A Study of Streamwise Vortex Enhanced Mixing in Lobed Mixer Devices

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SUBMITTED TO THE DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
Doctor of Philosophy
in
Aeronautics and Astronautics
at the
Massachusetts Institute of Technology
June 1992

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Abstract

The effect of shed streamwise vorticity on mixing in a lobed mixer device has been examined computationally and experimentally. Computational models for assessing the mixing augmentation due to shed streamwise vorticity in laminar and turbulent flows have been developed. The basic idea is to track the flow development due to shed streamwise vorticity in a frame that convects at the mean flow velocity. Detailed parametric studies were carried out. It was found that there is a critical effective Reynolds number based on the strength of shed streamwise vorticity. Below the critical Reynolds number, the mixing increase per unit downstream distance due to streamwise vorticity is proportional to a non-dimensional shed circulation parameter $\frac{\Gamma}{U\lambda}$, where $\Gamma$ is the shed circulation, $U$ is the mean velocity, and $\lambda$ is the wavelength of the lobe. Above the critical Reynolds number, the mixing increase per unit downstream distance is proportional to $(\frac{\Gamma}{U\lambda})^{2/3}$. The computational results were shown to agree well with experimental results in terms of total pressure distribution and static pressure recovery due to mixing.

The mixing performance, in terms of static pressure recovery downstream of the lobe trailing edge, was measured to assess the relative importance of shed streamwise vorticity compared to lobe trailing edge length in providing momentum mixing enhancement. It was found that, for the configuration examined, the contribution of the streamwise vorticity to the mixing is roughly of the same order as that of the lobe trailing edge length. Flow visualization experiments and static temperature measurements were also carried out to investigate the mechanism by which streamwise vorticity enhances mixing. The effect of streamwise vorticity was identified as producing convective transport in the cross flow plane which increases the mean fluid interface area, thus leading to the mixing enhancement.

Thesis advisor: Edward M. Greitzer
H.N. Slater Professor of Aeronautics and Astronautics

Thesis advisor: Choon S. Tan
Principal Research Engineer
Acknowledgments

The completion of this thesis would have been impossible without the advice, assistance and friendship of a number of people and I wish to thank them all, in particular:

To Professor E. M. Greitzer, for his incisive suggestions, continual support and encouragement.

To Dr. C. Tan, for his encouragement, suggestions, friendship, and for getting me interested in the project in the first place (and for many dinner trips to Chinatown).

To Professor F. Marble, for his original ideas, insightful comments and warm words of encouragement. I am honored to have had the opportunity to discuss my research with him.

To Professor M. Landahl, for his suggestions and many valuable discussions.

To Professor I. Waitz, for his valuable suggestions. His patience in correcting grammar mistakes in the thesis is gratefully appreciated.

To Professor M. Martinez-Sanchez, for initially taking me in as a graduate student.

To Professor E. Covert, for his suggestions and constructive comments.

I would like to thank my fellow colleagues in lobed mixer research, past and present:

J. Elliott, for friendship and hours of discussions; T. Manning, for help in running experiments, etc; F. Kennedy, for building the air test facility; and G. Tillman of United Technologies Research Center, for many valuable discussions and kindness in providing the experimental data.

I would also like to thank many friends and fellow students in GTL who made my stay at MIT more enjoyable: Dan, Peter, Earl, Knox, Yang, Andreas, S.J. and to my Chinese friends Charlie, Dawei, Fei; also to Vicktor, for many interesting discussions on V.W.; to Holly, for keeping GTL running smoothly; and especially to Ling, for without her help the completion of this thesis would be impossible.

Finally to my parents and my sister, for their faith in me and love throughout my life. It is to them that I dedicate this thesis.

Support for this project was provided by Naval Air Systems Command, under contract N0001988-C-0029 and supervision of Mr. George Derderian and Dr. Lewis Sloter II. This support is gratefully acknowledged. Additional computational resources were provided by NASA Lewis Research Center.
Contents

Nomenclature 15

1 Introduction 18
   1.1 Introduction ........................................... 18
   1.2 Background .............................................. 19
   1.3 Objective .............................................. 21
      1.3.1 Introduction ........................................... 21
      1.3.2 Research Goals ....................................... 22
   1.4 Scope of Investigation and Contributions ................. 22
      1.4.1 Computational Study ................................... 22
      1.4.2 Experimental Study .................................... 23
      1.4.3 Contributions ......................................... 24
   1.5 Overview of Thesis ..................................... 24

2 Computational Study of Streamwise Vorticity Enhanced Mixing: Method of Approach 27
   2.1 Introduction ............................................. 27
   2.2 Slender Body Formulation .................................. 29
   2.3 Initial and Boundary Conditions .......................... 31
      2.3.1 Streamwise Vorticity at Trailing Edge ................. 31
      2.3.2 Scalar Distribution at Trailing Edge ................ 33
      2.3.3 Boundary Conditions .................................. 34
   2.4 Mixedness Parameter ..................................... 34
   2.5 Method of Solving Slender Body Equations ................ 34
   2.6 Summary .................................................. 35
3 Effect of Streamwise Vorticity on Mixing in Laminar Flow with Stream to Stream Velocity Ratio Close to Unity

3.1 Introduction ........................................... 39
3.2 Effect of Streamwise Vorticity Distribution on Mixing .................. 40
  3.2.1 Flow Field Development ............................ 40
  3.2.2 Effects of Initial Scalar Layer Thickness and Initial Streamwise Vorticity Thickness .......................... 41
  3.2.3 Effect of the Distribution of Circulation Per Unit Trailing Edge Length 42
3.3 Effect of Reynolds Number/Streamwise Vorticity Strength on Mixing . 43
  3.3.1 Effect of Reynolds Number \( R_e \) on Mixing .................. 43
  3.3.2 Mixing Augmentation and Marble’s Point Vortex Model .............. 43
  3.3.3 Effect of Strength of Streamwise Vorticity on Maximum Mixing Augmentation Rate .......................... 44
3.4 Estimation of Rotation Speed of Vortical Region ......................... 46
3.5 Summary .................................................................. 47

4 Effect of Streamwise Vorticity on Mixing in Turbulent Flow with Large Stream to Stream Velocity Difference

4.1 Introduction .................................................. 73
4.2 Computational Model for Turbulent Flow .............................. 74
  4.2.1 Computational Model .................................... 74
  4.2.2 Eddy Viscosity .......................................... 76
  4.2.3 Normalized Equations ................................. 77
4.3 Initial and Boundary Conditions ..................................... 78
  4.3.1 Streamwise Vorticity at Trailing Edge ...................... 78
  4.3.2 Axial Velocity Perturbation Distribution at Trailing Edge ......... 79
  4.3.3 Boundary Conditions .................................... 79
4.4 Momentum Mixedness Parameter ...................................... 80
4.5 Parametric Study ............................................. 80
  4.5.1 Flow Field Development .................................. 80
  4.5.2 Effect of Streamwise Velocity Ratio on Mixing ................. 81
  4.5.3 Effect of Lobe Height on Mixing .......................... 84
List of Figures

1-1 Schematic drawing of a lobed mixer device ........................................ 26

2-1 Schematic drawing of an “advanced lobed mixer” ................................. 36

2-2 Illustration of space and time analogy and definition of coordinates ...... 37

2-3 Initial streamwise vorticity distributions for slender body computation ... 38

3-1 Contours of streamwise vorticity at different time \( t = \frac{F}{U_{\lambda}} \) for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \)) ................................. 49

3-2 Contours of static pressure at different time \( t = \frac{F}{U_{\lambda}} \) for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \)) ................................... 51

3-3 Contours of scalar value at different time \( t = \frac{F}{U_{\lambda}} \) for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \)) .................................. 52

3-4 Mixedness as a function of time \( t = \frac{F}{U_{\lambda}} \) for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \)) ................................ 54

3-5 Comparison of mixedness as a function of downstream distance \( (\frac{x}{L}) \) for flows with streamwise vorticity (same as Figure 3.1) and without streamwise vorticity \( (Re_{\lambda} = 2000, \frac{F}{U_{\lambda}} = .39 \) and \( Sc=1.0 \)) .......................................................... 55

3-6 Effect of scalar thickness \( \sigma_{s} \) on mixedness for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \)) .................................. 56

3-7 Effect of streamwise vorticity thickness \( \epsilon \) on mixedness for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \)) .................................. 57

3-8 Contours of streamwise vorticity at different time \( t = \frac{F}{U_{\lambda}} \) for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (concentrated initial streamwise vorticity distribution and \( Sc=1.0 \)) .................................. 58

3-9 Contours of scalar value at different time \( t = \frac{F}{U_{\lambda}} \) for \( Re_{\Gamma} = 1000 \) and \( \frac{h}{\lambda} = .54 \) (concentrated initial streamwise vorticity distribution and \( Sc=1.0 \)) .................................. 60
3-10 Comparison of mixedness for uniform and concentrated initial streamwise vorticity distributions for $Re_\Gamma = 1000$, $\frac{h}{\lambda} = 0.54$ and $Sc=1.0$ ............ 62

3-11 Contours of scalar value at different time ($t = \frac{r}{\frac{C}{\lambda_\lambda}}$) for $Re_\Gamma = 4000$ and $\frac{h}{\lambda} = 0.54$ (uniform initial streamwise vorticity distribution and $Sc=1.0$) .... 63

3-12 Mixedness for $Re_\Gamma = 1000$ and $Re_\Gamma = 4000$ (uniform initial streamwise vorticity distribution, $\frac{h}{\lambda} = 0.54$ and $Sc=1.0$) ............. 65

3-13 Mixing augmentation for $Re_\Gamma = 1000$ and $Re_\Gamma = 4000$ (uniform initial streamwise vorticity distribution, $\frac{h}{\lambda} = 0.54$ and $Sc=1.0$) .................. 66

3-14 Mixing augmentation for $Re_\Gamma = 1000$ and rescaled $Re_\Gamma = 4000$ according to Equation 3.1 (uniform initial streamwise vorticity distribution, $\frac{h}{\lambda} = 0.54$ and $Sc=1.0$) ..................................... 67

3-15 Maximum scalar mixing augmentation rate as a function of Reynolds number (uniform initial streamwise vorticity distribution, $\frac{h}{\lambda} = 0.54$ and $Sc=1.0$) .... 68

3-16 Maximum scalar mixing augmentation per unit downstream distance as a function of $\frac{C}{\lambda_\lambda}$ (uniform initial streamwise vorticity distribution, $Re_\lambda = 2000$, $\frac{h}{\lambda} = 0.54$ and $Sc=1.0$) .................. 69

3-17 Maximum scalar mixing augmentation rate as a function of Reynolds number (uniform initial streamwise vorticity distribution, $\frac{h}{\lambda} = 1.0$ and $Sc=1.0$) .... 70

3-18 Schematic drawing of the behavior of the moment of vorticity ................. 71

3-19 Time required for 90 degree rotation (symbols are from computation and solid line is given by Equation 3.4) ........................................ 72

4-1 Contours of streamwise vorticity at different time ($\frac{C}{\lambda_\lambda}$) for $r = 0.5$, $\frac{C}{\lambda} = 0.39$ and $\frac{h}{\lambda} = 0.54$ ................. 87

4-2 Contours of axial velocity perturbation at different time ($\frac{C}{\lambda_\lambda}$) for $r = 0.5$, $\frac{C}{\lambda} = 0.39$ and $\frac{h}{\lambda} = 0.54$ .................. 88

4-3 Comparison of momentum mixedness for flows with and without streamwise vorticity ($r = 0.5$, $\frac{C}{\lambda} = 0.39$ and $\frac{h}{\lambda} = 0.54$) ......... 89

4-4 Momentum mixedness for $r=0.5$ and $r=0.67$ for flows with streamwise vorticity ($\frac{C}{\lambda} = 0.39$ and $\frac{h}{\lambda} = 0.54$) ............... 90

4-5 Momentum mixedness for $r=0.5$ and $r=0.67$ for flows without streamwise vorticity ($\frac{C}{\lambda} = 0.39$ and $\frac{h}{\lambda} = 0.54$) .................. 91
4.6 Momentum mixing augmentation for $\frac{t}{\lambda} = 0.54$ and $\frac{F}{U_\lambda} = 0.39$ ................................. 92
4.7 Momentum mixing augmentation rescaled for $\frac{t}{\lambda} = 0.54$ and $\frac{F}{U_\lambda} = 0.39$ ......................... 93
4.8 Momentum mixing augmentation for $\frac{t}{\lambda} = 0.54$ and $\frac{F}{U_\lambda} = 0.39$ ................................. 94
4.9 Momentum mixing augmentation for $\frac{t}{\lambda} = 1.0$ and $\frac{F}{U_\lambda} = 0.727$, showing that for $1.0 < t < 3.0$ the mixing augmentation rate is roughly independent of stream to stream velocity ratio ........................................... 95
4.10 Momentum mixedness for $\frac{t}{\lambda} = 0.54$ for flows with and without streamwise vorticity ($\alpha = 20^0$, $\frac{F}{U_\lambda} = 0.39$ and $r = 0.25$) .......................................................... 96
4.11 Momentum mixedness for $\frac{t}{\lambda} = 1.0$ for flows with and without streamwise vorticity ($\alpha = 20^0$, $\frac{F}{U_\lambda} = 0.727$ and $r = 0.25$) ................... 97
4.12 Ratio of shear layer growth rate for compressible shear layer to that of incompressible shear layer .................................................. 98
4.13 Momentum mixing augmentation for $M_c = 0.0$ and $M_c = 0.5$ ($\frac{F}{U_\lambda} = 0.39$ and $r = 0.67$) .......................................................... 99
4.14 Momentum mixing augmentation for $M_c = 0.0$ and rescaled $M_c = 0.5$ ($\frac{F}{U_\lambda} = 0.39$ and $r = 0.67$), showing that mixing augmentation for different Mach number can be scaled according to Reynolds number ratios .......... 100

5.1 Schematic drawing of a lobed mixer and a convoluted plate .............. 115
5.2 Schematic drawing of the air facility .............................................. 116
5.3 Lobe geometry .................................................................................. 117
5.4 Static pressure decrease along duct downstream of the convoluted plate and the lobed mixer at unity velocity ratio .................................. 118
5.5 Wall static pressure downstream of the lobed mixer for $\frac{U_\lambda}{U_1} = 0.20$ ............................................. 119
5.6 Ideal static pressure recovery for $\frac{U_\lambda}{U_1} = 0.13$ (fully mixed value $(\frac{AP}{\frac{1}{2}pU_1^2})_{max} = 0.374$) ......................................................... 120
5.7 Comparison of integral length of ideal static pressure recovery as a function of velocity ratio .................................................. 124
5.8 y-momentum at lobe trailing edge ..................................................... 125
5.9 Net y-momentum change as a function of velocity ratio (curve fit: $K_y = 0.33945 - 0.35472(\frac{U_2}{U_1})^2$) ................................................. 126
5-10 Effect of streamwise velocity on ideal static pressure recovery downstream of the lobed mixer .................................................. 127
5-11 Effect of streamwise velocity on ideal static pressure recovery downstream of the convoluted plate ............................................. 128
5-12 Effect of velocity ratio on ideal static pressure recovery downstream of the lobed mixer ......................................................... 129
5-13 Effect of velocity ratio on ideal static pressure recovery downstream of the convoluted plate .................................................. 130
5-14 Maximum slope of normalized ideal static pressure recovery ................................................................. 131
5-15 Temperature $T^*$ downstream of the convoluted plate for $\frac{U_2}{U_1} = 1.0$ ........................................ 132
5-16 Temperature $T^*$ downstream of the lobed mixer for $\frac{U_2}{U_1} = 1.0$ ................................................ 133
5-17 Temperature $T^*$ downstream of the convoluted plate for $\frac{U_2}{U_1} = 0.31$ .......................................... 134
5-18 Temperature $T^*$ downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.31$ ............................................... 135
5-19 Comparisons of mixedness downstream of the lobed mixer and the convoluted plate for $\frac{U_2}{U_1} = 0.31$ and $\frac{U_2}{U_1} = 1.0$ ............................................... 136

6-1 Schematic drawing of the water tunnel facility .................. 144
6-2 Lobe geometry .............................................. 145
6-3 Definition of viewing angles ........................................ 146
6-4 Flow structure in $\xi = 1$ plane downstream of the convoluted plate for $\frac{U_2}{U_1} = 0.5147$ ...................... 149
6-5 Flow structure in $\xi = 2$ plane downstream of the convoluted plate for $\frac{U_2}{U_1} = 0.5148$ ...................... 150
6-6 Flow structure in $\xi = 1$ plane downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.5$ ...................... 149
6-7 Flow structure in $\xi = 2$ plane downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.5$ ...................... 150
6-8 Flow structure in $\xi = 0$ plane downstream of the convoluted plate for $\frac{U_2}{U_1} = 0.5151$ ...................... 151
6-9 Flow structure in $\xi = 0.2$ plane downstream of the convoluted plate for $\frac{U_2}{U_1} = 0.5$ ...................... 152
6-10 Flow structure in $\xi = 0$ plane downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.5$ ...................... 153
6-11 Flow structure in $\xi = 0.2$ plane downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.5$ ...................... 154
6-12 Illustration of measurement of visual thickness of shear layer ................................................................. 155
6-13 Histogram for visual thickness growth rate ................................................................. 156
6-14 Flow structure at the peak of the lobed mixer trailing edge in $z$ plane for $\frac{U_2}{U_1} = 0.5$ ................................................................. 157
6-15 Flow structure at the trough of the lobed mixer trailing edge in z plane for 
\[ \frac{U}{U_1} = 0.5 \] .................................................. 158
6-16 Features of trailing edge boundary layer at lobe peak and trough .... 159

7-1 Comparison of total pressure downstream \( (\bar{x} = 3.1) \) of the lobed mixer (ex-
periment data from Presz, [25], 1986) .................................................. 164
7-2 Comparison of ideal static pressure recovery downstream of the lobed mixer 165
7-3 Comparison of ideal static pressure recovery downstream of the convoluted plate ............................................................. 166
7-4 Rotation of vortical region and fluid interface ................................ 167
7-5 Effect of compressibility on total temperature for \( M_c = .14 \) and \( M_c = .65 \): Symbols from experiment (Tillman, 1991) and lines from computation . . . 168

B-1 Schematic of trailing edge profile and cross flow velocity ............... 180

D-1 Trailing edge profile and strength of streamwise vorticity per unit length from Euler solver ................................................................. 186
D-2 Comparison of scalar field at \( \bar{x} = 5.6 \) ........................................ 187
D-3 Comparison of scalar field at \( \bar{x} = 9 \) ........................................ 188
D-4 Comparison of static pressure distributions at \( \bar{x} = 4 \) ................. 189

E-1 Comparison of momentum mixedness with pressure gradient term to that without pressure term for \( \frac{h}{\bar{x}} = 0.54, \frac{U}{U_1} = 0.39 \) and \( \frac{U}{U_1} = 0.67 \) ........ 192
E-2 Comparison of momentum mixedness with pressure gradient term to that without pressure term for \( \frac{h}{\bar{x}} = 0.54, \frac{U}{U_1} = 0.39 \) and \( \frac{U}{U_1} = 0.50 \) ........ 193

F-1 Two-dimensional shear layer .................................................. 196

G-1 Static pressure decrease along duct downstream of the flat plate splitter at unity velocity ratio .................................................. 198
G-2 Comparison of the normalized static pressure recovery downstream of the lobed mixer, convoluted plate and flat plate splitter for velocity ratio of 0.31 199
G-3 Comparison of the normalized static pressure recovery downstream of the lobed mixer, convoluted plate and flat plate splitter for velocity ratio of 0.20 200
I-1 Rankine Vortex ................................................................. 204
List of Tables

5.1 Temperature measurement positions .......................... 114

6.1 Visual thickness growth rate in y=0 plane ................. 143
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>cross sectional area of mixing duct</td>
</tr>
<tr>
<td>$A_1$</td>
<td>flow area of fast stream at trailing edge</td>
</tr>
<tr>
<td>$A_2$</td>
<td>flow area of slow stream at trailing edge</td>
</tr>
<tr>
<td>$a_1$</td>
<td>speed of sound in high stream</td>
</tr>
<tr>
<td>$C, C_1, C_2$</td>
<td>constant</td>
</tr>
<tr>
<td>$D$</td>
<td>scalar diffusion coefficient</td>
</tr>
<tr>
<td>$D_f$</td>
<td>friction loss</td>
</tr>
<tr>
<td>$D_v$</td>
<td>drug</td>
</tr>
<tr>
<td>$F(s) =$</td>
<td>strength of streamwise vorticity per unit trailing edge length</td>
</tr>
<tr>
<td>$f_c$</td>
<td>friction coefficient</td>
</tr>
<tr>
<td>$h$</td>
<td>lobe height</td>
</tr>
<tr>
<td>$h_t$</td>
<td>test section height</td>
</tr>
<tr>
<td>$J_y$</td>
<td>$y$ moment of streamwise vorticity</td>
</tr>
<tr>
<td>$K$</td>
<td>loss coefficient</td>
</tr>
<tr>
<td>$K_y$</td>
<td>net change of $y$-momentum across mixing duct</td>
</tr>
<tr>
<td>$L$</td>
<td>mixing duct length</td>
</tr>
<tr>
<td>$l_i$</td>
<td>integral length of ideal static pressure recovery</td>
</tr>
<tr>
<td>$l_t$</td>
<td>trailing edge length</td>
</tr>
<tr>
<td>$M$</td>
<td>scalar mixedness</td>
</tr>
<tr>
<td>$M_a$</td>
<td>scalar mixing augmentation</td>
</tr>
<tr>
<td>$M_c$</td>
<td>convective Mach number $\frac{U_1-U_s}{a_1}$</td>
</tr>
</tbody>
</table>
\( M_p \)  momentum mixedness
\( M_{pa} \)  momentum mixing augmentation
\( n \)  normal distance to trailing edge profile in cross flow plane
\( p \)  pressure
\( pt \)  total pressure
\( Re_t \)  Reynolds number \( \frac{L}{\nu} \)
\( Re_{t*} \)  Effective Reynolds number \( \frac{L}{\nu} \)
\( Re_{\lambda} \)  Reynolds number \( \frac{U_{e}}{\nu} \)
\( r \)  streamwise velocity ratio \( \frac{U_1}{U_2} \)
\( Sc \)  Schmidt number \( \frac{\nu}{\kappa} \)
\( Sc_t \)  Schmidt number in turbulent flow \( \frac{\nu}{\kappa} \)
\( s \)  coordinate along the trailing edge
\( T \)  temperature
\( T^{*} \)  normalized temperature
\( t \)  non-dimensional time \( \frac{L}{U_{e}} \)
\( t^{*} \)  convective time \( \frac{L}{U_{e}} \)
\( \overline{U} \)  average velocity \( \frac{U_1+U_2}{2} \)
\( U_1 \)  fast stream velocity at trailing edge
\( U_2 \)  slow stream velocity at trailing edge
\( U_c \)  convective velocity
\( u \)  velocity in the x direction
\( u' \)  axial velocity perturbation in the x direction
\( \nu \)  velocity
\( v \)  velocity in the y direction
\(w\) velocity in the \(z\) direction
\(x, y, z\) coordinates
\(\alpha\) lobe penetration angle
\(\epsilon\) streamwise vorticity layer thickness parameter at trailing edge
\(\omega\) vorticity
\(\omega_v\) streamwise vorticity
\(\Gamma\) strength of streamwise vorticity or circulation
\(\lambda\) lobe wavelength
\(\Delta P\) static pressure recovery
\(\Delta P_i\) ideal static pressure recovery due to mixing
\(\Delta P_f\) static pressure loss due to wall friction
\(\Delta P_w\) wall static pressure difference across mixing duct
\(\Delta U\) streamwise velocity difference \(U_1 - U_2\)
\(\delta\) thickness of two dimensional shear layer
\(\delta_v\) streamwise vorticity thickness at trailing edge
\(\delta_{vis}\) visual thickness of Kelvin-Helmholtz shear layer
\(\delta t\) time step size
\(\nu\) kinematic viscosity
\(\nu_t\) kinematic viscosity in turbulent flow
\(\phi\) scalar variable
\(\sigma_s\) scalar layer thickness parameter at trailing edge
\(\sigma_u\) axial velocity perturbation thickness parameter at trailing edge
\(\rho\) density
Chapter 1

Introduction

1.1 Introduction

Lobed mixers have been used extensively to improve mixing of co-flowing streams in air breathing propulsion systems as well as in chemical laser systems. A schematic drawing of a lobed mixer is shown in Figure 1.1. The geometry of such mixers is characterized by periodic convolutions of the trailing edge of the splitter between the streams being mixed. Due to the convolutions, the flow behavior downstream of lobed mixers differs from that in conventional flat plate splitters, because strong streamwise vorticity is shed at trailing edges resulting in periodic streamwise vortices in the downstream mixing field. Lobed mixers are used in a wide range of flow conditions, from laminar flow ($Re < 3000$) in chemical lasers to turbulent flow ($Re > 10^6$) in jet engines.

Despite this wide usage, the underlying mixing mechanism of lobed mixers is not understood in any depth. It is generally believed that streamwise vorticity is responsible for rapid mixing, but its role in the mixing process has yet to be clarified quantitatively. In addition existing lobe design procedures appear to be largely empirical, relying mainly on model testing. This can not only result in increased developmental cost, but also in decreased realization of the full potential benefit of the device. There is thus a need for a thorough study of the basic fluid mechanics of mixing downstream of the lobed mixers.

This thesis constitutes an experimental and computational study of the mixing mechanism in lobed mixers. Emphasis is on the clarification and quantification of the effect of streamwise vorticity on the mixing in the flow field downstream of the lobed mixer trailing edge. In particular, the relations between mixing augmentation and streamwise vorticity
strength and distribution are investigated in some detail.

1.2 Background

Investigation into the use of convoluted lobes to increase mixing can be traced back to 1941, the early days of jet engine development (Hawthorne, [9], 1990), when such devices were suggested to increase the air and fuel mixing rate. It was not until the 1960s, however, that lobed mixers were extensively used in jet engines to increase mixing of exhaust jets and to decrease jet noise.

Most of the early work on lobed mixers was concentrated on overall performance of the devices in air breathing propulsion systems, with main interest being in the net thrust augmentation. In the Energy Efficient Engine ($E^3$) development program, in the late 1970s and early 1980s, a number of studies were carried out to compare performances of lobed mixers to those of other types of mixers (Kozlowski et al., [15], 1980; Kuchar, [16], 1980; Shumpert, [31], 1980). Those studies found that lobed mixers were more effective in promoting mixing for the same length of mixing duct than other configurations tested.

More recently, lobed mixers in ejector configurations have been explored by several investigators (Presz et al., [25], 1986; Skebe et al., [32], 1988). An ejector is a device that uses a high speed flow (called the primary stream) to pump a low speed flow (called the secondary stream), converting a low volume high velocity flow to a high volume low velocity flow. One measure of the performance is the ratio of secondary to primary mass flow. For a constant area mixing duct, simple control volume analysis shows that the ideal (i.e. fully mixed) secondary to primary mass flow ratio achievable for incompressible flow depends only on the area ratio of the two streams. In practice, however, mixing may not be complete and the mass flow ratio can depend on the degree of mixing between the primary and secondary streams. The secondary to primary mass flow ratio for a given configuration can thus be used to provide a good indication of the effectiveness of the mixing device. Presz et al. ([25], 1986) have shown that, for the same mixing duct length, a higher secondary to primary mass flow ratio was obtained using a lobed mixer compared to a conventional mixer with non-convoluted splitter. For some of the configurations tested, the lobed mixer provided twice the mass flow ratio of the conventional splitter mixer.

To better assess why lobed mixers are so effective in promoting mixing, Povinelli et
al. ([24], 1981) measured total temperature and total pressure distributions downstream of lobed mixers. They found "horseshoe-shaped total temperature signatures" structures downstream of lobed mixers. To further identify the origin of those structures, Paterson ([23], 1982) measured the downstream velocity field of a model lobed mixer using a Laser Doppler Velocimeter (LDV). He established that there is a circulating flow in the cross flow plane downstream of the lobe trailing edge, which persists several lobe wavelengths downstream. This observation was further confirmed by measurements of total pressure and total temperature. He conjectured that the circulating flow provides the major mechanism for the observed increase in mixing of the lobed mixers over conventional mixers.

Flow visualization experiments have been reported by Werle et al. ([37], 1987). Using dye injection, they observed that downstream of the lobed mixer trailing edge, the shed streamwise vortex sheet undergoes "tight roll up into some sort of a core"; and further downstream, the flow goes through a process that could be characterized as "some form of vortex breakdown". They suggested that vortex breakdown might be the major mechanism responsible for streamwise vorticity enhanced mixing. However, it is not clear that vortex breakdown occurs in the flow downstream of the lobed mixer trailing edge. In their experiment, the two streams had the same mean stream velocity and therefore, a slightly favorable streamwise pressure gradient in the mixing duct existed due to wall friction. Such situations are generally not associated with vortex breakdown.

To relate lobed mixer geometry to the magnitude of the cross flow velocity downstream of the lobe trailing edge, Skebe et al. ([33], 1988) measured the downstream flow fields for mixers of different lobe geometries. They found that the ratio of maximum cross flow velocity, at the lobe trailing edge, to average axial stream velocity is approximately proportional to the tangent of the lobe penetration angle, and the magnitude of half lobe streamwise circulation $\Gamma$ (see Figure 1-1) is proportional to the lobe height.

The use of lobed mixers for mixing enhancement in chemical laser systems, where rapid mixing between reactants is desired, was investigated by Driscoll ([6], 1986). He studied the flow structure downstream of the lobed mixer using flow visualization methods and found that the fluid interface in the cross flow plane increased as a result of the cross flow. He then conjectured that interface increase due to the cross flow is the main mechanism responsible for rapid mixing downstream of the lobed mixer.

There have been several computations of the mixing downstream of the lobed mixer
trailing edge. Most of the computational work (Povinelli et al., [24],1981; Anderson et al., [1], 1980) has been based on the use of Reynolds averaged equations with turbulence models. With a "generic" representation of the distribution of shed streamwise vorticity at the trailing edge, Anderson et al. ([1], 1980) have computed the flow field downstream of the trailing edge of a lobed mixer. The predictions using the "generic" representation were found to be in better agreement with experiment data than those without any streamwise vorticity and they concluded that the inclusion of shed streamwise vorticity is critical in computational studies of the mixing downstream of a lobed mixer. More recently, Koutmos et al. ([14], 1989) have studied the flow both over the lobe surface and downstream of the trailing edge using Navier-Stokes equations with a turbulence model, and have obtained good agreement with the experimental results of Paterson ([23], 1982).

1.3 Objective

1.3.1 Introduction

The existing experimental results show that the lobed mixer is an effective mixing augmentation device. In comparing the overall performance of the lobed mixer with that of other types of mixers in air breathing propulsion systems, some researchers (Shumpert [31], 1980; Kuchar, [16],1980; Presz [25], 1986) have linked the observed mixing increase to the cross flow associated with the shed streamwise vorticity downstream of the lobed mixer trailing edge. However, there is also another cause for the observed mixing increase. Due to the trailing edge convolution, the initial fluid interface contact length at the lobe trailing edge can be as much as three times that of a conventional flat plate splitter. Since the amount of mixing is roughly proportional to the interface area, the increase in the trailing edge length could also account for much of the observed mixing increase. In view of this, one question that needs to be addressed is: what is the relative contribution of streamwise vorticity and initial fluid interface at the trailing edge to the mixing process?

In addition to the above question, the underlying mechanism of the mixing increase due to streamwise vorticity has not been clarified. There is no quantitative connection between the amount of mixing augmentation and streamwise vorticity strength/distribution. Elucidating the dependence of mixing on the flow parameters is, therefore, important for the formation of a rational lobe design procedure.
1.3.2 Research Goals

Based on the above considerations, the goals of this thesis were as follows:

- To define the relative contribution to mixing of streamwise vorticity and trailing edge length of lobed mixers in air-breathing propulsion systems.
- To delineate the underlying mechanism of streamwise vorticity enhanced mixing in the flow field downstream of lobed mixers.
- To determine the quantitative relations between the streamwise vorticity strength/distribution and mixing augmentation.

1.4 Scope of Investigation and Contributions

1.4.1 Computational Study

There are two related problems that arise in studying the effect of streamwise vorticity on mixing. One is the generation of streamwise vorticity, i.e. the relationship of shed streamwise vorticity with lobe geometry. This involves the study of lobe surface pressure distribution, boundary layer behavior, etc. The other problem is the flow field downstream of the lobe trailing edge. Here the issues involved are: a) what is the mechanism of the streamwise vorticity enhanced mixing; and b) how is the mixing increase linked to various flow parameters. Although the two problems are related, at this stage it is useful to address each separately. The area least understood is that of the flow downstream of the lobe trailing edge and the associated mixing process, and the computational efforts here are thus focussed on the effect of the streamwise vorticity on the mixing in the flow field downstream of the lobed mixer trailing edge.

With this goal in mind, a computational model based on tracking the flow in a mean velocity convective frame was developed, and the dependence of the mixing augmentation on the flow parameters was analyzed. The use of a lobed mixer in laminar flow was investigated first because, in addition to being relevant to applications such as chemical lasers, this can bring out essential features of the mechanism of streamwise vorticity enhanced mixing. The central idea is to capture the motion associated with the streamwise vorticity and to compute the corresponding mixing increase. A parametric study was then carried out to relate mixing augmentation to flow parameters.
The model was also extended to investigate the effect of streamwise vorticity on mixing in turbulent flow, since this regime is typical of that in air breathing propulsion systems. In this case, the aim of the investigation is to study the mixing enhancement associated with the streamwise vorticity when the flow is turbulent (rather than to investigate the turbulent mixing downstream of the lobed mixer trailing edge). Comparisons between computational results using the model developed and available experimental results have also been carried out.

The method adopted in this thesis differs from earlier studies (Povinelli et al., [24], 1981; Anderson et al., [1], 1980) using three-dimensional Navier-Stokes solvers, in that the emphasis here is on the mixing enhancement effect of the cross flow associated with the streamwise vorticity. This choice was motivated by experimental results (Paterson, [23], 1981; Skebe et al., [32], 1988), which suggest that the dominant mechanism for the observed mixing increase in lobed mixer devices is the cross flow convection due to the streamwise vorticity.

While three-dimensional Navier-Stokes methods are useful in understanding the flow downstream of the lobed mixer trailing edge, they are time-consuming when studying the parametric dependence of the lobed mixer flow field and the quantitative relations between the streamwise vorticity and the mixing augmentation. In addition, three-dimensional computations may also put a severe restriction on the flow field resolution for given computational resources. Therefore, for the goals outlined above, concentrating the computational efforts on the cross flow due to streamwise vorticity allows formulation of simplified computational models and carrying out of detailed parametric studies in an efficient manner, and facilitates examination of the basic mechanism associated with mixing augmentation.

1.4.2 Experimental Study

To assess the relative importance of streamwise vorticity and trailing edge length to the mixing in turbulent flow, as well as to isolate the effect of streamwise vorticity, the momentum mixing (i.e. bulk fluid mixing) for two different mixers was measured. The mixers were designed such that they had the same trailing edge profile but only one had strong streamwise vorticity in its downstream mixing field. Temperature fields were also obtained, to identify the mechanism of streamwise vorticity enhanced mixing. Flow visualization experiments were carried out to investigate the flow structure associated with the mixing.
1.4.3 Contributions

The major contributions of the thesis are summarized as follows:

- A model that captures the essential features of streamwise vorticity enhanced mixing is presented.
- The detailed flow structure downstream of the lobed mixer trailing edge is presented.
- Quantitative relations between mixing augmentation and shed streamwise vorticity strength/distribution are obtained.
- It is identified that the shed streamwise vorticity alone is able to provide strong momentum mixing augmentation downstream of the lobed mixer trailing edge. For the lobe geometry tested ( \( \frac{A}{l} = 2.0 \) and \( \lambda = 20^0 \)) and stream to stream velocity ratios ranging from .13 to .31, the contribution of the streamwise vorticity to the mixing enhancement downstream of the lobed mixer trailing edge is roughly the same magnitude as that of the trailing edge length.
- The increase of fluid mean interface in the cross flow plane due to streamwise vorticity is identified as the main mechanism of streamwise vorticity enhanced momentum mixing. While the stream to stream velocity ratio does influence the momentum mixing, its effect is reduced in the presence of streamwise vorticity compared to situations in which the streamwise vorticity is absent.

1.5 Overview of Thesis

The arrangement of the thesis is as follows:

Chapter 2 presents the flow model describing the mixing augmentation due to the streamwise vorticity in laminar flow. A mixedness parameter for scalar mixing is defined as a measure of merit for the mixing process.

In Chapter 3, computational results are discussed for laminar flow. The relations between mixing augmentation and strength/distribution of shed streamwise vorticity at the trailing edge of the lobed mixer are investigated.

In Chapter 4, a model for computing the effect of streamwise vorticity on mixing in
turbulent flow is formulated, and the momentum mixing parameter is defined. The computational results obtained are related to those of laminar flow computations.

Chapter 5 describes an experimental study of momentum mixing in the presence of streamwise vorticity. In particular, the mixing augmentation due to the streamwise vorticity is separated from that due to the increased trailing edge length. In addition, the temperature field in the cross flow plane is examined to explain the effect of streamwise vorticity on downstream mixing.

Chapter 6 presents flow visualization experimental results that reveal the mixing field structure downstream of the lobed mixer trailing edge. Comments are made relating the flow structure to the mechanism of the mixing enhancement.

Chapter 7 presents the comparison between the computational and experimental results in turbulent flow. The conclusions are then summarized in Chapter 8, and suggestions for future work and design guidelines are also presented.
Lobed mixer in a turbofan engine

Figure 1-1: Schematic drawing of a lobed mixer device

Lobed mixer as a mixing device

Figure 1-1: Schematic drawing of a lobed mixer device
Chapter 2

Computational Study of Streamwise Vorticity Enhanced Mixing: Method of Approach

2.1 Introduction

A critical feature that differentiates the flow field of a lobed mixer from that of other types of mixers is the streamwise vorticity shed at the lobed mixer trailing edge. The aim of this study was to capture and compute the effect of the cross flow associated with this vorticity on the mixing and to investigate the relations between the mixing augmentation and the distribution/width of shed streamwise vorticity.

In this chapter, a computational model for assessing the effect of shed streamwise vorticity on mixing in laminar flow with a stream to stream velocity ratio close to unity is presented. Although this has a direct application to the mixing of reactants in chemical laser systems where the Reynolds number is typically less than 3000, the main reason for the examination of laminar flow is that many flow features, particularly cross flow convection, exist in both laminar and turbulent flows. Study of the laminar case is thus helpful in bringing out essential features of streamwise vorticity enhanced mixing and in providing insight into mechanisms in the turbulent situation.

It should be stated that the quantitative results of the investigation are focussed on a particular type of lobe geometry known as "advanced lobed mixer" (Skebe [33], 1988),
although the results (and the method developed) apply equally well qualitatively to other geometries. As shown in Figure 2.1, the advanced lobed mixer is characterized by the parallel sides of trailing edge profile. This type of geometry has been found to be very effective in promoting mixing and is widely used in industry.

The approach taken here is similar to the slender body approximation used in external aerodynamics, in which a steady three-dimensional flow field is approximated by an incompressible, two-dimensional, unsteady one. This implies that the flow field development in the mean stream direction is treated as an unsteady process in successive cross flow planes as illustrated in Figure 2.2, with distance in the mean stream direction replaced by a time variable.

The use of this conceptual approach was based on several observations. First, the length scale associated with the diffusion of velocity or scalar quantities, for downstream regions close to the trailing edge, is generally smaller than the length scale associated with the cross flow due to the streamwise vorticity. While the length scale of the cross flow is the lobe wavelength, the diffusion length scale is of the same order as the mixing layer thickness, which is of the order of the displacement thickness of the flow leaving the trailing edge. The typical displacement thickness at the trailing edge for a well designed lobe (i.e. no boundary layer separation and low loss) is in the range of 1% to 5% of the lobe wavelength (Skebe, [33], 1988), which is over an order of magnitude smaller than the length scale associated with the cross flow.

Second, the cross flow velocity due to the streamwise vorticity is, for a large class of lobed mixers, small compared with the mean stream velocity. For a typical lobe, the ratio of maximum velocity in the cross flow plane to mean stream velocity is approximately \( \tan \alpha \), where \( \alpha \) is the penetration angle. The penetration angles of lobes are in the range of 5° to 20°, and the ratio of the maximum cross flow velocity to the mean stream velocity is thus in the range of 0.08 to 0.36. This also means that the Mach number of the cross flow is relatively low and the flow in the cross flow plane can be treated as approximately incompressible. For example, for a penetration angle of 10° and mean flow Mach number of 2, the maximum cross flow Mach number is about 0.35. Effects due to compressibility at this Mach number would not be significant.

Finally, the gradients of velocities and scalars in the cross flow plane are much greater than those in the mean stream direction. Because the mixing layer is "thin" (compared
to lobe wavelength), the dominant gradients are in the direction across the mixing layer. The direction across the mixing layer is roughly in the cross flow plane, with the maximum angle between the direction across the mixing layer and the cross flow plane being roughly $\alpha$, where $\alpha$ is the penetration angle of the lobe. The ratio of gradients of velocities and scalar quantities in the cross flow plane to those across the mixing layer is thus approximately $\cos \alpha$, i.e. close to unity for small values of $\alpha$. The gradients in the cross flow plane can thus be used to approximate the gradients across the mixing layer. Since the diffusion is directly related to the gradient, we can compute the mixing based on the gradients of velocities and scalar quantities in the cross flow plane.

Finally we again emphasize that the focus here is on the effect of streamwise vorticity on mixing and the detailed flow over the lobe surface is regarded as secondary. The computation is for the flow region downstream of the lobed mixer trailing edge where the mixing occurs, and the effect of a given distribution of the streamwise vorticity at trailing edge on mixing is the principal topic investigated.

### 2.2 Slender Body Formulation

As discussed earlier, the goal is to investigate the flow development in the mean stream direction by following a frame that convects at mean axial velocity. The coordinates used are shown in Figure 2.2. The x-axis is in the mean axial direction (or streamwise direction) and the z-axis is in the lobe periodic direction.

The axial velocity can be expressed as $u = \bar{U} + u'$, where $\bar{U}$ is the mean axial velocity at the lobe trailing edge, and $u'$ is the perturbation. For $\frac{u'}{\bar{U}} \ll 1$, the $u \frac{\partial}{\partial x}$ term in the equations of motion can be approximated by a time derivative $\frac{\partial}{\partial t}$ (details are given in Appendix A). For small penetration angle $\alpha$ and “thin” mixing layer of thickness $\delta$, 

$$\left( \frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial z^2} \right) = O((\frac{\delta}{L})^2, \tan^2 \alpha) \ll 1$$

(2.1)

where $L$ is a length scale of order of the lobe wavelength.

Our objective is to study the effect of cross flow on mixing and, therefore, it is reasonable to normalize flow variables based on parameters associated with the cross flow. Since the magnitude of the cross flow velocity is determined by the strength of streamwise vor-
ticity $\Gamma$ (where $\Gamma$ is the half lobe circulation as indicated in Figure 1-1), the normalization scale for the velocities is chosen to be $\Gamma/\lambda$. The resulting equations for cross flow velocity components, $v$ and $w$, can be written as (the detailed derivation is given in Appendix A)

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_\Gamma} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v$$  \hspace{1cm} (2.2)

$$\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_\Gamma} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) w$$  \hspace{1cm} (2.3)

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$  \hspace{1cm} (2.4)

where $Re_\Gamma = \frac{\Gamma}{\nu}$ is the Reynolds number, all length scales have been normalized by the lobe wavelength $\lambda$, all velocities by $\frac{\Gamma}{\lambda}$, and the non-dimensional time $t$ is defined as $\frac{t}{U/\lambda^2}$.

To compute the diffusion, a scalar equation can be written as

$$\frac{\partial \phi}{\partial t} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} = \frac{1}{Re_\Gamma Sc} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right)$$  \hspace{1cm} (2.5)

where $Sc$ is the Schmidt number $\frac{\nu}{\Gamma}$ and $\phi$ is a scalar quantity.

As a result of normalizing the cross flow velocities by $\Gamma/\lambda$ and the length by $\lambda$, the relevant downstream distance is represented by the non-dimensional time $t = \frac{t}{U/\lambda^2}$. Other important parameters are the Reynolds number $Re_\Gamma$ and Schmidt number $Sc$. This means that for two flows with the same Reynolds number, Schmidt number and trailing edge profile, the flow development due to streamwise vorticity is only a function of the non-dimensional time $t$.

It is also useful to compare the mixing for flow with streamwise vorticity to that without streamwise vorticity. For the latter case, because no cross flow exists and circulation $\Gamma$ is equal to zero, the only meaningful equation is the scalar diffusion equation. A different set of non-dimensional parameters is thus used to normalize the equation. The velocity scale is chosen to be the mean axial velocity $\bar{U}$ and the length scale is the lobe wavelength $\lambda$. The resulting scalar equation in the absence of cross flow is

$$\frac{\partial \phi}{\partial \xi/\lambda} = \frac{1}{Re_\lambda Sc} \left( \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right)$$  \hspace{1cm} (2.6)

where the Reynolds number $Re_\lambda = \frac{\bar{U}\lambda}{\nu}$ and Schimdt number $Sc = \frac{\nu}{\Gamma}$.  

30
2.3 Initial and Boundary Conditions

2.3.1 Streamwise Vorticity at Trailing Edge

To solve the slender body equations for the flow downstream of the lobe trailing edge, the initial conditions representing the flow at the lobe trailing edge must be specified. In particular, the distribution of streamwise vorticity at the lobe trailing edge must be given. The production of shed streamwise vorticity by a lobed mixer is achieved in a manner analogous to its generation by a wing of finite length. The direction along the lobe wavelength corresponds to the spanwise direction in a finite span wing. Due to the penetration of lobed mixer into the stream and the variation of lobe geometry, there is a pressure difference across the lobe surface, and this pressure loading and the associated bound vorticity strength on the lobe surface vary in the spanwise direction. Under these conditions, streamwise vorticity must be shed from the trailing edge. For a specific lobe geometry, the distribution of the shed streamwise vorticity could be found using an inviscid method, as shown by Elliott ([7], 1990). However, to understand how streamwise vorticity augments the mixing, knowing the precise distribution of the streamwise vorticity at the trailing edge is not necessary. In this study, therefore, an approximation to the distribution of the streamwise vorticity at the lobe trailing edge is used. Parametric studies were also carried out to investigate the effects of different initial streamwise vorticity specifications on the mixing.

There are several constraints on the choice of initial streamwise vorticity distribution at the lobe trailing edge. The streamwise vorticity must be confined to a thin layer of fluid because the vorticity is within the boundary layer leaving the trailing edge, the shape of the vortex layer must correspond to the trailing edge profile, and the streamwise vorticity must approach zero at the lobe peak and trough to satisfy symmetry conditions. Aside from the above conditions, there can be a variation in the distribution of the streamwise vorticity strength (circulation) per unit length along the trailing edge, although for the lobe of interest the form of the variation can be closely approximated.

To specify the distribution of the shed streamwise vorticity, it was assumed that the shed streamwise vorticity distribution may be described by two functions: one represents the variation of the strength of streamwise vorticity per unit length along the trailing edge and the other represents the distribution of streamwise vorticity in the direction normal to
the trailing edge profile. As stated earlier, the discussion will be limited to the geometry of the advanced lobed mixer (Figure 2.1).

Strength of Streamwise Vorticity per Unit Trailing Edge Length

As stated above, the precise shed streamwise vorticity distribution at the lobe trailing edge can only be obtained by solving the flow over the lobe surface. However, simple kinematic considerations (the detailed derivation is given in Appendix B), first put forward by Skebe et al. ([33], 1988), show that the shed streamwise vorticity strength per unit trailing edge length can be assumed to be constant along the parallel side of the trailing edge of the advanced lobed mixer (Figure 2.1). The relation between the circulation of the streamwise vorticity at the lobe trailing edge and the lobe height can be written as

$$\Gamma \approx 2h\bar{U}tana$$  \hspace{1cm} (2.7)

where $\Gamma$ is the half lobe circulation as indicated in Figure 1.1, $h$ is the lobe height and $\alpha$ is the penetration angle. This type of distribution of circulation per unit length at the trailing edge is defined as the "uniform" distribution. Most of the error between the assumed uniform distribution and the actual vorticity distribution occurs in the region close to the lobe peak and trough. Since the percentage of the total circulation near the lobe peak and trough is small, it is not expected to have a strong influence on the overall mixing. Equation 2.7 has been shown (Skebe et al., [33], 1988) to represent the shed circulation per half lobe wavelength for a stream to stream velocity ratio of unity very well, with experimental results indicating that

$$\Gamma = 1.95h\bar{U}tana$$  \hspace{1cm} (2.8)

In addition to studying the uniform distribution, we shall also investigate a different circulation distribution per unit trailing edge length. In this type of distribution, referred to as the "concentrated" distribution, the streamwise vorticity is concentrated near the $y=0.0$ point of the lobe trailing edge. Figure 2.3 illustrates the two different streamwise vorticity distributions.
Streamwise Vorticity Distribution Normal to Trailing Edge

In addition to the circulation per unit trailing edge length, the thickness of vortex sheet and the distribution of streamwise vorticity across the vortex sheet must be specified. It is assumed that the streamwise vorticity distribution along a line normal to the local trailing edge in the cross flow plane is given as

\[ \omega = F(s) - \frac{1}{\sqrt{\pi \epsilon}} e^{-\left(\frac{n}{\epsilon}\right)^2} \] (2.9)

where \( n \) is the local normal distance to the lobe trailing edge line in the cross flow plane, \( \epsilon \) is a parameter that defines the thickness of the vortex sheet and \( F(s) \) represents the strength of streamwise vorticity per unit length at the trailing edge. For simplicity, \( \epsilon \) is taken to be independent of the position along the trailing edge. Although Equation 2.9 is an approximation to the actual streamwise vorticity distribution, computations based on the above streamwise vorticity distribution show that, for the range of thickness parameters of practical interest, the effect of vortex sheet thickness on the mixing is small (see Section 3.2.2). Thus the approximation used to represent the streamwise vorticity distribution across the vortex sheet should not have a strong influence on the overall mixing.

2.3.2 Scalar Distribution at Trailing Edge

To investigate the mixing, it is useful to assign a scalar value \( \phi \) to each unmixed stream representing different fluid states. For convenience we can use numerical values of +1 for one stream before mixing and -1 for the other. Since the computational method used cannot resolve an infinite gradient, the initial scalar layer across the trailing edge in the cross flow plane is assumed to have a finite thickness. An error function is used to specify the distribution of scalar values across the trailing edge profile

\[ \phi(t = 0) = 2 \times erf\left(\frac{n}{\sigma_s}\right) - 1 \] (2.10)

where \( \sigma_s \) is the thickness parameter for the initial thickness of the scalar layer and \( n \) is the local normal distance to the trailing edge profile in the cross flow plane. For most of the computations, \( \sigma_s = 0.02 \) is used. A finite value of the scalar layer thickness means that the flow at the trailing edge is not totally unmixed. Effects of different initial scalar layer
thickness on the computational results obtained will also be investigated.

2.3.3 Boundary Conditions

Since we are only interested in the mixing, friction on the mixing duct wall is neglected. This is achieved with zero gradient of tangential velocity. The lobes investigated are periodic in the z-direction, so reflective boundary conditions (i.e. zero gradient of tangential velocity and zero normal velocity) are used, and the computational domain width is thus half of the lobe wavelength.

2.4 Mixedness Parameter

To assess the effect of streamwise vorticity, it is necessary to define an overall measure of the mixing. For the present purpose, it is sufficient to define scalar mixedness in terms of molecular mixing as the percentage of product generated in a diffusion limited, bi-molecular, reaction with an equivalence ratio of one. To be more precise, if each of the unmixed streams is assigned a scalar value of $\phi = 1$ or $\phi = -1$, the scalar mixedness at any station downstream can be defined as

$$M = \frac{1}{A} \int_A (1 - |\phi|) dA$$

(2.11)

where the integration is over the cross sectional area $A$ of the mixing duct. The value of $M$ is zero for unmixed flow, and unity for fully mixed flow, if the initial areas of the two streams are the same.

2.5 Method of Solving Slender Body Equations

To ensure accuracy in computing the diffusion and to avoid the problem of artificial diffusion associated numerics, a finite-element spectral method is used for solving the slender body equations. This method is based on a time splitting scheme that solves the convective, pressure and viscous effects separately. The computational domain is divided into small elements and within each element, the flow field variables are represented by $7 \times 7$ orders of Chebychev polynomials. The detailed formulation is given in Appendix C.
2.6 Summary

A computational model based on capturing the cross flow associated with streamwise vorticity in laminar flow with a stream to stream velocity ratio close to unity is presented. The aim is to explore the effect of the cross flow on mixing, and to relate the mixing augmentation to the distribution/strength of the shed streamwise vorticity at the trailing edge. An approximate method of specifying the shed streamwise vorticity distribution at the lobe trailing edge is given and is consistent with experimental observation. In addition, a mixedness parameter for scalar mixing is defined.
Figure 2-1: Schematic drawing of an “advanced lobed mixer”
Figure 2-2: Illustration of space and time analogy and definition of coordinates
Figure 2-3: Initial streamwise vorticity distributions for slender body computation
Chapter 3

Effect of Streamwise Vorticity on Mixing in Laminar Flow with Stream to Stream Velocity Ratio Close to Unity

3.1 Introduction

In this chapter, we investigate the relations between the distribution/strength of shed streamwise vorticity and the mixing augmentation in laminar flow. As indicated in Chapter 2, the following assumptions were made: 1). the difference in axial velocity between two streams is small compared with the average stream velocity; 2). the diffusion in the cross flow plane is much greater than the diffusion in the axial direction; 3). the cross flow velocity is small compared with the average stream velocity. These assumptions lead to a set of equations for cross flow velocities and an equation for a scalar field.

Scalar mixing is measured using the mixedness parameter ($M$) defined in Chapter 2. In the computations, the Schmidt number is kept at unity for all cases studied. The Reynolds number based on the circulation is limited to a maximum of $5 \times 10^4$ so that gradients in scalar and velocity fields can be satisfactorily resolved.

As stated previously, the discussion here will be limited to the geometry of the advanced-lobed mixer, which is characterized by parallel sides of the trailing edge profile. The method
outlined in Section 2.3 will be used to specify the initial conditions for the streamwise vorticity and scalar fields. However, the computational model developed can also be applied to lobes of different trailing edge geometries. An example is given in Appendix D, where a comparison between solutions of a three-dimensional Euler solver, and those of the computational model developed in Chapter 2, for a lobe of sinusoidal trailing edge profile, is presented.

3.2 Effect of Streamwise Vorticity Distribution on Mixing

3.2.1 Flow Field Development

To understand how the streamwise vorticity can increase the mixing, it is useful to investigate the development of the vortical field in the cross flow plane downstream of the lobed mixer. A lobe with a height to wavelength ratio of 0.54 was used. This lobe geometry was utilized in a United Technologies Research Center flow visualization test (McCormick, [21], 1988). The distance between the side walls of the mixing duct is $A$. The vortical field, as a function of time (or non-dimensional downstream distance), is shown in Figure 3.1a and Figure 3.1b, with the time, $t$, equals to $\frac{L}{\lambda \cdot \lambda}$. The uniform distribution of trailing edge vorticity (see Figure 2.3) is used as an initial condition with the thickness of the vortex sheet $\epsilon = 0.02$. The Reynolds number, $Re$, is $10^3$.

Figure 3.1a and Figure 3.1b show the time development process in which the initially distributed streamwise vorticity along the trailing edge tends to evolve into vortex cores. At time $t=0.4$, the vortical region is somewhat elliptical (Figure 3.1a), but the formation of vortex cores is nearly completed by time $t=1.0$ (Figure 3.1b). For a larger time, $t=2.9$, the shape of the vortex cores remains roughly unchanged while their size increases as a result of diffusion.

Another indication of vortex core formation can be seen by examining the static pressure field, as shown in Figure 3.2. At $t=0.4$, the pressure contours are elongated, whereas at $t=1.0$, they are represented by the approximately circular lines characteristic of a vortex core.

The scalar field development associated with vortex core formation is also investigated. The initial scalar distribution is specified using Equation 2.10 with a thickness parameter $\sigma_s = 0.02$. This specification is used in all subsequent studies in the rest of this chapter un-
less otherwise stated. Figure 3.3a and Figure 3.3b show the scalar distributions at different times. The scalar field reflects the effect of the cross flow due to streamwise vorticity. As the vortical region rotates, the fluid interface is stretched and winds around. As a result, the net interface increases. At $t=1.0$, considerably more interface area exists compared with that at $t=0.0$. By $t=2.9$, the combined effect of the interface winding and diffusion has resulted in an almost completely mixed scalar field.

The flow field development has a strong effect on the mixing enhancement. The mixedness parameter $M$ as a function of time ($\frac{U}{\alpha} \xi$) is shown in Figure 3-4. The non-zero value of $M$ at $t=0.0$ is due to the initial scalar field having a finite thickness across the trailing edge profile. We can characterize the mixedness behavior as consisting of three distinct regions. In the first region, $t < 1.0$, the slope of mixedness against $t$ increases gradually, corresponding to the initially distributed streamwise vorticity evolving into vortex cores. In the second region $1.0 < t < 2.0$, the slope of $M$ vs. $t$ is essentially constant. Here, the convective motion and the mixing are driven by vortex cores. For $t > 2.0$, with the scalar field nearly completely mixed, local diffusion becomes dominant and $M$ slowly approaches its asymptotic value of unity.

It is of interest to compare mixedness as a function of downstream distance $\xi$ for flow with streamwise vorticity to that without streamwise vorticity. In this, the mixing with no streamwise vorticity is computed based on the simple diffusion equation (Equation 2.6). This is shown in Figure 3-5 for a Reynolds number $\frac{U}{\nu} = 2000$ and $\frac{U}{\nu}$ = 0.39. The effect of the streamwise vorticity on the scalar mixing is evident; the slope of mixedness, $M$, for mixing with streamwise vorticity is roughly eight times larger than that without streamwise vorticity at $\xi = 2.5$.

### 3.2.2 Effects of Initial Scalar Layer Thickness and Initial Streamwise Vorticity Thickness

As stated before, it is assumed that at $t=0.0$, a small amount of mixing has already occurred (Figure 3-4). This is the result of the initial scalar distribution having a finite thickness. The effects of using different initial scalar layer thickness parameters on the mixedness are shown in Figure 3-6. As expected, the effect of the initial scalar layer thickness on the overall mixedness is limited to the region of small $t$, and thus for subsequent investigation, only $\sigma_s = 0.02$ is used.
The effects of different initial streamwise vortex sheet thickness on mixing can also be examined by varying the thickness parameter $\epsilon$ (defined in Equation 2.9). Figure 3-7 shows the mixedness, $M$, as a function of time with an initial vortex thickness of $\epsilon = 0.05$ and $\epsilon = 0.02$ for $Re_T = 1000$. The uniform streamwise vorticity distribution is used to specify the circulation per unit length along the trailing edge. The range of vorticity thickness investigated corresponds to the displacement thickness of the boundary layers observed in the experiment of Skebe ([33], 1988). For the range of $\epsilon$ of practical interest, the streamwise vortex sheet thickness has little effect on the mixedness parameter. This is again expected since, for the range of the thickness parameter investigated, the streamwise vorticity at the trailing edge is essentially contained in a “thin” vortex sheet; its evolution should not be strongly dependent on the thickness of the vortex sheet.

3.2.3 Effect of the Distribution of Circulation Per Unit Trailing Edge Length

The cases investigated so far employed a uniform distribution of the circulation along the trailing edge. It is of interest to examine the effect of different circulation distributions on the downstream mixing. The time development of the vortical field for a concentrated initial streamwise vorticity distribution (as shown in Figure 2.3) at the trailing edge is therefore shown in Figure 3.8a and Figure 3.8b. In this case, the distributed vorticity evolves into a concentrated vortex core at an earlier time of $t=0.4$ compared with (roughly) $t=1.0$ for the uniform initial streamwise vorticity distribution. The corresponding scalar field, shown in Figure 3.9a and Figure 3.9b, indicates that more mixing has occurred around the region of vorticity concentration.

A comparison of the mixedness parameter, $M$, as a function of time for the concentrated and uniform initial streamwise vorticity distributions is shown in Figure 3-10. As with the case of the uniform initial vorticity distribution, the mixedness, $M$, for concentrated initial vorticity distribution, shows three distinctive regions; the region for constant slope of mixedness, $M$, vs. time starts near $t=0.4$, compared with $t=1.0$ for the case of the uniform initial vorticity distribution. The mixing becomes diffusion dominated at $t=1.5$ compared with $t=2.0$ for the uniform initial distribution case. To reach the same level of mixedness, say $M=0.5$, the case with concentrated initial vorticity distribution takes about 15% less time. This is because the effect of the streamwise vorticity on mixing is the strongest when
the vorticity is in the form of a vortex core, and the flow with the concentrated initial vorticity distribution evolves into vortex cores at an earlier time compared to that of the uniform initial vorticity distribution. Therefore, for a given circulation, it is advantageous to have an initial streamwise vorticity distribution that is more concentrated and can evolve into a vortex core as quickly as possible.

3.3 Effect of Reynolds Number/Streamwise Vorticity Strength on Mixing

3.3.1 Effect of Reynolds Number $Re_T$ on Mixing

Keeping the Schmidt number constant and increasing Reynolds number is equivalent to reducing the diffusion coefficient. The time development of the scalar, for the uniform initial vorticity distribution and $Re_T = 4000$, is shown in Figure 3.11a and Figure 3.11b. The parameter $\epsilon = 0.02$ is used for the streamwise vortex sheet thickness. Because of the smaller diffusion, the scalar layer thickness is thinner than the case for $Re_T = 1000$ (see Figure 3.3a and Figure 3.3b). Mixedness, $M$, against time for $Re_T = 4000$ and $Re_T = 1000$ is shown in Figure 3-12. A comparison between the mixedness for $Re_T = 1000$ and that for $Re_T = 4000$ with the same initial vorticity distribution shows that the higher Reynolds number flow takes longer to achieve the same mixedness.

3.3.2 Mixing Augmentation and Marble's Point Vortex Model

The critical feature of the mixing increase due to the streamwise vorticity is the stretching and winding of the fluid interface. The mixing rate increases as the initially distributed vorticity evolves into vortex cores. After the formation of vortex cores, there is a time interval during which the mixing rate stays approximately constant. Because the main interest is in the mixing increase due to the presence of the streamwise vorticity, it is useful to study the mixing augmentation, $M_a$, defined as the difference between mixedness, $M$, for the mixing with streamwise vorticity and the mixedness, $M$, for the mixing without streamwise vorticity while keeping the diffusion coefficient constant.

The mixing augmentation, $M_a$, due to a single vortex has been worked out by Marble ([20], 1985). The fluid interface winds round the vortex and, if the viscous diffusion of
the vortex is small, the growth of a well mixed scalar core is much faster than that of the
viscous core. The vortex can, as far as mixing is concerned, be treated as a point vortex,
and the mixing augmentation is a function of the circulation. The scaling of the mixing
augmentation rate with the Reynolds number based on circulation has been found to be
(Marble, [20], 1985)

\[
\left( \frac{\partial M_a}{\partial t} \right)_1 \left( \frac{\partial M_a}{\partial t} \right)_2 = \left( \frac{(Re_G Sc)_1}{(Re_G Sc)_2} \right)^{-\frac{1}{2}}
\]

(3.1)

where Reynolds number \( Re_G \) is equal to \( \frac{\Gamma}{\nu} \) and Schmidt number \( Sc \) is equal to \( \frac{j}{\nu} \).

Although Marble's model applies only to a single point vortex, there is no qualitative
difference between a single vortex and an array of vortices. After the formation of vortex
cores and before scalar layers diffuse into each other, the downstream mixing should scale
in a similar way as that indicated by Equation 3.1. We can assess this by examining the
computed mixing augmentation, \( M_a \), for \( Re_G = 4000 \) and \( Re_G = 1000 \), shown in Figure
3-13. Multiplying the mixing augmentation, \( M_a \), for \( Re_G = 4000 \) by a scaling factor of
4\( \frac{1}{2} \) as suggested by Equation 3.1 (with Schmidt number constant), the rescaled mixing
augmentation, \( M_a \), for \( Re_G = 4000 \) is shown in Figure 3-14, together with the mixing
augmentation for \( Re_G = 1000 \). It can be seen that the rescaled mixing augmentation, \( M_a \),
for \( Re_G = 4000 \) has approximately the same slope as that of \( Re_G = 1000 \) in the range of
1.1 \( \leq t < 1.8 \); it is in this time interval that the mixing is driven by vortex core.

3.3.3 Effect of Strength of Streamwise Vorticity on Maximum Mixing
Augmentation Rate

The scaling relation given in Equation 3.1 is only valid after vortex cores are formed and
before the fluid field is completely mixed. However, whether concentrated vortex cores can
be formed also depends on the Reynolds number \( Re_G \). At very low Reynolds numbers,
diffusion dominates (for fixed Schmidt number) and the flow is more likely to be mixed out
before vortex cores can be formed. If so, the relationship between mixing augmentation and
strength of the streamwise vorticity will be different from that indicated in Equation 3.1.

To investigate the effect of the strength of the streamwise vorticity on mixing augmentation
for large diffusion, it is useful to use the maximum value of the mixing augmentation
rate \( \left( \frac{\partial M_a}{\partial t} \right) \) as a relative measure since it indicates the upper limit of the effect of the stream-
wise vorticity on mixing. A number of computations were carried out for different Reynolds
numbers \((\text{Re}_t)\) and the corresponding maximum mixing augmentation rates were obtained. The lobe geometry was the one used in earlier computations with a height to wavelength ratio of 0.54. The uniform vorticity distribution was used as an initial condition and the vorticity thickness parameter \(\varepsilon\) was set to 0.02.

The computed maximum mixing augmentation rates are shown in Figure 3-15 as a function of Reynolds number \(\text{Re}_t\). The behavior of the maximum mixing augmentation rate as a function of Reynolds number can be divided into two regions. For high Reynolds number \(\text{Re}_t > 500\), the maximum mixing rate is governed by the similarity law for a point vortex (Equation 3.1), indicating that there is a time range where the mixing is driven by vortex cores. For \(10 < \text{Re}_t < 500\), however, the maximum mixing augmentation rate shows a different and weaker dependence on \(\text{Re}_t\). In fact, for a change of Reynolds number of a factor of 25, from \(\text{Re}_t = 20\) to \(\text{Re}_t = 500\), the maximum mixing augmentation rate varies only 30%. The parameter that separates the two different behaviors is characterized by a critical Reynolds number \((\text{Re}_t)_c\). For the geometry investigated \((\text{Re}_t)_c\) is about 500.

In practice, one is also interested in the mixing augmentation per unit downstream distance. Figure 3-16 shows the maximum mixing augmentation per unit downstream distance as a function of the strength of the streamwise vorticity based on Figure 3-15. For a flow with fixed Reynolds number based on mean axial velocity, \(\text{Re}_\lambda = \frac{U_\lambda}{\nu}\), the strength of the streamwise vorticity can be changed by increasing \(\frac{f}{U_\lambda}\). Figure 3-16 suggests that, for low strength of streamwise vorticity (or small \(\text{Re}_t = \frac{f}{\nu}\)), the maximum mixing augmentation per unit downstream distance is proportional to \(\frac{f}{U_\lambda}\); for high strength of streamwise vorticity, the maximum mixing augmentation per unit downstream distance is proportional to \(\left(\frac{f}{U_\lambda}\right)\frac{1}{2}\).

The same type of Reynolds number effect on maximum mixing augmentation rate happens for lobes of higher amplitude. Because initially distributed vorticity tends to evolve into a vortex core, the maximum mixing augmentation rate for a higher lobe amplitude at a sufficiently high Reynolds number is expected to be governed by the Marble scaling law. However, for a given strength of streamwise vorticity, it is also true that the rotation speed of the vortical region for a higher lobe amplitude is slower than that for a lower lobe amplitude, and there is more time for the scalar field to diffuse before the formation of the vortex core. The critical Reynolds number \((\text{Re}_t)_c\) is thus expected to increase with the increase of the lobe height.

The maximum mixing augmentation rate has been calculated as a function of Reynolds
number for a lobe of different height, \( \frac{h}{\lambda} = 1.0 \), with distance \( 2\lambda \) between the side walls of the mixing duct. The maximum scalar mixing augmentation rate for this lobe is shown in Figure 3-17. The uniform initial vorticity distribution is used and the vorticity thickness parameter is \( \varepsilon = 0.02 \). It can be seen that the critical Reynolds number dividing the regions of viscous dominated behavior from that of vortex core behavior is about 2000, compared with 500 for lobe of \( \frac{h}{\lambda} = 0.54 \). This reflects the fact that the distributed vorticity of the higher amplitude lobe requires a longer time to evolve into vortex cores.

### 3.4 Estimation of Rotation Speed of Vortical Region

Since the mixing augmentation rate is a maximum when the streamwise vorticity evolves into a vortex core, it is useful to determine how far downstream the vortex core will form for a given streamwise vorticity distribution. However, what constitutes a vortex core is hard to define. As seen earlier, the evolution from distributed vorticity to a concentrated vortex core is associated with the rotation of the vortical region, and the rotation speed of the vortical region thus provides a good indication of the time required for the formation of the vortex core. For a given circulation distribution, the larger the lobe height, the longer it takes for the vortical region to rotate through a fixed angle.

To measure the rotation of the vortical region, the y-moment of the vorticity is defined as

\[
I_y = \int \frac{\omega}{\lambda} y dA
\]  

(3.2)

where \( \omega \) is the streamwise vorticity, \( A \) is the cross sectional area. All length scales are normalized by the lobe wavelength \( \lambda \). As the streamwise vorticity is convected downstream, the y-moment of vorticity will go through a set of minimum and maximum values as illustrated in Figure 3-18. When the y-moment of vorticity goes through the first minimum, the vortical region can be considered to have rotated through \( 90^\circ \) angle. The time required for \( 90^\circ \) rotation thus provides a relative measure of the rotation speed of the vortical region.

For the purpose of this investigation, we shall only consider the advanced lobed mixer because of its practical interest. The circulation per unit trailing edge length is assumed to be constant along the parallel sides of the trailing edge (uniform distribution, see Figure 2.3) with the thickness of streamwise vortex sheet fixed at \( \varepsilon = 0.02 \). Computations were carried out for lobes of different heights (from \( \frac{h}{\lambda} = 0.5 \) to 1.5). Figure 3-19 shows the time...
required for the vortical region to rotate through 90° as a function of the lobe height. It can be seen that the time required for a 90° rotation scales roughly with the square of the lobe height.

This relationship is expected because Lamb ([17], 1945) has shown that, for an elliptical area of constant vorticity, the time required for a 90° rotation is

\[ t_{90} = \frac{\Gamma x}{U\lambda^2} = \pi^2 \frac{a^2 + b^2}{2\lambda^2} \]  

where \( \Gamma \) is the circulation, \( a \) and \( b \) are the semi-major and semi-minor axis of the ellipse, and \( \lambda \) is the lobe wavelength. For the uniform vorticity distribution, lobe height, \( h \approx 2a \), and \( \epsilon \approx b \) which is much smaller than \( h \). Therefore, the time required for a 90° rotation based on Equation 3.3 is

\[ t_{90} = \frac{\pi^2 b^2}{8(\frac{h}{\lambda})^2} \]  \( (3.4) \)

The Equation 3.4 is also shown in Figure 3-19, and although it strictly applies to a single vortical region of elliptical area, the agreement between the computed value and predicted one based on Equation 3.4 is good.

### 3.5 Summary

The main results of this chapter are summarized as follows:

- Distributed streamwise vorticity at the lobe trailing edge tends to evolve into concentrated vortex cores. The convective flow due to the streamwise vorticity increases the fluid interface.

- The mixing augmentation is found to be dependent on the distribution of the shed streamwise vorticity along the lobe trailing edge: the more concentrated the streamwise vorticity, the higher the mixing augmentation rate. The distribution of the shed streamwise vorticity normal to the lobe trailing edge has only a small effect for the range of parameters investigated,

- After the formation of the vortex core, the mixing augmentation rate, \( \frac{\partial M}{\partial t} \), for a fixed Schmidt number scales with \( Re^{-\frac{1}{2}} \).

47
For a given initial distribution of shed streamwise vorticity and fixed Reynolds number ($Re_{\lambda} = \frac{U_{\lambda}}{\nu}$) and Schmidt number ($Sc = \frac{\nu}{\gamma} = 1.0$), the maximum mixing augmentation per unit downstream distance $\frac{\partial M}{\partial x}$ is approximately proportional to $\frac{1}{Re_{\lambda}}$ for low $Re_{\lambda}$ and proportional to $(\frac{1}{Re_{\lambda}})^{\frac{3}{2}}$ for high $Re_{\lambda}$. The parameter that separates regions is given by $Re_{\Gamma}$, and is about 500 for $\frac{h}{\lambda} = 0.54$ and 2000 for $\frac{h}{\lambda} = 1.0$.

For a given strength of shed streamwise vorticity, the rotation speed of the vortical regions scales with the lobe height squared.
Figure 3.1a: Contours of streamwise vorticity at different time \( t = \frac{L}{U \lambda} \) for \( Re_L = 1000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \))
Figure 3.1b: Contours of streamwise vorticity at different time ($t = \frac{L}{\lambda} \frac{f}{u_\lambda}$) for $Re_T = 1000$ and $\frac{L}{\lambda} = .54$ (uniform initial streamwise vorticity distribution and $Sc = 1.0$)
Figure 3-2: Contours of static pressure at different time ($t = \frac{L}{U\lambda}$) for $Re_T = 1000$ and $\frac{h}{\lambda} = 0.54$ (uniform initial streamwise vorticity distribution and $Sc = 1.0$)
Figure 3.3a: Contours of scalar value at different time \((t=\frac{\tau}{\lambda})\) for \(Re_f = 1000\) and \(\frac{A}{\lambda} = .54\) (uniform initial streamwise vorticity distribution and \(Sc=1.0\))
Figure 3.3b: Contours of scalar value at different time ($t=\frac{r}{\bar{u}}$) for $Re_T = 1000$ and $\frac{h}{\bar{x}} = .54$ (uniform initial streamwise vorticity distribution and $Sc=1.0$)
Figure 3-4: Mixedness as a function of time \( t = \frac{\Gamma x}{U\lambda \lambda} \) for \( Re_\Gamma = 1000 \) and \( k_\lambda = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \))
Figure 3-5: Comparison of mixedness as a function of downstream distance ($\frac{x}{\lambda}$) for flows with streamwise vorticity (same as Figure 3.1) and without streamwise vorticity ($Re_\lambda = 2000$, $\frac{\Gamma}{U_\lambda} = .39$ and $Sc=1.0$)
Figure 3-6: Effect of scalar thickness $\sigma_s$ on mixedness for $Re_T = 1000$ and $\frac{h}{\lambda} = .54$ (uniform initial streamwise vorticity distribution and Sc=1.0)
Figure 3-7: Effect of streamwise vorticity thickness $\epsilon$ on mixedness for $Re_{\theta} = 1000$ and $\frac{\lambda}{\lambda'} = .54$ (uniform initial streamwise vorticity distribution and $Sc=1.0$)
Figure 3.8a: Contours of streamwise vorticity at different time ($t = \frac{\tau}{U_\lambda}$) for $Re_T = 1000$ and $\frac{h}{\lambda} = .54$ (concentrated initial streamwise vorticity distribution and $Sc=1.0$)
Figure 3.8b: Contours of streamwise vorticity at different time \((t = \frac{\xi}{U_A})\) for \(Re_T = 1000\) and \(\frac{\xi}{L} = .54\) (concentrated initial streamwise vorticity distribution and \(Sc=1.0\))
Figure 3.9a: Contours of scalar value at different time ($t = \frac{D}{UL}$) for $Re_T = 1000$ and $\frac{h}{\lambda} = .54$ (concentrated initial streamwise vorticity distribution and $Sc=1.0$)
Figure 3.9b: Contours of scalar value at different time \((t = \frac{r}{U\lambda})\) for \(Re = 1000\) and \(h = 0.54\) (concentrated initial streamwise vorticity distribution and \(Sc = 1.0\))
Figure 3-10: Comparison of mixedness for uniform and concentrated initial streamwise vorticity distributions for $Re = 1000$, $\frac{X}{\lambda} = 0.54$ and $Sc=1.0$
Figure 3.11a: Contours of scalar value at different time \((t = \frac{r}{U\lambda})\) for \(Re = 4000\) and \(\frac{h}{\lambda} = .54\) (uniform initial streamwise vorticity distribution and \(Sc=1.0\))
Figure 3.11b: Contours of scalar value at different time \( t = \frac{L}{U_h \lambda} \) for \( Re_l = 4000 \) and \( \frac{h}{\lambda} = .54 \) (uniform initial streamwise vorticity distribution and \( Sc=1.0 \))
Figure 3-12: Mixedness for $Re_{\Gamma} = 1000$ and $Re_{\Gamma} = 4000$ (uniform initial streamwise vorticity distribution, $\frac{\Gamma}{\lambda} = 0.54$ and $Sc=1.0$)
Figure 3-13: Mixing augmentation for $Re_T = 1000$ and $Re_T = 4000$ (uniform initial stream-wise vorticity distribution, $\frac{b}{\lambda} = 0.54$ and $Sc=1.0$)
Figure 3-14: Mixing augmentation for $Re_T = 1000$ and rescaled $Re_T = 4000$ according to Equation 3.1 (uniform initial streamwise vorticity distribution, $\frac{4}{\lambda} = 0.54$ and $Sc=1.0$)
Figure 3-15: Maximum scalar mixing augmentation rate as a function of Reynolds number (uniform initial streamwise vorticity distribution, $\frac{\Delta x}{l} = 0.54$ and $Sc=1.0$)
Figure 3-16: Maximum scalar mixing augmentation per unit downstream distance as a function of $\log\left(\frac{\Gamma}{U_\lambda}\right)$ (uniform initial streamwise vorticity distribution, $Re_\lambda = 2000$, $\frac{\lambda}{\lambda} = 0.54$ and $Sc=1.0$)
Figure 3-17: Maximum scalar mixing augmentation rate as a function of Reynolds number (uniform initial streamwise vorticity distribution, $\frac{h}{\lambda} = 1.0$ and $Sc=1.0$)
Figure 3-18: Schematic drawing of the behavior of the moment of vorticity
Figure 3-19: Time required for 90 degree rotation (symbols are from computation and solid line is given by Equation 3.4)
Chapter 4

Effect of Streamwise Vorticity on Mixing in Turbulent Flow with Large Stream to Stream Velocity Difference

4.1 Introduction

The preceding chapters showed some of the overall features of the lobed mixer flow field. These mixers, however, are widely used in situations where the flow is turbulent and the mean velocity difference between the unmixed streams is not small, and it is necessary to examine this situation. As emphasized before, the goal here is not to study the turbulent mixing downstream of lobed mixers, but rather to investigate the mixing augmentation effect of the shed streamwise vorticity; in other words, for a given turbulent flow field, we are interested in how much additional mixing can be achieved by the introduction of the shed streamwise vorticity. This is an important distinction, which has driven the development of the model described herein.

There are several important features that exist in both turbulent and laminar flows downstream of the lobed mixer: there is a cross flow due to the shed streamwise vorticity; the mixing layer close to and downstream of the trailing edges is "thin" compared to the lobe wavelength (because the trailing edge boundary layer is thin); and the flow gradient
in the downstream direction is smaller than that in the cross flow direction.

4.2 Computational Model for Turbulent Flow

4.2.1 Computational Model

The aim here is to investigate the effect of the cross flow due to the shed streamwise vorticity on the mixing. To capture the effect of the cross flow on the mixing in turbulent flows, we shall use the same method as that in laminar flow and follow the development of the cross flow in a frame that convects at a "mean" velocity (the definition of the mean velocity will be given later). The downstream distance, \( x \), is replaced by a time variable. The effect of turbulence is represented by an eddy viscosity \( \nu_t \). The equations for the cross flow velocities, \( v \) and \( w \), can be written as

\[
\frac{\partial v}{\partial t^*} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu_t \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v \quad (4.1)
\]

\[
\frac{\partial w}{\partial t^*} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu_t \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) w \quad (4.2)
\]

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (4.3)
\]

where the time \( t^* = \frac{x}{U} \), \( U \) is the mean velocity, and \( \nu_t \) is an effective eddy viscosity for turbulent diffusion (and its value will be defined later).

With this convecting frame of reference, the equation for the axial velocity perturbation can be written as

\[
\frac{\partial u'}{\partial t^*} + v \frac{\partial u'}{\partial y} + w \frac{\partial u'}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_t \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) u' \quad (4.4)
\]

where \( u' = u - \bar{U} \) is the axial velocity perturbation.

The idea of using a convective time to represent the downstream distance in the case of large stream to stream velocity difference cannot be justified rigorously because nonlinear terms, such as \( u' \frac{\partial u'}{\partial x} \), are neglected. However, similar methods have been used in other applications (Batchelor, [2], 1954; Sowerby and Cooke, [34], 1953; Rayleigh, [27], 1911) with reasonable success. For example, the laminar boundary layer flow over a flat plate can be obtained with reasonable accuracy by solving a diffusion equation with a convective
time \( t = x/U \) (see White, [38], 1974), where \( U \) is the free stream velocity.

In the present case, it is reasonable to choose the average convective speed of the shed streamwise vorticity as the mean velocity. Since the streamwise vorticity is shed at the lobe trailing edge, the average convective velocity of the streamwise vorticity is approximately

\[
\overline{U} = \frac{U_2 + U_1}{2}
\]  

(4.5)

where \( U_1 \) and \( U_2 \) are the mean streamwise velocities of the two streams at the lobe trailing edge.

Our main interest is in the mixing of two co-flowing streams, and wall friction is neglected. If so, integration of Equation 4.4 over the cross sectional area of a constant area mixing duct suggests that (assuming that \( \nu_t \) is independent of \( y \) and \( z \) coordinates and making use of Equation 4.3), in order to satisfy the axial mass flow conservation, the pressure term satisfies,

\[
\frac{\partial}{\partial x} \int_A p dy dz = 0
\]

(4.6)

This means that the mean value of the pressure in Equation 4.4 does not change from one downstream station to another and thus the formulation outlined above can not be used to compute the static pressure recovery due to mixing explicitly. In consistency with the axial mass flow conservation, we shall neglect the pressure gradient term in Equation 4.4, and write the equation for the axial velocity perturbation as

\[
\frac{\partial u'}{\partial \tau^*} + v \frac{\partial u'}{\partial y} + w \frac{\partial u'}{\partial z} = \nu_t (\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) u'
\]

(4.7)

where \( u' \) is the axial velocity perturbation. This simplified model can not be rigorously justified, however the computational results based on the model appear to predict flow features observed in experiments.

\(^1\text{We have also neglected pressure variation in the cross flow plane due to the swirl. This variation in pressure can, in fact, be obtained from Equations 4.1-4.3. As shown in Appendix E, including the pressure variation due to the swirl has a small effect on the computed mixedness parameters for stream to stream velocity ratios investigated.}\)
4.2.2 Eddy Viscosity

To obtain a reasonable estimate of eddy viscosity, we consider a flow without streamwise vorticity (or for lobe of zero penetration angle). In the absence of streamwise vorticity, the mixing layer downstream of the lobe trailing edge is convoluted. The thickness of the mixing layer, at least for the first few wavelengths downstream of the trailing edge, is thin compared to the lobe wavelength. We can approximate the convoluted mixing layer as a quasi-two-dimensional shear layer that starts at the trailing edge of the lobe. Far downstream of the trailing edge, the adjacent mixing layers interact and the quasi-two-dimensional assumption breaks down, but the mixing at that location is nearly complete and the details of the flow field are thus of less interest.

The growth rate of a two-dimensional shear layer as a function of the axial velocity ratio is well documented (Brown and Roshko, [4], 1974; Lin, [18], 1984; Dimotakis, [5], 1989; Hermanson, [10], 1989; Sabin, [29], 1965). The vorticity thickness of a two-dimensional shear layer of constant density has been found to be (Dimotakis, [5], 1989),

$$\frac{\delta}{x} = C \frac{1 - r}{1 + r}$$

(4.8)

where $\delta$ is the vorticity thickness of the two-dimensional shear layer, $x$ is the streamwise distance and $r = \frac{U}{U_t}$ is the streamwise velocity ratio. The constant, $C$, varies between 0.125 and 0.225 (Dimotakis, [5], 1989), depending on flow conditions of the experiment. In this investigation, we take an average value of $C=0.175$. The above relation between the shear layer thickness growth rate and the velocity ratio can also be used in compressible flow if the constant $C$ is taken as a function of the convective Mach number (Papamoschou and Roshko, [22], 1988). The relationship between the constant $C$ and the convective Mach number for compressible flow will be presented later.

From Equation 4.3, we can obtain an effective eddy viscosity as a function of convective time, as presented in Appendix F. The effective eddy viscosity is

$$\nu_t = \frac{1}{2\pi} \left( C \frac{1 - r}{1 + r} \right)^2 U^2 t^*$$

(4.9)

where $t^*$ is equal to $\frac{\delta}{\bar{U}}$.

The above eddy viscosity is for flows without streamwise vorticity. However, in the
present study, we shall assume that the same eddy viscosity can be used for flows with streamwise vorticity. Although the investigation of the dependence of the eddy viscosity on the streamwise vorticity is beyond the scope of this thesis, there is some evidence suggesting that the presence of the shed streamwise vorticity will not change the quasi-two-dimensional shear layer behavior downstream of a lobed mixer. A primary mechanism of the growth of a two-dimensional shear layer is Kelvin-Helmholtz instability. If we consider the two-dimensional shear layer as in the x-z plane, the cross flow due to the streamwise vorticity stretches the two-dimensional shear layer in the y direction, but it has been shown (Lin, [18], 1984) that the growth rate of the Kelvin-Helmholtz instability is only marginally affected by the stretching for the range of parameters examined here.

The above representation of the effect of turbulence by a single eddy viscosity is a very simplified approximation. The interest, however, is on the effect of cross flow convection, due to streamwise vorticity, on the mixing in a given turbulent flow field, rather than the turbulent mixing in general. In this sense, the modeling of the turbulent flow is less critical.

4.2.3 Normalized Equations

The equations for turbulent flow are normalized in a similar manner as that for laminar flow: the cross flow velocities are normalized by the $\frac{\Gamma}{\lambda}$, and the length by $\lambda$. With this normalization, the resulting equations for the cross flow velocities in turbulent flow can be written as

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{\partial p}{\partial y} + \frac{1}{Re_{\Gamma t}} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v$$  \hspace{1cm} (4.10)

$$\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z} + \frac{1}{Re_{\Gamma t}} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) w$$  \hspace{1cm} (4.11)

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$  \hspace{1cm} (4.12)

where $Re_{\Gamma t} = \frac{\Gamma}{\nu t}$ and $t = \frac{\Gamma}{U\lambda \xi}$.

The equation for the axial velocity perturbation, $u'$, is,

$$\frac{\partial u'}{\partial t} + v \frac{\partial u'}{\partial y} + w \frac{\partial u'}{\partial z} = \frac{1}{Re_{\Gamma t}} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) u'$$  \hspace{1cm} (4.13)
where the effective Reynolds number is

$$Re_{el} = \frac{2\pi}{(C(1-r)\frac{U}{L})^2}$$  \hspace{1cm} (4.14)

The resulting equations of motion are similar to those in laminar flow and the equation of the axial velocity perturbation has the same form as that of a scalar in laminar flow. The difference between the equations in laminar flow and the equations in turbulent flow is that in the latter the effect of diffusion is represented by an eddy diffusion coefficient $\nu_t$. The same computational method for laminar flow is thus used to solve the equations for turbulent flow.

The mixing of the axial velocity perturbation in the absence of streamwise vorticity is also of interest. In this case, the only meaningful equation is the axial velocity perturbation equation, given by

$$\frac{\partial u'}{\partial \eta} = \frac{1}{Re_{\lambda}} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) u'$$  \hspace{1cm} (4.15)

where $Re_{\lambda}$ = $U\lambda/\nu_t$.

In addition we should state that one of the drawbacks of the method is that the scalar diffusion in flow with large velocity difference cannot be computed using the current slender body model, because the conservation of scalar in downstream direction is not satisfied if we assume the scalar diffusion is determined by a two-dimensional diffusion equation.

### 4.3 Initial and Boundary Conditions

#### 4.3.1 Streamwise Vorticity at Trailing Edge

As with the laminar flow computations, we are interested in the flow phenomena downstream of the lobe trailing edge. The flow condition at the lobe trailing edge must thus be specified and the method outlined in Section 2.3 is used to specify the initial streamwise vorticity distribution. For the flow investigated subsequently, a uniform streamwise vorticity distribution (Figure 2.3) is used as the initial condition. It should be pointed out that although the uniform initial streamwise vorticity distribution is a good approximation to the shed streamwise vorticity at the lobe trailing edge for an advanced lobed mixer with a stream to stream velocity ratio of unity, its application to a flow with large stream to stream velocity difference is not strictly justified. However, comparisons of computational
and experimental results presented in Chapter 7 suggest that the assumed uniform initial vorticity distribution also appears to be a good approximation for flows with large stream to stream velocity difference.

4.3.2 Axial Velocity Perturbation Distribution at Trailing Edge

The initial axial velocity distribution is specified in a similar manner as the initial scalar distribution in the laminar flow computation. We assume that each stream at the trailing edge has a velocity perturbation of \( \pm \frac{\Delta U}{2} \) or \( -\frac{\Delta U}{2} \). More precisely, the axial velocity distribution is given by

\[
\frac{u'}{\Delta U/2} = 2 \times \text{erf}\left(\frac{n}{\sigma_u}\right) - 1
\]  
(4.16)

where \( \Delta U = U_1 - U_2 \), \( \sigma_u \) is a thickness parameter and \( n \) is the normal distance to the lobe trailing edge profile in the cross flow plane. To resolve the gradient with good accuracy, the thickness parameter \( \sigma_u \) is chosen to be 0.02 for the computational results presented below. The non-zero thickness parameter for the initial axial velocity perturbation has the same effect as a non-zero thickness parameter for the initial scalar distribution in laminar flow and means that the flow at the trailing edge is partially mixed. If the thickness parameter \( \sigma_u \) is small, the percentage of partially mixed flow is small and the computational results obtained are independent of the value of \( \sigma_u \) used.

4.3.3 Boundary Conditions

We are interested in the mixing of two streams and thus friction at the wall boundaries is neglected. This is achieved with zero gradient of tangential velocity at the wall. Since the lobe trailing edge profile is periodic, we compute the flow over an area of half a lobe wavelength width; the flow over the area of the other half lobe wavelength width can be obtained using reflective boundary conditions.
4.4 Momentum Mixedness Parameter

As a measure of the mixing, it is useful to define an integrated parameter \( M_p \) for axial velocity perturbation, given by

\[
M_p = \frac{1}{A} \int_A \left( 1 - \left( \frac{u'}{A U} \right)^2 \right) dA
\]  

(4.17)

where \( \Delta U = U_1 - U_2 \) is the mean axial velocity difference at the trailing edge. The above parameter has a similar form to that for scalar mixedness used in Chapter 2. Although the same form of scalar mixedness parameter could equally be used to assess the mixing of the axial velocity perturbation in turbulent flow, it is more meaningful to take the integral of velocity squared (as a measure of momentum) rather than the integral of its absolute value.

For one-dimensional, constant area mixing, the static pressure recovery is directly related to the integral of axial velocity perturbation squared. As discussed earlier, the formulation presented in this chapter cannot give the static pressure recovery explicitly. However, for any convective time (or any downstream location from the trailing edge), the distribution of the axial velocity perturbation can be obtained by solving Equation 4.13. Substituting this distribution of axial velocity perturbation into the integral relation between static pressure recovery and axial velocity perturbation (obtained from one-dimensional continuity and momentum conservation equations), the static pressure recovery can be computed. For a given \( u' \) distribution, the relation between the static pressure recovery and momentum mixedness parameter defined above can thus be written as

\[
\frac{\Delta P_i}{\frac{1}{2} \rho U_t^2} = \frac{1}{2} (1 - r)^2 (\Delta M_p)
\]  

(4.18)

where \( r \) is the axial velocity ratio at the trailing edge and \( \Delta M_p \) is the change of the momentum mixedness between two downstream locations.

4.5 Parametric Study

4.5.1 Flow Field Development

The equations for the cross flow velocities in turbulent flow are of the same form as those in the laminar flow, and the equation for the axial velocity perturbation is similar to that
for a scalar in laminar flow, except that the Reynolds number is now time dependent. We thus expect that the streamwise vorticity has a similar effect on mixing of the axial velocity perturbation as that for the scalar in laminar flow. To illustrate this, computations of turbulent flow mixing, for a lobe of height $h = 0.54$, have been carried out. The lobe used has the same trailing edge geometry as the one investigated in Section 3.2 in laminar flow. The uniform initial vorticity distribution is used with a vorticity thickness parameter $\epsilon = 0.02$. The relative magnitude of the strength of the streamwise vorticity, $\frac{\Gamma}{U\lambda}$, is taken as 0.39, corresponding to a lobe penetration angle of 20°. The axial velocity ratio, $r$, is equal to 0.5.

The time development of the streamwise vorticity is shown in Figure 4-1. The initially distributed vorticity at the trailing edge ($t=0.0$) is rotated and developed into concentrated vortex cores by $t=1.0$. Compared with the laminar case (Figure 3.1b) of the same initial vorticity distribution, the size of the vortex core for turbulent case is larger at $t=1.0$ because the turbulent flow is more diffusive. The general shape of the vortical region, however, is similar to that in laminar flow.

The axial velocity perturbation field, Figure 4-2, reflects the convective motion associated with the streamwise vorticity. The general shape of the velocity perturbation field in turbulent flow is similar to that of the scalar in the laminar flow (Figure 3.3b). The velocity perturbation layer is thicker than the scalar layer of the laminar flow because of the difference in the diffusion coefficients.

A comparison of the momentum mixedness as a function of time for flows with and without streamwise vorticity is shown in Figure 4-3. The strong effect of the streamwise vorticity on mixing in turbulent flow can be seen: at $\frac{\xi}{h} = 2.5$ the momentum mixedness for mixing with streamwise vorticity is about twice that without streamwise vorticity. This indicates that even in turbulent flow where the local diffusion is strong, considerable mixing augmentation can be achieved using streamwise vorticity.

4.5.2 Effect of Streamwise Velocity Ratio on Mixing

For the same lobe penetration angle, a change in axial velocity ratio is equivalent to a change in the effective Reynolds number. Figure 4-4 shows the effect of velocity ratio on the momentum mixedness for the lobe geometry investigated in Section 4.5.1. As expected, a decrease in the velocity ratio is accompanied by an increase in the momentum mixedness.
(for a given $\bar{f}$). For the same velocity ratios, the momentum mixedness for a lobe of the same trailing edge profile but with no shed streamwise vorticity (zero penetration angle) is also computed, as shown in Figure 4-5. Comparison of Figure 4-4 and Figure 4-5 shows that the relative change of the momentum mixedness due to a given change in axial velocity ratio is reduced when the streamwise vorticity is present. For the flows without streamwise vorticity, at $\bar{f} = 3.0$, a change of velocity ratio from 0.67 to 0.50 is accompanied by a change of the momentum mixedness of 55%. For the flows with the streamwise vorticity, this change in velocity ratio only introduces roughly a 33% change in the momentum mixedness at the same $\bar{f}$. With streamwise vorticity, then the overall mixing is less dependent on the local eddy viscosity.

It is useful to investigate the momentum mixing augmentation ($M_{pa}$), i.e. the momentum mixedness for flow with streamwise vorticity minus that for flow without streamwise vorticity. Because the equations of motion for turbulent flow are of the same form as those for laminar flow, and the equation for the axial velocity perturbation is similar to the scalar equation for laminar flow, we expect that scaling relations between the scalar mixedness augmentation and Reynolds number in laminar flow should also be valid for momentum mixedness augmentation in turbulent flow.

In laminar flow, we found that below a critical Reynolds number ($\%$)c, the scalar mixing augmentation rate was roughly independent of Reynolds number $\%$. Above the critical Reynolds number, the scalar mixing augmentation rate was proportional to $(\%)^{-\frac{1}{2}}$. Although the Reynolds number in turbulent flow changes as a function of time, the ratio of Reynolds numbers for the same lobe geometry (or the same $\%$) is independent of convective time and is only a function of axial velocity ratio as given by

$$\frac{(Re_{v1})_1}{(Re_{v1})_2} = \left(\frac{1-r}{1+r}\right)^2 \left(\frac{1-r}{1+r}\right)^{-2} \quad (4.19)$$

The effect of axial velocity ratio on the momentum mixing augmentation rate for a given lobe geometry should thus scale according to the ratio of effective Reynolds numbers in the same manner as that indicated in laminar flow.

To illustrate this, we have computed momentum mixing augmentations for axial velocity ratios $r=0.67$ and 0.8, which are shown in Figure 4-6. The lobe geometry used is the one investigated in Section 3.2. For a velocity ratio $r=0.67$, the Reynolds numbers are in
the range of 1585 to 528 for $0.5 < t < 1.5$ according to Equation 4.14. For flow with $r=0.8$ and the same geometry, the Reynolds numbers are in the range of 5316 to 1772 for $0.5 < t < 1.5$. These Reynolds numbers are larger than the critical Reynolds number of 500, which is found in laminar flow (Figure 3.15). The momentum mixing augmentation rates should scale with their Reynolds number ratio to the power of one-third within this time interval. This is shown in Figure 4-7 with the momentum mixing augmentation for $r=0.8$ multiplied by a factor of 1.48, which is the ratio of Reynolds numbers to the power of one-third. It can be seen that the rescaled momentum mixing augmentation for $r=0.8$ has roughly the same slope as that for $r=0.67$. It should be noted that the scaling relation only applies to the downstream region where the effective Reynolds number is greater than the critical Reynolds number. At low velocity ratios, say $r=0.25$ and $r=0.125$, for the time interval $0.5 < t < 1.0$, the effective Reynolds numbers are in the ranges of 176 to 88 and 108 to 54, respectively. These Reynolds numbers are below the critical Reynolds number and the mixing augmentation rate is thus roughly independent of Reynolds number as shown in Figure 4-8. For large time, say $t > 1.0$, the mixing is nearly complete and mixing augmentation rate decreases.

For a lobe of height $h = 1.0$, the laminar flow results show that the critical Reynolds number $(Re)_c$ is 2000 (Figure 3-17). For a penetration angle of $20^\circ$, the parameter $\frac{L}{\lambda}$ is 0.727 (according to Equation 2.7). The effective Reynolds numbers for velocity ratios of 0.25 and 0.5, for $1.0 < t < 3.0$, are between 311 to 103 and 975 to 325, respectively. These are smaller than the critical Reynolds number of 2000 for this trailing edge geometry. Therefore, the mixing augmentation rate should be roughly the same for the two flows. The computed momentum mixing augmentations for velocity ratios of 0.25 and 0.5 are shown in Figure 4-9. It can be seen from Figure 4.8 that the two curves of momentum mixing augmentation as a function of the non-dimensional time $t$ have about the same slope for $1 < t < 3.0$.

In summary, for a given lobe geometry the momentum mixing augmentation is a function of the effective Reynolds number based on the strength of streamwise vorticity and eddy viscosity. If the effect of stream to stream velocity ratio is represented by a turbulent eddy diffusion coefficient, mixing augmentations for different velocity ratios can be scaled according to their effective Reynolds number ratios.
4.5.3 Effect of Lobe Height on Mixing

It is also useful to compare the performance of lobes of different height. Here we must emphasize that there are two causes of mixing enhancement downstream of a lobed mixer: the shed streamwise vorticity and the lobe trailing edge length. As we have seen in laminar computation, the initial rotation speed of the streamwise vorticity region decreases with the increase of the lobe height. This decrease in the rotation speed of the vortical region also reduces the stretching of material interface and thus reduces the effectiveness of streamwise vorticity as a mixing enhancement mechanism. Figure 4-10 and Figure 4-11 show the computed momentum mixedness for lobes of different height, $\frac{h}{\lambda}$, 0.54 and 1.0, for flows with and without streamwise vorticity. The stream to stream velocity ratio is kept at 0.25. The effect of lobe height on mixing can be measured in terms of momentum mixing augmentation due to streamwise vorticity (see Section 4.5.2 for definition) at a particular location. For the lobe with height $\frac{h}{\lambda} = 0.54$, the mixing augmentation due to streamwise vorticity is roughly 45% of the momentum mixedness for the flow with streamwise vorticity at downstream location $\xi = 2.5$. For the lobe with height $\frac{h}{\lambda} = 1.0$, despite the fact that the strength of streamwise vorticity is almost twice that of the lobe of height $\frac{h}{\lambda} = 0.54$, the mixing augmentation is roughly 33% of the momentum mixedness for the flow with streamwise vorticity at downstream location $\xi = 2.5$. This suggests that increasing the lobe height may have increased the streamwise vorticity strength, since the latter is roughly proportional to the lobe height, but the relative contribution to the mixing due to streamwise vorticity can actually be reduced.

4.6 Application to Compressible Flow

The thickness growth rate of a shear layer is known to decrease at large convective Mach numbers. Devices that rely on the shear layer mixing become less effective at high Mach number. It is thus desirable to have a mixing device that can provide strong mixing at high Mach number. Lobed mixers can be a good potential candidate for augmenting the mixing in high Mach number flow, since part of the mixing is due to the cross flow associated with the streamwise vorticity. It is thus useful to relate the mixing augmentation at high Mach number to that of incompressible flow in lobed mixer devices.

The ratio of the thickness growth rate for a given Mach number to that of incompressible
flow has been shown to be a function of convective Mach number (Dimotakis, [5], 1989), where the convective Mach number is defined as

\[ M_c = \frac{U_1 - U_c}{a_1} \]  

(4.20)

\[ U_c = \frac{a_2 U_1 + a_1 U_2}{a_1 + a_2} \]  

(4.21)

In Equations 4.20 and 4.21, \( a_1 \) and \( a_2 \) are the speeds of sound of the fast and slow streams, and \( U_c \) is the convective velocity of the large scale structures in shear layer. The convective Mach number thus measures the relative free stream Mach number as seen from the Galilean frame of the large scale structure in two-dimensional shear layer (see Dimotakis, [5], 1989). The growth rate of the shear layer drops to about 20% of the incompressible value when the convective Mach number becomes near unity.

The effect of decreasing mixing layer growth as a function of convective Mach number can be incorporated into the computational model by assuming the constant, \( C \), in Equation 4.8, to be Mach number dependent. The variation of \( C \) with Mach number is taken from Figure 4-12 (Dimotakis, [5], 1989). As example, we consider a lobe of height of \( \frac{h}{L} = 0.54 \) and penetration angle of 20°. The computed momentum mixing augmentation for flow with Mach number \( M_c = 0.5 \) is shown in Figure 4-13, together with the case \( M_c = 0.0 \). As can been seen, the mixing is reduced for the case of \( M_c = 0.5 \) as compared with the incompressible case.

We can also apply the scaling relation of momentum mixing augmentation rates for the above two flows. At \( t=1.0 \), the effective Reynolds number for a convective Mach number of 0.5 and velocity ratio of 0.67 is 2400. This Reynolds number is above the critical Reynolds number of 500 obtained for this geometry from the laminar flow computation. We thus expect that the momentum mixing augmentation rate for \( M_c = 0.5 \) should scale with that of \( M_c = 0.0 \) according to ratio of Reynolds numbers to power of \(-\frac{1}{3}\) or \( (\frac{Re_{M_c=0.5}}{Re_{M_c=0.0}})^{\frac{1}{3}} \). This is shown in Figure 4-14, where the momentum mixing augmentation for the case of \( M_c = 0.5 \) is rescaled. As can be seen, the rescaled momentum mixing augmentation slope for \( M_c = 0.5 \) is close to that for the incompressible case.
4.7 Summary

A model for computing the mixing augmentation due to streamwise vorticity in turbulent flow has been formulated. The effect of turbulence is represented by an effective eddy viscosity, which is a function of axial velocity ratio. The resulting equations have the same form as those of the slender body approximation used in laminar flow.

Although the turbulent flow is much more diffusive than laminar flow, the effect of the streamwise vorticity on the mixing augmentation is still strong. The effect of velocity ratio, or Mach number, on the mixing augmentation for lobes of the same lobe geometry can be approximately scaled according to the ratio of the effective Reynolds numbers and the scaling relations are the same as those obtained in laminar flow. Keeping penetration angle constant and increasing lobe height may reduce the relative contribution of streamwise vorticity to the mixing process downstream of the lobed mixer.
Figure 4-1: Contours of streamwise vorticity at different time $(\frac{\partial \xi}{\partial x})$ for $r = 0.5$, $\frac{\partial}{\partial y} = 0.39$ and $\frac{h}{\chi} = 0.54$
Figure 4-2: Contours of axial velocity perturbation at different times \( \frac{r}{U \lambda} \) for \( r = 0.5 \), \( \frac{r}{U \lambda} = 0.39 \) and \( \frac{h}{\lambda} = 0.54 \)
Figure 4-3: Comparison of momentum mixedness for flows with and without streamwise vorticity ($r = 0.5$, $\frac{x}{\lambda} = 0.39$ and $\frac{\lambda}{\lambda} = 0.54$)
Figure 4-4: Momentum mixedness for $r=0.5$ and $r=0.67$ for flows with streamwise vorticity ($\frac{r}{U_\lambda} = 0.39$ and $\frac{h}{\lambda} = 0.54$)
Figure 4-5: Momentum mixedness for $r=0.5$ and $r=0.67$ for flows without streamwise vorticity ($\frac{\frac{D}{D\lambda}}{\frac{D}{D\lambda}} = 0.39$ and $\frac{\lambda}{A} = 0.54$)
Figure 4-6: Momentum mixing augmentation for $\frac{k}{\lambda} = 0.54$ and $\frac{\Gamma}{\bar{U}\lambda} = 0.39$
Figure 4-7: Momentum mixing augmentation rescaled for $\frac{h}{\lambda} = 0.54$ and $\frac{\Gamma}{U\lambda} = 0.39$
Figure 4-8: Momentum mixing augmentation for $\frac{\lambda}{\chi} = 0.54$ and $\frac{\Gamma}{U\lambda\lambda} = 0.39$
Figure 4-9: Momentum mixing augmentation for $\frac{L}{\lambda} = 1.0$ and $\frac{\Gamma}{U\lambda \lambda} = 0.727$, showing that for $1.0 < t < 3.0$ the mixing augmentation rate is roughly independent of stream to stream velocity ratio.
Figure 4-10: Momentum mixedness for $\frac{L}{\lambda} = 0.54$ for flows with and without streamwise vorticity ($\alpha = 20^\circ$, $\frac{x}{\nu\lambda} = 0.39$ and $r = 0.25$)
Figure 4-11: Momentum mixedness for $k = 1.0$ for flows with and without streamwise vorticity ($\alpha = 20^0$, $\Gamma_{\lambda} = 0.727$ and $r = 0.25$)
symbols are experimental data

curve fit: $f(M_c) = 0.8 \exp^{-3M_c^2} + 0.2$

Figure 4-12: Ratio of shear layer growth rate for compressible shear layer to that of incompressible shear layer
Figure 4-13: Momentum mixing augmentation for $M_c = 0.0$ and $M_c = 0.5$ ($\frac{\Gamma}{U\lambda} = 0.39$ and $r = 0.67$)
Figure 4-14: Momentum mixing augmentation for $M_c = 0.0$ and rescaled $M_c = 0.5$ ($\frac{\Gamma x}{\overline{U}\lambda \lambda}$ = 0.39 and $r = 0.67$), showing that mixing augmentation for different Mach number can be scaled according to Reynolds number ratios.
Chapter 5

Experimental Studies of Momentum and Scalar Mixing Downstream of a Lobed Mixer and a Convoluted Plate

The effect of streamwise vorticity on the mixing of co-flowing streams was also investigated experimentally. The experiments were designed to separate the effects of: 1) lobe trailing edge length and 2) shed streamwise vorticity. Momentum mixing measurements were taken to assess the importance of streamwise vorticity on mixing enhancement. The temperature distribution in the cross flow plane was also determined to provide further information on the mechanism of the streamwise vorticity enhanced mixing. The major effect of the streamwise vorticity was found to increase the mean fluid interface area, on a scale of lobe wavelength, in the cross flow plane.

5.1 Introduction

As discussed in Chapter 1, there are two causes of the mixing increase downstream of a lobed mixer compared with a conventional flat plate splitter: increased trailing edge length and shed streamwise vorticity. Although it is widely believed that the streamwise vorticity plays a major role (and this is in agreement with the computational results presented in
Chapter 3 and Chapter 4), there are no experimental data that isolate the two effects so that their relative importance can be assessed. The purposes of the present study were thus to investigate the effect of streamwise vorticity alone on mixing, to quantify the relative contributions of streamwise vorticity and trailing edge length to the mixing process, and to obtain information about the mechanism of streamwise vorticity enhanced mixing.

To achieve these objectives, the mixing downstream of two lobes of the same trailing edge profile was measured. One lobe was a typical advanced lobed mixer used in turbofan engines. The other, referred to hereafter as the "convoluted plate", is a lobed mixer with a straight extension at the trailing edge (Figure 5-1). The straight extension tends to make the flow at the trailing edge parallel along the downstream direction, i.e. it removes the net streamwise vorticity. A comparison of the performance of the two lobes thus provides an assessment of the effect of the streamwise vorticity on mixing.

As stated previously, to measure the effectiveness of a mixing device requires a mixedness parameter and the definition of the mixedness parameter used depends on applications of mixer devices. One key application of lobed mixers is in turbofan engines where momentum mixing is of most interest. For the present study, a measure of momentum mixing, or more specifically, the ideal static pressure recovery due to mixing in a constant area mixing duct was used.

To further relate the mixing to the flow field development, scalar fields (temperature) downstream of the trailing edge of the lobed mixer and the convoluted plate in the cross flow plane were determined. These measurements were used to explain the mechanism of streamwise vorticity enhanced mixing.

5.2 Experimental Facility

The mixing experiments were conducted in a low speed wind tunnel. A schematic drawing of the test facility is shown in Figure 5-2. A blower supplies air flow to both streams with speeds up to $60m/s$ at the test section. Flow conditioning is provided by a set of honeycomb screens and a 3:1 contraction nozzle. The mean velocity ratio of the two streams can be varied by blocking one flow with a perforated plate. A set of resistance heaters is installed.

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1The mixing downstream of a conventional flat plate splitter was also obtained, and a comparison with the lobed mixer and the convoluted plate is presented in Appendix G.
in one stream so that the temperature of one stream can be increased.

The lobed mixer and the convoluted plate were constructed from fiberglass. The lobe geometry was based on an advanced lobed mixer tested at United Technologies Research Center (UTRC) and a detailed drawing is shown in Figure 5-3. The lobed mixer has a constant penetration angle, $\alpha$, of $20^\circ$, a wavelength, $\lambda$, of 1.25 inch and a lobe height to wavelength ratio of 2.0. The trailing edge geometry is the same for both the lobed mixer and the convoluted plate. The length of the parallel extension for the convoluted plate is 1.4 times as long as the lobe height, which is sufficient\(^2\) to make the flow at the trailing edge be essentially parallel, so that there is little streamwise vorticity shed downstream of the convoluted plate trailing edge. It should also be stated that the construction of the lobe was carried out before the computational results were obtained, so that the relative contribution of the streamwise vorticity to the mixing for this lobe geometry with $\frac{h}{\lambda} = 2.0$ may not be as strong as that for a smaller amplitude lobe.

The test section is rectangular in cross-section, 11.5 inches high by 4.0 inches wide by 40 inches long. The height of the test section allows both the lobed mixer and the convoluted plate to contain nine wavelengths so that end effects are small. Wall static pressure taps are placed at different downstream distances from the trailing edge. The positions of the taps are given in Appendix H. At each axial measuring station along each side wall, two static pressures, corresponding to the lobe peak and trough positions on the side walls, were measured.

The speed of flow in each stream was measured with a total pressure probe (Kiel probe) placed at the end of the contraction nozzle and with a wall static pressure tap. For the experiments described below, the area-averaged speed of the fast stream at the lobe trailing edge, $U_1$, was $40\text{m/s}$ unless otherwise stated, while the speed of the low velocity stream was varied by inserting perforated plates to provide different stream to stream velocity ratios, $r (\equiv \frac{U_2}{U_1})$, from 0.13 to 1.0. The area ratio, $\frac{A_2}{A_1}$, is equal to 1.25, with $A_1$ being the fast stream flow cross section at the trailing edge. The flow Reynolds number, $Re_\lambda = \frac{U_1 \lambda}{\nu}$, based on the fast stream velocity and the lobe wavelength, is $10^6$. Reynolds number, $Re_\Gamma = \frac{\Gamma}{\nu}$, based on half lobe circulation, is estimated to be $1.4 \times 10^5$.

\(^2\)The length scale associated with the flow over lobe surface is the lobe height or the mixing duct half-width. Potential effects due to the penetration angle change should thus largely decay over a distance of the lobe height, since the ratio of the lobe height to the mixing duct half-width is 1.25.
5.3 Momentum Mixing Parameter

5.3.1 Ideal Static Pressure Recovery

The ideal static pressure recovery due to mixing at a distance $x$ from the trailing edge for a frictionless, constant area duct can be derived by considering the $x$-momentum and mass flow conservations, and can be written as

$$\frac{\Delta \bar{P}_i}{\frac{1}{2} \rho U_1^2} = \frac{\bar{P}_i(x) - \bar{P}_i(x = 0)}{\frac{1}{2} \rho U_1^2} = \frac{1}{\frac{1}{2} AU_1^2} \int_A (U^2(x) - U^2(x = 0))dA \tag{5.1}$$

where $U(x)$ is the axial velocity at distance $x$ downstream of the trailing edge, $U_1$ is the mean fast stream speed at the lobe trailing edge and $A$ is the total cross sectional area of the mixing duct. Because it is a function of the streamwise velocity alone and is related to the velocity squared, the ideal static pressure recovery $\Delta \bar{P}_i$ provides a convenient measure of the momentum mixing. The value of $\Delta \bar{P}_i$ from unmixed to fully mixed state is

$$\frac{\Delta \bar{P}_i}{\frac{1}{2} \rho U_1^2} = 2 \frac{A_2}{A_1} \frac{(1 - \frac{U_2}{U_1})^2}{(1 + \frac{A_2}{A_1})^2} \tag{5.2}$$

where $U_1$ and $U_2$, $A_1$ and $A_2$ are the velocities and areas of the two unmixed streams respectively.

For real flow with two incompressible parallel streams entering a constant area mixing duct, the static pressure increases as a function of downstream distance as mixing occurs. The ideal static pressure recovery $\Delta \bar{P}_i$ due to the mixing can be written as

$$\Delta \bar{P}_i = \Delta \bar{P} + \Delta \bar{P}_f \tag{5.3}$$

where $\Delta \bar{P}_f$ is the static pressure loss due to the wall friction, and $\Delta \bar{P}$ is the static pressure recovery at a given axial location.

To determine the ideal static pressure recovery $\Delta \bar{P}_i$ based on Equation 5.3 for a given lobe geometry and flow conditions, the static pressure recovery $\Delta \bar{P}$ and static pressure loss $\Delta \bar{P}_f$ due to wall friction must be obtained. For the present investigation, the static pressure recovery $\Delta \bar{P}$ can be obtained by measuring the wall static pressure.

It should be pointed out that the above argument is based on one-dimensional analysis. In a three-dimensional flow, the local wall static pressure can be different from the averaged
static pressure obtained from one-dimensional analysis. For flow downstream of the lobed mixer trailing edge, the streamwise vortices can create a strong swirl so that the static pressure increases as one moves away from the center of the vortex core. An estimate of the error involved due to this effect is presented in Appendix I. For the data presented, the difference between the wall static pressure and the area averaged static pressure is small (less than 5% of ideal static pressure recovery).

### 5.3.2 Friction Loss

The static pressure loss due to wall friction depends on the wall shear stress and hence on the local flow field near the wall, but, a rough estimate can be obtained by assuming that the overall pressure loss is proportional to the average dynamic head. For flow with an axial velocity ratio of \( \frac{U_1}{U_2} \) and an area ratio of \( \frac{A_1}{A_2} \) at the trailing edge, the static pressure loss due to wall friction can be written as

\[
\Delta P_f = K \frac{1}{2} \rho \bar{U}^2 \frac{x}{\lambda} = K \frac{1}{2} \rho \left( \frac{U_1 A_1 + U_2 A_2}{A_1 + A_2} \right)^2 \frac{x}{\lambda}
\]

(5.4)

where \( K \) is the loss coefficient, \( \frac{x}{\lambda} \) is the normalized downstream distance from the lobe trailing edge and \( \lambda \) is the lobe wavelength. As a first order approximation, the friction loss coefficient \( K \) can be assumed to be independent of the velocity ratio of the two streams.

To obtain the value of the friction factor \( K \), the static pressures downstream of the convoluted plate and the lobed mixer were measured for the velocity ratio \( \frac{U_1}{U_2} = 1 \). Since the velocity ratio is unity, the measured static pressure reflects only the wall friction loss\(^3\). Figure 5-4 shows the static pressure losses of the convoluted plate and the lobed mixer as a function of the downstream distance from the trailing edge. The friction factor, \( K \), given by the slope of a linear curve fit of the static pressure vs. downstream distance, is 0.0033 for the convoluted plate and 0.0055 for the lobed mixer. These values of \( K \) obtained are consistent with pipe friction loss coefficients (Schlichting, [30], 1983). The corresponding pipe friction coefficients are found to be 0.016 and 0.027. A higher friction factor for the lobed mixer is expected since the cross flow introduced by the streamwise vorticity should result in larger wall shear stress.

For the experimental results presented, the ideal static pressure recovery is computed

\(^3\)Since the boundary layer is thin, the pressure recovery due to wake mixing is small.
based on the experimentally obtained wall static pressure and the friction loss from Equation 5.4. A constant friction factor $K=0.0033$ is used for the convoluted plate and $K=0.0055$ for the lobed mixer. Because the static pressure loss due to friction is roughly 10% of the ideal static pressure recovery for velocity ratios from 0.13 to 0.31, the choice of friction factor does not appreciably affect the overall conclusions presented in this chapter.

5.3.3 Effect of Local Flow Field Near Wall

Since the distance from the lobe peak to the mixing duct wall is about one half wavelength, it is not clear that the static pressure at any isolated location is representative of wall static pressures at other locations. To assess this, static pressures at two locations, corresponding to the lobe peak and trough, were measured. Figure 5-5 shows representative wall static pressures downstream of the lobed mixer trailing edge for a velocity ratio $\frac{U}{U_1}$ of 0.2. Along each side wall, the difference between the lobe peak and trough wall static pressures is small, indicating that the effect of the local flow on the wall static pressure is negligible. Thus, it is sufficient to present only one pressure measurement, corresponding to the lobe peak position.

5.4 Results and Discussions of Momentum Mixing Measurements

5.4.1 Effect of Streamwise Vorticity on Mixing

The ideal static pressure recoveries downstream of the lobed mixer and the convoluted plate are shown in Figure 5-6. The trailing edge is at $\frac{x}{L} = 0.0$. The ideal static pressure recovery was computed based on Equation 5.3 from the measured wall static pressure and the friction loss (Equation 5.4). The streamwise velocity ratio varies from 0.13 to 0.55. Ideal static pressure recoveries corresponding to both fast and slow stream side walls are presented. For all velocity ratios investigated, the ideal static pressure recovery downstream of the lobed mixer increases faster than that of the convoluted plate. For example, for velocity ratio $\frac{U}{U_1}$ from 0.13 to 0.31, the same amount of the ideal static pressure rise for the lobed mixer is achieved in less than half of the convoluted plate.

To assess the effect of the improvement on mixing, we can also define an integral recovery
length as

\[ l_i = \frac{\int \Delta P_i d\xi}{(\Delta P_i)_{t.e.}} \]  

(5.5)

where \((\Delta P_i)_{t.e.}\) is the pressure difference between the trailing edge and the mixing duct exit. The integration is taken from the trailing edge to the mixing duct exit. The integral recovery length, \(l_i\), as a function of velocity ratio is shown in Figure 5-7. Only the average integral recovery length of two side walls is shown (the difference between two side wall pressure recovery is not related to mixing, as will be explained in the next section). Note that the data for a velocity ratio of 0.55 is less reliable because the pressure rise due to the mixing at this velocity ratio is small. It can be seen from Figure 5-7 that, for the range of velocity ratios from .13 to .31, the integral recovery length for the lobed mixer is about half of that for the convoluted plate.

The difference in the mixing performance observed between the lobed mixer and the convoluted plate is somewhat larger than that given by the computational model of Chapter 4. This perceived difference may be a result of not including in the model the influence of the flow condition at the trailing edge on the downstream mixing. Nevertheless, both the experimental and the computational results strongly suggest that the streamwise vorticity is a major contributor to the mixing enhancement process downstream of the lobed mixer. The contribution of the streamwise vorticity to the mixing is of the same order as that of the trailing edge length.

5.4.2 Difference between the Fast and Slow Stream Wall Static Pressures

For the lobed mixer, there is a large difference between the fast and slow stream side wall pressures across the mixing duct (Figure 5-6), for all velocity ratios. This can be explained by considering the momentum across the mixing duct. We define the y-direction as perpendicular to the lobe wavelength direction in the cross flow plane (see Figure 5-8). At the trailing edge, each stream has a y-component of momentum because of the penetration angle. Since the velocity and the cross sectional area of each stream are generally different, the net y-component of momentum of the two streams at the trailing edge is not zero. At the exit of mixing duct, the mixing is essentially complete and the flow is in the downstream direction with no net y momentum. There is thus a change of y-component of momentum from lobed mixer trailing edge to mixing duct exit which must be balanced by a wall static
pressure difference across the mixing duct.

A relationship between the change of the $y$-component of momentum from the lobe trailing edge to the mixing duct exit and the axial velocity ratio can be formulated as follows. The $y$-component of velocity at the trailing edge is roughly $U_t \tan \alpha$, where $\alpha$ is the penetration angle. If so, the flux of $y$-component of momentum at the lobe trailing edge for the fast stream is $C_1 A_1 \rho U_1^2 \tan \alpha$, and for the slow stream is $C_2 A_2 \rho U_2^2 \tan \alpha$, where $C_1$ and $C_2$ are constants. The net change of $y$-component of momentum from the lobe trailing edge to the mixing duct exit can be written as

$$K_y = \int \frac{\Delta P_w}{\frac{1}{2} \rho U_1^2} \frac{x}{\lambda} = 2C_1 \tan \alpha \frac{A_1}{\lambda h_t} [1 - \frac{C_2 A_2}{C_1 A_1} \left( \frac{U_2}{U_1} \right)^2]$$

(5.6)

where $\Delta P_w$ is the wall static pressure difference between the slow and fast stream walls at a given downstream location, and $h_t$ is the height of the test section.

For a given geometry, $K_y$ is thus a linear function of the velocity ratio squared according to Equation 5.6. Integrating the measured static pressure difference between the slow and fast stream side walls, $K_y$ as a function of velocity ratio squared is shown in Figure 5-9. For the convoluted plate, since there is no penetration angle at the trailing edge, little difference between the slow and fast stream wall static pressure is observed (see Figure 5.6).

### 5.4.3 Effect of Mean Axial Velocity on Mixing

It is also of interest to examine the effect of mean axial velocity $\bar{U}$, i.e. Reynolds number, on the ideal static pressure recovery. Figure 5-10 shows the ideal static pressure recovery downstream of the trailing edge of the lobed mixer for different axial velocities. A similar plot for the convoluted plate is shown in Figure 5-11. For clarity, only the curve fit for the experimental data is shown. The fast stream $U_1$ varies from 20 to 60 m/s. The velocity ratio is chosen at a low value of 0.20, so the influence of friction loss is small. It can be seen that the effect of axial velocity on the downstream mixing is small as far as the ideal static pressure recovery is concerned. This is consistent with the results obtained by Manning ([19], 1991), who found that at Reynolds numbers ($Re_\lambda = \frac{U_1 \lambda}{v}$) $> 2.0 \times 10^4$, the molecular mixing rate as a function of the downstream distance from the lobe trailing edge was independent of the Reynolds number.
5.4.4 Effect of Axial Velocity Ratio on Mixing

In two-dimensional shear flow, decreasing the velocity ratio \( \left( \frac{U_2}{U_1} \right) \), with \( U_2 < U_1 \), increases the mixing (Brown and Roshko, [4], 1974). Since the flow field at the trailing edge is (locally at least) a two-dimensional shear layer, the axial velocity ratio must influence mixing. Figure 5-12 and Figure 5-13 show the normalized ideal static pressure recovery for the lobed mixer and the convoluted plate for different axial velocity ratios. The normalization factor is the maximum pressure recovery for the same streamwise velocity ratio as given by Equation 5.2. As discussed earlier, the difference in static pressure across the mixing duct downstream of the lobed mixer is due to the non-zero y-momentum at the trailing edge and this is not related to mixing, therefore only the averaged pressure of two side walls is presented. For both the lobed mixer and the convoluted plate, the ideal static pressure recovery in the region close to the lobe trailing edge can be seen to increase with the decrease of velocity ratio. This is consistent with the two-dimensional shear layer results.

The maximum slope of the normalized ideal static pressure recovery as a function of the downstream distance is shown in Figure 5-14. The effect of axial velocity ratio on the momentum mixing downstream of the lobed mixer is not as strong as that of the convoluted plate. For example, for a velocity ratio change from 0.13 to 0.2, the relative change in the maximum slope is for the lobed mixer is 7% compared with a 21% change in the convoluted plate. This is another indication that the mixing downstream of the lobed mixer is not purely driven by the axial velocity difference as that of a two-dimensional shear layer: the streamwise vorticity provides strong mixing enhancement.

The reduced influence of the axial velocity ratio on momentum mixing in the presence of the streamwise vorticity is opposite to the observation of Manning ([19], 1991) for molecular mixing. By comparing the molecular mixing downstream of a lobed mixer and a convoluted plate, Manning ([19], 1991) has found that the percentage increase of the molecular mixing downstream of the lobed mixer is somewhat greater than that downstream of the convoluted plate for the same changes in the stream to stream velocity ratio. This may not be totally the result of the action of the streamwise vorticity. The molecular mixing could be sensitive to upstream conditions, especially different boundary layer flows. The boundary layer flow over a lobe with a large penetration angle is different from that of a lobe with zero penetration angle. Unlike molecular mixing, the static pressure recovery is a bulk mixing measure and is likely to be less influenced by small scale upstream disturbances and boundary layer flows.
5.5 Scalar Mixing: Temperature Measurements

The ideal static pressure recovery gives a global measure of the mixing downstream of the lobed mixer trailing edge. The distributions of a passive scalar (temperature) in the cross flow plane at several stations downstream of the trailing edge of the lobed mixer and the convoluted plate were also investigated.

5.5.1 Experimental Technique

The measurements of temperature were obtained with an infra-red camera with one stream heated using a bank of resistance heaters. The same lobed mixer and convoluted plate used in momentum measurement experiments were used in this temperature measurement experiment. Attachments of different lengths were made so that the length of the mixing duct could be varied. At the exit of the mixing duct, a nylon screen was placed in the cross flow plane. The nylon screen is porous so that the air can pass through with minimum resistance. The temperature of nylon screen, which is directly related to the local time averaged fluid temperature, was determined by measuring the infra-red light emission.

Because of the difficulty in supplying heat, experiments were done at a relatively low velocity with the fast stream speed fixed at $20 m/s$, corresponding to a Reynolds number $5 \times 10^4$ based on the fast stream velocity and the lobe wavelength. This provided a stream to stream temperature difference of about $20^\circ C$. To avoid seeing the upstream heat source and the heat radiation due to the lobe surface, the camera axis was placed approximately at $60^\circ$ from the downstream direction and the first measuring station was placed at $\xi = 2.0$ downstream of the trailing edge (see Table 5.1).

A problem in using the infra-red light method is the background wall radiation because the camera also detects radiation from the test section side walls. To minimize this effect, the sides of the test section were painted to keep the emissivity in the range of 0.02 to 0.06, compared with a typical nylon screen emissivity of about 0.6. As a further check, the measurements were taken with the camera placed on each side of the test section. This corresponds to the camera seeing a cold or hot background. A comparison of the results of the measurements with the camera at two positions showed that the maximum error caused by the background wall temperature was approximately 5% of the measured temperature range.
Since the camera was placed at an angle to the test section, the image obtained was distorted. To correct the distortion, a set of reference points was used to determine the size and the position of the image relative to the test section and these were used to remove the image distortion in the data reduction process. Each image consisting of 30 frames taken at 4 frames/sec was digitized and recorded directly on a computer storage disk. In the results presented below, the temperature is normalized by the maximum and minimum temperature at \( \xi = 2.0 \); the normalized temperature is given by

\[
T^* = \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}}
\]

where \( T_{\text{min}} \) and \( T_{\text{max}} \) are the maximum and minimum measured temperature at \( \xi = 2.0 \) station.

Although spatial resolution of the infra-red detection technique is, in theory, only limited by the camera display (500x300), this spatial resolution is degraded by the resolution of the temperature which is limited to 10% of the full scale due to the camera internal hardware, so that the overall accuracy is not as good as the pressure measurements. However, the technique has the advantage of measuring the temperature of the whole cross section in one pass, providing a rapid way to obtain the scalar field downstream of the lobed mixer trailing edge.

### 5.5.2 Results and Discussions

Figure 5-15 shows the temperature contours downstream of the convoluted plate as a function of downstream distance for \( \frac{U_2}{U_1} = 1.0 \). At station \( \xi = 2.0 \), the temperature contours show a convoluted mixing layer, resembling the shape of the convoluted plate trailing edge. Further downstream at \( \xi = 5.2 \), the general shape of the diffusion layer is unchanged while its thickness increases.

The temperature contours downstream of the lobed mixer trailing edge (Figure 5-16) show different features. The diffusion layer downstream can still be seen at \( \xi = 2.0 \), but it does not resemble the lobe trailing edge profile. Instead, it is mushroom shaped, with more hot fluid pushing into the otherwise cold fluid region. There is also more interface area between the hot and cold streams over which diffusion can take place. We can take the length of contour line \( T^* = 0.5 \) as a measure of the mean interface area. For the convoluted
plate, at downstream location $\bar{\xi} = 2.0$ the length of the contour line $T^* = 0.5$ is $4.8\lambda$, which is about the same as the trailing edge length. For the lobed mixer, the length of the contour line $T^* = 0.5$ is $6.4\lambda$ at the downstream location $\bar{\xi} = 2.0$, about 33% more than that of the convoluted plate. And further downstream, at $\bar{\xi} = 3.6$, the length of the contour line $T^* = 0.5$ for the lobed mixer is about 80% more than the length of the trailing edge. This clearly indicates that the convection in the cross flow plane increases the mean interface area.

Decreasing the streamwise velocity ratio to $\frac{U_2}{U_1} = 0.31$ increases the diffusion of the flow downstream of both the convoluted plate and the lobed mixer, as shown in Figure 5-17 and Figure 5-18. We can again estimate the length of mean interface using the length of the contour line $T^* = 0.5$. It is $4.75\lambda$ for the convoluted plate and $6.4\lambda$ for the lobed mixer at $\bar{\xi} = 2.0$. For the lobed mixer, the ratio of the length of the contour line $T^* = 0.5$ to the trailing edge length is about 1.33 and this is about the same as that for $\frac{U_2}{U_1} = 1.0$. For downstream location $\bar{\xi} = 5.2$, the flow downstream of the the lobed mixer is well mixed. This is consistent with the pressure recovery measurement, in that the majority of the pressure recovery occurs within about five wavelengths downstream of the lobed mixer trailing edge.

The effect of the convection due to the streamwise vorticity in this experiment ($\frac{h}{\lambda} = 2.0$) is less than that of the cases studied in Chapter 3 and Chapter 4, where $\frac{h}{\lambda}$ varies from 0.54 to 1.0. One measure of the strength of the convection is the rotation speed of the region containing streamwise vorticity. As shown in Chapter 3, the rotation speed of the regions of streamwise vorticity scales with the lobe amplitude squared. Based on Equation 3.4, the estimated downstream distance $\bar{\xi}$ required for a $90^\circ$ rotation for the geometry tested in this experiment is 3.5. As can be seen from Figure 5-16, by that location, considerable diffusion has already occurred and the effect of the convection due to streamwise vorticity is thus reduced.

We can also assess the effect of streamwise vorticity using the scalar mixedness used in Chapter 3 and define the mixedness as

$$M = \frac{1}{A} \int_A (1 - |2T^* - 1|)dA$$

(5.8)

where $A$ is the cross sectional area of the mixing duct and $T^*$ is the normalized temperature.
Comparisons of the mixedness downstream of the lobed mixer and the convoluted plate are shown in Figure 5-19 for the axial velocity ratios of 1.0 and 0.31. For a given stream to stream velocity ratio, the mixedness downstream of the lobed mixer is higher than that downstream of the convoluted plate, again suggesting that the streamwise vorticity has a strong mixing augmentation effect. The amount of the mixedness increase due to the streamwise vorticity is less than that predicted by the pressure recoveries. This may be due to the fact that the temperature measurements were not as accurate as the pressure measurements.

To compare the mixedness of flows with different axial velocity ratios, one needs to compute the fully mixed state for a given velocity ratio. As a rough estimate, the fully mixed state temperature is taken to be the mass averaged temperature. Assuming that the maximum and minimum measured temperatures at station $\xi = 2.0$ are also the maximum and minimum temperatures at the lobe trailing edge, the mixedness for fully mixed flow based on the mass averaged temperature for $r=1.0$ is computed to be $M = 0.9$ and $M = 0.58$ for $r=0.31$. Therefore, in terms of the percentage of the mixing relative to the fully mixed state, the flow for $r=0.31$ mixes better than the flow for $r=1.0$.

5.6 Summary

Experiments were carried out to assess the effect of streamwise vorticity on mixing. The ideal static pressure recovery downstream of a lobed mixer and a convoluted plate was determined. The streamwise vorticity is a significant contributor to the mixing enhancement downstream of a lobed mixer. The effect of the stream to stream velocity ratio on the mixing increase is reduced in the presence of the streamwise vorticity. The effect of streamwise vorticity on mixing was also examined by measuring the temperature distribution in the cross flow plane using an infra-red camera. It is shown that, in the cross flow plane downstream of the lobed mixer, the length of the mean diffusion interface is increased by the convection due to streamwise vorticity. It appears that the increased diffusion interface area is the main mechanism of the streamwise vorticity enhanced mixing.
Table 5.1: Temperature measurement positions

<table>
<thead>
<tr>
<th>station no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance (in.)</td>
<td>2.5</td>
<td>4.375</td>
<td>6.25</td>
</tr>
</tbody>
</table>
Lobed Mixer

Convoluted Plate

Figure 5-1: Schematic drawing of a lobed mixer and a convoluted plate
Figure 5-2: Schematic drawing of the air facility
Figure 5-3: Lobe geometry

lobed mixer

convoluted plate
Figure 5-4: Static pressure decrease along duct downstream of the convoluted plate and the lobed mixer at unity velocity ratio.
Figure 5-5: Wall static pressure downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.20$
Figure 5.6a: Ideal static pressure recovery for $\frac{U^2}{U_1} = 0.13$ (fully mixed value $(\frac{\Delta P}{\frac{1}{2} \rho U_1^2})_{max} = 0.374$)
Figure 5.6b: Ideal static pressure recovery for $\frac{U_2}{U_1} = 0.20$ (fully mixed value $(\frac{\Delta \bar{F}_1}{\frac{1}{2} \rho U_1^2})_{max} = 0.316$)
Figure 5.6c: Ideal static pressure recovery for $\frac{U_2}{U_1} = 0.31$ (fully mixed value $(\frac{\Delta P_i}{\frac{1}{2}\rho U_1^2})_{max} = 0.235$)
Figure 5.6d: Ideal static pressure recovery for $\frac{U_2}{U_1} = 0.55$ (fully mixed value $(\frac{\Delta \bar{P}}{\frac{1}{2}\rho U_1^2})_{max} = 0.10$)
Figure 5-7: Comparison of integral length of ideal static pressure recovery as a function of velocity ratio
Figure 5-8: y-momentum at lobe trailing edge
Figure 5-9: Net y-momentum change as a function of velocity ratio (curve fit: $K_y = 0.33945 - 0.35472(U_2/U_1)^2$)
Figure 5-10: Effect of streamwise velocity on ideal static pressure recovery downstream of the lobed mixer
Figure 5-11: Effect of streamwise velocity on ideal static pressure recovery downstream of the convoluted plate
Figure 5-12: Effect of velocity ratio on ideal static pressure recovery downstream of the lobed mixer
Figure 5-13: Effect of velocity ratio on ideal static pressure recovery downstream of the convoluted plate
Figure 5-14: Maximum slope of normalized ideal static pressure recovery
Figure 5-15: Temperature $T^*$ downstream of the convoluted plate for $\frac{U}{U_1} = 1.0$
Figure 5-16: Temperature $T^*$ downstream of the lobed mixer for $\frac{U}{U_1} = 1.0$
Figure 5-17: Temperature $T^*$ downstream of the convoluted plate for $\frac{\bar{u}}{u_l} = 0.31$
Figure 5-18: Temperature $T^*$ downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.31$
Figure 5-19: Comparisons of mixedness downstream of the lobed mixer and the convoluted plate for $\frac{U_2}{U_1} = 0.31$ and $\frac{U_2}{U_1} = 1.0$
Chapter 6

Visualization of Flow Structures Downstream of a Lobed Mixer and a Convoluted Plate

6.1 Introduction

Most of the existing experimental studies of lobed mixers report the time average flow field. The local mixing, however, is an unsteady process and there is thus a need to identify this unsteady flow structure. The goals of the flow visualization experiment reported here were to obtain information about the unsteady flow structure associated with mixing and to investigate its dependence on the streamwise vorticity and the axial velocity ratio. To accomplish these objectives, the flow structures downstream of a lobed mixer and a convoluted plate (a lobe with zero penetration angle) were examined.

For a convoluted plate, the two streams at the trailing edge are roughly parallel, and the flow is approximately in the downstream direction. Because the shear layer is thin (at least close to the trailing edge), the mixing layer can be approximately taken as quasi-two-dimensional. The flow structure associated with the axial velocity difference should thus be qualitatively the same as that of the two-dimensional shear layer (Brown and Roshko, [4], 1974; Ho, [11], 1984; Jimenez, [12], 1985). For a lobed mixer, strong streamwise vorticity is shed at the lobe trailing edge, resulting in a highly three-dimensional flow field downstream. It is not clear that the structure of two-dimensional shear layer, as commonly found in other
mixing devices, will thus exist in the flow field downstream of the lobed mixer. It is to be stressed that the streamwise vorticity downstream of the lobed mixer trailing edge is much stronger than the streamwise vorticity that results from the three-dimensional instability of the two-dimensional shear layer. In fact, for a penetration angle of $20^\circ$ and an axial velocity ratio of 0.5, the strength of shed streamwise vorticity of a lobed mixer is comparable to the strength of the vorticity (normal vorticity) associated with the axial velocity difference. We thus investigate how this streamwise vorticity alters the flow and increases the mixing.

6.2 Experimental Facility and Measurement Technique

The flow visualization experiments were conducted at the MIT gas turbine laboratory blow-down water tunnel. A detailed description of the facility is given by Manning ([19], 1991). The tunnel is a gravity driven apparatus with each stream supplied from a different reservoir. A schematic drawing of the test facility is shown in Figure 6-1. The test section is constructed from plexiglass, with a cross section of $8.5 \text{cm} \times 40 \text{cm}$ and length of $100 \text{cm}$. The flow speed is controlled by a set of valves upstream with the pressure drop across the valves calibrated for the flow rate. After passing a set of perforated plates, the streams are accelerated though a 4:1 contraction nozzle to reduce turbulence and boundary layer thickness.

The geometry of the lobed mixer was based on the UTRC advanced lobe concept. A detailed drawing of the lobe geometry is shown in Figure 6-2. The penetration angle is $22^\circ$ and the lobe height to wavelength ratio is 1.0 (this lobe geometry is different from that in momentum mixing experiment, because the lobes were designed independently. However, for the purpose of understanding flow structures associated with the mixing, the difference in geometry is not important). A convoluted plate with the same trailing edge profile as that of the lobed mixer was also constructed (Figure 6-2).

For flow visualization, fluorescent dye was pre-mixed with one stream before the flow enters the test section. The concentration of fluorescein solution was set to 0.0125g/l. A laser sheet about 2mm in thickness was created by passing a laser beam through a cylindrical lens, and a reflective lens system was set up so that the flow field can be viewed from three different angles. Still pictures of flow structures were recorded using a SLR camera with shutter speeds between $\frac{1}{125} \text{sec}$ to $\frac{1}{500} \text{sec}$, depending on the viewing angle.
To avoid unacceptable image blur on film, the average speed of the two streams was limited to 10cm/s, corresponding to a flow Reynolds number \((Re = \frac{UL}{\nu})\) of \(5 \times 10^3\), based on the averaged velocity of the two streams and the lobe wavelength. The image obtained thus only reflects large scale fluid motion, which should not change at higher Reynolds numbers. The range of the axial velocity ratio, \(\frac{U_2}{U_1}\), investigated was from 0.5 to 0.33.

### 6.3 Results and Discussions

In studying the flow structure downstream of the lobed mixer and convoluted plate, we have chosen to view the flow in three mutually perpendicular planes, as indicated in Figure 6.3. The interpretation of the flow field is thus somewhat limited by this choice. In the discussions below, the streamwise direction is \(x\) and the lobe periodic direction is \(z\), as shown in Figure 6.3.

#### 6.3.1 Flow Structure in the Cross Flow Plane

Two still pictures taken at different instants of time in the \(\frac{x}{L} = 1.0\) plane downstream of the trailing edge of the convoluted plate are shown in Figure 6-4. The axial velocity ratio, \(\frac{U_2}{U_1}\), is 0.5. The flow direction is out of the picture. The dark part corresponds to the fast stream and the bright part, the slow stream. The walls of the mixing duct are at the left and right edges of the pictures. The mean fluid interface can be seen to resemble the trailing edge profile of the convoluted plate. Further downstream at \(\frac{x}{L} = 2.0\), in Figure 6-5, smaller structure appears but the mean interface still resembles the trailing edge profile.

The flow visualization pictures in the \(\frac{x}{L} = 1.0\) plane downstream of the trailing edge of the lobed mixer are shown in Figure 6-6. Although there are small scale structures along the interface, the overall shape of the interface can be seen to be "mushroom like", indicating the existence of rotation of the fluid interface. Further downstream at \(\frac{x}{L} = 2.0\), in Figure 6-7, the size of the mushroom structure appears enlarged in a manner that is consistent with the influence of the cross flow due to the streamwise vorticity. A comparison between Figure 6-6 and Figure 6-7 shows that the "mean" interface between two streams downstream of the lobed mixer has increased. In addition, the fast stream (dark part) in the cross flow plane downstream of the lobed mixer penetrates into a region closer to the side wall than that of downstream of the convoluted plate.
0.3.2 Flow Structure in the \( \chi = 0 \) plane

Figure 6-8 shows flow structures in the \( \chi = 0 \) plane downstream of the convoluted plate. The dark part of the picture corresponds to the fast stream and the bright part of the picture represents the slow stream. The flow is from right to left. The trailing edge is at the right edge of the picture. The axial velocity ratio, \( U_2/U_1 \), is equal to 0.5. Figure 6-8 shows alternating dark and bright regions because the laser sheet cuts through the center plane of the convoluted plate. Along the interface of the two streams, the flow structure associated with the Kelvin-Helmholtz instability and subsequent roll-up of two-dimensional shear layer can be observed. These structures persist until about two to three lobe wavelengths downstream of the trailing edge. Further downstream, adjacent shear layers start to interact, resulting in the generation of smaller scale flow structures. A similar pattern can be seen as one moves away from the \( \chi = 0 \) plane to the \( \chi = 0.2 \) plane in Figure 6-9.

The flow downstream of the lobed mixer trailing edge is shown in Figure 6-10 for the \( \chi = 0 \) plane and in Figure 6-11 for the \( \chi = 0.2 \) plane. The axial velocity ratio, \( U_2/U_1 \), is equal to 0.5, as that in Figure 6-8. Similar to the flow field of the convoluted plate, along the fluid interface, the structures resembling those of the Kelvin-Helmholtz instability of the two-dimensional shear layer can be observed. Compared to that of the convoluted plate, the Kelvin-Helmholtz structure appears to develop at a location closer to the trailing edge. The flow downstream of the lobed mixer exhibits strong Kelvin-Helmholtz structure in spite of the fact that the strong streamwise vorticity exists in the flow field downstream of the lobed mixer. This is because the time scale associated with the development of the Kelvin-Helmholtz instability is much shorter than the convective time scale associated with the streamwise vorticity. The local mixing layer appears to be bounded by the growth of the Kelvin-Helmholtz structure, consistent with the eddy diffusivity model presented in Chapter 4.

The effect of the streamwise vorticity on the Kelvin-Helmholtz structure can be assessed by comparing the growth rate of the shear layer in the \( \chi = 0 \) plane downstream of the trailing edge of the lobed mixer with that of the convoluted plate. The \( \chi = 0 \) plane is chosen because the shear due to streamwise vorticity is strongest in this plane. A "visual thickness" can be defined by drawing a tangent line to the outer edges of the shear layer starting from the lobe trailing edge (Brown and Roshko, [4], 1974), as illustrated in Figure 6-12, with the
origin of the shear layer assumed to be at trailing edge. Such a comparison is not strictly valid since the flow downstream of the lobed mixer is three-dimensional, however, as stated before, the time scale associated with the Kelvin-Helmholtz instability is much smaller than that associated with the streamwise vorticity. Thus the comparison provides an assessment of relative effect of the streamwise vorticity on the growth of two-dimensional shear layer structure.

Histograms of the visual thickness slope for the mixing layer in the $\xi = 0$ plane downstream of the trailing edge of the lobed mixer and the convoluted plate are shown in Figure 6-13. Each histogram contains data from 30 pictures. Although there is a large spread, the histogram shows that the growth rate has a most probable value, which is defined as the average growth rate for the shear layer. The average growth rates for the shear layer downstream of the lobed mixer and the convoluted plate are tabulated in Table 6.1. It can be seen that the difference between the growth rates of the shear layer for the lobed mixer and convoluted plate decreases with the decrease of the axial velocity ratio, and the development of the shear layer at low velocity ratio is thus less dependent on the presence of the shed streamwise vorticity. This is consistent with the fact that the time scale associated with the Kelvin-Helmholtz instability is reduced at low velocity ratio due to increased shear.

6.3.3 Flow Structure at the Peak and Trough of the Lobed Mixer Trailing Edge in the z-plane

The flow at the lobe peak and trough of the lobed mixer trailing edge can be examined using a laser sheet in the z plane, which cuts through the lobe peak or the lobe trough, as shown in Figure 6-3b. The terms peak and trough are relative to the fast stream (as shown in Figure 6-16). Figure 6-14 and Figure 6-15 show the flow structures at the lobe peak and trough for an axial velocity ratio $\frac{U_2}{U_1} = 0.5$. The side walls are at the top and bottom edges of the pictures, the trailing edge is at the right edge and the flow is from right to left. The dark part corresponds to the fast stream. As can be seen, the interface between the fast and slow streams (dark and bright parts) at the lobe peak, Figure 6-14, shows a pronounced Kelvin-Helmholtz structure. However, at the trough, Figure 6-15 shows that such structure is less developed.

The reason for the weaker Kelvin-Helmholtz instability in the trough can be understood
from the boundary layer behavior of the flow over the lobe surface. The development of the instability depends on the local velocity shear. For a given wave number, the higher the velocity shear, the more unstable the shear layer. The local velocity shear is dominant by the fast stream boundary layer. The difference in the boundary layers at the lobe peak and trough is illustrated in Figure 6-16. At the lobe peak, the fast stream goes through a favorable pressure gradient, so the trailing edge boundary layer is thin and results in a higher velocity shear downstream of the trailing edge. At the lobe trough, the fast stream goes through an adverse gradient and the resulting trailing edge boundary layer is thus thick. This results in a lower shear so that the instability develops at a much slower rate than that at the lobe peak.

6.4 Summary

Flow visualization experiments were carried out to compare the flow structures downstream of the trailing edge of the convoluted plate and the lobed mixer. Conclusions obtained are as follows:

- The effect of the streamwise vorticity is most evident in the cross flow plane downstream of the lobed mixer. While the shape of the mean interface downstream of the convoluted plate stays roughly the same as that of the trailing edge profile, the mean interface downstream of the lobed mixer develops into a “mushroom” shaped pattern, in a manner consistent with the action of the streamwise vorticity. The size of the “mushroom” increases along the downstream direction and as a result, the area of the mean interface is also increased.

- Flow structures similar to those of a two-dimensional shear layer have been observed in the y-plane downstream of the lobed mixer and the convoluted plate. The local mixing appears to be governed by the Kelvin-Helmholtz instability.

- Kelvin-Helmholtz structures can be seen at the peak of the lobed mixer trailing edge, while they are less developed at the trough. It appears that, in these regions, the boundary layer at the lobe trailing edge plays a more significant role in the development of the mixing layer.
Table 6.1: Visual thickness growth rate in \( y=0 \) plane

<table>
<thead>
<tr>
<th>velocity ratio ( \frac{U_2}{U_1} )</th>
<th>0.50</th>
<th>0.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>lobed mixer</td>
<td>0.15</td>
<td>0.26</td>
</tr>
<tr>
<td>convoluted plate</td>
<td>0.19</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Figure 6-1: Schematic drawing of the water tunnel facility
lobed mixer

convoluted plate

Figure 6-2: Lobe geometry
Figure 6-3: Definition of viewing angles
Figure 6-4: Flow structure in $\xi = 1$ plane downstream of the convoluted plate for $\frac{U_r}{c} = 0.5$
Figure 6-5: Flow structure in $\frac{y}{l} = 2$ plane downstream of the convoluted plate for $\frac{U_x}{U_1} = 0.5$
Figure 6-6: Flow structure in $\xi = 1$ plane downstream of the lobed mixer for $\frac{U}{U_1} = 0.5$
Figure 6-7: Flow structure in $\xi = 2$ plane downstream of the lobed mixer for $\frac{U_2}{U_1} = 0.5$
Figure 6-8: Flow structure in $\frac{y}{X} = 0$ plane downstream of the convoluted plate for $\frac{U_z}{U_1} = 0.5$
Figure 6-9: Flow structure in $\zeta = 0.2$ plane downstream of the convoluted plate for $\frac{U}{U_1} = 0.5$
Figure 6-10: Flow structure in $Y = 0$ plane downstream of the lobed mixer for $U_2/U_1 = 0.5$
Figure 6-11: Flow structure in $\frac{y}{h} = 0.2$ plane downstream of the lobed mixer for $\frac{U_y}{U_x} = 0.5$
Figure 6-12: Illustration of measurement of visual thickness of shear layer
Figure 6-13: Histogram for visual thickness growth rate
Figure 6-14: Flow structure at the peak of the lobed mixer trailing edge in z plane for $\frac{U_T}{U_s} = 0.5$
Figure 6-15: Flow structure at the trough of the lobed mixer trailing edge in z plane for $\frac{U_2}{U_1} = 0.5$
Figure 6-16: Features of trailing edge boundary layer at lobe peak and trough
Chapter 7

Comparison between Computational and Experimental Results

7.1 Introduction

Comparisons between the computational and existing experimental results of flow downstream of lobed mixers are presented to assess the usefulness of the computational model developed in Chapter 4. Most of the previous experimental work is focussed on the overall performance of lobed mixers as mixing devices and there are few data sets about the detailed flow field development downstream of lobed mixers. The scope of the comparison is thus limited.

7.2 Comparison of Total Pressure Distribution

The use of lobes in an ejector configuration has been tested by Presz et al. ([25], 1986). In that experiment, the total pressure distribution in the cross flow plane downstream of the lobe trailing edge was determined. The total pressure from the computational model can be written as

\[ p_t = (p_t)_{2D} + \frac{1}{2} \rho (U + u')^2 \]  

(7.1)
where \( (p_t)_{2D} \) is the computed total pressure of the cross flow and \( u' \) is the streamwise velocity perturbation.

For the trailing edge profile of the lobe tested in the experiment (Presz et al., [25], 1986), the flow field downstream of the trailing edge can be computed using the model presented in Chapter 4. For the initial condition of the computation, uniform initial vorticity distribution is assumed with a thickness parameter \( \epsilon = .02 \). The comparison of the total pressure contours between the experiment (Presz et al., [25], 1986) and the computation is shown in Figure 7-1. Both experimental and computational total pressure contours show the cross flow convection due to the streamwise vorticity. The overall total pressure patterns of the two results in the cross flow plane are similar, suggesting that the method developed in Chapter 4 does capture the cross flow convection due to the streamwise vorticity well. Comparison of contour values in Figure 7-1 also suggests that the diffusion is stronger in the experiment than that predicted using the computational model.

### 7.3 Comparison of Ideal Static Pressure Recovery

The ideal static pressure recoveries downstream of a lobed mixer and a convoluted plate were experimentally obtained as described in Chapter 5. As stated in Chapter 4, the model can not compute the ideal static pressure recovery explicitly, but an equivalent ideal static pressure recovery can be obtained based on the computed axial velocity distribution. Applying one-dimensional momentum and mass conservations, the relation between the ideal static pressure recovery and momentum mixedness \( M_p \) is given as (Chapter 4),

\[
\frac{\Delta \bar{P}_i}{\frac{1}{2} \rho U_1^2} = \frac{1}{2} (1 - r)^2 (M_p - M_p(x = L))
\] (7.2)

where \( M_p \) is the momentum mixedness, \( M_p(x = L) \) is a reference value chosen to be the momentum mixedness at the exit of the mixing duct and \( r \) is the velocity ratio.

Figure 7-2 shows a comparison between the computed ideal static pressure recovery based on Equation 7.2 and the measured ideal static pressure recovery for the lobed mixer. The predicted values using the model presented in Chapter 4 agree with the experimental values very well, for the stream to stream velocity ratios in the range of 0.13 to 0.31.

A similar comparison between the measured and the computed ideal static pressure recovery of the convoluted plate is shown in Figure 7-3. The agreement between computation
and experiment is still reasonable, but a larger discrepancy between the experimental and computational results can be seen compared with the case of the lobed mixer (Figure 7-2). In particular, the slope of the ideal static pressure recovery measured experimentally is less than that predicted using the computational model for region close to the trailing edge. The cause for the discrepancy is not clear. One possible reason is that, for the convoluted plate, the parallel extension increases the boundary layer thickness at the trailing edge as compared to that of the lobed mixer. The computational model assumes that the turbulent mixing is determined by the two-dimensional shear layer flow behavior. However, the mixing layer at the trailing edge of the convoluted plate may behave more like a wake because of the thicker boundary layer, and thus mix at a slower rate than predicted.

7.4 Rotation of Mean Fluid Interface

In Chapter 3, the initially distributed shed streamwise vorticity is shown to rotate as it is convected downstream. We have provided the time (or downstream distance) required for the initially distributed streamwise vorticity to rotate through a $90^\circ$ angle (see Equation 3.4). The rotation of the vorticity is also associated with a rotation of the mean fluid interface.

From the flow visualization pictures presented in Chapter 6, we can interpret the approximate rotation of the mean fluid interface as a function of downstream distance. This is shown in Figure 7-4. As can be seen, the originally vertical interface at the trailing edge has rotated through angles of less than $90^\circ$ at $\xi = 1.0$ and more than $90^\circ$ at $\xi = 2.0$.

For this particular lobe geometry, we can also estimate the time required for a $90^\circ$ rotation from Equation 3.4. For the lobe penetration angle of $22^\circ$, $\beta = 1.0$ and $\frac{1}{\theta} = 0.81$ according to Equation 2.6, the downstream distance $\xi$ required for a $90^\circ$ rotation based on Equation 3.4 is roughly 1.5. This is consistent with the flow visualization observations.

7.5 Effect of Compressibility in Supersonic Flow

The effect of compressibility on the mixing downstream of a lobed mixer has been measured experimentally by Tillman ([36], 1991). The total temperature at "the aerodynamic center" downstream of a lobed mixer device as a function of downstream distance from the trailing edge is shown in Figure 7-5. The position of the aerodynamic center relative to the trailing edge is...
edge is also shown in Figure 7-5. It can be seen that the total temperature decreases slower for high Mach number flow than that for low Mach number one.

This effect can be explained using the model developed in Chapter 4. Although the cross flow is approximately incompressible and the mean interface increase due to the cross flow is independent of the axial flow Mach number (Elliott, [7], 1990), the high axial flow Mach number can have a strong effect on the local diffusion across the mean interface. For a two-dimensional shear layer, the growth rate of the thickness is shown to be a function of the convective Mach number (Papamoschou [22], 1988 and Figure 4.19). Since the local turbulent diffusion in the computational model is based on the growth rate of the two-dimensional shear layer, this effect of the convective Mach number on the turbulent mixing in the lobed mixer can be assessed.

Computations of the flow field using model developed in Chapter 4 for different Mach number have been carried out for the lobe geometry used in Tillman's experiment ([36], 1991). The total temperature here is treated as a scalar and Schmidt number is assumed to be unity. The computed total temperature for different convective Mach number is shown in Figure 7-5, together with experimental data. The computed total temperature reflects the fact that the effect of high Mach number reduces the rate of the temperature decrease and is consistent with the experimental data.

7.6 Summary

Limited comparisons between the experimental and computational results are presented. Despite the simplicity of the model for turbulent flow, good agreement is obtained between the experimental and computational values of ideal static pressure recovery due to the mixing downstream of the lobed mixer and the convoluted plate trailing edges. A larger discrepancy between the computational and experimental ideal static pressure recovery is observed for the flow downstream of the convoluted plate; this may be due to the thicker boundary layer at the trailing edge of the convoluted plate and so the flow does not behave as a quasi-two-dimensional shear layer. Comparison between computed and measured total pressure distributions shows that the model presented does capture the effect of cross flow convection well.
Figure 7-1: Comparison of total pressure downstream ($\xi = 3.1$) of the lobed mixer (experiment data from Presz, [25], 1986)
Figure 7-2: Comparison of ideal static pressure recovery downstream of the lobed mixer
Figure 7-3: Comparison of ideal static pressure recovery downstream of the convoluted plate
Figure 7-4: Rotation of vortical region and fluid interface
Figure 7-5: Effect of compressibility on total temperature for $M_c = .14$ and $M_c = .65$; Symbols from experiment (Tillman, 1991) and lines from computation
Chapter 8

Conclusions and Recommendations

The effect of shed streamwise vorticity on mixing downstream of a lobed mixer has been examined experimentally and computationally. The conclusions obtained are summarized as follows:

8.1 Experimental Results

- Measurements of static pressure recovery have been carried out to identify the relative effects of streamwise vorticity and of lobe trailing edge length on momentum mixing downstream of the lobed mixer trailing edge. It is shown that the streamwise vorticity is a significant contributor to mixing enhancement. For the lobe geometry tested ($\alpha = 20^\circ$ and $\frac{b}{h} = 2.0$) and stream to stream velocity ratios ranging from $0.13$ to $0.31$, the contribution of the streamwise vorticity to the momentum mixing appears to be of the same order as that of the trailing edge length. A decrease in the axial velocity ratio also increases the rate of momentum mixing, but this effect is reduced in the presence of the streamwise vorticity.

- The temperature distributions in the cross flow plane downstream of a lobed mixer and a convoluted plate have been measured using an infra-red camera. It is found that the effect of the streamwise vorticity is to increase the mean interface in the cross flow plane downstream of the lobed mixer trailing edge.
Flow visualization using laser-induced fluorescence has been carried out to investigate the detailed flow field downstream of a lobed mixer and a convoluted plate. The effect of the streamwise vorticity on the flow field is most evident in the cross flow plane downstream of the lobed mixer trailing edge. While the shape of the mean interface downstream of the convoluted plate trailing edge stays roughly the same as the trailing edge profile, the mean interface downstream of the lobed mixer trailing edge develops into a "mushroom" shaped structure. The mixing layer in regions close to the lobe trailing edge is found to have a roll-up structure similar to that observed in two-dimensional shear layers.

8.2 Computational Results

An approximate method for analyzing the mixing augmentation due to streamwise vorticity was formulated. The objectives were to track the mean fluid interface in the cross flow plane and to investigate the parametric dependence of the mixing increase with the shed streamwise vorticity.

8.2.1 Mixing in Laminar Flow with a Stream to Stream Velocity Ratio Close to Unity

- The distributed streamwise vorticity at the trailing edge tends to evolve into a vortex core as it is convected downstream.

- The mixing augmentation depends on the distribution of the shed streamwise vorticity at the trailing edge. The more concentrated the shed streamwise vorticity at the trailing edge, the higher the mixing augmentation rate downstream of the trailing edge.

- For a given initial distribution of the shed streamwise vorticity and a fixed Reynolds number ($Re_{\lambda} = \frac{UL}{\nu}$), the maximum mixing augmentation per unit downstream distance, $\frac{\partial M}{\partial \lambda}$, is approximately proportional to $\frac{L}{U\lambda}$ for low $Re_{\lambda}$ and proportional to $\left(\frac{L}{U\lambda}\right)^{\frac{3}{2}}$ for high $Re_{\lambda}$. The parameter that separates two regions is given by $Re_{\lambda}$ and is about 500 for $\frac{L}{\lambda} = 0.54$ and 2000 for $\frac{L}{\lambda} = 1.0$. 

170
• For a given strength of the shed streamwise vorticity, the rotation speed of the vortical region scales with the lobe height squared.

• For the parameter range of practical interest, the thickness of the initial streamwise vortex sheet at the lobe trailing edge only has a marginal effect on the downstream mixing.

• Comparison between the computational model presented and a three-dimensional Euler solver shows that the computational model can be used to track the fluid interface if the streamwise vorticity distribution at the trailing edge is given.

8.2.2 Mixing in Turbulent Flow with a Large Stream to Stream Velocity Difference

• A computational model for assessing the effect of streamwise vorticity on mixing in turbulent flow with a large stream to stream velocity difference is formulated. The effect of turbulence is modeled as an eddy viscosity and the value of the eddy viscosity is obtained from two-dimensional shear layer results.

• It is found that although the turbulent diffusion is strong, considerable mixing augmentation can be achieved with streamwise vorticity. For the lobe investigated with $\frac{H}{\lambda} = 0.54$, the streamwise vorticity can reduce the required mixing duct length by at least half.

• For a given lobe geometry, the mixing augmentation rates can be scaled according to the ratio of their effective Reynolds numbers, with the detailed scaling relations being the same as those obtained from the laminar flow computations.

• Increasing the strength of streamwise vorticity by increasing the lobe height for a fixed penetration angle can reduce the relative contribution of the streamwise vorticity to the mixing enhancement process.

• Comparisons between the computational and available experimental results have been carried out. The prediction of static pressure recovery due to mixing based on the computational model for turbulent flow is found to be in good agreement with the experimental results. The comparison between the experimental and computational
total pressure distributions shows that the computational model presented does capture the cross flow convection well.

8.3 Recommendations for Future Work

There are several improvements and extensions that follow directly from the current work.

1. The computational studies presented in this thesis have provided relations between the mixing augmentation and the streamwise vorticity strength. The mixing augmentation per unit downstream distance was found to scale with \( \frac{L}{Ux} \). It would be interesting to carry out an experimental investigation to further elucidate relations between the mixing augmentation per unit downstream distance and the strength of the streamwise vorticity. This can be done using lobes of different penetration angles but the same trailing edge profile.

2. The comparison of mixing performance between the lobed mixer and convoluted plate showed that the lobed mixer can reduce the mixing duct length requirement by a factor of two. The amount of mixing increase was attributed to the streamwise vorticity. However, apart from the streamwise vorticity, flow at the lobed mixer trailing edge is very different from that of the convoluted plate. In particular, the relative effect of the trailing edge flow conditions on the downstream mixing should be further investigated.

3. The detailed distribution of shed streamwise vorticity at the trailing edge for a given lobe geometry is another area that requires additional investigation. The current understanding of the shed streamwise vorticity draws heavily from the existing knowledge of flow over a three-dimensional wing. However, the velocities of the two streams over the lobe surface are generally different, whereas the velocity of incoming flow over a wing is same. The effect of the velocity difference on the shed streamwise vorticity needs to be clarified. In addition, the knowledge of the exact distribution of the shed streamwise vorticity for a given lobed mixer can remove uncertainty in applying the computational model developed in this thesis. Therefore, it would be helpful that an Euler solver be used to obtain the flow over the lobe surface, with the main emphasis on the determination of the relation between the shed streamwise vorticity distribution and the streamwise velocity ratio.
4. In the computational model, the effect of turbulence is assumed to be a function of the streamwise velocity ratio only. Such turbulence model is very simplified. In addition, the flow with strong streamwise vorticity is highly three-dimensional. It would thus be of interest to carry out three-dimensional Navier Stokes computations with a turbulence model to further verify the relations between the shed streamwise vorticity and the mixing augmentation obtained in this thesis.

5. The question of the optimum lobe design requires one to address the loss associated with the mixing process. Although either increase in the strength of the streamwise vorticity or the trailing edge length can improve the mixing, the loss mechanisms are different and must be investigated. As a rough estimation, we can characterize the loss as drag on the lobe surface. The drag on the lobe surface due to the trailing edge length can be considered as friction loss and can be written as

\[ D_t \propto f_c \frac{1}{2} \rho \bar{U}^2 \times l_t \times X_{lc} \]  

(8.1)

where \( f_c \) is a characteristic surface friction coefficient, \( l_t \) is the trailing edge length and \( X_{lc} \) is a characteristic length representing the length of the lobe in the streamwise direction. The drag due to the streamwise vorticity can be written as (from kinetic energy loss consideration)

\[ D_v \propto \frac{1}{2} \rho \bar{U}^2 (\tan \alpha)^2 \times A \]  

(8.2)

where \( \alpha \) is a lobe penetration angle and \( A \) is the cross section area of mixing duct. While the drag on the lobe varies linearly with the trailing edge length, the drag due to the streamwise vorticity varies with square of the tangent of penetration angle and hence square of the streamwise vorticity strength. For minimum loss, the relative amount of mixing augmentation due to the streamwise vorticity and the trailing edge length must be determined. This requires the understanding of the loss due to the friction over the lobe surface, and could be solved using a three-dimensional Navier-Stokes solver with a turbulence model for the flow field over the lobe surface and downstream of the lobed mixer trailing edge.
8.4 Some Suggestions for Lobe Design

The following recommendations are made based on the limited scope of the current investigation and only address the distribution of streamwise vorticity at the lobe trailing edge that is most beneficial to induce rapid mixing.

The parametric study indicates that for a fixed Reynolds number $\left( \frac{UL}{v} \right)$ and fixed trailing edge profile, the mixing augmentation per unit downstream distance is roughly proportional to $\frac{\Gamma}{UA}$ for small value of the circulation and proportional to $\frac{\Gamma}{UA}^{(2/3)}$ when the circulation is large. The parameter that separates the two behaviors is a critical Reynolds number $(\frac{\Gamma}{UA})_c$. This means that, when the strength of the streamwise vorticity is weak, increasing one percent of the strength of streamwise vorticity will provide one percent of the mixing increase; when the strength of streamwise vorticity is strong (or $Re_\Gamma > Re_\Gamma_c$), increasing one percent of the strength of streamwise vorticity will roughly result in two-third of one percent of the mixing increase. Therefore, increasing lobe penetration angle (with fixed trailing edge profile) is an effective way of increasing the mixing. The biggest penetration angle is limited by the flow separation over the lobe surface. For low Mach number flow, $20^0$ can be regard as an upper limit.

Increasing the strength of the streamwise vorticity by increasing the lobe height may reduce the effectiveness of the streamwise vorticity as a mixing enhancement contributor. The effectiveness of the streamwise vorticity in enhancing mixing rests upon the relative value of the new mean interface generated in the cross flow plane compared with the total trailing edge length. For a fixed penetration angle, a large normalized lobe height ($\frac{h}{\lambda}$) decreases the rotation speed of the distributed streamwise vorticity in the downstream region close to the lobe trailing edge and results in the low percentage of the new interface, and thus reduces the contribution of the streamwise vorticity to the mixing enhancement process. Further, for an array of vortices of alternating signs, the convective velocity is strongest over a distance of half wavelength. Therefore, the normalized lobe height should be around unity or less and the distance between the two side walls should also be about the same as the lobe wavelength.
Appendix A

The Equations for Slender Body Approximation

For three dimensional, steady and incompressible flow, the equations of mass and momentum conservation are

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \tag{A.1} \\
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) u \tag{A.2} \\
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) v \tag{A.3} \\
u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) w \tag{A.4}
\end{align}

where \( u, v \) and \( w \) are velocities in \( x, y \) and \( z \) directions respectively.

The scalar equation is

\begin{equation}
\frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} = D (\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}) \tag{A.5}
\end{equation}

where \( D \) is diffusion coefficient.

The axial velocity component \( u \) can be written as

\begin{equation}
u = \bar{U} + u' \tag{A.6}
\end{equation}

where \( \bar{U} \) is the mean axial velocity, and \( u' \) is the axial velocity perturbation.
As discussed in Chapter 2, for a stream to stream velocity ratio close to unity,

\[
\frac{u'}{U} << 1 \tag{A.7}
\]

In addition, the characteristic length in the y and z directions is the mixing layer thickness, \(\delta\), and is much smaller than the lobe wavelength, \(\lambda\), or

\[
\delta << \lambda \tag{A.8}
\]

The length scale associated with the flow development in the x direction is the mixing duct length and is of order of the lobe wavelength \(\lambda\). Therefore,

\[
\frac{\partial^2}{\partial x^2} << \frac{\partial^2}{\partial y^2} \tag{A.9}
\]

\[
\frac{\partial^2}{\partial x^2} << \frac{\partial^2}{\partial z^2} \tag{A.10}
\]

The dominant terms in Equations A.1-A.5 are thus

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{A.11}
\]

\[
\frac{U}{\partial x} \frac{\partial v}{\partial y} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v \tag{A.12}
\]

\[
\frac{U}{\partial x} \frac{\partial w}{\partial y} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) w \tag{A.13}
\]

\[
\frac{U}{\partial x} \frac{\partial \phi}{\partial y} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} = D \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi \tag{A.14}
\]

The wavelength \(\lambda\) of the lobe is chosen to be the length scale. Since the cross flow velocities scale with the strength of the streamwise vorticity \(\Gamma\), it is useful to normalize the cross flow velocity by \(\frac{\Gamma}{\lambda}\). The normalized equations are

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{A.15}
\]

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{\partial p}{\partial y} + \frac{1}{Re \Gamma} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) v \tag{A.16}
\]
\[
\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z} + \frac{1}{Re_T} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) w 
\] (A.17)

\[
\frac{\partial \phi}{\partial t} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} = \frac{1}{Re_T \cdot Sc} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi 
\] (A.18)

where \( Re_T = \frac{\Gamma}{\nu} \) and \( Sc = \frac{\gamma}{\nu} \). As a result of velocity normalization, \( dt = \frac{\Gamma}{u_\lambda^2} dx \).
Appendix B

Estimation of Circulation per Unit Length at the Trailing Edge for an Advanced Lobed Mixer

This appendix describes an approximate, one-dimensional, inviscid method for computing the shed streamwise vorticity distribution of an advanced lobed mixer with a stream to stream velocity ratio of unity. We approximate the lobe geometry as rectangular and neglect rounded corners at the lobe peak and trough, as shown in Figure B. A more detailed derivation is given by Skebe ([33], 1988).

Consider a control volume bounded on the top by the lobe surface and on the bottom by the plane $y=\text{constant}$, and of width $dx$. For constant axial velocity ($\bar{U}$), the continuity requires

$$v \frac{dS_2}{dx} = \bar{U} \frac{dS_1}{dx} \quad (B.1)$$

For advanced lobed mixer of parallel sides and constant penetration angle,

$$\frac{dS_1}{dx} = btan\alpha \quad (B.2)$$

$$\frac{dS_2}{dx} = b \quad (B.3)$$

where $\alpha$ is the penetration angle.
Combining Equations B.1, B.2 and B.3 gives

\[ v = \bar{U} \tan \alpha \quad (B.4) \]

At the trailing edge, the strength of streamwise vorticity per unit length along the vertical leg is thus \( 2v = 2\bar{U} \tan \alpha \). The total circulation along the dashed line is

\[ \Gamma = 2vh = 2\bar{U}htan \alpha \quad (B.5) \]

where \( h \) is the lobe height.

Although the above model is very simplified, the computed total circulation of the streamwise vorticity at the lobe trailing edge for the advanced lobed mixer has been shown to agree well with experimental results (Skebe et al., [33], 1988).
Figure B-1: Schematic of trailing edge profile and cross flow velocity
Appendix C

Method of Solving Slender Body Equations

The governing equations for the slender body approximation are

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re} \nabla^2 v \tag{C.1}
\]

\[
\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \nabla^2 w \tag{C.2}
\]

\[
\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{C.3}
\]

where \( \nabla = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \).

With the initial \( \tilde{V} = (0, v, w) \) velocity field, the equations are solved using the fractional time-stepping scheme that consists of a nonlinear convective step, a pressure correction step and a viscous correction step (Korczak and Patera, [13], 1986; Tan, [35], 1985).

The convective step applies explicit third order Adams-Bashforth scheme,

\[
\tilde{V}^{n+1} - \tilde{V}^n = \frac{\delta t}{12} (23(\tilde{V} \times \tilde{\omega})^n - 16(\tilde{V} \times \tilde{\omega})^{n-1} + 5(\tilde{V} \times \tilde{\omega})^{n-2}) \tag{C.4}
\]

where \( \tilde{\omega} = (0, 0, \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}) \), the superscript \( n \) and \( n + 1 \) denote time level, \( \delta t \) is the time step size.

Once the \( \tilde{V}^{n+1} \) is obtained, the pressure correction step consists of
The above equations can be reformulated as

\[ \nabla^2 p_t = \nabla \frac{\ddot{V}^{n+1}}{\delta t} \]  

(C.7)

and boundary condition is

\[ \frac{\partial p_t}{\partial \vec{n}} = 0 \]  

(C.8)

where \( \vec{n} \) is normal to the boundary. The final viscous correction step imposes the appropriate boundary conditions for the velocity field. This step is discretized in time by using the implicit Crank-Nicholson scheme, i.e.,

\[ (\nabla^2 - \frac{2Re\Gamma}{\delta t})(\ddot{V}^{n+1} + \ddot{V}) = -\frac{2Re\Gamma}{\delta t}(\dot{V}^{n} + \dot{V}) \]  

(C.9)

where zero normal velocity and zero gradient of tangential velocity are applied at the boundaries.

The spatial discretization in the y-z plane uses multi-domain spectral method (Renaud, [26], 1991; Gottlieb, [8], 1977; Tan, [35], 1985). For each subdomain, the flow variables are expanded as follows

\[ \zeta = \sum_{N_y} \sum_{N_z} \zeta_{jk} h_j^f(\xi) h_k^f(\mu) \]  

(C.10)

where \( h_m(S) \) are high order local Lagrangian interpolants in terms of Chebyshev polynomials and can be written as

\[ h_m(S) = \frac{2}{M} \sum_{n=0}^{M} \frac{1}{\overline{C}_m \overline{C}_n} T_n(S_m) T_n(S) \]  

(C.11)

\[ \overline{C}_m = 1 \quad \text{for } m \neq 0 \text{ or } m \neq M \]

\[ \overline{C}_m = 2 \quad \text{for } m = 0 \text{ or } m = M \]
The collocation points $S_m$ in each subdomain are given as

$$(\zeta, \mu)_{jk}^i = (\cos(\frac{\pi j}{J}), \cos(\frac{\pi k}{K}))$$

(C.12)

In the computation, the order of Chebyshev is limited to 7 and the number of elements is 128.
Appendix D

Comparison between the Slender Body Approximation and a Three-dimensional Euler Solver

In using the slender body approximation for viscous flow, an elliptic problem is approximated by a parabolic one and the computational domain contains only the region downstream of the trailing edge. This implies that the interaction between the flow over the lobe surface and the flow downstream of the trailing edge is neglected. To assess the error involved, a comparison between the results obtained from the slender body approximation and that of a full three-dimensional Euler equation solver was carried out.

The solver for the three-dimensional Euler equations, was developed by Elliott ([7], 1990), is based on the flux-corrected transport method (Boris et al., [3], 1973). The flow over the lobe surface and downstream of the trailing edge is computed. At the trailing edge, the Kutta condition is applied. Although the method is inviscid and not directly applicable to mixing, it can provide a measure of the fluid interface area as a function of downstream distance.

For the comparison, a lobe with sinusoidal trailing edge profile is used (Skebe et al., [33], 1988). The lobe has an amplitude to wavelength ratio of 0.5 and a penetration angle $\alpha$ of 5.7°. This particular lobe geometry was used because of the difficulty in generating computational grids for lobes with higher penetration angles and more complex geometry. The stream to stream velocity ratio is unity. The Mach number is taken as 0.1 in the Euler
computation, so the flow can be considered to be approximately incompressible.

The streamwise vorticity distribution at the lobe trailing edge obtained from the Euler computation is shown in Figure D-1, together with the lobe trailing edge profile, and this streamwise vorticity distribution is used as the initial condition in the slender body approximation.

For the slender body computation, a Reynolds number of \( Re = 1000 \) based on the circulation of the streamwise vorticity and the lobe wavelength is used. The use of a low Reynolds number is to avoid gradient resolution problems in the spectral-element computational method. To mark the interface, an initial scalar value of +1 or -1 is assigned to each stream. On the boundaries of the computational domain, conditions of zero gradient of tangential velocity and zero normal velocity are imposed.

Comparisons of the scalar fields from the Euler computation with those of the slender body computation are shown in Figure D-2 and Figure D-3. The contours represent the distribution of the scalar in the cross flow plane at different downstream locations. The fluid interface (represented by scalar contours) computed from the slender body approximation is seen to be similar to that of the three-dimensional Euler computation.

Comparison of the static pressure distributions (normalized by mean stream dynamic head) is presented in Figure D-4. Although the resolution of the Euler solution leaves much to be desired, good agreement is also obtained.

Based on the above observation, it is concluded that, at least for the range of the strength of the streamwise vorticity investigated, the slender body method can be used to compute the fluid interface in the cross flow plane accurately if the exact distribution of the streamwise vorticity at the trailing edge is used. Further, the effect of the flow over the lobe surface on the downstream flow evolution appears to be small.

Note that the similarity in the scalar layer thickness between the two computations does not mean that the Euler computation has captured the diffusion. The thickness of the scalar layer downstream of the lobe trailing edge in the Euler solution is purely a result of artificial damping.
Figure D-1: Trailing edge profile and strength of streamwise vorticity per unit length from Euler solver
Figure D-2: Comparison of scalar field at $\frac{Z}{X} = 5.6$
Figure D-3: Comparison of scalar field at $\frac{\xi}{\lambda} = 9$
Figure D-4: Comparison of static pressure distributions at $\frac{x}{\lambda} = 4$
Appendix E

Effect of Pressure Variation Due to Swirl on Mixedness Parameter

In Chapter 4, the axial velocity perturbation is written as

\[
\frac{\partial u'}{\partial t^*} + v \frac{\partial u'}{\partial y} + w \frac{\partial u'}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu_l \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) u'
\]  

(E.1)

Because of the axial mass conservation, the pressure term must satisfy

\[
\frac{\partial}{\partial z} \int _A \rho dydz = 0
\]

(E.2)

where the integration is taken over the cross section of the mixing duct. However, there is a static pressure non-uniformity in the cross flow plane due to swirl. The non-uniformity is such that the mean value of static pressure does not change from one downstream station to another according to Equation E.2. Note that the axial velocity perturbation is only affected by the axial gradient of static pressure, but not by static pressure itself.

In fact, the static pressure non-uniformity due to swirl can be computed from Equations 4.1-4.3. To assess the effect of axial pressure gradient due to swirl on the axial momentum mixing, we have computed the momentum mixedness for the lobe geometry investigated in Chapter 4, for \( \frac{A}{\lambda} = .54 \) and \( \frac{r}{\lambda} = .39 \), with the pressure variation term included. Figure E.1 and Figure E.2 show the computed momentum mixedness for stream to stream velocity ratios of 0.67 and 0.50. The inclusion of pressure variation term has only slight effect on the momentum mixedness at stream to stream velocity ratios of 0.67 and 0.5. Therefore,
neglecting the effect of static pressure gradient due to swirl in computing the momentum mixing is justified.
Figure E-1: Comparison of momentum mixedness with pressure gradient term to that without pressure term for $\frac{\lambda}{\lambda} = 0.54$, $\frac{U}{U\lambda} = 0.39$ and $\frac{U}{U_1} = 0.67$
Figure E-2: Comparison of momentum mixedness with pressure gradient term to that without pressure term for $\frac{\lambda}{\lambda} = 0.54$, $\frac{U}{U_{\lambda}} = 0.39$ and $\frac{U_{\lambda}}{U_{t}} = 0.50$
Appendix F

Effective Eddy Viscosity for a Two-dimensional Turbulent Shear Layer

The behavior of a two-dimensional shear layer is well documented, and the maximum vorticity thickness of the shear layer as a function of downstream distance (See Dimotakis, [5], 1989; Brown and Roshko, [4], 1974) is given as

\[
\frac{\delta}{x} = C \frac{1 - r}{1 + r} \tag{F.1}
\]

where the maximum vorticity thickness is defined as

\[
\delta = \frac{U_1 - U_2}{(\frac{\partial U}{\partial y})_{\text{max}}} \tag{F.2}
\]

Equation F.1 is based on the argument that the growth of the shear layer in a frame that convects at the mean velocity 0.5\((U_1 + U_2)\) is proportional \((U_1 - U_2)\) (Reynolds, [28], 1974).

The growth of the vorticity thickness with the downstream distance \(x\) is the result of turbulent diffusion across the mixing layer. Here we shall use an eddy viscosity to describe the behavior of the growth of the shear layer. To do this, we follow a frame convecting at the mean velocity 0.5\((U_1 + U_2)\) and write the perturbation velocity as

\[
u' = U(y) - \overline{U} \tag{F.3}
\]
where $\bar{U} = 0.5(U_2 + U_1)$ is the speed of the convective reference frame and $U(y)$ is the velocity distribution of the two-dimensional shear layer, shown in Figure F. Assuming that the equation governing the turbulent diffusion of the perturbation velocity in a frame convecting at the mean velocity can be approximated by a diffusion equation,

$$\frac{\partial u'}{\partial t^*} = \nu_t \frac{\partial^2 u'}{\partial y^2}$$  \hspace{1cm} (F.4)

where $t^*$ is convective time $\frac{\bar{U}}{U}$, and $\nu_t$ is an effective eddy viscosity. In order to be consistent with the equation F.1 of the vorticity thickness dependence on $x$, the eddy viscosity is assumed to be a linear function of convective time

$$\nu_t = Bt^*$$  \hspace{1cm} (F.5)

where $B$ is a constant. Making substitution $t^* dt^* = \frac{1}{2} d(t^*)^2$, the solution for $u'$ from equation F.4 is

$$u' = \frac{U_1 - U_2}{2} (2 \times er\text{f} \left( \frac{y}{\sqrt{2Bt^*}} \right) - 1)$$  \hspace{1cm} (F.6)

Therefore, the maximum vorticity thickness as a function of downstream distance (from Equation E.2, E.3 and E.6) is

$$\delta = \sqrt{\frac{2}{\pi}} B \frac{x}{\bar{U}}$$  \hspace{1cm} (F.7)

where relation $t^* = \frac{\bar{U}}{U}$ is used.

From Equation F.1 and F.7 to obtain the coefficient $B$, the eddy viscosity is given as

$$\nu_t = \frac{1}{2\pi} \left( C \frac{1 - r^2}{1 + r} \right)^2 \bar{U}^2 t^*$$  \hspace{1cm} (F.8)

where $C$ is a constant.
Figure F-1: Two-dimensional shear layer
Appendix G

Comparisons of Static Pressure Recovery Downstream of a Flat Plate Splitter, a Lobed Mixer and a Convoluted Plate

In Chapter 5, we presented comparisons of the ideal static pressure recovery downstream of a lobed mixer and a convoluted plate. It is also useful to compare the performance of the lobed mixer to that of a conventional flat plate splitter.

Similar to the lobed mixer case, the ideal static pressure recovery due to mixing downstream of the flat plate splitter is determined based on Equation 5.3. The friction loss is computed based on Equation 5.4. The static pressure loss for a unit velocity ratio is shown in Figure G.1, and from which the friction loss coefficient $K$ was determined to be 0.006.

Figure G.2 and Figure G.3 show normalized ideal static pressure recovery downstream of the flat plate splitter, together with that of the lobed mixer and the convoluted plate. The ideal static pressure recovery downstream of the flat plate splitter is only about 20% to 30% of that downstream of the lobed mixer or convoluted plate, indicating that both the lobed mixer and the convoluted plate are much better mixing devices than the conventional flat plate splitter.
Figure G-1: Static pressure decrease along duct downstream of the flat plate splitter at unity velocity ratio
Figure G-2: Comparison of the normalized static pressure recovery downstream of the lobed mixer, convoluted plate and flat plate splitter for velocity ratio of 0.31
Figure G-3: Comparison of the normalized static pressure recovery downstream of the lobed mixer, convoluted plate and flat plate splitter for velocity ratio of 0.20
Appendix H

Static Pressure Tap Positions for Momentum Mixing Experiment

Table H. Static pressure tap positions downstream of the trailing edge

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Appendix I

Effect of Swirl on Wall Static Pressure

In Chapter 5, the measured static pressure at the wall is assumed to be the same as the averaged static pressure. Because of strong swirl due to the streamwise vortex, the measured wall static pressure could be higher than the pressure inside the mixing duct. We can estimate the difference between the wall static pressure and the averaged static pressure by considering a single Rankine vortex of circulation $\Gamma$ with a viscous core of radius $r_0$, as shown in Figure I. It is easily shown that the static pressure distribution at radius $r$ inside vortex core is given as

$$ p = -\rho \left( \frac{\Gamma}{2\pi r_0} \right)^2 \left( 1 - \frac{r^2}{2r_0^2} \right) $$

(I.1)

and at radius $r$ outside the vortex core is

$$ p = -\rho \frac{1}{2} \left( \frac{\Gamma}{2\pi r} \right)^2 $$

(I.2)

For area of radius $R > r_0$, the averaged static pressure can be found by integrating Equation I.1 and I.2 over an area $\pi R^2$, and it is given as

$$ \bar{p} = \rho \frac{1}{R^2} \left( \frac{\Gamma}{2\pi} \right)^2 \left( -\frac{3}{4} - \ln \frac{R}{r_0} \right) $$

(I.3)
The difference between the static pressure at $R$ and the averaged static pressure is

\[
\frac{p_R - \bar{p}}{\frac{1}{2} \rho \bar{U}} = \left(\frac{1}{2\pi}\right)^2 \frac{\Gamma}{(UR)^2} \left(1 + \frac{1}{4} + \ln\frac{R}{r_0}\right)
\] (I.4)

The static pressure at $R$ can be used to represent the measured wall static pressure. For the geometry tested in experiment, $\frac{\Gamma}{UR} = \frac{24\tan\alpha}{\pi} \approx 0.5$, and $\frac{R}{r_0} = 2$. The difference between the averaged static pressure and wall static pressure is about 0.01 of the mean dynamic head, and is thus small compared with the measured static pressure rise due to mixing which is in the range of 0.2 to 0.3 of the mean dynamic head.
Figure I-1: Rankine Vortex
Bibliography


