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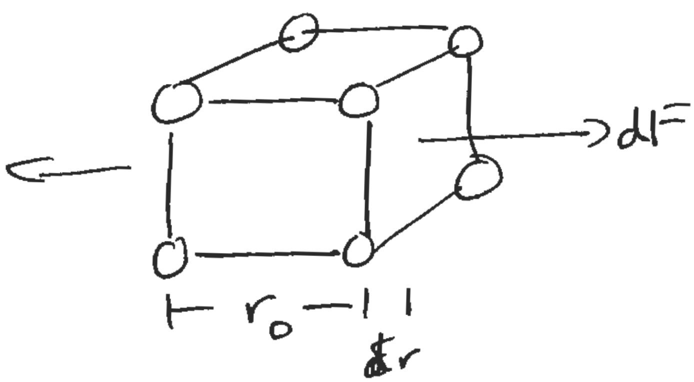
SOLUTIONS

$$U = -\frac{A}{r^m} + \frac{B}{r^n} \qquad (0)$$

long range electrostatic attraction

Short range repulsion Inner electron orbitals, nuclei.

Cubic unit cell



Young's modulus

$$= \frac{dF}{A} \cdot \frac{dr_0}{dr} = \frac{dF}{dr}\frac{r_0}{r_0^2}$$

$$= \frac{1}{r_0}\frac{dF}{dr} = \frac{1}{r_0}\left.\frac{d^2U}{dr^2}\right|_{r=r_0} \qquad (1)$$

$$\frac{dU}{dr} = mAr^{(-m-1)} - nBr^{(-n-1)}, \quad \cancel{\frac{d^2U}{dr^2}=}$$

at $r = r_0$ $\quad \frac{dU}{dr} = 0$

$$\Rightarrow B = \frac{m}{n}Ar_0^{n-m} \qquad \text{Substitute into } (0)$$

$$\Rightarrow U = -Ar^{-m} + \frac{m}{n}Ar_0^{n-m}r^{-n} \qquad (2)$$

$$\therefore U(r_0) = -Ar_0^{-m} + \frac{m}{n}Ar_0^{(n-m-n)} = Ar_0^{-m}\left(\frac{m}{n} - 1\right) = Ar_0^{-m}\left(\frac{m-n}{n}\right)$$

From definition in question

$$-KT_m = A r_0^{-m}\left(\frac{m-n}{n}\right)$$

$$\therefore\ A = \frac{n}{(m-n)} K T_m r_0^{m}, \qquad B = -\frac{m}{m-n} K T_m \cancel{r_0^{m}} . r_0^{\cancel{n-m}} = \frac{m}{m-n} K T_m r_0^{n}$$

$$U = +\frac{n}{m-n} K T_m \frac{r_0^{m}}{r^{m}} - \frac{m}{m-n} K T_m \frac{r_0^{n}}{r^{n}}$$

$$\therefore \frac{dU}{dr} = -\frac{mn}{m-n} K T_m r_0^{m} r^{(-m-1)} + \frac{nm}{m-n} K T_m r_0^{n} r^{(-n-1)}$$

$$\frac{dF}{dr} = +\frac{(m+1)mn}{m-n} K T_m r_0^{m} r^{-(m+2)} - \frac{(n+1)nm}{m-n} K T_m r_0^{n} r^{-(n+2)}$$

$$\text{for } r=r_0,\ \frac{1}{r_0}\frac{dF}{dr} = +\frac{(m+1)mn}{m-n} K T_m \cancel{r_0^{m}} \cdot \cancel{r_0^{-m-2}} . r_0^{-1} - \frac{(n+1)nm}{m-n} K T_m \cancel{r_0^{n}}\ \cancel{r_0^{-n-2}}\ r_0^{-1}$$

$$= \frac{mnKT_m}{\cancel{(m-n)}\, r_0^{3}}\Big(\underbrace{+(m+1) - (n+1)}_{=\ \cancel{m-n}}\Big), \qquad r_0^{3} = \Omega$$

$$\therefore\ E = \frac{1}{r_0}\frac{dF}{dr}\bigg|_{r=r_0} = \frac{mnKT_m}{\Omega}\ !$$

The purpose of this question is to demonstrate intrinsic link between moduli & T_m. Diamond, SiC have high E high T_m, Polymers have low E, low T_m.