

M15) i) Consider element of initial sides 1 unit in length

deformed element has sides $1 + \epsilon_1, 1 + \epsilon_2, 1 + \epsilon_3$

$$\therefore \text{Volume (deformed)} = (1 + \epsilon_1)(1 + \epsilon_2)(1 + \epsilon_3)$$

neglecting high order terms ($\epsilon_1 \epsilon_2, \epsilon_1 \epsilon_3, \epsilon_2 \epsilon_3, \epsilon_1 \epsilon_2 \epsilon_3$)

$$\text{deformed volume} = 1 + \epsilon_1 + \epsilon_2 + \epsilon_3$$

$$\therefore \frac{V_{\text{def}} - V_{\text{undef}}}{V_{\text{undef}}} = \epsilon_1 + \epsilon_2 + \epsilon_3 \quad \Leftarrow$$

ii)

$$\text{ii a) } \varepsilon_{11} = \frac{\partial U_1}{\partial x_1} = (x_1 + 0.5x_2) \times 10^{-3}$$

$$\varepsilon_{22} = \frac{\partial U_2}{\partial x_2} = (0.5x_2 - x_1) \times 10^{-3}$$

$$\varepsilon_{33} = \frac{\partial U_3}{\partial x_3} = 0$$

$$\begin{aligned} \varepsilon_{12} &= \frac{1}{2} \left(\frac{\partial U_1}{\partial x_2} + \frac{\partial U_2}{\partial x_1} \right) = \frac{1}{2} \left(-x_2 + 0.5x_1 + 0.5x_1 - x_2 \right) \\ &= \frac{1}{2} (x_1 - 2x_2) \times 10^{-3} \end{aligned}$$

$$\varepsilon_{23} = \frac{1}{2} \left(\frac{\partial U_2}{\partial x_3} + \frac{\partial U_3}{\partial x_2} \right) = 0$$

$$\varepsilon_{13} = \frac{1}{2} \left(\frac{\partial U_3}{\partial x_1} + \frac{\partial U_1}{\partial x_3} \right) = 0$$

6) Rigid body rotation

$$\Theta = \frac{1}{2} \left(\frac{\partial U_1}{\partial x_2} - \frac{\partial U_2}{\partial x_1} \right)$$

$$= \frac{1}{2} \left(-x_2 + 0.5x_1 - 0.5x_1 - x_2 \right) \times 10^{-3}$$

$$= \frac{1}{2} (-2x_2) = -x_2 \times 10^{-3}$$

c) at $x_1 = 5\text{mm}$, $x_2 = 7\text{mm}$.

$$\epsilon_{11} = \left(5 + \frac{7}{2}\right) \times 10^{-3} = 8.5 \times 10^{-3}$$

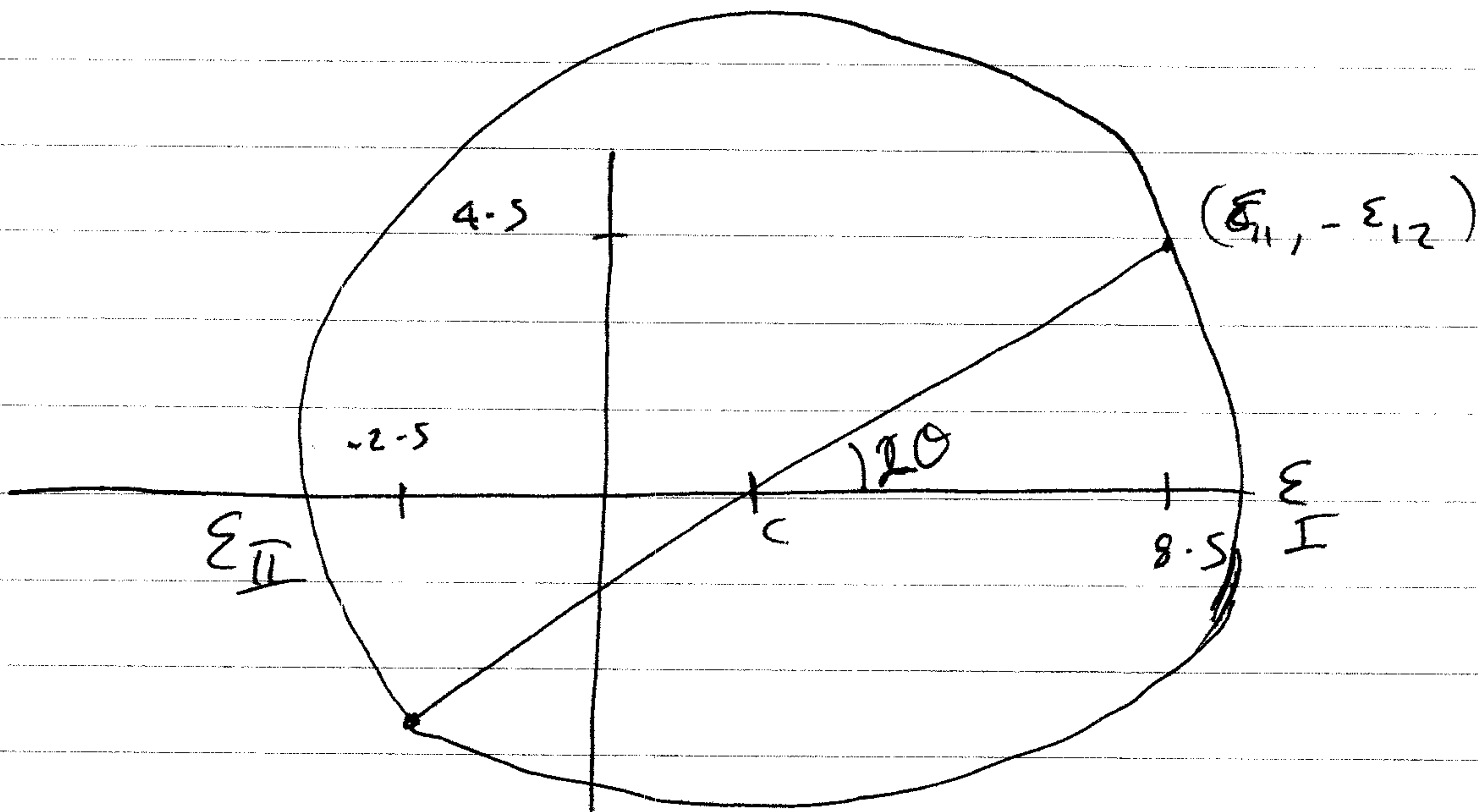
$$\epsilon_{22} = \left(\frac{7}{2} - 5\right) \times 10^{-3} = -2.5 \times 10^{-3}$$

$$\epsilon_{12} = \frac{1}{2} (5 - 14) \times 10^{-3} = -4.5 \times 10^{-3}$$

$$\epsilon_{33} = 0$$

d) Volumetric strain = $(8.5 + (-2.5) + 0) \times 10^{-3} = 1.5 \times 10^{-3}$

c)



Center @ $\frac{1}{2} [8.5 + (-2.5)] \times 10^{-3} = +3 \times 10^{-3}$

Radius = $10^3 \sqrt{(4.5)^2 + (5.5)^2} = 7.11 \times 10^{-3}$

$\epsilon_I = 3 \times 10^{-3} + 7.11 \times 10^{-3} = 10.11 \times 10^{-3}$

$\epsilon_{II} = 3 \times 10^{-3} - 7.11 \times 10^{-3} = -4.11 \times 10^{-3}$

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{4.5}{5.5} \right) = 19.6^\circ \text{ clockwise from } x,$$

d) Volumetric strain =

$$10.11 + (-4.11) + 0 = 6 \times 10^{-3} \text{ } \equiv$$