

1.061/1.61: Homework # 4 [10 pt total]**DUE October 7, 2002**

BOTH PROBLEMS SHOULD BE DONE WITH EXCEL OR MATLAB

Problem 1 [5 pts]

A mass, $M = 1$ kg, is instantaneously released at mid-depth in a very wide pond that is $h = 1$ m deep. Assume isotropic diffusion $D = 0.001$ m²/sec. The substance released is not volatile (no-flux at surface) but absorbs perfectly to the bed material (perfect sink at bed).

- Write an expression describing the evolution of concentration in the pond, $C(x,y,z,t)$
- Plot the concentration profile versus depth, z , through the horizontal center of the evolving cloud at the time steps: $t = 5, 20, \text{ and } 100$ sec.
- Plot the horizontal concentration profile, $C(x)$ through the vertical center of the cloud at the same time intervals given above.
- How would your solution to parts a and b differ if instead of isotropic diffusion, the vertical diffusion coefficient was $D_z = 0.00001$ m²/s while the horizontal diffusion coefficient was the same as above. Specifically, compare the vertical and horizontal length-scale of the cloud for times before the cloud encounters any boundaries.

Problem 2 [5 pts]

Combined sewers are municipal wastewater collection systems in which sanitary sewerage and storm water are combined. During normal weather, all of the flow goes to a wastewater treatment plant. During storm events, however, there is too much water in the system, and the flow exceeds the capacity of the sewer pipes. In former times, these systems were designed to release excess water via discharge gates to rivers and streams. These discharges are known as combined sewer overflows (CSOs).

Fecal coliform bacteria used as an indicator of the presence of pathogenic organisms associated with human waste. Samples of 100 ml are collected and passed through a permeable filter that is then incubated with growth media to see how many colonies form. The fecal coliform concentration is reported as organisms per 100 ml. The concentration of fecal coliform bacteria in combined sewer overflow water is typically 6 million organisms per 100 ml.

Consider a CSO that is located at the shoreline 1 kilometer upstream of a public beach. There is a prevailing along-shore current of 0.1 m/s from the CSO to the beach. Under Massachusetts's regulations, if the fecal coliform count at the beach exceeds 200 organisms per 100 ml, then swimming is prohibited. Consider a storm that causes 1000 m³ of wastewater to discharge from the CSO. Assume the discharge occurs instantaneously and instantly mixes over the depth of the water, which is 2 meters. Assume a horizontal diffusivity of 1 m²s⁻¹.

- Write an expression that describes the fecal coliform concentration, $C(x,y,t)$.
- Plot the shoreline concentration of fecal coliform at the beach as a function of time.
- When does the concentration of fecal coliform at the beach reach its maximum value, and what is that value in organisms per 100 ml?
- At what time after the storm will it be safe to swim at the beach?
- Repeat b. and c. using a horizontal diffusivity of 0.5 and 2 m²s⁻¹. Discuss the sensitivity of your results in b. and c. to the assumed value of horizontal diffusion coefficient.