Answer 3.2

What governing equation describes the evolution of the gas concentration in the hall?
For isotropic diffusion, the governing equation is:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right]$$

Align the coordinate x to the length of the hallway and define the y-z plane as the cross-section. Since the gas "mixes rapidly" in the vertical and horizontal, we assume $\partial C/\partial y = \partial C/\partial z = 0$. The problem statement gives no information about air currents in the hallway, so we assume they are negligible, u = 0. With these assumptions, the governing equation is reduced to,

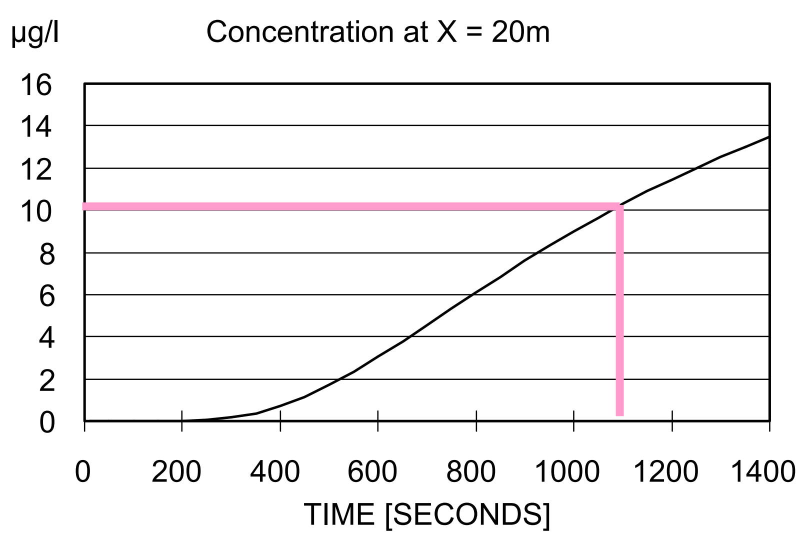
$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} \quad .$$

If the spill location is taken as x = 0, the initial condition is then, $M = \delta(x)$

At what time after the spill do you smell the gas?
The governing equation and initial condition above describe an instantaneous, point release diffusing in one-dimension. The concentration field is described by Equation 10 in chapter 3.

3.10 $$C(x,t) = \frac{M}{A_{yz}\sqrt{4\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right) = [M/L^3]$$

Use 3.10 to find the time at which the concentration at your door, C(x=20m, t) is 10 $\mu g l^{-1}$. As time appears in both the exponential and leading terms, it is simpler to use a graphical solution.



You will smell the gas at your office approximately 1100 seconds after the spill.

When does the smell, as perceived by humans, disappear from the hallway?
To answer this question we need information on the end conditions of the hallway. Lets first consider that the hallway is open at both ends, so that odor can diffuse beyond the end of the hall. Then, we need only consider the evolution of the maximum concentration, located at the site of the spill, x = 0. We seek the time for which

$$C_{max}(t) = \frac{M}{A_{yz}\sqrt{4\pi Dt}} < 10\mu g l^{-1},$$

or simply

$$t > \left(\frac{10g}{2m \times 3m \times 0.01gm^{-3}}\right)^2 \Big/ \left(4\pi \times 0.05m^2s^{-1}\right) = 44{,}232 \text{ seconds} = 12.3 \text{ hrs}$$

At this time, the length of the cloud will be $4\sigma = 4\sqrt{2Dt} = 266m$, indicating that the cloud has diffused beyond the length of the hallway. The above time scale is correct, only if the hallway is open at both ends. If the hallway is shut off by fire doors at both ends, then in reality the cloud cannot diffuse beyond the length of the hall. Under these conditions, the final concentration in the hallway set the maximum possible dilution, which is determined by distributing the total mass released over the total volume of the hallway (2m x 3m x 100m).

$$C_{final} = 10g \big/ (2m \times 3m \times 100m) = 0.016gm^{-3} = 16\mu g l^{-1}.$$

Since the final concentration is above the detection limit, the smell will not disappear until the fire doors are opened.