Solution to Problem 1.4

(i) Air

The concentration we are interested in is given by:

$$\frac{C}{\rho_{air}} = 1 \text{ ppm} \Rightarrow C = \frac{1.23 \text{ kgm}^{-3}}{10^6} = 1.23\times10^{-9}\,\text{gcm}^{-3}$$

The normal distribution of concentration is:

$$C = \frac{M}{A\sqrt{4\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right)$$

Therefore, for a mass fraction of 1 ppm,

$$1.23\times10^{-9}\,\text{gcm}^{-3} = \frac{0.1 \text{ g}}{\left(\pi(5 \text{ cm})^2\right)\sqrt{4\pi(0.14 \text{ cm}^2\text{s}^{-1})t}}\exp\left(-\frac{(50 \text{ cm})^2}{4(0.14 \text{ cm}^2\text{s}^{-1})t}\right)$$

With t in seconds, the dimensions cancel, and we are left with the equation:

$$1.23\times10^{-9} = \frac{9.599\times10^{-4}}{\sqrt{t}}\exp\left(-\frac{4.464\times10^3}{t}\right)$$

$$\Rightarrow \quad \exp\left(\frac{4.464\times10^3}{t}\right) = \frac{7.804\times10^4}{\sqrt{t}}$$

Solving this equation by trial and error gives us the solution that C = 1 ppm at x = 50 cm when ***t* = 423 s.**

(ii) Water

Following the same solution method,

$$\frac{C}{\rho_{water}} = 1 \text{ ppm} \Rightarrow C = \frac{1000 \text{ kgm}^{-3}}{10^6} = 1.00\times10^{-6}\,\text{gcm}^{-3}$$

Therefore, for a mass fraction of 1 ppm,

$$1.00 \times 10^{-6}\,\mathrm{g\,cm^{-3}} = \frac{0.1\ \mathrm{g}}{\left(\pi(5\ \mathrm{cm})^2\right)\sqrt{4\pi(1.71\times 10^{-5}\ \mathrm{cm^2 s^{-1}})t}} \exp\left(-\frac{(50\ \mathrm{cm})^2}{4(1.71\times 10^{-5}\ \mathrm{cm^2 s^{-1}})t}\right)$$

With t in seconds, the dimensions cancel, and we are left with:

$$1.00 \times 10^{-6} = \frac{8.686 \times 10^{-2}}{\sqrt{t}} \exp\left(-\frac{3.655 \times 10^{7}}{t}\right)$$

$$\Rightarrow \quad \exp\left(\frac{3.655 \times 10^{7}}{t}\right) = \frac{8.686 \times 10^{4}}{\sqrt{t}}$$

Solving this equation by trial and error gives us the solution that $C = 1$ ppm at $x = 50$ cm when $\boldsymbol{t} \mathbf{= 1.12 \times 10^7}$ **s (= 130 days)**, much longer than when the fluid is air.