Answer 9.3
We are interested in the vertical diffusion of PCE from its source at z = 0 upward into the aquifer. The groundwater is given to be stagnant, so that u = v = w = 0. In addition, all groundwater is laminar, so we will use the laminar (slow mixing) model of dissolution. That is, the boundary source is modeled by a fixed concentration boundary condition. We assume that the DNAPL pool is large in lateral extent and uniform in concentration, so that the lateral gradients of PCE are negligible ($\partial C/\partial x = \partial C/\partial y = 0$). No additional sources or sinks are named, so that S = 0. The system cannot be in steady state, because there is a source at the lower boundary and no sink, so that $\partial M/\partial t > 0$ within the aquifer. Directly at the boundary, z = 0, the concentration is assumed to be at the solubility concentration, $C_o = 150\ mgl^{-1} = 150$ ppm. We assume that the total PCE in the DNAPL pool is sufficiently large that C_O remains constants, even as PCE dissolves out of the pool. With these assumptions we may write the transport equation and boundary conditions,

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2},$$

C = 0, for all z, at t < 0

$C = C_o = 150$ ppm at z = 0 for $t \geq 0$.

The PCE concentration profile evolves according to,

$$C(z,t) = C_o\ \mathrm{erfc}\left(\frac{z}{2\sqrt{Dt}}\right).$$

The maximum concentration in the aquifer will be at the source (z = 0) and the minimum concentration at z = 2m, until a uniform concentration is reached throughout the aquifer. Thus, to find C > 5 ppb throughout the aquifer, we need only find when C(z=2 m) = 5 ppb. From the above equation, we need to find when

$$\mathrm{erfc}\left(\frac{z}{2\sqrt{Dt}}\right) = \frac{0.005}{150} = 0.000033.$$

Interpolating from the table at the end of chapter 9,

$$\mathrm{erfc}(2.94) \approx 0.000033$$

$$\frac{2\mathrm{m}}{2\sqrt{\left(4.4\ \mathrm{x}10^{-9}\,\mathrm{m}^2\mathrm{s}^{-1}\right)t}} = 2.94$$

$t = 2.6 \times 10^7$ s = 304 days