Answer 3.1

Estimate the coefficient of diffusivity within this region of atmosphere.

Hint 1 - Can you assume uniform concentration in any direction?
The gas is effectively released into an unbounded domain, so that one cannot expect it to mix rapidly to uniform conditions in any direction, i.e. $\partial C/\partial y \neq 0$, $\partial C/\partial z \neq 0$, $\partial C/\partial y \neq 0$. All three dimensions must be retained in the governing equation.

Hint 2 - What assumption can you make about the air currents?
The atmosphere is stagnant, which implies that there are no ambient air currents. You may assume that u=v=w=0. The passage of each plane will create air movement. This movement may locally and temporarily enhance the dilution of the cloud. However, this effect dies out within a few minutes, so that over an hour time frame, the effect is negligible.

Hint 3 - What assumption must you make about the coefficient of diffusion?
Since you are only given one value of concentration, you can solve uniquely for only one value of diffusion. Thus, to make the problem tractable, you must assume that the diffusion is isotropic.

With the above assumptions, the transport equation reduces to,

$$\frac{\partial C}{\partial t} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right].$$

Letting the initial release point be at (x, y, z)=0, the initial condition can be written,

$C(x,y,z,t=0) = M\,\delta(x)\,\delta(y)\,\delta(z)$.

The solution to the above equation and initial condition is Equation 25 in Chapter 3.

$$C(x,y,z,t) = \frac{M}{(4\pi D t)^{3/2}} \exp\left(-\frac{x^2+y^2+z^2}{4Dt}\right).$$

The maximum concentration within the cloud is given in Equation 26 in Chapter 3.

$$C_{MAX} = \frac{M}{(4\pi D t)^{3/2}}.$$

Using the information given in the problem statement, we can find D,

$$D = \left(M/C_{MAX}\right)^{2/3} \Big/ 4\pi t = \left(10\text{kg}/3\times10^{-5}\ \text{kgm}^{-3}\right)^{2/3} \Big/ (4\ \pi\ 3600\ \text{s}) = 0.1\text{m}^2\text{s}^{-1}$$