

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \left(\frac{u}{V_\infty}\right)^2 - \left(\frac{v}{V_\infty}\right)^2$$

a) Source:  $u = V_\infty + \frac{\Lambda}{2\pi} \frac{x}{x^2+y^2}$   
 $v = \frac{\Lambda}{2\pi} \frac{y}{x^2+y^2}$

Along  $-x, y=0$ :  $C_p = 1 - \frac{u^2}{V_\infty^2} = 1 - \left(\frac{V_\infty}{V_\infty} - \frac{\Lambda}{2\pi V_\infty} \frac{1}{x}\right)^2 = \left[\frac{\Lambda}{\pi V_\infty} \frac{1}{x} - \left(\frac{\Lambda}{2\pi V_\infty}\right)^2 \frac{1}{x^2}\right]$   
 Along  $y, x=0$ :  $C_p = 1 - \left(\frac{V_\infty}{V_\infty}\right)^2 - \left(\frac{\Lambda}{2\pi V_\infty}\right)^2 \frac{1}{y^2} = -\left(\frac{\Lambda}{2\pi V_\infty}\right)^2 \frac{1}{y^2}$

*dominant* *much smaller for large x*

b) Vortex:  $u = V_\infty + \frac{\Gamma}{2\pi} \frac{y}{x^2+y^2}$   
 $v = \frac{\Gamma}{2\pi} \frac{-x}{x^2+y^2}$

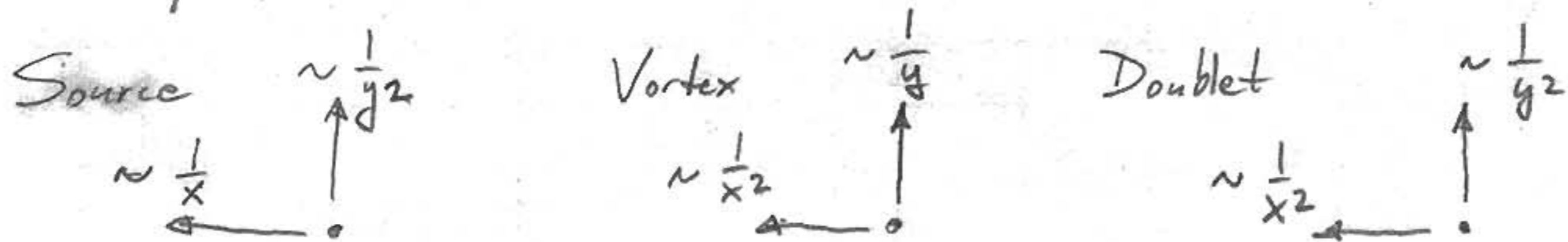
Along  $-x, y=0$ :  $C_p = 1 - \left(\frac{V_\infty}{V_\infty}\right)^2 - \left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \frac{1}{x^2} = -\left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \frac{1}{x^2}$   
 Along  $y, x=0$ :  $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} + \frac{\Gamma}{2\pi V_\infty} \frac{1}{y}\right)^2 = \left[-\frac{\Gamma}{\pi V_\infty} \frac{1}{y} - \left(\frac{\Gamma}{2\pi V_\infty}\right)^2 \frac{1}{y^2}\right]$

*dominant*

c) Doublet:  $\phi = V_\infty x + \frac{K}{2\pi} \frac{x}{x^2+y^2}$   $\left\{ \begin{aligned} u &= \frac{\partial \phi}{\partial x} = V_\infty + \frac{K}{2\pi} \frac{y^2-x^2}{(x^2+y^2)^2} \\ v &= \frac{\partial \phi}{\partial y} = \frac{K}{2\pi} \frac{-2xy}{(x^2+y^2)^2} \end{aligned} \right.$

Along  $-x, y=0$ :  $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} - \frac{K}{2\pi V_\infty} \frac{1}{x^2}\right)^2 = \frac{K}{\pi V_\infty} \frac{1}{x^2} - \left(\frac{K}{2\pi V_\infty}\right)^2 \frac{1}{x^4}$   
 Along  $y, x=0$ :  $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} + \frac{K}{2\pi V_\infty} \frac{1}{y^2}\right)^2 = -\frac{K}{\pi V_\infty} \frac{1}{y^2} - \left(\frac{K}{2\pi V_\infty}\right)^2 \frac{1}{y^4}$

The  $C_p$  fields decrease with distance as follows:



Far away the  $\frac{1}{x}$  and  $\frac{1}{y}$  terms dominate ( $\frac{1}{x^2}$  and  $\frac{1}{y^2}$  die off much faster)

A lifting airfoil has a nonzero  $\Gamma$ , so it looks mostly like a vortex far away. Largest  $C_p$  is above & below.

