

**Massachusetts Institute of Technology**  
**Department of Aeronautics and**  
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**Cambridge, MA 02139**

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**16.01/16.02 Unified Engineering I, II**  
**Fall 2003**

**Problem Set 15**

Name: \_\_\_\_\_

Due Date: Not Due

	<b>Time Spent (min)</b>
<b>F21</b>	
<b>F22</b>	
<b>M23</b>	
<b>M24</b>	
<b>M25</b>	
<b>Study Time</b>	

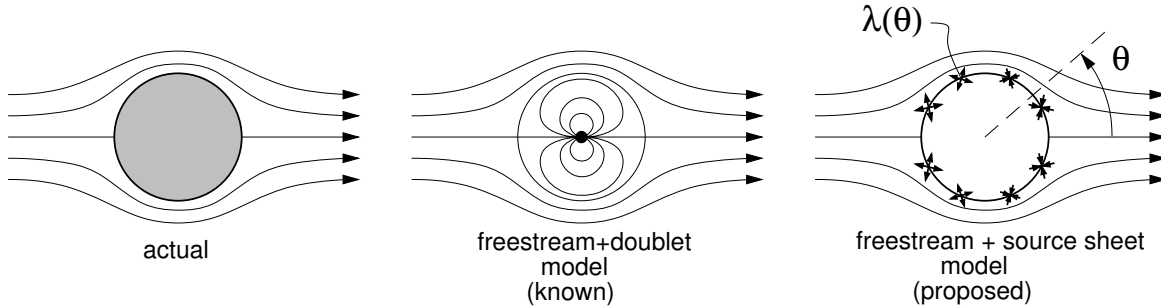
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Announcements: Good luck on your finals. Reminder: The unified final is on Monday, 12/15 at 9am

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F21. As shown in class, the nonlifting irrotational flow past a circular cylinder can be represented by superimposing the uniform freestream flow and a doublet. An alternative representation is proposed using a source sheet placed on the cylinder surface as shown. The proposed sheet strength distribution about the cylinder is  $\lambda(\theta) = -2V_\infty \cos \theta$ . There is no vortex sheet, so on the surface,  $\gamma = 0$ .



You are to determine whether the new model is correct.

a) Determine the velocity at point A from the known exterior surface velocities for the cylinder.

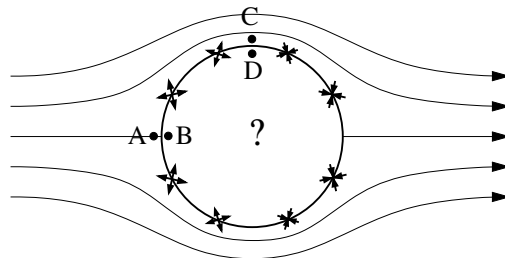
$$V_\theta(\theta) = -2V_\infty \sin \theta \quad , \quad V_r = 0$$

Using the sheet jump relations,

$$\Delta V_n = \lambda \quad , \quad \Delta V_s = \gamma$$

determine the interior velocity at point B.

- b) Again using the known exterior  $V_\theta(\theta), V_r$  result at point C, use the sheet jump condition to determine the velocity at point D.
- c) Compare velocities at B and D. What appears to be the fictitious velocity inside the cylinder?
- d) Is the source sheet jump  $\Delta V_n = \lambda$  consistent with the exterior and interior normal flows everywhere on the cylinder surface? Is the proposed source-sheet model correct?



F22. A long rectangular wing has span  $b$  and chord  $c$ , and hence the wing area is  $S = bc$ .

a) The wing airfoil has certain lift and drag coefficients  $c_\ell$  and  $c_d$  which are constant across the span. Determine how these relate to the wing's overall  $C_L$  and  $C_D$ .

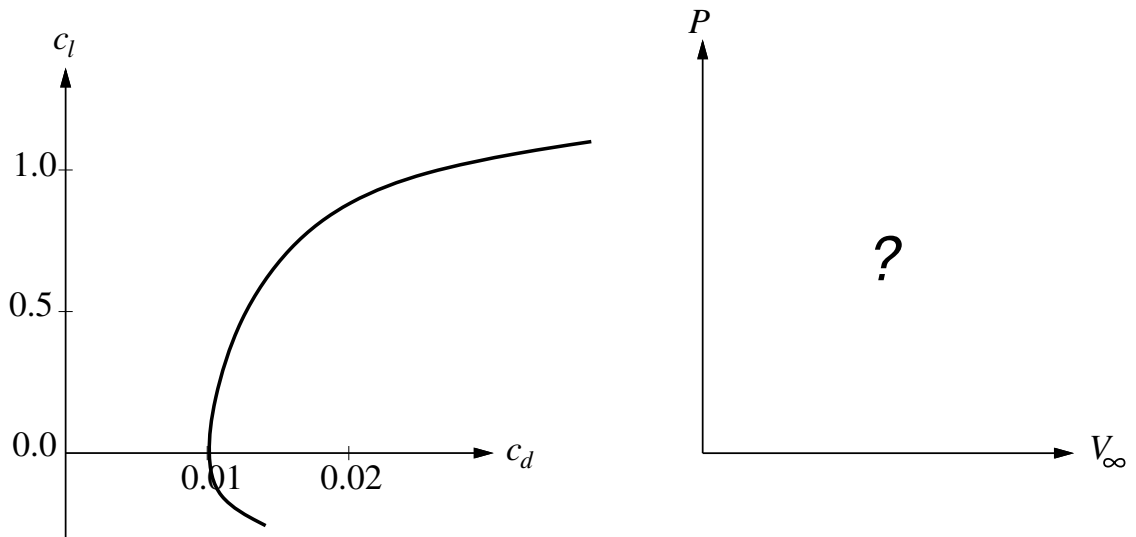
(Hint: Determine  $L'$  and  $D'$ , then get  $L$  and  $D$ , then from these determine  $C_L$  and  $C_D$ ).

The wing airfoil has a drag polar which can be approximated by

$$c_d \simeq 0.01 + 0.015 c_\ell^3$$

in the range  $c_\ell = 0.1 \dots 1.2$ . The propulsive power  $P$  needed to overcome drag  $D$  at flight speed  $V_\infty$  is given by

$$P = DV_\infty$$



b) Determine the form of the  $P(V_\infty)$  relation in level flight, and plot it for the range  $c_\ell = 0.1 \dots 1.2$ . Any constant multiplicative factors on the  $P$  and  $V_\infty$  axes are not important – only the shape of the curve is of interest. Hint: Simplest approach is to plot  $P(c_\ell)$  versus  $V(c_\ell)$  with  $c_\ell$  as a dummy parameter.

(Note: Using only the airfoil's  $c_d$  ignores other contributions such as induced drag, which become especially significant at low flight speeds!)

**Problem M23 (Materials and Structures)**

The potential energy,  $U$  of a pair of atoms in a solid can be written as:

$$U = \frac{A}{r^m} + \frac{B}{r^n}$$

where  $r$  is the separation of the atoms and  $A$ ,  $B$ ,  $m$  and  $n$  are positive constants. Indicate the physical significance of the two terms in this equation.

A material has a simple cubic unit cell with atoms placed at the corners of the cubes. Show that, when the material is stretched in a direction parallel to one of the cube edges, Young's modulus  $E$  is given by:

$$E = \frac{mnkT_M}{\Omega}$$

Where  $\Omega$  is the mean atomic volume,  $k$  is Boltzmann's constant and  $T_M$  is the absolute melting temperature of the solid. You may assume that  $U_0(r_0) = \Omega k T_M$ , where  $r_0$  is the equilibrium separation of a pair of atoms.

**Problem M24**

Two metals of current and historical interest for aerospace applications, nickel and magnesium, have face centered cubic and close packed hexagonal structures respectively.

- Assuming that the atoms can be represented as hard spheres, calculate the percentage of the volume occupied by atoms in each material.
- Calculate, from first principles, the dimensions of the unit cell in nickel and in magnesium. (The densities of nickel and magnesium are  $8.90 \text{ Mgm}^{-3}$  and  $1.74 \text{ Mgm}^{-3}$  respectively, the atomic weight of Nickel is 58.69, Magnesium is 24.31, Avogadro's number is  $6.023 \times 10^{23}$ ).

### **Problem M25**

In addition to chapters 4-7 of Ashby and Jones Engineering Materials, you may also find the chapters on polymers in Ashby and Jones, Engineering Materials 2, helpful (this is a green covered book, available in the Aero-Astro library).

- a) Define the term *polymer* and list three engineering polymers.
  
  
  
  
  
  
  
  
  
  
- b) Define a *thermoplastic* and a *thermoset*.
  
  
  
  
  
  
  
  
  
  
- c) Distinguish between a cross-linked and a non-cross-linked polymer.
  
  
  
  
  
  
  
  
  
  
- d) What is the glass transition temperature?
  
  
  
  
  
  
  
  
  
  
- e) Explain the change in moduli of polymers at the glass transition temperature.
  
  
  
  
  
  
  
  
  
  
- f) What is the range of temperature in which  $T_G$  lies for most engineering polymers?
  
  
  
  
  
  
  
  
  
  
- g) How would you increase the modulus of a polymer?