M13

Need $\frac{\partial \sigma_{mn}}{\partial x_m} + f_n = 0$ $\qquad f_n = 0$

a)

① $\frac{\partial \sigma_{11}}{\partial x_1}^{=0} + \frac{\partial \sigma_{21}}{\partial x_2}^{=0} + \frac{\partial \sigma_{31}}{\partial x_3} = 0$

Since $\frac{\partial \sigma_{11}}{\partial x_1} = 0$, $\sigma_{21} = 0 \Rightarrow \frac{\partial \sigma_{31}}{\partial x_3} = 0$

② $\frac{\partial \sigma_{12}}{\partial x_1}^{=0} + \frac{\partial \sigma_{22}}{\partial x_2}^{=0} + \frac{\partial \sigma_{32}}{\partial x_3}^{=0} = 0$

Since $\sigma_{12} = \sigma_{22} = \sigma_{32} = 0$ no additional informati

③ $\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3}$

Since $\sigma_{23} = \sigma_{33} = 0 \Rightarrow \frac{\partial \sigma_{13}}{\partial x_1} = 0$

$\frac{\partial \sigma_{31}}{\partial x_3} = \frac{\partial \sigma_{13}}{\partial x_1} = 0$

† Since $\sigma_{13} = 0$ @ $\pm h$

$\therefore \sigma_{13} = 0$ ~~everywhere~~ in x_3

σ_{13} = Constant in x_1 ∈

from (1)

6) $\sigma_{11} = C\left(\frac{M}{I}\right) x_3 x_1$

$$\frac{\partial \sigma_{11}}{\partial x_1} = \left(\frac{C M}{I}\right) x_3$$

$$\Rightarrow \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{31}}{\partial x_3} = 0 \Rightarrow \frac{\partial \sigma_{31}}{\partial x_3} = -\left(\frac{CM}{I}\right) x_3$$

$$\sigma_{31} = -\left(\frac{CM}{I}\right)\frac{x_3^2}{2} + D$$

but $\sigma_{31} = 0$ for $\pm h$.

$$\therefore D = \frac{CM}{I}\frac{h^2}{2} \Rightarrow \sigma_{31} = \frac{CM}{2I}\left(h^2 - x_3^2\right) \Longleftarrow$$

from (3) $\frac{\partial \sigma_{13}}{\partial x_1} = 0$ i.e. Shear stress constant along length.