependence, e.g. if \( \phi(t) \) goes through a maximum or a minimum during the pulse, a frequency chirp of \( d\phi(t)/dt = 2t\omega_0/\tau \). The physics involved in the frequency shifting and chirping of a device similar to the gyro-twt, the FEL, has been done by Shvets[79], and FEL measurements have been made by Conde[24].
Figure 7-28: Measured and best-fit TE_{31} radiation patterns from the gyro-twt operation on September 13, 1993 (top) and September 24, 1993 (bottom). The parameters are similar to the September 30 case in Table 7.2. The bottom plot, with a beam voltage and current of 380 kV and 115 A, corresponds to an efficiency of 8%. The filled circles show measurement of E_{\phi}, and the open triangles show measurement of E_{\theta}. Each measured value is an average over several gyro-twt pulses. The mode mix and total power corresponding to the theory curves are calculated by the FFMATCH program.
Figure 7-29: Measured and theoretical $\text{TE}_{31}$ amplified rf power and phase plotted against interaction length for the September 30, 1993 run (see Table 7.2). The peak measured power is 3 MW. The filled circles represent the measured amplified signal level. The open squares represent the measured phase. Since the absolute phase is arbitrary, the magnetic field axis was also used for the phase, except in degrees rather than Gauss. The theory curves are generated by CRM32 with $N = 4096$ particles, a beam radius of 5 mm, and initial values of $\alpha = 0.9$, $\sigma_\gamma/(\langle \gamma \rangle) = 3\%$, and $\sigma_{pz}/(p_z) = 12\%$. The absolute error range of the measured power is ±2 dB, and the relative error is ±0.5 dB.
Figure 7-30: Measured TE$_{31}$ amplified power levels plotted against the drive power level for runs on September 29, 1993 (open circles) and on September 20, 1993 (filled squares). The unsaturated September 29 case was measured by turning off the last two X-coils to shorten the interaction length. Both measurements were done for typical TE$_{31}$ run parameters similar to the September 30 case in Table 7.2. Measurements here have an absolute error margin of $\pm 2$ dB for both the input and output power, but the relative error between points is much smaller.
Because the gyro-twt was originally being considered as the rf source for a 17.1 GHz photocathode rf gun experiment, the spectral quality of the gyro-twt rf pulses is a critical issue. The photocathode rf gun has strict tolerances on phase variance and pulse width necessary to generate a high quality electron beam. Aside from the YIG-tuned filter already mentioned, three additional diagnostics were used to gauge the fast-time frequency response of the amplified gyro-twt pulses: a frequency mixing system, a commercial phase discriminator, and a narrow bandwidth 40 MHz YIG-tuned filter.

The ability of the LeCroy 7200 oscilloscope to sample at 2 GS/sec and to do real-time fast Fourier transforms on the sampled signals kept the frequency mixing system very simple. It consisted of a local oscillator (L.O.) and a mixer, with the intermediate frequency (I.F.) being sampled directly by the LeCroy 7200. The schematic is shown in Fig. 7-31. The mixer works by taking two input signals, \( V_1(t) \) and \( V_2(t) \), and generating an output signal, \( V_{mix}(t) \), that has a term proportional to the square of the two added input signals:

\[
V_{mix}(t) = a_0 + a_1 [V_1(t) + V_2(t)] + a_2 [V_1(t) + V_2(t)]^2 + \cdots \tag{7.16}
\]

If \( V_1(t) \) is a signal oscillating at frequency \( \omega \) and \( V_2(t) \) is a signal oscillating at \( \omega_{LO} \), then the term proportional to \( a_2 \) in Eq. 7.16 will have frequency components at \( 2\omega, 2\omega_{LO}, \omega + \omega_{LO}, \) and \( \omega - \omega_{LO} \). With respect to the gyro-twt experiments, \( \omega \) represents the unknown gyro-twt frequency and \( \omega_{LO} \) represents a known local oscillator frequency. If \( \omega_{LO} \) is close to \( \omega \), then the only frequency component in Eq. 7.16 detectable by the LeCroy 7200 will be the one at \( \omega - \omega_{LO} \). Analysis of this signal then reveals information about the real-time frequency of the gyro-twt.

If the phase of the gyro-twt amplified signal does not vary too rapidly, it can be measured with a phase discriminator. The phase discriminator is based on a square-law detection scheme similar to

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**Figure 7-31:** A schematic of the frequency mixing system. A local oscillator (HP #8671B) signal and the gyro-twt amplified pulse are fed into a mixer (Watkins Johnson #WJ-M14A) which generates an intermediate frequency (I.F.) equal to the difference of the two input frequencies. The local oscillator is tunable from 2–18 GHz.
the mixer. The difference is that the signals are carefully split, balanced, and phased so as to result in four outputs: two in-phase signals, \(I_1(t)\) and \(I_2(t)\), and two quadrature signals, \(Q_1(t)\) and \(Q_2(t)\). These signals have a D.C. component that is removed by differencing them, as shown in Fig. 7-32, where a schematic of the phase discriminator setup for the gyro-twt is presented. After removing the D.C. component, the signals are directly proportional to the sine and the cosine of the phase variation. Calculating the phase can then be done by numerically processing the digital traces. The phase discriminator was used to measure only \(\text{TE}_{31}\) phase variations starting in September, 1993.

7.4.1 Frequency Chirp

The first substantial effort to analyze the gyro-twt pulse frequency was made with the 1.905 cm I.D. tube installed and the gyro-twt acting as a second harmonic, \(\text{TE}_{21}\) mode amplifier. A frequency chirp was consistently present on the high power amplified pulses. A measurement of the chirp using the 350 MHz bandwidth YIG-tuned filter was taken on August 13, 1993. As shown in Fig. 7-33, the beginning of the amplified rf pulse had a substantially lower frequency component than the end of the pulse. Three YIG-filtered traces are shown, each at a different center frequency, and they are superimposed on top of each other. An upchirp of \(\sim 200\) MHz (allowing for the bandwidth of the filter) is indicated over the entire width of the \(\sim 20\) ns pulse. This corresponds to a chirp rate of \(\sim 10\) MHz/ns. The same upchirp was also evident in the I.F. signal when the frequency mixing system was used to measure the amplified pulse frequency. An I.F. signal and its digital Fourier transform (DFT) are shown in Fig. 7-34, the local oscillator frequency being 17.46 GHz. The zero-crossings of the I.F. signal give a real-time dynamic frequency estimate for
Figure 7-33: A high power (> 1 MW) amplified TE_{21} gyro-twt pulse filtered through a 350 MHz bandwidth YIG filter tuned to three different center frequencies. The pulse peak shifts in time as the center frequency increases, indicating a frequency chirp on the pulse. This measurement was taken on August 13, 1993. The run settings were similar to those for August 3 in Table 7.2, though at slightly higher voltage.
the pulse. A line of best fit applied to the zero-crossing frequency curve in Fig. 7-34 predicts a phase chirp of $\sim 9$ MHz/ns, which compares favorably to the rough estimate of $\sim 10$ MHz/ns for the chirp based on Fig. 7-33.

During September 1993, frequency analysis of the gyro-twt amplified pulses continued, but now for third harmonic TE$_{31}$ pulses. Both the TE$_{21}$ and TE$_{31}$ pulses were analyzed using the 40 MHz narrow bandwidth YIG-tuned filter. This filter was used to measure the amplified pulse spectrum in detail. Fig. 7-35 shows a comparison of the spectrum of a TE$_{31}$ high power amplified pulse as measured by the 40 MHz YIG filter and as measured by a DFT of the I.F. signal from the frequency mixing system. The two spectra were measured at the exact same settings within one hour of each other.

### 7.4.2 Phase Variability

A physical explanation of the frequency chirp seen on the TE$_{21}$ pulses and, as will be shown, on the TE$_{31}$ pulses, begins to manifest itself when the actual phase variation of the gyro-twt pulses is examined. This phase examination was done with the use of the phase discriminator. The first phase discriminator measurements were taken on September 17, 1993. The phase variation of the TE$_{31}$ amplified pulse for the September 30, 1993 case (Table 7.2) is shown in Fig. 7-36 (bottom). As is evident in the figure, the phase variation has an approximately parabolic shape. Moreover, when compared to the diamagnetic loop time-integrated pulse, the phase signal and the diamagnetic loop signal (roughly scaling like the beam pitch) are shown to have similar parabolic shapes. The phase variation in the bottom plot in Fig. 7-36, when fit to a parabola, yields the straight-line frequency chirp shown in the top plot. This chirp of $\sim 8$ MHz/ns agrees well with the chirp predicted by the zero-crossing frequencies of the I.F. signal. The zero-crossing frequencies and the I.F. signal are shown in Fig. 7-36 (top).

As mentioned, the diamagnetic loop signal appears similar in shape to the phase signal in Fig. 7-36 (bottom). To see if the shape of the diamagnetic loop signal explains the phase variation, we start by assuming that the diamagnetic loop signal scales like the square of the transverse velocity of the beam, which is a good assumption for small beam pitch (see Eq. 6.6). We must also assume that the beam pitch for the September 30 case is 0.6. A parabola is then fit to the diamagnetic loop pulse in Fig. 7-36 over the range of phase shown in the figure. The fit, given the assumptions, results in the following time profile of $\beta_\perp$ near the peak of the pulse:

$$\beta_\perp \approx \beta_{\perp MAX} - 0.0015 \beta_{\perp MAX} \left( \frac{t}{1 \text{ ns}} \right)^2 \quad (7.17)$$

If it is then assumed (see Fig. 3-12) that $\phi(t)$ changes $5^\circ/\%$ for changes in $\beta_\perp$, the phase variation resulting from Eq. 7.17 would be (for all other parameters fixed)

$$\phi(t) \approx \phi_{MAX} - 0.75^\circ \left( \frac{t}{1 \text{ ns}} \right)^2 \quad (7.18)$$

The resulting frequency chirp is $1.5^\circ/\text{ns}^2 = 4$ MHz/ns. Such a calculation is very simplified, but still results in a chirp value of the same order as the measured 8–10 MHz/ns chirps. It should be noted that the voltage variation during the time when the phase variation was measured is much
Figure 7-34: Frequency chirp on a TE$_{21}$ pulse. The top figure shows the I.F. signal (dashed curve) for a $\sim$ 2 MW amplified TE$_{21}$ gyro-twt pulse mixed with a 17.46 GHz local oscillator. The corresponding rf pulse is shown in Fig. 7-24. The zero-crossings of the I.F. signal are used to estimate the frequency chirp on the gyro-twt signal. The solid curve in the top figure shows the frequency corresponding to the zero-crossings, which are spaced farther apart as time increases. A digital Fourier transform (DFT) of the I.F. signal is shown in the bottom figure. Like the zero-crossing frequency, the DFT is shifted in frequency to account for the local oscillator frequency.
Figure 7-35: The dashed curve shows the frequency spectrum of a TE$_{31}$ amplified pulse as measured by filtering the pulse through a 40 MHz bandwidth YIG filter (Dorado #YF074) tuned to different center frequencies. The bar graph shows the DFT of the I.F. signal resulting from mixing the gyro-twt pulse with a 17.45 GHz local oscillator. The spectra are both normalized to unity and plotted in arbitrary units of rf power.
Figure 7-36: Phase variation and frequency chirp measured on a high power TE_{31} pulse. The top figure shows the I.F. signal (dashed curve) for a ~ 3 MW amplified TE_{31} gyro-twt pulse mixed with a 17.45 GHz local oscillator. This pulse is from the September 30 run parameters (Table 7.2). The bottom figure shows the actual rf pulse, its measured phase variation, and a diamagnetic loop time-integrated pulse. The diamagnetic loop pulse, recorded at similar run parameters, was measured two days earlier. A parabola was fit to the phase signal in the bottom figure. It was then differentiated to predict the frequency chirp which is shown by the straight line in the top figure. For comparison, the zero-crossing frequency is also shown on the top figure.
less significant than the diamagnetic loop variation. A fit to the voltage pulse from September 30, 1993 yields $V(t) \approx V_{MAX} - 0.0004V_{MAX}(t/1\text{ ns})^2$.

Two of the best phase stability results for the TE$_{31}$ amplifier operation are shown in Fig. 7-37. The bottom plot is from the September 30 run parameters. The top is from September 28 at similar settings. The phase stability in these cases, as discussed, is likely limited by the changing beam pitch. The phase reproducibility between gyro-twt pulses on a shot-to-shot basis was very good, typically varying by $< 5^\circ$ for the same time point on two consecutive shots.

### 7.4.3 Competing Instabilities

The diamagnetic loop pulse shown in Fig. 7-36 has a narrow profile because the wiggler guide field (1780 G in this case—see Table 7.2) is set high above the resonant value (1525 G). As shown in Fig. 7-8, when the wiggler guide field is set above resonance, the beam pitch profile narrows, has a more rounded top, and decreases in overall amplitude. Unfortunately, only this operating point consistently yielded high power pulse amplification. A typical example of the results for operating with the wiggler guide field closer to resonance, i.e. operating with a wider beam pitch pulse, is shown in Fig. 7-38. For each of the four curves in each plot, the wiggler guide field is tuned to a different value, ranging from 1770 G for the curves marked with a '1,' to 1900 G for the curves marked with a '4.' Even at 1770 G, the wiggler guide field is still much higher than the resonant value ($\sim 1500$ G) for the given beam voltage of this run ($\sim 370$ kV). As the wiggler guide field is lowered, the diamagnetic loop pulses show the increasing and widening beam pitch profile. The pulse spectra, measured through the 40 MHz bandwidth YIG-tuned filter, show a significant increase in power at 16.9 GHz as the beam pitch increases, and the center frequency of the main amplified mode shifts from 17.2 GHz to 17.13 GHz. Both the amplified signal and the parasitic 16.9 GHz signal were only present when a drive signal was applied (at 17.1 GHz). The 16.9 GHz signal shows up clearly in the rf pulses as the second "hump" in rf traces 1–3 in Fig. 7-38. This growth of signal power at frequencies other than the drive frequency is typical of both the TE$_{21}$ and TE$_{31}$ amplifier operation. In some cases, the parasitic signal was clearly superradiant and was present even without the drive signal. In other cases, it was present only with the drive signal and at later time than the amplified pulse. The parasitic signal may have been the result of a single pass reflection or of a temporally growing (absolutely unstable) mode. Possibly the beam pitch and voltage passed through the primary resonance at a lower value than the peak voltage, leading to a double-peaked rf pulse. The frequency shift of the 16.9 GHz signal may in fact be due to its occurrence late in the pulse, where a decreasing voltage and beam pitch may have resulted in a corresponding frequency shift. Likewise, The frequency shift of the amplified mode in Fig. 7-38 (top) from 17.2 GHz (trace 1) to 17.13 GHz (trace 4) may be due to the corresponding shift of the amplified pulse in time. In Fig. 7-38 (bottom), trace 4 shows the amplified mode shifting in time from the other amplified pulses. This shift relative to the voltage pulse may move the amplified pulse to a region of flatter beam energy and correspondingly less phase variability. A large linear phase shift during the rise of the voltage pulse would account for the frequency shift observed in Fig. 7-38 (top). Moving the wiggler axial guide field close to resonance and also lowering the transverse wiggler field, thereby trying to keep the peak beam pitch constant, always resulted in reduced rf output power. We believe this sort of parasitic mode was a significant limiting factor in
Figure 7-37: Measured phase variation of two high power TE$_{31}$ amplified gyro-twt pulses. Both pulses are $\sim$ 3 MW peak power pulses. The phase variation of each pulse is shown by the dashed curves. The top figure shows the best 15 ns result, measured on September 28, 1993. The parameters for this case are similar to those for the bottom figure, which is from the September 30 run (see Table 7.2). The rf pulses were detected by a horn in the far field at the peak TE$_{31}$ angle.
Figure 7-38: Growth of a parasitic mode as wiggler resonance is approached. Each plot in the figure shows four curves numbered 1 through 4. Each number corresponds to a different wiggler guide field setting, 1 being closest to resonance, 4 being farthest from resonance. The top figure shows the amplified rf pulse spectrum as measured with the 40 MHz bandwidth YIG tuned filter. Each spectrum is normalized to a peak value of unity and plotted on a power scale. The middle figure shows the time-integrated diamagnetic loop signal. The bottom figure shows the rf pulse shape (not normalized) detected by a horn in the far field at the peak $TE_{31}$ angle ($27^\circ$). The parasitic mode is shown growing at 16.9 GHz (top figure) late in the rf pulse (bottom figure).
Because superradiant emission was more prevalent for the TE$_{21}$ pulses, the superradiant power level was measured at the settings for a high power TE$_{21}$ amplified mode on August 5, 1993. The settings for this case are similar to the August 3 case, but the beam voltage is higher (360 kV) and the wiggler guide field is higher (1600 G). Fig. 7-39 shows the rf power level of the superradiant rf as a function of the beam pitch, $\alpha$. The beam pitch is derived by using WIGGLE32 to predict the $\alpha$ value based on the run parameters. The transverse wiggler field was increased from 0 A to 40 G to effect the change in $\alpha$. The amplified pulse power, based on the measured intensity at the peak

Figure 7-39: Growth of a TE$_{21}$ superradiant mode shown as the rf power level of the mode plotted against beam pitch. The power is plotted as a percentage of the amplified pulse power level (2 MW). Filled circles show the power for a flat interaction field, and filled squares show the power for a tapered interaction field. The beam pitch was increased by tuning the wiggler transverse field from 0 to 40 G. The value of $\alpha$ is then predicted by WIGGLE32 based on the run parameters of $V = 360$ kV, $I = 135$ A, and $B_0 = 0.365$ T. The axial guide field at the wiggler is $B_{\text{w,z}} = 1600$ G, and the resonant guide field was measured to be 1550 G, which corresponds to the predicted resonance value.

of the TE$_{21}$ pattern ($\theta = 15^\circ$), is estimated at $\sim 2$ MW. The superradiant mode had a frequency near 16 GHz (TE$_{21}$ cut-off is at 15.3 GHz), and the radiation pattern had the characteristic shape of a TE$_{21}$ mode. The critical $\alpha$ value for the onset of absolute instability, as calculated by the method of Sec. 3.3.1, is $\alpha_{\text{CRIT}} = 1.45$. An interesting feature seen in this run was the suppression of a competing superradiant mode by the application of the drive signal and resulting amplified
pulse. Fig. 7-40 shows two I.F. signals with the local oscillator set near TE_{21} cut-off at 16 GHz. The I.F. signal without the rf drive applied shows a mode at a frequency near 16 GHz. When the drive is applied, the mode is suppressed. Suppression of self-excited modes is a well documented phenomenon in gyro-twt operation [60, 5].

7.5 RF Gun Transmission Line Power Measurement

From October 1993 until the experimental termination of the gyro-twt, an rf gun transmission line was attached to the output of the gyro-twt. This involved replacing the gyro-twt window with a 2 m section of 5.08 cm I.D. waveguide followed by a downtaper to 2.54 cm I.D.. Connected to the downtaper, in order, are the two converters shown in Fig. 7-14 to convert the gyro-twt TE_{31} signal to TE_{11}, a circular-to-rectangular transition, a -60 dB forward wave pickoff coupler, and finally the rf gun itself. Essentially, the rf gun transmission line is the calibration setup in Fig. 7-14 run in reverse order. The 2 m waveguide section was used to provide isolation distance between the

![Figure 7-40: Suppression of a TE_{21} superradiant mode with an applied drive signal. The dashed curve I.F. signal, mixed with a local oscillator at 16 GHz, shows a superradiant mode with the drive off. When the drive is applied, shown by the solid curve, the superradiant mode is suppressed. The amplified power level is estimated at ~ 2 MW, and the drive power is estimated at ~ 1 kW. The run settings are similar to the August 3 case in Table 7.2.](image-url)
rf gun and the gyro-twt. The rf transmission line allowed an independent measurement of the rf power based on calibrated diode detection at the $\text{-60 dB}$ coupler. Because the rf gun transmission line employs the rippled-wall $\text{TE}_{31}$-to-$\text{TE}_{11}$ converter, the gyro-twt was only operated in the third harmonic, $\text{TE}_{31}$ mode (2.54 cm I.D. interaction waveguide).

The peak rf power of amplified pulses using the rf gun transmission line was $\sim 2.5 \text{ MW}$ at parameter settings consistent with the high power measurements from September. Considering that the estimated transmission line loss is $\sim 1 \text{ dB}$, this value corresponds to $\sim 3 \text{ MW}$, which is in good agreement with the far field power measurements that were made in September. A $2 \text{ MW}$ rf pulse measured at the end of the transmission line on the $\text{-60 dB}$ coupling port is shown in Fig. 7-41.

![Graph showing RF pulse](image)

Figure 7-41: A $\sim 2 \text{ MW}$ $\text{TE}_{31}$ amplified pulse measured at the end of the rf gun transmission line. The power was detected by a calibrated diode at a $\text{-60 dB}$ coupling port. The diode voltage is converted to a power scale using the diode calibration. The beam voltage and current for this case are 390 kV and 150 A.

### 7.5.1 Side Wall Input Coupler

Just before the termination of the gyro-twt experiment, in early March, 1994, a side-wall hole coupler replaced the wire-mesh input coupler of the gyro-twt. The side-wall coupler couples rectangular $\text{TE}_{10}$ radiation from WR62 guide into $\text{TE}_{31}$ radiation in 2.54 cm I.D. circular guide. The measured coupling from this input coupler was substantially better than the wire-mesh coupler; however, it was not characterized in the far field due to time constraints. In addition, no improvement in gyro-twt performance was measured due to the installation of the new coupler. It was expected
that removing the wire-mesh from the beam path might improve the quality of the electron beam; however, the beam did partially destroy the mesh in the original coupler, vaporizing the center portion with a hole diameter of \( \sim 1 \text{ cm} \). Fig 7-12 shows the radiation pattern from the wire-mesh coupler after the beam has already made the hole in the mesh.

7.6 Summary

The data collected from the gyro-twt brings up several important issues. The requirement that the wiggler guide field has to be far above wiggler resonance to achieve high output power is one important empirical result of the data. This requirement, which leads to narrow rf pulses and frequency chirping, must be understood. Other questions remain about the theoretical matches to the gain profiles of the amplifier. The low energy spreads needed to obtain the theoretical matches are not in agreement with the TRAJ predictions in Table 5.1. These issues and the limitations on the efficiency, phase stability, and pulse width of the amplifier will be addressed in Chapter 8.
Chapter 8
Discussion and Summary

8.1 Introduction

Any self-respecting theorist claims that disagreements between theory and experiment point out inadequacies in the experimental measurement technique. Any red-blooded experimentalist claims that disagreements between theory and experiment highlight flaws in the theoretical analysis. The truth usually lies somewhere between these two extremes. The data collected from the gyro-twt experiments and presented in Chapter 7 leaves many apparent discrepancies between theory and measurement to be explained. Predictions from simulations are not entirely consistent with the experimental results. It is the goal of this chapter to plausibly explain these findings.

8.2 Simulation Results

One issue arising from the data measurements made in Chapter 7 is that they are not consistent with an “end-to-end” simulation of the gyro-twt. An end-to-end simulation involves using EGUN (Sec. 4.5.1) to simulate the propagation of the beam from the cathode to the wiggler entrance. Next, TRAJ (Sec. 5.7) simulates the propagation of the beam through the wiggler and up to the beginning of the interaction region. The TRAJ simulation predicts the beam pitch, \( \alpha \), and the energy and axial momentum spreads, \( \sigma_e/\langle \gamma \rangle \) and \( \sigma_{pz}/\langle p_z \rangle \), as the beam enters the gyro-twt interaction region. Each of these three initial parameters is critical to the performance of the gyro-twt, and none of them were directly measured. Finally, CRM32 (Sec. 2.6) begins with the initial conditions on the beam provided by TRAJ and simulates the growth of the rf power in the interaction region. The results of such end-to-end simulations applied to one of the specific cases in Chapter 7 are now presented in detail. The conclusion is that taken together, these simulations are inconsistent with the measured rf power and growth rate. At least one (if not all three, to some degree) of the simulations does not appear to be correctly modeling the gyro-twt experiment, and we believe the most suspect of these simulations is the TRAJ simulation.

For a case study, the May 12, 1993 case is chosen. This case has two key aspects representative of all four cases listed in Table 7.2, but not included in the simulations in Chapter 5. The first aspect is that the beam current measured on May 12 and on most other run dates is not the value predicted by EGUN. The second aspect is that the matching coil (Sec. 6.2.1) was not used for any of the cases.
in Table 7.2. The low beam current is explained by the long duration of use for the primary cathode of interest, A_r. Referring to Table 7.1, this cathode was used for over 1.5 years. Over that lifetime, the cathode was exposed to atmospheric pressure (nitrogen gas) several times. In addition, when the new cathode C was installed on January 13, 1994, a marked increase in current was measured, much closer to the value predicted by EGUN. The low current is simulated in EGUN by setting a current density emission limit on the cathode. The matching coil, though it improves the axial field transition region, was not found to improve the overall gyro-twt output power. In fact, many settings resulted in higher power with the matching coil turned off, perhaps due to a non-adiabatic increase in the perpendicular momentum of the beam. With the matching coil not used, a “dip” exists in the axial guide field that likely affects the beam pitch and the beam spreads (see Fig. 6-4) for the cases in Table 7.2. TRAJ simulations have the ability to take into account the full effect of this transition region.

Figure 8-1 shows the magnetic profile for the May 12 case over the region of interest, from the cathode to the beginning of the interaction region where the rf drive signal is injected. The magnetic profile in the interaction region is shown in Fig. 7-22. The magnetic profile in Fig. 8-1 shows the on-axis field strength calculated by POISSON. The full 2D (r and z) field is calculated by POISSON and used by both EGUN and TRAJ to propagate the beam. The field in the wiggler region is almost flat at $\sim$ 1865 G. For the interaction, it then increases (after the dip in the transition region) to a peak of 3000 G before decreasing to achieve a tapered field in the interaction region.

Two EGUN simulations were done using the magnetic field profile in Fig. 8-1. One was allowed no density limit on the cathode. The other restricted the cathode density so as to limit the current from the original value of 360 A to a value of 200 A, which is much closer to the measured value of 160 A. The two simulations are shown in Fig. 8-2. Because the outer five rays in the density limited simulation deviate so largely from the rest of the beam, they have been dropped from the problem in order to focus on the distribution of the majority of the beam without skewing the spread numbers due to the few deviant rays. Without the outer rays, the current drops to 160 A, the measured value on May 12. The energy and axial momentum spreads predicted by EGUN are actually lower in the density limited case due to less voltage depression across the beam. The average pitch at the end of the EGUN simulation (before the wiggler), however, is nearly twice as high for the density limited case due to the poor beam focus and resulting scalloping.

The results from the TRAJ simulations that continue the two EGUN simulations into the wiggler region are shown in Figs. 8-3 and 8-4. There are noticeable differences between the simulations. For the density limited ($I = 160$ A) case, the final beam pitch of $\alpha = 0.51$ is 10% lower than for the full current case, $\alpha = 0.61$. Both plotted beam spreads in the density limited case end up with smaller values, $\sigma_\gamma / \langle \gamma \rangle = 3.9\%$ and $\sigma_{pz} / \langle p_z \rangle = 3.2\%$, than in the full current case, $\sigma_\gamma / \langle \gamma \rangle = 4.0\%$ and $\sigma_{pz} / \langle p_z \rangle = 4.8\%$. The beam spreads in Fig. 8-3 and 8-4 show oscillations at the cyclotron frequency, corresponding to mixing and de-mixing on opposite halves of the cyclotron orbit as the beam travels its helical path. The dip in the magnetic field and subsequent compression region are seen to have a significant impact on the beam spreads in each case.

If the TRAJ results of $\alpha \approx 0.6$, $\sigma_\gamma / \langle \gamma \rangle \approx 3\%$, and $\sigma_{pz} / \langle p_z \rangle \approx 4\%$ are now passed to CRM32, the prediction from CRM32 is significantly less than 1 MW output power, in complete disagreement with the measured power of 4 MW. Either TRAJ is predicting inconsistent initial beam parameters, or CRM32 is predicting inconsistent rf power. CRM32, however, is a well benchmarked program.
Figure 8-1: The magnetic profile for May 12, 1993. The magnetic coil positions and corresponding current settings are shown in the figure. The power from the gyro-twt was improved by turning off the matching coil and creating the non-adiabatic transition in the 75–90 cm region. The axial position is referenced to the left pole face of the focusing coils. The wiggler magnet generates the transverse field necessary to spin-up the beam.
Figure 8-2: EGUN simulations for May 12, 1993. The full 2D magnetic profile predicted by POISSON is used by EGUN to simulate the propagation of the electron beam. The top figure shows a 360 A beam drawn from a cathode with no restrictions on its emission current density. EGUN predicts an average voltage of 375 kV for the beam, which originates from a cathode-anode gap of 390 kV. The average pitch of the beam at the end of the simulation is \( \langle \alpha \rangle = 0.04 \), the energy spread is \( \sigma_{\gamma}/\langle \gamma \rangle = 0.4\% \), and the axial momentum spread is \( \sigma_{p_z}/\langle p_z \rangle = 0.6\% \). In the bottom figure, the current density from the cathode is limited. The total current emitted is 200 A. The reduced space charge causes significant beam scalloping. With the outer five beam rays not included, the total beam current is 160 A, the average beam energy is 383 keV, the average pitch at the end of the simulation is \( \langle \alpha \rangle = 0.08 \), the energy spread is \( \sigma_{\gamma}/\langle \gamma \rangle = 0.2\% \), and the axial momentum spread is \( \sigma_{p_z}/\langle p_z \rangle = 0.4\% \).
Figure 8-3: TRAJ simulation of beam propagation through the wiggler, drift, and compression regions of the gyro-twt experiment for the run parameters of May 12, 1993. In this case, the total beam current is 360 A (see Fig. 8-4). Average beam pitch is plotted in the top figure. Beam spreads (dashed curves) and the on-axis axial magnetic field profile are plotted in the bottom figure. Parameters at the end of the simulation are $\langle \alpha \rangle = 0.61$, $\sigma_{\gamma}/\langle \gamma \rangle = 4.0\%$, and $\sigma_{p_z}/\langle p_z \rangle = 4.8\%$. 
Figure 8-4: TRAJ simulation of beam propagation through the wiggler, drift, and compression regions of the gyro-twt experiment for the run parameters of May 12, 1993. In this case, the total beam current is 160 A (see Fig. 8-3). Average beam pitch is plotted in the top figure. Beam spreads (dashed curves) and the on-axis axial magnetic field profile are plotted in the bottom figure. Parameters at the end of the simulation are $\langle \alpha \rangle = 0.51$, $\sigma_\gamma/\langle \gamma \rangle = 3.9\%$, and $\sigma_{pz}/\langle p_z \rangle = 3.2\%$. 
using well established equations of motion for the CRM interaction. TRAJ, on the other hand, makes significant approximations and has never been benchmarked against a more sophisticated trajectory code. If we assume that the CRM32 result is more reliable, then the TRAJ simulation is predicting beam pitch values too low and/or beam energy spreads too high.

In order to better understand the correlation between beam pitch and beam spreads predicted by TRAJ, several more TRAJ simulations were run. In the first set of simulations, the May 12, 1993 parameters are used, but the guide field in the wiggler region is different for each set of runs, keeping all other parameters from the May 12 case fixed. By lowering the wiggler guide field, the predicted pitch of the beam increases. For each different wiggler guide field, POISSON was used to generate the magnetic field data, EGUN was used to predict the beam propagation up to the wiggler entrance, and TRAJ was used to predict the final beam pitch, energy spread, and axial momentum spread. For each case, TRAJ was run with the matching coil on, with the matching coil off, with a density limited cathode, and without a density limited cathode. For all four TRAJ runs, the final predicted value of each parameter is plotted in Fig. 8-5 (top) with an error bar corresponding to the maximum and minimum values from all different TRAJ runs at each wiggler guide field setting. In general, reducing the dip in the magnetic field profile by adding the matching coil to the simulations decreased the axial momentum spread by 1–2% and decreased the beam pitch by 5–10%. The matching coil had little effect on the energy spread of the beam. Also in Fig. 8-5 (bottom) are TRAJ results at beam energies other than 390 kV, keeping all other parameters from the May 12 case fixed. In both the top and bottom plots of Fig. 8-5, the spreads are shown to clearly correlate to the beam pitch. Higher beam pitch predictions correspond to higher spread predictions. Significantly, none of these TRAJ predictions result in CRM32 simulations that yield 4 MW of output power. For each given beam pitch predicted by TRAJ in Fig. 8-5, the corresponding energy spread is too high, and the corresponding CRM32 simulation results in < 1 MW predicted rf power.

With the TRAJ simulation suspect, we turn primarily to the CRM32 results shown in chapter 7 to estimate a beam pitch and beam spreads consistent with the experimental measurements. In Fig. 8-6, contours of constant initial $\alpha$ are plotted against initial beam energy spread and axial momentum spread for 4 MW of predicted output power by CRM32 for the May 12 case. The calculations are done for $N = 4096$ particles. As Fig. 8-6 demonstrates, the 4 MW of rf power occurs both for low $\alpha$ and low spreads or for high $\alpha$ and high spreads. This was also alluded to in Chapter 7 by analysis of theoretical matches to the gain history curves. The apparent limit on energy spread, according to Fig. 8-6, is $\sim 4\%$ for a value of $\alpha (1.1)$ that is twice what TRAJ predicts for the May 12 case. On the other hand, at the TRAJ predicted $\alpha$ value of 0.6, the required energy spread of 1% (from Fig. 8-6) differs significantly from the TRAJ prediction of 4%.

### 8.2.1 Estimated Beam Pitch

It is our belief that the beam pitch predicted by TRAJ is too low. While a beam pitch of $\alpha = 0.6$ can be consistent with the CRM32 results in Fig. 8-6, the beam spreads required are quite small. We feel it is more likely that $\alpha \sim 0.9$ due to the good matches between the CRM32 gain curves for $\alpha = 0.9$ and every measured gain history curve. An $\alpha$ of 0.9 consistently results in predictions from CRM32 that match the measured growth rate and saturated power level. The low beam spreads that CRM32 requires for $\alpha = 0.6$ or $\alpha = 0.7$ to yield 4 MW are inconsistent with the day-to-day
Figure 8-5: Beam pitch, energy spread, and axial momentum spread predicted by TRAJ for different wiggler axial guide fields and beam energies. For each different wiggler guide field (top) or beam energy (bottom), TRAJ was run with the matching coil on and off, and with both a density limited and a full current case. The error bars show the range of each parameter for all of these cases. Filled circles show TRAJ predictions of α. Solid curves show WIGGLE32 predictions. Open triangles show axial momentum spread, and open squares show energy spread. The May 12, 1993 case is at 1860 G in the top figure and at 390 keV in the bottom figure. All other parameters except wiggler guide field (top) or beam energy (bottom) remain fixed at the May 12 run settings.
Figure 8-6: Curves of constant beam pitch plotted against energy spread and axial momentum spread for 4 MW of rf output power at the May 12 run settings. The curves are predicted by CRM32 (nonlinear CRM theory) using \( N = 4096 \) particles and a beam radius of 5 mm. The figure demonstrates that a wide range of \( \alpha \) and beam spread values result in 4 MW predicted output power. As \( \alpha \) decreases, the beam spreads required also decrease. The experimental measurement error of \( \pm 2 \) dB corresponds approximately to \( \pm 0.5\% \) in \( \sigma_{1}/\langle \gamma \rangle \) and \( \pm 3\% \) in \( \sigma_{pz}/\langle p_z \rangle \). For example, if \( \sigma_{1}/\langle \gamma \rangle = 3\% \), then the actual range is 2.5–3.5\%.
operation of the gyro-twt. If the spreads were actually as low as $\sigma_\gamma/\langle \gamma \rangle \sim 1\%$ and $\sigma_{pz}/\langle p_z \rangle \sim 5\%$, the operation of the gyro-twt likely would have been less stable and more sensitive to fluctuations in any of the system parameters. Higher spread enhances the reproducibility and stability of a system, and the gyro-twt showed very good pulse-to-pulse reproducibility. Low spreads are also inconsistent with the absence of any measurable $\text{TE}_{11}$ gain for the CARM experimental parameters.

Finally, higher $\alpha$ than predicted by TRAJ is consistent with another MIT result from a CARM oscillator experiment[1]. In this experiment, beam pitch consistent with the rf power measurements was nearly twice that predicted by TRAJ. This discrepancy is still not fully understood. Perhaps it is the result of beam instabilities in the wiggler region or in the compression region. Also, TRAJ simulations do not include the effect of the axial magnetic self-field due to the corkscrewing motion of the electron beam in the wiggler. This self-field would oppose the axial guide field, and for the run parameters of May 12, a lower guide field correlates to a higher $\alpha$. A simple estimation based on modeling the beam as a large-diameter, helically-wound conductor predicts that inclusion of the longitudinal magnetic self-field for the May 12 case increases the predicted $\alpha$ by $\sim 5\%-10\%$.

8.2.2 Estimated Energy Spread

If a beam pitch value of $\alpha = 0.9$ is assumed, TRAJ does not predict beam spreads consistent with CRM32. From Fig. 8-5, the spreads predicted for cases where TRAJ predicts $\alpha \sim 0.9$ are in the range $\sigma_\gamma/\langle \gamma \rangle = 6\%$ and $\sigma_{pz}/\langle p_z \rangle = 8\%$. The axial momentum spread of 8\% is consistent with the measured rf power, but the energy spread is much higher than a consistent value of $\sim 3\%$. Here, we suspect that the neglect of longitudinal (axially directed) space-charge by TRAJ chiefly explains the inconsistency. Fig. 8-7 demonstrates the inadequacy of using only transverse forces for a beam with a high pitch ratio. Fig. 8-7 shows a local region of a beam traveling with a pitch of $\alpha = 1$, i.e. at 45°. The TRAJ assumed field, which neglects the axial component, has the wrong amplitude (different by a factor of $\sqrt{2}$ from the actual amplitude) and the wrong direction. While the TRAJ field results in a net $v \cdot E$ on the outer particles, the actual electric self-field does not. We believe this explains the inflated energy spreads. A full 3D time-dependent simulation may be necessary to fully understand the effects of the bifilar helical wiggler, particularly if an instability in the wiggler region is responsible for larger than predicted beam pitch values. Values of $\alpha = 0.9, \sigma_\gamma/\langle \gamma \rangle = 3\%, \text{ and } \sigma_{pz}/\langle p_z \rangle = 12\%$ are the most consistent initial beam parameters based on the CRM32 results shown in chapter 7. They result in excellent matches between the CRM32 predictions for gain-history curves and the measured data in every case. Based on this fact and on the previous arguments given, we believe $\alpha = 0.9, \sigma_\gamma/\langle \gamma \rangle = 3\%, \text{ and } \sigma_{pz}/\langle p_z \rangle = 12\%$ are the best estimates for the beam parameters upon entering the gyro-twt interaction region.

8.3 Narrow Pulses—Operating Far From Wiggler Resonance

In chapter 7 it was noted that in most cases the highest gyro-twt rf output power occurred for cases where the wiggler axial guide field was set far above the wiggler resonance value (see Table 7.2). Theoretically, an equivalent operating beam pitch can be obtained by operating with the transverse wiggler field lower and the axial wiggler guide field closer to resonance. The cases of operating close to wiggler resonance and far from wiggler resonance were discussed in Sec. 5.7.4
Figure 8-7: Self electric field forces on the outer particles of a beam traveling with a pitch of $\alpha = 1$. TRAJ, by using only transverse self-field forces, ignores a substantial component of the electric force. Moreover, the portion of the force that TRAJ uses results in a net $\mathbf{v} \cdot \mathbf{E}$ on the outer particles, which will change the energy of these particles. The actual force is perpendicular to the particles and would not result in a net $\mathbf{v} \cdot \mathbf{E}$. This may result in an artificial inflation of the beam energy spread by TRAJ.

with the conclusion that operating with the wiggler guide field above resonance results in lower predicted beam spreads for equivalent values of $\alpha$. In Figs. 8-3 and 8-4, the $\alpha$ profile in the wiggler region is typical of running above wiggler resonance. It rises to a peak before the end of the wiggler and then begins to decrease until exiting. Perhaps this reduction of interaction with the wiggler before exiting improves the transport of the beam as it exits the wiggler. Fig. 8-8 shows the predicted energy spreads from TRAJ for operation of the wiggler near and far from wiggler resonance. The resulting beam pitch is nearly equivalent in each case, but the energy spread is consistently higher in the near-resonance case. Though TRAJ is believed not to predict energy spread accurately, the correlation shown in Fig. 8-8 is consistent with experimental results. A 1–2% increase in energy spread (or even a 0.5–1% increase) is enough to significantly reduce rf output power and to encourage operation with the wiggler axial guide field set above the resonant value. As was discussed in chapter 7, operating far from wiggler resonance results in narrowing of both the diamagnetic loop pulse and the rf pulse due to the beam pitch having a higher sensitivity to the beam energy (see Fig. 7-18). The significantly rounded profile of the diamagnetic loop pulse resulting from operation far from wiggler resonance is, in turn, responsible for the frequency chirping and limited phase stability of the high power gyro-twt pulses.

Some cases presented in Chapter 7 were run with the operating parameters close to wiggler resonance. For the $\text{TE}_{21}$ run on April 3, the wiggler axial guide field was set almost precisely at wiggler resonance, and the $\text{TE}_{21}$ pulse width from that run and similar runs (Figs. 7-24 and 7-33) is correspondingly wider. Lower beam spread may not be the only reason that operation above wiggler resonance gave the best gyro-twt rf power results. In nearly every experimental run, increasing and widening the beam pitch profile by operating closer to wiggler resonance resulted
Figure 8-8: Predicted energy spread near wiggler resonance and far from wiggler resonance. The predictions are from TRAJ cases similar to those used for Fig. 8-5, except here the wiggler axial guide field and the wiggler transverse field were adjusted to give equivalent beam pitch values with the guide field at wiggler resonance (solid curve, filled circles, bottom axis) and far above wiggler resonance (dashed curve, open circles, top axis). Operation near resonance consistently results in higher predicted energy spread.
in the growth of competing instabilities and parasitic modes.

8.4 Space Charge Effects

The highest efficiency cases from the gyro-twt experiments all have relatively low measured beam currents—160 A for the May 12, 1993 case and 115 A for the May 24 case that achieved the top efficiency of 8%. These results are inconsistent with the CRM32 simulations, which predict relatively constant efficiency over a range of beam current up to at least 500 A for the experimental TE31 gyro-twt operating parameters. This leads to speculation that at currents > 200 A, the space-charge forces of the beam begin to have a deleterious effect. The condition under which space-charge effects can be neglected, \( s_e \ll s_e^0 \) (Eq. 2.59), appears to be well satisfied in all cases in Table 7.2 except perhaps for the April 30 case. The assumption is made for these cases that the electron beam in the interaction region has a uniform 5 mm radius. The TRAJ result shown in Fig. 5-13 confirms this approximation, though the beam cross-section upon entering the interaction region has a highly non-uniform distribution. For Fig. 5-13, however, the magnetic field is still tapering upwards to the peak value in the interaction region, which will compress the beam further. Indeed, the markings on the CVR copper strike plate used to measure the beam current (Sec. 7.2.2) show a 3–4 mm radius, which would increase the estimated \( s_e \) values in Table 7.2 by a factor of 1.5–2.5. Equation 2.59 is the conclusion made for a free-space CRM dispersion relation with no wave-guide effects and no cyclotron harmonics included. Inclusion of these effects would be an important step to understanding how important space-charge effects are for the gyro-twt operating parameters. The lower efficiencies at higher beam currents are suggestive of a deleterious effect from increased space charge.

8.5 Summary

In this thesis, results from the first multi-megawatt (4 MW, \( \eta = 8\% \)) harmonic relativistic gyrotron traveling-wave tube amplifier experiment have been presented. The first ever third harmonic gyro-twt results are reported, and the first detailed phase measurements of gyro-twt operation are also shown. The purpose of these experiments was to demonstrate high power, high gain amplification at 17.1 GHz. The gyro-twt experiments were driven by SNOMAD-II, an all solid-state linear induction accelerator with nominal beam parameters of 400 kV, 350 A, 30 ns flat-top, and capable of running at repetition rates up to 1 kHz. Simulations of SNOMAD-II and the magnetic focusing system used by the gyro-twt experiments predict beam spreads of \( \sigma_{\gamma}/\langle \gamma \rangle = 0.6\% \) and \( \sigma_{p_z}/\langle p_z \rangle = 0.4\% \) and a beam radius of \( \sim 6 \) mm after 40 cm of propagation. The beam is then imparted with perpendicular momentum by a three period bifilar helical wiggler magnet which generates a spatially rotating transverse on-axis magnetic field. With proper settings of axial guide field in the wiggler region, beam energy, and wiggler period, the beam resonates with the transverse wiggler field and begins corkscrewing around the axis of propagation with significant pitch, \( \alpha \).

After exiting the wiggler region, the beam is compressed into a region of higher axial field and then begins copropagating with an injected rf drive signal. The interaction field is tuned to achieve the appropriate gyro-twt interaction. Two different circular waveguide sizes were used for
the interaction circuit. A 2.54 cm (1 in) I.D. tube was used for third harmonic TE\textsubscript{31} amplification. A 1.905 cm (3/4 in) I.D. tube was used for second harmonic TE\textsubscript{21} amplification. The amplified pulse exits through a vacuum tight rf window into free space where the angular radiation pattern is measured with a calibrated horn, attenuator, and diode. This calibrated far field measurement and, independently, the use of a TE\textsubscript{31}-to-TE\textsubscript{11} in-guide converter are the basis for the gyro-twt amplified rf power results. A sliding kicker magnet was used to measure rf power versus interaction length in the gyro-twt experiments.

High power amplification was observed for both the third harmonic and the second harmonic experiments. The third harmonic experiment generated 4 MW of amplified rf power and 50 dB measured gain with ±2 dB absolute error attributable to measurement inaccuracies. The measured beam parameters for this case were $V = 380$ kV and $I = 160$ A. Other estimated parameters are $\alpha = 0.9$, $\sigma_\gamma/\langle \gamma \rangle = 3\%$, and $\sigma_{pz}/\langle p_z \rangle = 12\%$. The highest measured efficiency was 8% at 3.5 MW of rf power and 115 A beam current. The second harmonic experiment yielded 2 MW rf power and 4% efficiency with a measured gain of 40 dB. Superradiant modes were more noticeable in the second harmonic experiment. The measured rf power in the absence of the rf drive signal was often 10–15 dB lower than the amplified power. For the third harmonic experiment, the rf power in the absence of an rf drive signal was typically > 30 dB lower than the amplified power. The poor beam quality of $\sigma_\gamma/\langle \gamma \rangle = 3\%$ and $\sigma_{pz}/\langle p_z \rangle = 12\%$ results from the impartation of transverse momentum to the beam by the bifilar helical wiggler. A MIG gun may be more suitable for producing high pitch, high quality electron beams.

Frequency and phase of the amplified pulses were measured with YIG-tuned filters, a frequency mixing system, and a phase discriminator. The measured rf pulses are narrow in comparison to the voltage pulse, with the typical width being 10–15 ns. In addition, a characteristic frequency upchirp of $\sim 10$ MHz/ns was measured on the rf pulses and is attributable to the rounded top of the voltage pulse and, correspondingly, the beam pitch profile. The best measured phase stability of the high power TE\textsubscript{31} amplified pulses was $\pm 10^\circ$ over a 9 ns duration and $\pm 20^\circ$ over a 15 ns duration. These measurements were made on 3 MW pulses. The beam pitch profile was especially sensitive to voltage fluctuations due to operation of the gyro-twt far from wiggler resonance. This operating regime is predicted to have better beam quality than operating near wiggler resonance, and the narrower pulses also kept competing instabilities from growing and deleteriously affecting the amplified pulse. A longer pulse length would necessarily have required a sectioned interaction region with a sever to suppress oscillations.

In addition to the experimental work, several theoretical accomplishments were made in the effort to better explain the gyro-twt measurements. An existing program based on the nonlinear, single particle CRM equations of motion, CRM32 (Sec. 2.6), was improved to include full Bessel function coupling factors for waveguide problems, arbitrary axial magnetic field profiles, and arbitrary waveguide radius profiles. (The program was actually rewritten from scratch in C to improve functionality and portability). The arbitrary magnetic field profile and the Bessel function coupling terms are critical to explaining the measured data from the harmonic gyro-twt experiments.

An existing program to simulate particle orbits in the wiggler region, TRAJ (Sec. 5.7), was found to give inconsistent results when compared to the measured data from the gyro-twt results. The program, TRAJ, ignores longitudinal self-fields attributable to the electron beam. These fields should not be ignored for cases of high beam pitch, $\alpha \sim 1$. By ignoring the longitudinal self-fields,
we believe TRAJ overestimates beam energy spread. TRAJ also predicts beam pitch values that are lower than those consistent with CRM32 simulations based on the gyro-twt output power. This inconsistency was observed in a previous MIT experiment[1]. Because of the 3D nature of the wiggler problem, a full 3D particle-in-cell simulation may explain some of these discrepancies.

Finally, a program, FFMATCH (Sec. 7.2.3), was written using Powell's search algorithm to quickly determine best-fit mode mixes and total power corresponding to measured radiation patterns from the gyro-twt. This program is able to find very good fits to the gyro-twt data and forms the basis for many of our quoted power results. The power measurements were verified independently by using an in-guide TE_{31}-to-TE_{11} mode converter.

The experimental results of this thesis demonstrate the unique combination of stability and high power at high frequencies promised by the harmonic gyro-twt, even when beam quality is not ideal. The most promising area for the harmonic gyro-twt is likely at the 95 GHz frequency, where a high-gain, high-power amplifier is aggressively being pursued by both industry and the government due to the 95 GHz propagation window in the earth's atmosphere. A third (or higher) harmonic gyro-twt could conceivably generate a 95 GHz amplified pulse without the use of a superconducting magnet. The results from this thesis suggest that to be a successful contender, such a harmonic gyro-twt should not use a Pierce-wiggler beam formation system due to the poor resulting beam quality, particularly the high energy spread. Also, a multi-sectioned interaction with severs would reduce competing instabilities for long pulses.
Bibliography


