

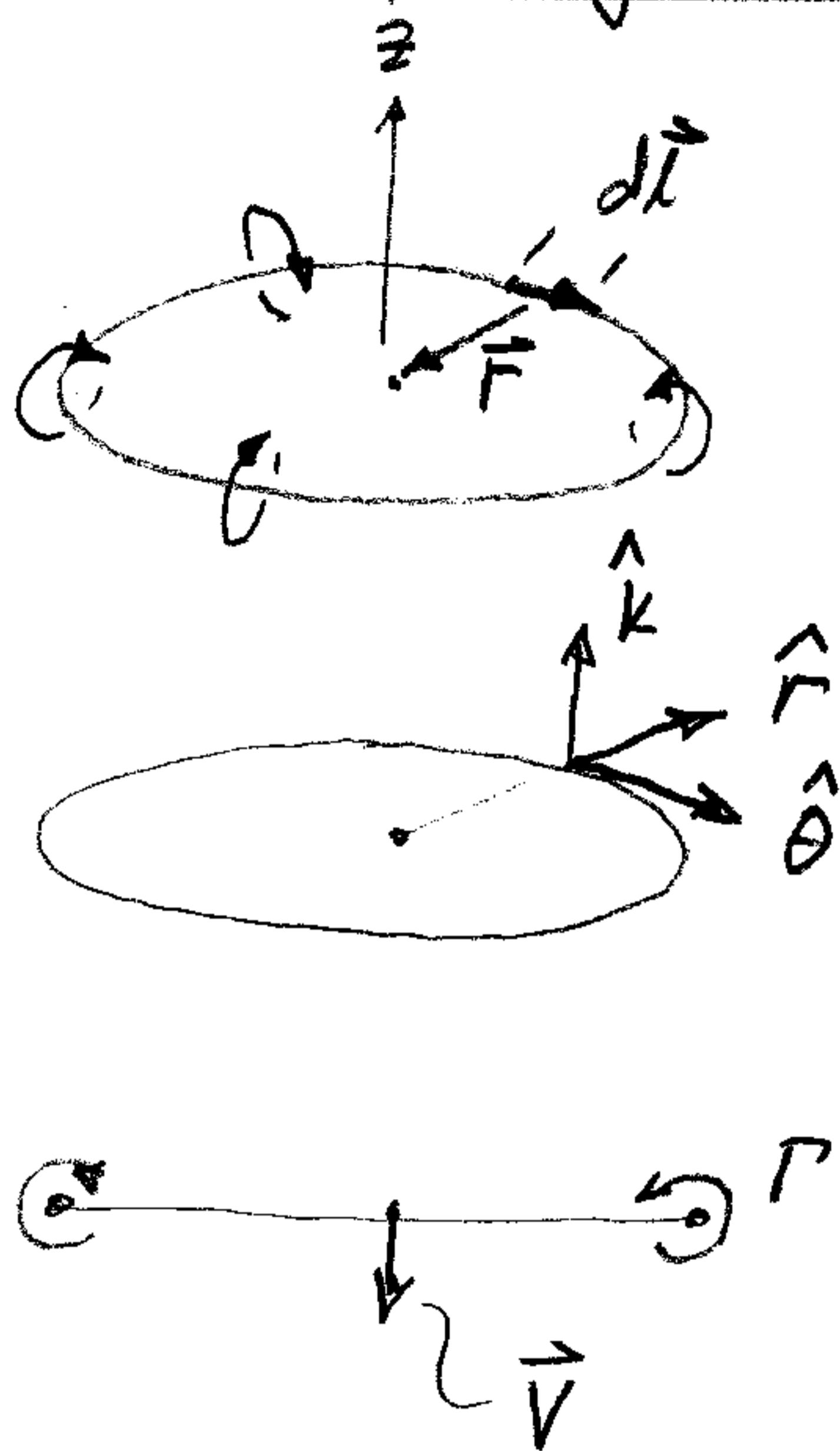
(Anderson p 416)

$$1. \vec{V} = \frac{\Gamma}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$d\vec{l} = R d\theta \hat{\theta}, \quad \vec{r} = -R \hat{r}, \quad r = R$$

$$d\vec{l} \times \vec{r} = -R^2 d\theta \hat{\theta} \times \hat{r} = -R^2 d\theta \hat{k}$$

$$\vec{V} = \frac{\Gamma}{4\pi} \int_0^{2\pi} \frac{-R^2 d\theta \hat{k}}{R^3} = -\frac{\Gamma}{2R} \hat{k}$$



2. Now we have

$$d\vec{l} = R d\theta \hat{\theta}, \quad \vec{r} = -R \hat{r} + A \hat{k}$$

$$r^3 = (R^2 + A^2)^{3/2}$$

$$d\vec{l} \times \vec{r} = (-R^2 (\hat{\theta} \times \hat{r}) + RA (\hat{\theta} \times \hat{k})) d\theta = (-R^2 \hat{k} - RA \hat{r}) d\theta$$

Note that \hat{k} is constant, but \hat{r} depends on θ (varies around circle)

$$\vec{V} = \frac{\Gamma}{4\pi} \int_0^{2\pi} \frac{-R^2 \hat{k} - RA \hat{r}}{(R^2 + A^2)^{3/2}} d\theta$$

$$\vec{V} = -\frac{\Gamma}{2} \frac{R^2}{(R^2 + A^2)^{3/2}} \hat{k} + \frac{\Gamma}{4\pi} \frac{-RA}{(R^2 + A^2)^{3/2}} \int_0^{2\pi} \hat{r} d\theta$$

But we note that $\int_0^{2\pi} \hat{r} d\theta = 0$, since \hat{r} cancels when integrated around perimeter.

$$\vec{V} = -\frac{\Gamma}{2} \frac{R^2}{(R^2 + A^2)^{3/2}} \hat{k}$$

