Answer 9.1

For a continuously operating smokestack and steady climatic conditions, we can assume a steady concentration field, *i.e.* $\partial C/\partial t = 0$. The wind is given as u = 5 m/s, implying v = w = 0. We are told to assume a uniform wind, *i.e.* no shear, so we neglect shear-dispersion. For the length-scale of interest, L_X = 10,000m, the Peclet number is (5m/s)(10,000m)/(1m^2/s) = 50,000 >>> 1. With this high value of Pe, the longitudinal diffusion term is negligible relative to longitudinal advection, and we drop it. With the above assumptions, the transport equation

$$\text{(a)}\quad \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \frac{\partial}{\partial x}D_x\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}D_y\frac{\partial C}{\partial y} + \frac{\partial}{\partial z}D_z\frac{\partial C}{\partial z} \pm S,$$

becomes

$$\text{(b)}\quad u\frac{\partial C}{\partial x} = D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2} - S,$$

with S = 0 for Freon and S = $k_{TCE}C$ for the TCE.

For a continuous release $\dot{m}$ = 5 kg/min = 83 gs^{-1} at (x,y,z) = (0, 0, H), the solution to (b) is

$$\text{(c)}\quad C(x,y,z) = \frac{\dot{m}}{4\pi\sqrt{D_y D_z}\,x}\exp\left(-\frac{uy^2}{4D_y x} - \frac{u(z-H)^2}{4D_z x}\right)\exp\left(-\frac{kx}{u}\right),$$

where k = 0 for Freon and k = k_{TCE} for the TCE. To account for the no-flux boundary we add a positive image source at (x,y,z) = (0, 0, -H).

$$\text{(d)}\quad C(x,y,z) = \frac{\dot{m}}{4\pi\sqrt{D_y D_z}\,x}\left[\underbrace{\exp\left(-\frac{uy^2}{4D_y x} - \frac{u(z-H)^2}{4D_z x}\right)\exp\left(-\frac{kx}{u}\right)}_{\text{real source}} + \underbrace{\exp\left(-\frac{uy^2}{4D_y x} - \frac{u(z+H)^2}{4D_z x}\right)\exp\left(-\frac{kx}{u}\right)}_{\text{image source}}\right]$$

In any transverse dimension for which the plume is unbounded (here the y-direction), the maximum concentration is at the centerline of the plume (here y = 0). The vertical coordinate, however, is bounded by a no-flux boundary at the ground, z = 0. Once the plume reaches the ground, concentration will build up at the no-flux boundary. Because the upper edge of the plume is not bounded, the vertical concentration field will eventually become asymmetric with the maximum concentration at the ground. We estimate the distance at which this will occur using the time-scale for the edge of the plume (the 2σ contour) to reach the ground,

$$\text{(e)}\quad T_{2\sigma} = H^2/(8D_Z) = (20\text{m})^2 / (8 \times 0.1\ \text{m}^2\text{s}^{-1}) = 500\ \text{s}.$$

Thus, for $x >> u\, T_{2\sigma} = 2500$ m, which includes the point of interest, we expect the maximum concentration to be at the ground. Therefore, the maximum concentration at x = 10,000 m will be $C_{max} = C$ (x =10000 m , y =0, z = 0). Evaluating (d) for Freon and TCE we find,

FREON:

$$C_{max} = \frac{83 gs^{-1}}{2\pi\sqrt{(1m^2s^{-1})(0.1m^2s^{-1})}\ 10{,}000m} \exp\left(- \frac{5ms^{-1}(20m)^2}{4(0.1m^2s^{-1})10{,}000m} \right) = 2.5 \text{ mg m}^{-3}$$

TCE:

$$C_{max} = \frac{83 gs^{-1}}{2\pi\sqrt{(1m^2s^{-1})(0.1m^2s^{-1})}\ 10{,}000m} \exp\left(- \frac{5ms^{-1}(20m)^2}{4(0.1m^2s^{-1})10{,}000m} \right) \exp\left(- \frac{10000m}{5ms^{-1}}(0.1d^{-1})(d/86400s) \right)$$

$$= 2.5 \text{ mgm}^{-3} \times 0.998 \approx 2.5 \text{ mgm}^{-3},$$

Very little degradation of TCE occurs over the 10,000m distance.