Answer 2.1

Case1. Phosphorus is removed at a constant rate, with removal completed in T_R.

The removal rate is $\partial C/\partial t = (C_O/T_R)$, such that $C(t) = C_o(1 - t/T_R)$ for $t \le T_R$, and $C(t) = 0$ for $t > T_R$. From Chapter 2.5 equation (21), $C_e = \int_0^\infty RTD(t)\, C(t)\, dt$.

For Plug-Flow, $RTD(t) = \delta(t-T_R)$, then $C_e = \int_0^{T_R} \delta(t - T_R)\, C_0 (1 - t/T_R) dt = 0.$

Note the integration is only be carried to $t = T_R$, because for $t > T_R$, $C = 0$.

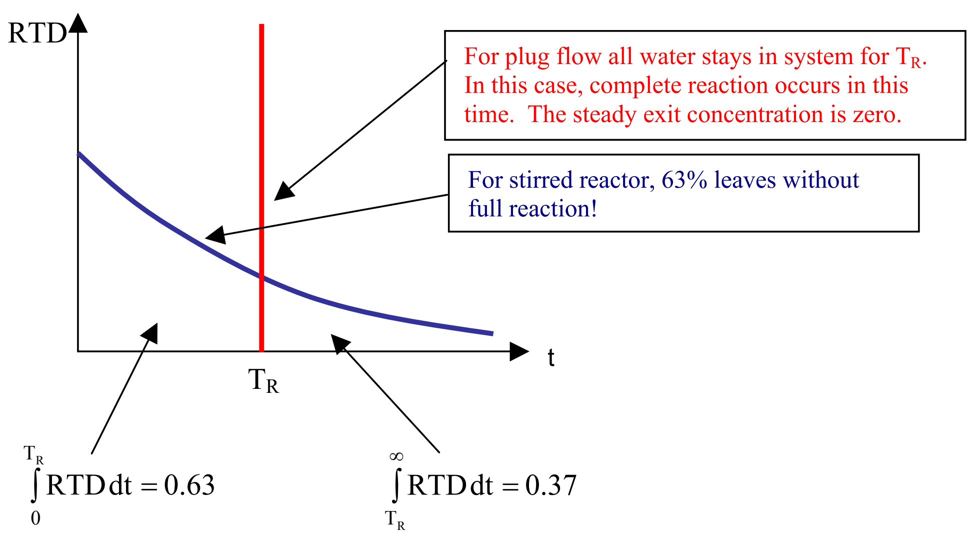
For a Stirred Reactor $RTD(t) = \exp(-t/T_R)/T_R$, then

$$C_e = \int_0^{T_R} \exp(-t/T_R) \frac{C_0}{T_R}(1 - t/T_R) dt = \frac{C_0}{T_R}\int_0^{T_R}\left(\exp(-t/T_R) - \frac{t}{T_R}\exp(-t/T_R)\right) dt$$

$$= \frac{C_0}{T_R}\left[-T_R \exp(-t/T_R) - T_R \exp(-t/T_R)(-t/T_R - 1)\right]\Bigg|_0^{T_R}$$

$$= \frac{C_0}{T_R}\left[-T_R(\exp(-1) - \exp(0)) - T_R(\exp(-1)(-2) - \exp(0)(-1))\right] = C_0 \exp(-1) = 0.37C_0$$

Again the integration is carried out only to $t = T_R$, because $C = 0$ for $t > T_R$.
For Plug-Flow the steady exist concentration is zero, indicating 100% removal. For the Stirred Reactor the steady exit concentration is 0.37 C_O, indicating only 63% removal. So, clearly the Plug-Flow circulation provides better removal.



Case2. **Phosphorus is removed in a first-order reaction with rate, $k = 1/T_R$.**

Following the same proceedure above, but with $C(t) = C_0 exp(-kt)$.

Plug Flow: $$C_e = \int_0^{\infty} \delta(t - T_R)\, C_0 \exp(-kt) dt = C_O exp(-kT_R) = 0.37\, C_0$$

Stirred Reactor:

$$C_e = \frac{C_0}{T_R} \int_0^{\infty} \exp(-\,(1/T_R + k)\, t\,)\, dt = -\frac{C_0}{T_R} \frac{1}{(1/T_R) + k} \exp(-(1/T_R + k)t)\Big|_0^{\infty} = \frac{C_0}{1 + kT_R}$$

$$C_e = C_o / (1 + kT_R) = 0.5\, C_0$$

Again, Plug-Flow yields a lower exit concentration, and thus achieves higher removal.