The node equations are:

\[ e_1 : \left( c_2 \frac{d}{dt} + 6 \right) e_1 - c_2 \frac{d}{dt} e_2 = 0 \]

\[ -c_2 \frac{d}{dt} e_1 + \left( c_1 \frac{d}{dt} + c_2 \frac{d}{dt} + G_5 \right) e_2 - c_1 \frac{d}{dt} e_3 = 0 \]

\[ -c_1 \frac{d}{dt} e_2 + \left( c_1 \frac{d}{dt} + G_3 \right) e_3 = 0 \]

Plugging in component values,

\[ \left( 2 \frac{d}{dt} + 1 \right) e_1 - 2 \frac{d}{dt} e_2 = 0 \]

\[ -2 \frac{d}{dt} e_1 + \left( 3 \frac{d}{dt} + 1 \right) e_2 - \frac{d}{dt} e_3 = 0 \]

\[ -\frac{d}{dt} e_2 + \left( \frac{d}{dt} + 0.5 \right) e_3 = 0 \]

To find the solution, assume

\[ e_1(t) = E_1 e^{st} \]

\[ e_2(t) = E_2 e^{st} \]

Then

\[ (2s + 1) E_1 - 2s E_2 = 0 \]

\[ -2s E_1 + (3s + 1) E_2 - s E_3 = 0 \]

\[ -s E_2 + (s + 0.5) E_3 = 0 \]
In matrix form,

\[
\begin{bmatrix}
2s+1 & -2s & 0 \\
-2s & 3s+1 & -s \\
0 & -s & s+0.5
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} = 0
\]

= \mathbf{M}(s) \mathbf{E}

For there to be a nontrivial solution,

\[
\det(\mathbf{M}(s)) = 0
\]

= 5s^2 + 3.5s + 0.5

This equation can be solved by using the quadratic formula, or a polynomial solver. The roots are

\[
s_1 = -0.2 \text{ sec}^{-1} \\
s_2 = -0.5 \text{ sec}^{-1}
\]

Solve for \( \mathbf{E} \) in each case:

\[
s_1 = -0.2 \quad \Rightarrow \quad \mathbf{M}(s) = \begin{bmatrix}
0.6 & 0.4 & 0 \\
0.4 & 0.4 & 0.2 \\
0 & 0.2 & 0.3
\end{bmatrix}
\]

Normally, would solve by row reduction. Because of the zeros in \( \mathbf{M} \), can solve as follows: Set \( E_3 = 1 \). From last row of \( \mathbf{M} \),

\[
0.2E_2 + 0.3E_3 = 0
\]

\[
\Rightarrow \quad E_2 = -1.5
\]
From row 1 of $M$,
\[0.6E_1 + 0.4E_2 = 0\]
\[\Rightarrow E_1 = 1\]

So
\[E_1 = \begin{bmatrix} 1 \\ -1.5 \\ 1 \end{bmatrix}\]

(Of course, any multiple of this is also a solution.)

\[S_2 = -0.5 \quad \Rightarrow \quad M(s) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -0.5 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix}\]

From row 1 (or row 3),
\[+E_2 = 0 \quad \Rightarrow \quad E_2 = 0\]

Arbitrarily choose $E_3 = 1$. Then from row 2,
\[+E_1 - 0.5E_2 + 0.5E_3 = 0\]
\[\Rightarrow E_1 = -0.5\]

Therefore,
\[E_2 = \begin{bmatrix} -0.5 \\ 0 \\ 1 \end{bmatrix}\] (or any multiple)

Total solution

The total solution is given by
\[
\begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{pmatrix} = aE_1 e^{s_1 t} + bE_2 e^{s_2 t}
\]
From the circuit,
\[ U_1(t) = e_3(t) - e_2(t) \]
\[ U_2(t) = e_1(t) - e_2(t) \]

To match the initial conditions,
\[ U_1(0) = 10 \, \text{V} = a \left( 1 + 1.5 \right) e_0 + b \left( 1 - 0 \right) e_0 \]
\[ = 2.5a + b \]
\[ U_2(0) = 0 \, \text{V} = a \left( 1 + 1.5 \right) e_0 + b \left( -0.5 - 0 \right) e_0 \]
\[ = 2.5a - 0.5b \]

Therefore,
\[
\begin{cases}
2.5a + b = 10 \\
2.5a - 0.5b = 0
\end{cases}
\Rightarrow \begin{cases}
a = 1.333 \\
b = 6.667
\end{cases}
\]

The final solution is then
\[
U_1(t) = \left( 3.333 \, e^{-0.2t} + 6.667 \, e^{-0.5t} \right) \, \text{V}
\]
\[
U_2(t) = \left( 3.333 \, e^{-0.2t} - 3.333 \, e^{-0.5t} \right) \, \text{V}
\]

N.B.: Corrected lines are marked with an asterisk.