M11 Torsion of circular cross-section shafts

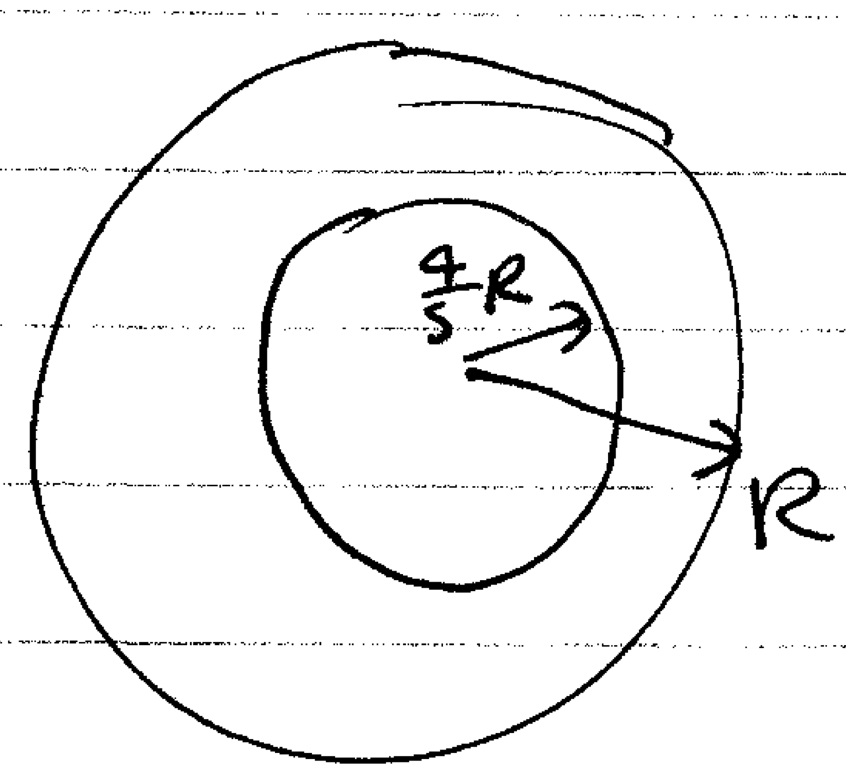
a) Torque shear stress relation: $\tau = \dfrac{Tr}{J}$

for solid circular cross section $J = \dfrac{\pi R^4}{2}$

$$\therefore \quad R^3 = \frac{2T\cancel{R}}{\pi \tau_y} \Rightarrow R = \sqrt[3]{\frac{2T}{\pi \tau_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 200 \times 10^6}}$$

$= 0.086\,m \Rightarrow$ ~~R~~ diameter $= 0.172\,m \Leftarrow$

b) Hollow shaft



$$J = \frac{\pi R^4}{2} - \frac{\pi}{2}\left(\frac{4}{5}\right)^4 R^4$$
$$= \frac{\pi R^4}{2}\left(1 - \left(\frac{4}{5}\right)^4\right)$$
$$= \frac{\pi R^4}{2}\left(1 - \frac{256}{625}\right)$$

$$\therefore \quad \frac{\pi R^{3\cancel{4}}}{2}\left(0.5904\right) = \frac{T\cancel{R}}{\tau_y}$$

$$R = \sqrt[3]{\frac{2T}{\pi(0.5904)\tau_y}} = \sqrt[3]{\frac{2 \times 200 \times 10^3}{\pi \times 0.5904 \times 200 \times 10^6}} = 0.103\,m$$

$\therefore$ diameter $= 0.205\,m. \Leftarrow$

Fractional weight $= \frac{\cancel{\pi}\left(0.086^2 - (0.103)^2\left(1-\left(\frac{4}{5}\right)^2\right)\right)}{\cancel{\pi}(0.086)^2}$

$= 1 - \left(\frac{0.103}{0.086}\right)^2\left(1-\left(\frac{4}{5}\right)^2\right)$

$= 0.484 \quad = 48.4\%$ weight saving.

c) Twist angle. from $T = GJ\frac{d\phi}{dx}$

Since Torque is constant, twist angle $= L\frac{d\phi}{dx}$

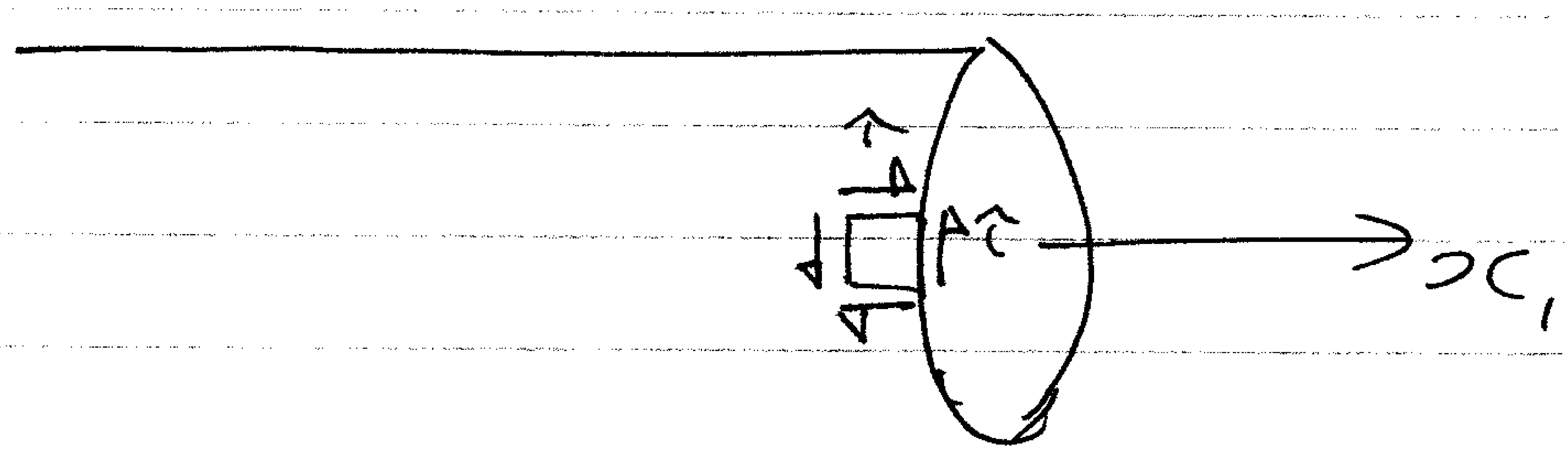
$\therefore$ ratio of twist angles is ratio of J

$= \frac{\pi(0.103)^4}{x}\left(1-\frac{256}{625}\right) \times \frac{x}{\pi(0.086)^4}$

$= \frac{(0.103)^4}{(0.086)^4}\left(1-\frac{256}{625}\right) = 1.215$

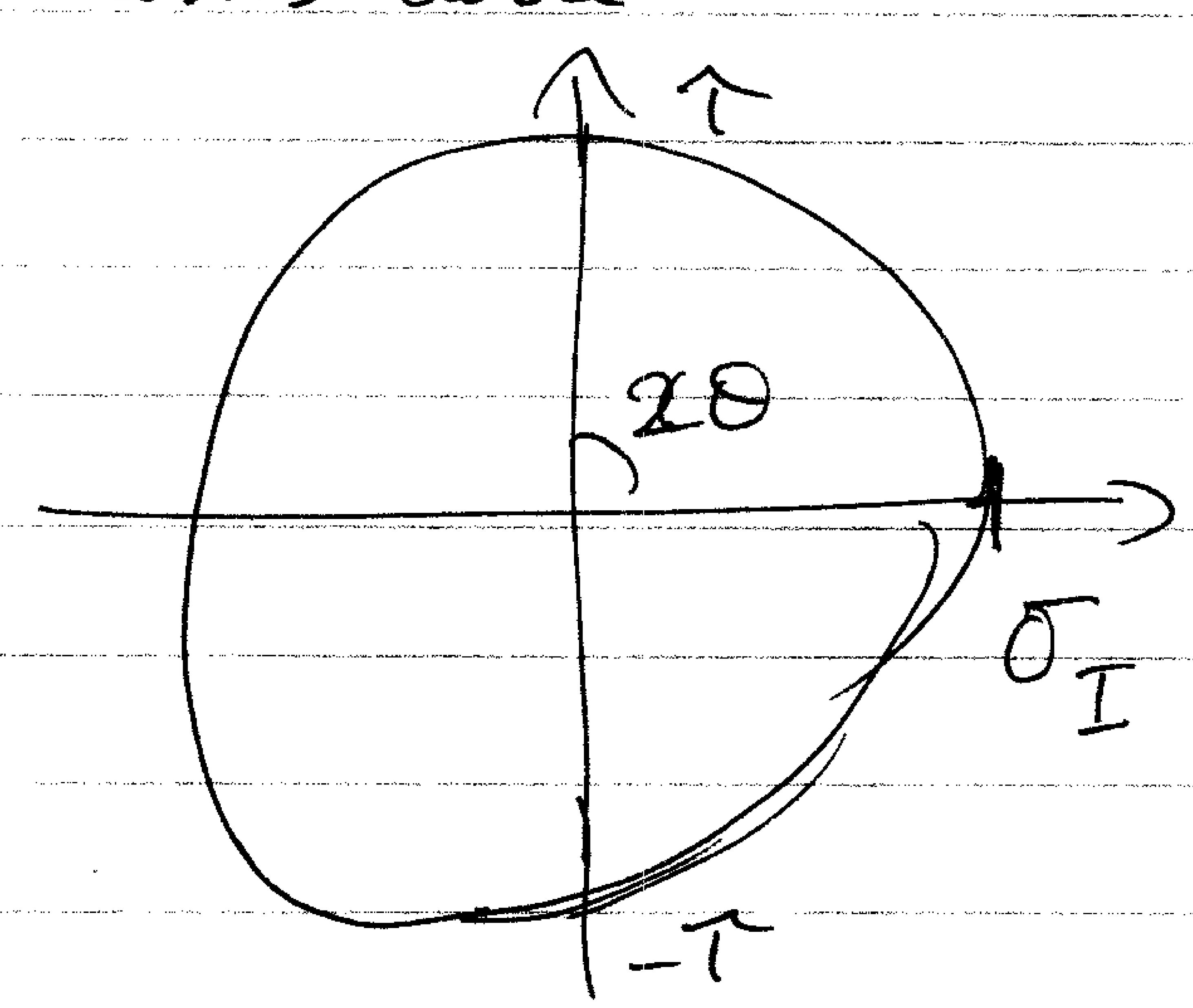
ie. shaft (a) will twist 21.5% more than shaft (b).

d.



in case b (or a) shear stress τ acts in plane of section.

Drawing Mohrs circle



max tensile stress, σ_I will act at 45° to max shear ie.