Three Essays in Law and Economics

by

Joshua B. Fischman

A.B., Princeton University (1994)
J.D., Yale University (1999)

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2006

© Joshua B. Fischman 2006

The author hereby grants to Massachusetts Institute of Technology permission to reproduce and
to distribute copies of this thesis document in whole or in part.

Signature of Author

Department of Economics
20 September 2006

Certified by

Glenn Ellison
Professor of Economics
Thesis Supervisor

Accepted by

Peter Temin
Elisha Gray II Professor of Economics
Three Essays in Law and Economics

by

Joshua B. Fischman

Submitted to the Department of Economics
on 20 September 2006, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Abstract

The first chapter presents a model of legal interpretation in a hierarchical court. Using a
two-level court in which judges have spatial preferences over doctrine, the model examines how
appeals, panels, and other structural features of the court affect the incentives of judges and
promote uniform interpretation of the laws. The threat of appeal has a moderating influence on
judges in the lower court. When the cost of appeal is low, this effect will be stronger, but the
lower court will also have less influence on the final decision. Hence, under many conditions,
overall uniformity will be maximized at an intermediate cost of review. Factors that may
increase the predictability of rulings on the higher court, such as panel size, may weaken the
incentives toward moderation on the lower court.

The second chapter analyzes judicial decision making in three-judge appellate panels. When
judges are ideological but have a preference for consensus, there will be negotiation among the
three judges in an effort to reach agreement. This paper constructs a model of judicial negotia-
tion, where judges have preferences on an ideological spectrum and disutility from disagreement.
The parameters of the negotiation model and the judges' ideological inclinations are then es-
timated on a data set of sex discrimination cases using maximum likelihood estimation. The
results find strong evidence that judges' votes are influenced by their panel colleagues, but that
this influence mostly takes the form of outvoted judges joining the majority. However, judges
in the minority appear to have a small but significant effect on case outcomes.

The third chapter examines the impact of liability law on firms' investments in product
safety when such investments take the form of fixed costs and liability does not apply equally
to competing products. Using a model with one innovative good and one competitively supplied
good, the paper finds that asymmetric liability deters safety innovation when the administra-
tion of the tort system is inefficient. When inefficiencies in the tort system are small, however,
incentives to develop safer products may be stronger under asymmetric liability.

Thesis Supervisor: Glenn Ellison
Title: Professor of Economics
Acknowledgments

I began graduate school five years ago with very little knowledge of economics. I owe a huge debt to so many people who have helped me get where I am today.

I am especially grateful to Glenn Ellison, who has been an insightful, patient, and dedicated thesis advisor. Much of what I know about economics I learned from him. I thank him for his constant support and encouragement during my time at MIT.

Stephen Ryan provided much guidance for the second chapter of this thesis. I thank him for his insights, his enthusiasm, and for always being patient when I had another question. I thank Jim Snyder for offering a useful perspective and for so many helpful conversations and suggestions.

I thank the Department of Economics at MIT for giving me a chance when I barely knew anything about economics and for its generous support during the first two years of graduate school.

I have been fortunate to have been part of an extraordinary intellectual environment at MIT and to have had so many wonderful classmates. I have learned so much from all them. I am grateful to Jackie Chou, Liz Ananat, and Sarah Siegel, who helped me get through my first two years of classes. David Matsa has been a wonderful roommate, office-mate, study partner and friend. I would also like to thank David Abrams, Marek Pycia, J.J. Prescott, Dominique Lauga, and Alan Grant for numerous helpful discussions.

I am deeply grateful to my wife, Polly, who gave me so much love and support during these last five years. She encouraged me to attend graduate school when it still seemed like a crazy idea, and made many sacrifices so that I could do it. She has endured all of the ups and downs of graduate life with me, and has been a constant source of strength.

I thank my daughters, Maisie and Josie, for bringing so much joy into my life and for understanding when I had to spend a lot of time at “Daddy’s office.” Finally, I wish to thank my parents, my brother, my sister, and my in-laws for their love, support, and encouragement. I could not have done this without all of you.
Contents

1 Uniformity of Interpretation in a Hierarchical Court .......................... 6
  1.1 Description of the Model .................................................................. 9
  1.2 General Results ............................................................................ 11
  1.3 Implications of the Model .............................................................. 14
    1.3.1 Cost of Review ........................................................................ 15
    1.3.2 Uncertainty ............................................................................. 17
    1.3.3 Aversion to Reversal ............................................................... 18
  1.4 Effects of Court Structure on Interpretation ................................... 19
  1.5 Conclusion ..................................................................................... 22
  1.6 Appendix ....................................................................................... 24

2 Collegial Decision Making in the Courts of Appeals: An Empirical Analysis 39
  2.1 Model ............................................................................................ 43
  2.2 Estimation ...................................................................................... 47
  2.3 Data ................................................................................................ 50
  2.4 Results ........................................................................................... 53
    2.4.1 Judge Preferences .................................................................... 53
    2.4.2 Costs of Disagreement ............................................................ 58
  2.5 Simulations .................................................................................... 59
    2.5.1 Frequency of Compromise ...................................................... 59
    2.5.2 Individual Judge vs. Three-Judge Panels ................................. 61
  2.6 Conclusion ....................................................................................... 62
3 Asymmetric Liability and Incentives for Innovation

3.1 Homogeneous consumers ........................................ 65
   3.1.1 Asymmetric Liability ...................................... 67
   3.1.2 Symmetric Liability .................................... 68
3.2 Heterogeneous consumers ..................................... 70
   3.2.1 Asymmetric Liability ...................................... 70
   3.2.2 Symmetric Liability .................................... 71
   3.2.3 Analysis .................................................. 72
3.3 Conclusion ...................................................... 76
Chapter 1

Uniformity of Interpretation in a Hierarchical Court

Most systems of adjudication feature a hierarchical organization and a process for appeals. While appeal is usually characterized as a procedural safeguard for the benefit of litigants, it clearly also serves as a monitoring device to enforce restraint among judges in the lower courts. This is especially important in an independent judiciary, where there are few constraints on judges' exercise of authority. This article develops a model of judicial interpretation to examine how the appeals process influences judges' incentives. When judges have different ideologies and preferences over methods of interpretation, the model shows that appeals can have the effect of promoting uniformity and ideological moderation in the interpretation of the law. The model explores how changes in appellate structure, such as panel size and frequency of review, can have an effect on the uniformity of legal interpretation.

Viewing judges as having preferences on an ideological spectrum, the model shows how the appeals process will lead to rulings that are clustered in the middle of the spectrum. This is not only because of "correction" of extreme rulings by an appeals court, but also because the prospect of appellate review leads lower courts to moderate their rulings. Appeals have the effect of depoliticizing judicial rulings by mitigating the influence of individual judges' ideologies in the outcome of cases. Thus the outcome of a case will be less dependent on the identity of the judge hearing the case.
This is important for several reasons. First, basic conceptions of fairness and rule of law require similar outcomes for similar litigants. If a judge's idiosyncratic preferences were a major factor in each decision, the application of the law would be viewed as arbitrary and illegitimate. Second, uniformity of interpretation promotes predictability of the law, allowing people and firms to make better plans in the face of legal uncertainty. Third, when there is uncertainty regarding how a law will be applied, compliance will be uneven, and the law will be less effective at achieving its stated goals. Finally, uniformity reduces litigation: parties will be less likely to have conflicts and more likely to settle if they have similar expectations about how the law will be applied.

This is not to suggest that absolute uniformity is possible, or even desirable. Variability is a necessary consequence of judicial discretion. Disagreements among judges leads courts to reexamine and modernize precedent and facilitates the evolution of the law. Also, uncertain laws could potentially lead to better compliance among risk-averse agents. The point to be made here is simply that idiosyncratic interpretation by judges can be harmful to the rule of law, and that the organization of courts has an impact on how ideologically heterogeneous judges harmonize their rulings.

Previous models of hierarchical courts have either focused on the role of error correction, or examined how higher courts enforce compliance with doctrine in the lower courts. The error correction models, e.g., Shavell (1995), Daughety and Reinganum (2000), Cameron and Kornhauser (2005), view judges as being part of a "team" who share the same objective: maximizing the number of correctly decided cases. While these models explain one essential purpose of the appellate system, they are less applicable to the interpretive, or "lawmaking" function of courts. In areas where the law is indeterminate, determining what the law means is an exercise in judicial discretion. In this context, ideological differences among judges are more significant, and the assumption that judges share the same objective is weakest.

To analyze the lawmaking context, this article employs the "political" model of judging\(^1\), in which rulings are viewed as ideological decisions that can be mapped into a one-dimensional "policy space." Using a two-level hierarchy in which there is uncertainty about the higher courts' preferences, the model shows how lower court judges will strategically shift their rul-

\(^1\)This is also referred to as the "attitudinal" model. (Segal & Spaeth 2002)
ings toward the center of the spectrum. Previous models of political judging in a hierarchy have investigated various ways in which higher courts monitor lower courts in order to enforce doctrinal compliance. (McNollgast 1995, Cameron, Segal and Songer 2000, Spitzer and Talley 2000, Mialon, Rubin and Schrag 2004) The contribution of this article is in examining how the interaction between courts promotes uniformity in the interpretation of the law. This has been an important concern in the legal literature, especially in the context of analyzing proposed structural changes in the courts. By developing a formal model with testable implications, this article seeks to bring the insights of economics and positive political theory to the analysis of court structure and judicial interpretation.

Although the article focuses on courts, the insights of the model can be applied to any situation involving ideological decision-making and serial review. Thus it could also apply to relations between administrative agencies and courts, agencies and legislatures, or civil servants and political appointees.

We summarize the main conclusions of the model here. First, judges on the lower court will shift their rulings toward the center of the distribution of interpretations. This shift represents a balance between two interests: conformity with the judge's own preferences and likelihood of surviving review. Thus the appeals process can increase the predictability of interpretation, even when appeal occurs infrequently. Second, more frequent review will create stronger incentives for the lower court to rule moderately, but will also result in more cases being decided by the higher court, which is unconstrained. Thus it will always be optimal to limit appeals to some degree. Third, increasing the consistency of rulings in the higher court (for example, by using large panels) may increase the consistency of the lower court, but this effect is nonmonotonic: too much consistency in the higher court will have the opposite effect. When the higher court is too predictable, the lower court judges can “game” the appeals process: they will know more precisely how much they can adhere to their preferences while still avoiding appeal.

Section I provides the setup of the model. Section II provides some general results and

\(^2\) Examples include the debate over splitting the Ninth Circuit Court of Appeals (e.g., Hug 2000, O'Scannlain 1999), changes to the en banc review process (Banks 1997), procedural reforms for immigration appeals (ABA Commission on Immigration Policy 2003), and a now-shelved proposal to establish an Intercircuit Panel to resolve conflicts of law between circuits (Ginsburg & Huber 1987), have focused on the how these plans would affect consistency and coherence in the law.
shows that under general assumptions, all lower court judges will avoid rulings that are outside a bounded set within the spectrum of interpretations. In section III, we make parametric assumptions regarding judges’ preferences, and derive some implications of the model. In particular, we can show that a very high frequency of appeal or a high court that is too predictable can be suboptimal for the uniformity of final rulings. Finally, section IV also provides some numerical calculations and graphs to illustrate the effects of court structure and the interplay between appeals and panels.

1.1 Description of the Model

This model focuses on the process of legal interpretation. Hence, we may take issues of fact as having been predetermined by the court. We will assume that it is a dominant strategy for the losing party to appeal, and that appeals are at the discretion of the higher court.\footnote{Although these assumptions are not formally true in every court system, they are still reasonable in situations where the costs of appeal will be small compared to the amounts of money and the importance of the legal issues at stake. Also, even when appeals are automatic, the higher court may only provide perfunctory review in cases where there is no disagreement with the lower court. For example, in the federal system, circuit courts do not have discretion to deny appeals, but they may dispose of cases in unpublished, non-precedential opinions. In the federal circuit courts, an overwhelming majority of cases are in fact decided in unpublished opinions. (Merritt & Brudney 2001)}

Following other spatial models of hierarchical courts (McNollgast 1995, Cameron, Segal and Songer 2000, Spitzer and Talley 2000, Segal and Spaeth 2002) and legislature-agency interactions (e.g. Ferejohn and Shipan 1990, Eskridge and Ferejohn 1992), we use a one-dimensional spatial model of judging. Each ruling is represented as a number on the real line. We can think of this spectrum as representing the range of plausible interpretations of the law in question, with $+\infty$ and $-\infty$ representing the extremes. For example, the spectrum could represent “liberal” vs. “conservative” preferences, or “strict” vs. “loose” construction of laws. Although courts are typically hierarchical with multiple levels, we consider a two-level court for simplicity. The model assumes that judges’ preferences are based strictly on doctrine – how the law is interpreted – and not on how this doctrine impacts the particular litigants in the case.

The model does not explicitly account for how precedent influence judges’ decisions. Since we are only considering cases where the law is indeterminate, we may assume that no precedent is dispositive. If there are multiple precedents that are relevant, then judges’ ideal points may
reflect the amount of weight that they place on each of these precedents. For example, a judge
with an ideal point on the “liberal” side of the spectrum might place more weight on a “liberal"
precedent than a judge on the “conservative” side of the spectrum. Also, the assumption that
judges’ utility is based on the about the final ruling in a case (and not merely which side wins)
implies that judges expect that their rulings will have precedential value in subsequent cases.

The model assumes that judges are concerned only with the final disposition of a case, and
derive no utility from “posturing.” A judge with preferences outside the mainstream would
therefore prefer to moderate his opinions to reduce the risk of being overruled by an appellate
court, if he expects that the appellate court would deviate even further from his own preferences.

We consider two courts, a lower court consisting of a single judge, and a higher court
consisting of a single judge or a panel of judges. In the first stage, the lower court judge issues
a ruling $x \in \mathbb{R}$, representing her interpretation of the law. In the second stage, the higher court
decides whether to review the ruling; if it does, it issues a new ruling $y$. If the higher court
reviews the lower court’s ruling, it will incur an effort cost $e > 0$, and the lower court will incur
a disutility $d \geq 0$. This disutility represents a reputational cost to the lower court judge from
being overruled.

We assume that the lower court judge’s utility function is

$$U_l = \begin{cases} 
-(q_l - x)^2, & \text{if it is not overruled} \\
-(q_l - y)^2 - d, & \text{if it is overruled}
\end{cases}$$

where $q_l$ is the lower court judge’s ideal point. Similarly, the higher court’s utility function is

$$U_h = \begin{cases} 
-(q_h - y)^2 - e, & \text{if it overrules the lower court} \\
-(q_h - x)^2, & \text{if it does not overrule the lower court}
\end{cases}$$

where $q_h$, the higher court’s ideal point, is unknown to the lower court judge. We can view
this as reflecting random assignment of judges to panels, so that the identity of the appellate
judges is unknown to the trial judge, or as uncertainty about the higher court’s preferences
on this question of law. We model $q_h$ as a random variable with density function $f^h$ and
cumulative distribution function $F^h$, where $f^h$ is continuous, symmetric, and unimodal with
mean $\mu_h$. Note that because of symmetry, $\mu_h$ will also be the median and the mode of the
distribution of \( q_h \). We assume that both distributions have full support over the spectrum of interpretations.

### 1.2 General Results

Our first two results explain the basic effect illustrated by our model: that judges on the lower court will strategically shift their rulings toward the center of the distribution, in order to balance their own preferences with the likelihood of appeal.

Our first theorem establishes a simple rule for determining when the higher court will hear an appeal, and characterizes the lower court’s ruling implicitly as a function of \( g \).

**Theorem 1** If \( x \in [q_h - c, q_h + c] \), where \( c = \sqrt{\epsilon} \), the higher court judge will decline to review the case.

If \( x \notin [q_h - c, q_h + c] \), the higher court judge will review the case and issue a ruling \( y = q_h \).

The lower court judge will issue a ruling \( x \) that satisfies

\[
x = g^{-1}(q_l)
\]

where

\[
g(z) = z + \frac{1}{2c} \left[ (c^2 + d) \left[ f^h(z + c) - f^h(z - c) \right] \right]
\]

**Proof.** See Appendix. □

We can think of \( c \) as the amount of latitude that the higher court will accept in the lower court’s interpretation. Remember that \( q_h \) is unknown to the lower court, so that the lower court knows the size, but not the position, of the interval of permissible rulings.

Although theorem 1 only defines \( x \) implicitly as a function of \( q_l \), we can use it to understand the shape of the lower court’s choice function.

**Theorem 2** The lower court’s ruling \( x \) will always be between its own ideal point \( q_l \) and the center of the appeals court’s distribution \( \mu_h \). If \( q_l = \mu_h \), then \( x = \mu_h \).

**Proof.** See Appendix. □
This result follows from the fact that the fractional part of $g(x)$ will be strictly positive for $x > \mu_h$ and strictly negative for $x < \mu_h$. Thus when $x > \mu_h$, we have $q_l = g(x) > x > \mu_h$. Similarly, when $x < \mu_h$, we have $q_l = g(x) < x < \mu_h$.

This theorem demonstrates one of the basic results of the model: that judges will shift their rulings from their own preference points toward $\mu_h$, the center of the appeals court's distribution. This shift is motivated by the tradeoff between issuing a ruling close to the judge's preference point and reducing the chance of being overruled by ruling close to the center. Since $g$ is continuous, and $g(\mu_h) = \mu_h$, the amount of shift will be small when $q_l$ is close to $\mu_h$.

Even though the lower court judges' ideal points are distributed over the entire real line, the incentives created by the appeals process will limit their choice of rulings to a bounded range. The intuition for this is that if the lower court judge chooses a ruling that is extreme, it will have a very high probability of being overruled. Outside the interval $(x_0, x_1)$, this effect will dominate the benefit to the lower court judge of choosing a ruling close to her ideal point. Instead, the judge would prefer a ruling closer to $\mu_h$ that has a greater chance of surviving review. We state this result formally in the following theorem:

**Theorem 3** There exist $x_0$ and $x_1$ such that the lower court's ruling will always be bounded by the interval $(x_0, x_1)$, regardless of $q_l$.

**Proof.** See Appendix.

The proof relies on the fact that the denominator of the fractional part of $g$ will be negative at $\mu_h$, but will be positive in some range on either side of $\mu_h$. Hence $g$ will have asymptotes on both sides of $\mu_h$. Even though the lower court judges' ideal points are distributed over the entire real line, the incentives created by the appeals process will limit their choice of rulings to a bounded range.

Theorem 3 shows that there is a subset of interpretations – those outside the interval $(x_0, x_1)$ – that will never be chosen by a lower court judge, even though they are preferred by a non-trivial subset of lower court judges and have a positive chance of being upheld.

These theorems show how the appeals process leads to more uniform interpretation of the law by the lower court. First, instead of ruling at his own ideal point, the lower court judge will choose a ruling that is closer to $\mu_h$; this shift will typically bring judges with different
preferences closer together. Second, the set of possible rulings issued by the lower court will be bounded. This will eliminate the possibility of the most extreme interpretations being issued by the lower court. Note that in our model, there are no constraints on the higher court, so that its rulings will be unbounded if \( f^h \) has full support on the real numbers. Nevertheless, the likelihood of the most extreme rulings will be reduced significantly.

Although this result seems counterintuitive – the lower court is always centrist, while the appeals court may be extreme – it captures the fact that major changes in doctrine usually issue from the highest court. Lower courts do not have the authority to dramatically alter the interpretation of the law, as higher courts can.

Theorems 2 and 3 also show how judicial restraint derives from the incentives facing the lower court. The lower court judge’s interest in influencing the law, and its awareness that it may be overruled if it strays too far from the center, provide strong incentives to suppress its personal views. Note that both theorems hold irrespective of \( d \), even if there is no additional disutility from being overruled.

Theorem 2 shows how the appeals process causes lower court rulings to be clustered around \( \mu_h \). The use of panels will also cause rulings from the higher court to be more tightly clustered around \( \mu_h \). Thus both appeals and panels lead rulings to be closer to the ideal point of the median appellate judge. This focal point has several appealing consequences. For example, if we assume that there exists an optimal interpretation, which each judge observes with error, then the median will closely approximate the optimum. One the other hand, if we view interpretations of the law as political choices, then the median represents the most democratic outcome. Finally, the view of the median judge has the benefit of legitimacy, in that represents a consensus decision among judges.

The following theorem states the density function of the final outcome \( y \). We will use this in Section IV of the paper to graph some results of the model when both courts’ ideal points are normally distributed.

**Theorem 4** Let \( F^h \) and \( F^l \) denote the cumulative density functions of \( q_h \) and \( q_l \), respectively, and \( f^h \) and \( f^l \) denote the density functions. If \( g(x) \) is an increasing function, then the density
function $f^v$ of $y$ satisfies

$$f^v(y) = \left[F^h(y+c) - F^h(y-c)\right] f^i(g(y)) g'(y)$$
$$+ \left[1 - F^d(g(y+c)) + F^d(g(y-c))\right] f^h(y)$$

**Proof.** See Appendix.

We provide an outline of the proof. There are two ways of reaching a final ruling of $y$: a lower court ruling of $y$ that is not overruled, or a different lower court ruling, followed by a higher court ruling of $y$.

In the first case, we have $q_l = g(y)$. The probability that the higher court will not review is

$$\Pr(|q_h - y| \leq c) = \left[F^h(y+c) - F^h(y-c)\right].$$

This represents the first term in the right-hand side of theorem 4.

In the second case, we have $q_l = g(x)$, $q_h = y$. The probability that the higher court will review is

$$\Pr(|x - y| > c) = \Pr(y - c < x < y + c)$$
$$= \Pr(g(y - c) < q_l < g(y + c))$$
$$= 1 - F^d(g(y+c)) + F^d(g(y-c))$$

The contribution from the second way of reaching a ruling of $y$ represents the second term in theorem 4.

**1.3 Implications of the Model**

In this section, we will explore some of the implications of the model. By making parametric assumptions – normally distributed preferences on both levels of the court – we can examine the impact of changes in the cost of review and the predictability of the higher court. In part (a), we show that the cost of review will have nonmonotonic effects on the variance of higher court rulings. This results from two competing effects: lower court judges will be more restrained when the cost of review is low, but since they are more likely to be overruled, their decisions will have less impact on the final ruling. Thus the variance of final rulings will always
be minimized at $c > 0$.

In part (b) we prove another nonmonotonicity theorem: that the predictability of the higher court $\sigma$ will have nonmonotonic effects on the range of lower court decisions. When the higher court's rulings are highly variable, the lower court will have less incentive to be moderate, since the likelihood of being overruled will decrease less sharply toward the center. On the other hand, if the higher court is very predictable, the lower court can more easily "game" the higher court. Since there will be a fairly well-defined "safe" region, in which the likelihood of review is low, the lower court can shift toward the center only as much as necessary to avoid review.

These results are presented in terms of parameters $c$ and $\sigma$, representing the latitude granted to the lower court and the predictability of the higher court, respectively. Since $c = \sqrt{e}$, where $e$ is the equilibrium effort cost of review, we can use these results to inform our understanding of the effects of structural changes in the court. Changes that reduce the equilibrium cost of review, such as appointing additional judges, or reducing the total caseload, could be modeled as decreasing $c$. Increasing the size of appellate panels or employing a more rigorous screening process for judicial appointments could be modeled as reducing $\sigma$.

1.3.1 Cost of Review

We assume that

$$q_l \sim N(0, 1)$$

and

$$q_h \sim N(0, \sigma^2)$$

Implicit in these distributional assumptions is that the preferences of judges on both courts have equal means. We assume that there is no ideological tension between the courts in order to focus on the effects of court structure on uniformity of interpretation.

Recall that the lower court's ruling is determined by $q_l = g(x)$, where

$$g(x) = x + \frac{1}{2c} \frac{(c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right]}{\left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right]}.$$ 

where $f^h(x) = \frac{1}{\sigma} \phi \left( \frac{x}{\sigma} \right)$. Recall from theorem 3 that the bounds of $x$ (the possible rulings of
the lower court) are determined by the denominator of the above expression. By symmetry, we may denote the lower and upper bounds of $x$ as $-\lambda$ and $\lambda$. In this section, $\lambda$ will be endogenously determined as a function of $c$ and $\sigma$.

The following theorem shows how the cost of appeal affects the range of rulings issued by the lower court.

**Theorem 5** The range of lower court rulings $(-\lambda, \lambda)$, where the cost of review is $c$ and the variance of the higher court rulings is $\sigma$, has the following properties with respect to $c$:

a) $\lambda(c, \sigma)$ is finite for all $c, \sigma$

b) $\lim_{c \to 0} \lambda(c, \sigma) = \sigma$.

c) $\lambda(c, \sigma) \sim c$ as $c \to \infty$.

d) $\lambda$ is strictly increasing in $c$.

**Proof.** See Appendix.

Note that Part (a) is a direct result of theorem 3. Part (b) shows that the range will contract to $[-\sigma, \sigma]$ as $c \to 0$, but even when the probability of review approaches 1, the range of rulings will not contract completely.

Part (c) holds because as $c$ gets large relative to $\sigma$, any lower court judge with $|q_1| < c$ will rule very close to his ideal point, since the probability of review will be very small. Judges with $|q_1|$ close to $c$ will shift just enough to reduce the probability of review to be very small. Hence the outer bound $\lambda$ will be close to $c$ in magnitude.

Part (d) is an intuitive result: the bounds of possible rulings will expand when the cost of review increases.

Taken together, these results show the relationship between $c$ and the bounds of the lower court's rulings.

Theorem 5 showed a monotonicity result on lower court rulings: that the lower court becomes more predictable when $c$ decreases. However, if appeal is too frequent, then the lower court will only have a small impact on the final ruling, since its decision will usually be reviewed. Thus a low value of $c$ provides strong incentives for the lower court to be rule moderately, but the predictability of the lower court will not matter for the final result. The following theorem
formalizes the above intuition, showing that the variance of the final ruling is minimized at a positive cost of review.

**Theorem 6** The variance of the final ruling $y$ will be decreasing in $c$ as $c \to 0$, and will therefore be minimized at some $c > 0$.

**Proof.** See Appendix.  

1.3.2 Uncertainty

In this section, we will study the effect of $\sigma$ on the rulings of the lower court. Since $\sigma$ is the standard deviation of higher court rulings, we can use it to model structural changes in the court, such as panel size, changes in the process for appointing judges, or the impact of external constraints such as legislative override or judicial elections. As the following theorem the effect of $\sigma$ on the lower court rulings is nonmonotonic: the bounds of rulings will be decreasing in $\sigma$ for small $\sigma$ and increasing in $\sigma$ for large $\sigma$.

**Theorem 7** The range of lower court rulings $(-\lambda, \lambda)$ has the following properties with respect to $\sigma$:

a) $\lim_{\sigma \to 0} \lambda(c, \sigma) = c$.

b) $\lambda$ is nonmonotonic with respect to $\sigma$: it is decreasing for small $\sigma$ and increasing for large $\sigma$.

c) For any fixed $c$, $\lambda$ reaches a global minimum at some $\sigma > 0$.

**Proof.** See Appendix.  

Part (a) examines the bounds when $\sigma \to 0$, i.e., when the higher court’s decision approaches certainty. In this case, any ruling $x$ satisfying $|x| \leq c$ will not be reviewed by the higher court. Therefore the lower court judge will rule sincerely if $|q| \leq c$. Any judge with $q > c$ will choose $x = c$, and any judge with $q < -c$ will choose $x = -c$. Thus the range of lower court rulings will be $[-c, c]$.

Part (b) shows that the range of the lower court’s rulings will decrease in $\sigma$ for low values of $\sigma$ and increase for high values of $\sigma$. This is a somewhat counterintuitive result: for low values of $\sigma$, as the higher court rulings become less uniform, the lower court rulings become more uniform. The intuition for this is as follows: when $\sigma = 0$, the higher court’s position is
known, so the lower court will know exactly how much to moderate its opinion, if necessary, in order to avoid appeal. In particular, if the lower court’s preference point $q_l$ satisfies $|q_l| > c$, then the lower court will choose $x = \pm c$. If we increase $\sigma$ very slightly, then there is a very small amount of uncertainty about the higher court’s preferences. If the lower court now chose $x = \pm c$, there would be a $\frac{1}{2}$ probability of review. However, by moving slightly toward the center, the probability of review drops dramatically; at $x = \pm (c - \sigma)$, the probability of review goes to 0. The decreased likelihood of review results in a first-order gain, while the shift from the preference point results in a second-order loss. Thus the lower court will shift slightly toward the center in such cases.

Part (c) is an immediate consequence of part (b).

This means that reducing the uncertainty of the appellate judges’ decisions will increase the outer bound of the lower court’s ruling when the level of uncertainty is already very low.

1.3.3 Aversion to Reversal

**Theorem 8** The range of lower court rulings $(-\lambda, \lambda)$ is independent of $d$ the disutility from being overruled, but the variance of lower court rulings is strictly decreasing in $d$.

**Proof.** See Appendix. ■

The result shown in theorem 8 is unsurprising: that lower court judges will rule more moderately when they are averse to being overruled by the higher court. What is most significant is that the model does not require any individual disutility from reversal to show that appeals promote moderation in the lower courts. The desire to influence the outcome of cases, coupled with strategic anticipation of the appellate court’s ruling, is sufficient to ensure moderation in the lower court.

Although it is frequently believed that judges do not like to be overturned, i.e., $d > 0$, this assumption has been challenged. (Klein and Hume 2003) Thus, although incorporating reversal aversion strengthens the predictions of the model, it is not necessary for any of the results in this section.
1.4 Effects of Court Structure on Interpretation

In this section, we use some of the results from Section III to explore the effects of court structure on interpretation. As in Section III, we assume that we have a two-level court system, and we normalize the variance of the lower court judges' ideal points to 1. Let $\sigma^2$ be the variance of the higher court, where typically $\sigma^2 < 1$. Finally, we assume that $d = 0$.

Appeals courts usually consist of a panel of judges. Formally, panels rule by majority vote, but the deliberative process and collegial decision-making among the judges may lead to a more nuanced process of preference aggregation. (Sunstein, et. al. 2004) In a purely independent voting model, the higher court ruling would be the median ideal point among the judges in the panel; under a joint utility maximization model, the ruling would be the mean.

There are other factors that would influence $\sigma^2$. The appointment and confirmation process could lead to greater scrutiny of potential judges, and hence more moderation, or a highly politicized process could have the opposite effect. Influences from outside the judiciary – such as the threat of legislative override, greater media scrutiny, and in some case, judicial elections – may have a stronger influence on the appeals court. Without attempting to model each of these effects explicitly, we may simply observe that the predictability of the higher court should be increasing in panel size, and that the variance of a court with panel size $n$ should decrease proportionately by a factor of order $n$.

For simplicity, we assume in the following discussion that $\sigma^2 = \frac{1}{n}$. This would occur, for example, if all judges' preferences on both courts are drawn from the same distribution, and the higher court chooses a ruling that maximizes the sum of utilities of all judges on the panel.

First, we analyze the simplest model: a higher court with a single judge, so that $\sigma^2 = 1$. At $c = 0$, the lower court has no latitude and appeal is costless, so that the outcome will always coincide with the higher judge's ideal point. At $c = \infty$, appeal will never occur, and the trial judge will always rule at her ideal point. Thus, in either of these cases, the outcome will be normally distributed with variance 1. For $0 < c < \infty$, the variance will be strictly less than 1, as shown in Figure 1. Notice that the probability of review decreases in $c$, as expected.

Figure 1 shows that the variance in this case is minimized at $c \approx 2.3$, which corresponds to a likelihood of review of around 10%. At this level, the variance of the final outcome is 0.52. When both courts have the same variance, uniformity is optimized at a relatively low likelihood.
Figure 1-1: Variance and Probability of Review as Functions of c, One-Judge Appeals Court

![Graph showing variance and probability of review as functions of c, One-Judge Appeals Court.](image-url)
Next, we consider the effect of using 3-judge panels in the higher court, so that $\sigma^2 = \frac{1}{3}$. Figure 2 shows the impact of $c$ on the variance of the outcome and the probability of review. Here, the minimum variance is approximately 0.17 at $c \approx 1.1$; the probability of review is approximately 0.34. At this level, the variance is reduced by a factor of almost 6. Since the probability of review is 0.34, and each appeal requires 3 judges, the court system will require the same level of resources ("judge-hours") on the higher court as on the lower court.

The degree of latitude is not directly controlled; it is determined in equilibrium by the marginal effort cost of the higher court judges. Suppose, for example, that there are equal numbers of judges on both courts, and that each case on the higher court requires a panel of three judges. In equilibrium, if each higher court judge is participating in as many cases as each lower court judge, then the probability of review must be $\frac{1}{3}$. This would mean that $c \approx 1.1$, which is close to the optimum.
Figure 3 illustrates how lower variances can be achieved with larger panels and a greater mass of judges. It compares the variance against the measure of judges for panels of one, three, and five judges. The x-axis in Figure 3 measures "Ratio of Higher/Lower Court Workload," which is scaled under the assumption that each judge's participation in an appeal requires the same amount of effort as a lower court judge requires to decide a case.

Note that when $c = \infty$, the variance will be equal one in all cases, since there will never be appeal. This corresponds to the point in the upper-left corner of the graph. In the case of a one-judge higher court, the variance goes to one as $c \to 0$; this corresponds to the scenario where every case is decided by the appeals court (when the workload ratio is 1).

The superimposed variances for different panel sizes shows that larger reductions in variance are possible with larger panels, but that more appellate judges are necessary to achieve these reductions. Thus when few appellate judges are used, smaller panels with more frequent review will yield more uniformity than larger panels with less frequent review. For example, if the ratio of higher court to lower court judges is 0.5, Figure 3 shows that 3-judge panels achieve the greatest degree of uniformity. When many judges are used, it is possible to achieve large reductions in variance when there are both large panels and frequent review.

It is worth noting that, at least in theory, significant reductions in variance may occur even when the likelihood of review is very small. For example, the U.S. Supreme Court agrees to hear fewer than 3% of cases that are petitioned for review. (Epstein, et. al. 1996) According to the model, this could still have a significant impact on judges in the appeals courts.

Figure 3 presents a very simplified graph illustrating how appeals may increase the uniformity of final rulings. The reduction in variance would be greater when judges are averse to being overruled ($d > 0$). The graph also does not consider the effect of a multiple-tiered hierarchy, which could achieve even greater reductions in variance. Nevertheless, it provides a sense of how choice of court structure can affect how judges interpret the law.

1.5 Conclusion

Formal modeling in this context provides several insights that would be less obvious from casual observation. When there is uncertainty about the ideologies of judges on both tiers of the court,
Figure 3: Variance of Final Rulings, by Panel Size

1-judge court
$\sigma^2 = 1$

5-judge court
$\sigma^2 = 1/5$

3-judge court
$\sigma^2 = 1/3$

Figure 1-3:
then the strategic interaction between the lower court and the higher court can result in greater consistency of interpretation that either court could achieve independently. Thus, even when the higher court is much more consistent, it is still optimal to limit review to some degree.

In the legal literature, it is often assumed that more frequent review enhances predictability in the interpretation of the law by providing closer monitoring of lower court judges and resolving conflicts arising from different cases. This assumption neglects the fact that higher court itself introduces some uncertainty. For example, while the Supreme Court has been criticized for not reviewing enough cases, commentators have also complained of doctrinal incoherence in areas of the law in which the Supreme Court has been active. (Hellman 1996)

Similarly, questions of panel size are typically framed as a trade-off between consistency and the use of judicial resources. As the model shows, however, using panels to increase consistency in appellate courts may be counterproductive if it weakens incentives for moderation in the lower court. Whether this effect is observable in practice is a question for future empirical research.

There are several theoretical questions arising from this model that could be explored in future research. The model could be extended to consider multiple-tiered hierarchies, which could potentially provide strong incentives for restraint in the lower court, while minimizing concentration of authority in the highest court. A model that endogenizes the role of precedent could help explain how appeals judges select cases for review, and also explore how the structure of courts affects the evolution of precedent.

1.6 Appendix

Proof of Theorem 1:

First, note that for a given ruling $x$, the higher court judge has utility $-(q_h - x)^2$ if he does not review the case, and utility $-(q_h - y)^2 - e$ if he does review. Clearly, this latter term is minimized at $y = q_h$, so the judge will review the case if $-e > -(q_h - x)^2$. Hence the ruling will be reviewed if $|x - q_h| > \sqrt{e} = c$. 

24
Now for a given ruling $x$, the lower court judge's utility will be

$$
U_t = \begin{cases} 
-(q_l - q_h)^2, & \text{if the case is reviewed} \\
-(q_l - x)^2, & \text{if the case is not reviewed}
\end{cases}
$$

Thus,

$$
EU_t = \int_{-\infty}^{x-c} -(q_h - q_l)^2 f^h(q_h) dq_h + \int_{x-c}^{x+c} - (x - q_l)^2 f^h(q_h) dq_h + \int_{x+c}^{\infty} -(q_h - q_l)^2 f^h(q_h) dq_h
$$

Applying Leibnitz's Rule,

$$
\frac{\partial EU_t}{\partial x} = -(x - q_l - c)^2 f^h(x - c) - 2(x - q_l) \left[ F^h(x + c) - F^h(x - c) \right] \\
- (x - q_l)^2 \left[ f^h(x + c) - f^h(x - c) \right] + (x - q_l + c)^2 f^h(x + c) \\
= 2(x - q_l) \left[ c \left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right] \right] \\
+ (c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right]
$$

Hence the first-order condition yields

$$
x = q_l - \frac{1}{2} \frac{(c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right]}{c \left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right]}
$$

To simplify our notation, let

$$
g(x) = x + \frac{1}{2} \frac{(c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right]}{c \left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right]} \\
n(x) = \frac{1}{2} \frac{(c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right]}{c \left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right]}, \text{ and} \\
d(x) = c \left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right].
$$
so that
\[ q_l = g(x) = x + \frac{n(x)}{d(x)} \]

Proof of Theorem 2:
Let \(n(x)\) and \(d(x)\) denote the numerator and denominator, respectively, of \(g(x)\):
\[
\begin{align*}
  n(x) &= \frac{1}{2}(c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right], \text{ and} \\
  d(x) &= c \left[ f^h(x + c) + f^h(x - c) \right] - \left[ F^h(x + c) - F^h(x - c) \right]
\end{align*}
\]
so that
\[ q_l = g(x) = x + \frac{n(x)}{d(x)} \]

Then \(g(x), n(x),\) and \(d(x)\) satisfy the following properties:

a) \(n(x) \geq 0\) for \(x \leq \mu_h\)

b) \(n(x) \leq 0\) for \(x \geq \mu_h\)

c) \(d(\mu_h) < 0\)

\[ d(\mu_h) = \mu_h \]

First, we will show that \(n(x) \geq 0\) for \(x \leq \mu_h\):

For \(x < \mu_h - c\), the result follows from the fact that \(f^h\) is increasing on \((-\infty, \mu_h)\). For \(\mu_h - c < x < \mu_h\), note that
\[
\begin{align*}
  n(x) &= \frac{1}{2}(c^2 + d) \left[ f^h(x + c) - f^h(x - c) \right] \\
  &= \frac{1}{2}(c^2 + d) \left[ f^h(2\mu_h - x - c) - f^h(x - c) \right] \\
  &> 0, \text{ since } x - c < 2\mu_h - x - c < \mu_h.
\end{align*}
\]

Similarly, we can show that \(n(x) \leq 0\) for \(x \geq \mu_h\). This also implies that \(n(\mu_h) = 0\).

Now we show that \(d(\mu_h) < 0\):

Note that \(f^h(\mu_h - c) = f^h(\mu_h + c)\), and \(f^h\) is increasing on \((\mu_h - c, \mu_h)\) and decreasing on \((\mu_h, \mu_h + c)\). Thus \(f^h(x) \geq f^h(\mu_h - c)\) for all \(x \in (\mu_h - c, \mu_h + c)\), with strict inequality in a
neighborhood of $\mu_h$. Hence,
\[
d(\mu_h) = c \left[ f^h(\mu_h + c) + f^h(\mu_h - c) \right] - \left[ F^h(\mu_h + c) - F^h(\mu_h - c) \right]_{\mu_h+c}^{\mu_h-c} \\
= c \left[ f^h(\mu_h - c) + f^h(\mu_h - c) \right] - \int_{\mu_h-c}^{\mu_h+c} f^h(t) dt \\
= \int_{\mu_h-c}^{\mu_h+c} \left[ f^h(\mu_h - c) - f^h(t) \right] dt \]
< 0.

Also, this implies that $\frac{n(\mu_h)}{d(\mu_h)} = 0$ so that $g(\mu_h) = \mu_h$.

Now $d$ must be negative over the range of $x$. If $d(x_1) = 0$ for some value $x_1 > \mu_h$, then $g$ will asymptote to $+\infty$ at $x_1$. Similarly, $g$ will asymptote to $-\infty$ if $d(x_0) = 0$ for some value $x_0 > \mu_h$. Hence for all $x$ for which $g(x)$ is defined, $d(x) < 0$.

Since $n(x) \leq 0$ for $x > \mu_h$ and $d(x) < 0$, it follows that $g(x) > x$, and therefore, $\mu_h < x < q_1$. Similarly, for $x < \mu_h$, we have $q_1 < x < \mu_h$. Therefore the judge chooses a ruling $x$ between $q_1$ and $\mu_h$.

**Proof of Theorem 3:**

We will show that $g$ has asymptotes on both sides of $\mu_h$. First, note that
\[
\int_{-\infty}^{\infty} d(x) dx = \int_{-\infty}^{\infty} c \left[ f^h(x + c) + f^h(x - c) \right] dx - \int_{-\infty}^{\infty} \int_{x-c}^{x+c} f^h(t) dt dx \\
= 2c - 2c \int_{-\infty}^{\infty} f^h(t) dt \\
= 0
\]
Since $d$ is continuous, there exists some $x_0$ such that $d(x_0) = 0$.

Also, $x_0 \neq \mu_h$, since $d(\mu_h) < 0$, as shown in theorem 2. If there exist multiple choices for $x_0$, choose the value that minimizes $|\mu_h - x_0|$. Since $f$ is symmetric about $\mu_h$, it follows that $d$ is symmetric about $\mu_h$, so that $d(x_1) = 0$, where $x_1 = 2\mu_h - x_0$. For simplicity, we
can assume that $x_0 < \mu_h < x_1$. Now $n(x_1) < 0$ and $n(x_0) > 0$, so $\lim_{x \to x_0^+} \frac{n(x)}{\frac{d(x)}{dx}} = -\infty$ and $\lim_{x \to x_1^-} \frac{n(x)}{\frac{d(x)}{dx}} = +\infty$. Thus $g$ has asymptotes at $x_0$ and $x_1$ and has a range covering the entire real line on $(x_0, x_1)$. Since $q_I = g(x)$, it follows that $x_0 < x < x_1$. Thus the lower court’s choice of ruling will always be bounded by the interval $(x_0, x_1)$.

**Proof of Theorem 4:**

Consider any value $y_0$. Then $F^y(y_0) = \Pr(y < y_0)$. Now there are two conditions on $q_I$ and $q_h$ that will achieve $y < y_0$: either $x < y_0$ and the case is not reviewed or $y < y_0$ and the case is reviewed. Thus

$$\Pr(y < y_0) = \Pr(q_h < y_0 \text{ and } |g^{-1}(q_I) - q_h| > c)$$

$$+ \Pr(g^{-1}(q_I) < y_0 \text{ and } |g^{-1}(q_I) - q_h| \leq c)$$

We may sum the two probabilities on the right-hand side because the events are disjoint. We consider each of these terms separately.

$$\Pr(q_h < y_0 \text{ and } |g^{-1}(q_I) - q_h| > c) = \Pr(q_h < y_0 \text{ and } (q_I) \notin [q_h - c, q_h + c])$$

$$= \int_{-\infty}^{y_0} \left[1 - F^I(g(q_h + c)) + F^I(g(q_h - c))\right] f^h(q_h) dq_h$$

28
And

\[
\Pr \left( g^{-1}(q_l) < y_0 \text{ and } |g^{-1}(q_l) - q_h| \leq c \right) = \Pr (q_l < g(y_0) \text{ and } (q_l) \in [q_h - c, q_h + c])
\]

\[
= \int_{\min\{g(y_0), g(q_h+c)\}}^{\max\{g(y_0), g(q_h+c)\}} f^h(q_h) f^l(q_l) dq_l dq_h
\]

\[
= \int_{\min\{g(y_0), g(q_h+c)\}}^{\max\{g(y_0), g(q_h+c)\}} f^h(q_h) f^l(q_l) dq_l dq_h
\]

\[
+ \int_{g(q_h-c)}^{g(q_h+c)} f^h(q_h) f^l(q_l) dq_l dq_h
\]

\[
= \int_{-\infty}^{\max\{g(y_0), g(q_h+c)\}} \left[ F^l(g(q_h+c)) - F^l(g(q_h-c)) \right] f^h(q_h) dq_h
\]

\[
+ \int_{g(q_h-c)}^{\max\{g(y_0), g(q_h+c)\}} \left[ F^l(g(y_0)) - F^l(g(q_h-c)) \right] f^h(q_h) dq_h
\]

Hence

\[
F^y(y_0) = \int_{-\infty}^{\min\{g(y_0), g(q_h+c)\}} \left[ 1 - F^l(g(q_h+c)) + F^l(g(q_h-c)) \right] f^h(q_h) dq_h
\]

\[
+ \int_{\min\{g(y_0), g(q_h+c)\}}^{\max\{g(y_0), g(q_h+c)\}} \left[ F^l(g(q_h+c)) - F^l(g(q_h-c)) \right] f^h(q_h) dq_h
\]

\[
+ \int_{g(q_h-c)}^{\max\{g(y_0), g(q_h+c)\}} \left[ F^l(g(y_0)) - F^l(g(q_h-c)) \right] f^h(q_h) dq_h
\]

By Leibnitz’s Rule,
\[ f^y(y) = \left[ 1 - F^l(g(y + c)) + F^l(g(y - c)) \right] f^h(y) \]
\[ + \int_{y-c}^{y+c} f^l(g(y)) g'(y) f^h(q_h) dq_h \]
\[ = \left[ 1 - F^l(g(y + c)) + F^l(g(y - c)) \right] f^h(y) \]
\[ + \left[ F^h(y + c) - F^h(y - c) \right] f^l(g(y)) g'(y) \]

**Lemma 9** There exists \( x_0 \) such that \( d(x_0) = 0 \).

**Proof.**
\[
\int_{-\infty}^{\infty} d(x) dx = \int_{-\infty}^{\infty} c \left[ f^h(x + c) + f^h(x - c) \right] dx - \int_{-\infty}^{\infty} \int_{x-c}^{x+c} f^h(t) dt dx \\
= 2c - 2c \int_{-\infty}^{\infty} f^h(t) dt \\
= 0 
\]

Since \( d \) is continuous, there must be some \( x_0 \) for which \( d(x_0) = 0 \).

**Lemma 10** If \( f^h \) is convex on the interval \( (x - c, x + c) \), then \( d(x) > 0 \). Similarly, if \( f^h \) is concave on the interval \( (x - c, x + c) \), then \( d(x) < 0 \).

**Proof.**
\[
d(x) = c \left[ f^h(x + c) + f^h(x - c) \right] - \int_{x-c}^{x+c} f^h(t) dt \\
= \int_{x-c}^{x+c} \left[ \frac{x-t+c}{2c} f^h(x-c) + \frac{t-x+c}{2c} f^h(x+c) - f(t) \right] dt \\
= \int_{x-c}^{x+c} \left[ \frac{x-t+c}{2c} f^h(x-c) + \frac{t-x+c}{2c} f^h(x+c) - f^h \left( \frac{x-t+c}{2c}, x-c \right) \right] dt \\
> 0 
\]
Lemma 11

\[ \frac{d\lambda}{dc} = \frac{(c^2 + c\lambda) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c^2 - c\lambda) \phi \left( \frac{\lambda - c}{\sigma} \right)}{(c\lambda + c^2 + \sigma^2) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c\lambda - c^2 - \sigma^2) \phi \left( \frac{\lambda - c}{\sigma} \right)} \] 

and

\[ \frac{d\lambda}{d\sigma} = \frac{\left[ \frac{c}{\sigma} (\lambda + c)^2 + c\lambda \right] \phi \left( \frac{\lambda + c}{\sigma} \right) + \left[ \frac{c}{\sigma} (\lambda - c)^2 - c\lambda \right] \phi \left( \frac{\lambda - c}{\sigma} \right)}{(c\lambda + c^2 + \sigma^2) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c\lambda - c^2 - \sigma^2) \phi \left( \frac{\lambda - c}{\sigma} \right)} \]

Proof.

\[ \frac{c}{\sigma} \left[ \phi \left( \frac{\lambda + c}{\sigma} \right) + \phi \left( \frac{\lambda - c}{\sigma} \right) \right] = \Phi \left( \frac{\lambda + c}{\sigma} \right) - \Phi \left( \frac{\lambda - c}{\sigma} \right) \]

Differentiating yields

\[ \begin{pmatrix} \frac{c}{\sigma} \phi' \left( \frac{\lambda + c}{\sigma} \right) (\frac{d\lambda}{dc} + 1) \\ \frac{c}{\sigma} \phi' \left( \frac{\lambda - c}{\sigma} \right) (\frac{d\lambda}{dc} - 1) \\ \frac{1}{\sigma} \left[ \phi \left( \frac{\lambda + c}{\sigma} \right) + \phi \left( \frac{\lambda - c}{\sigma} \right) \right] \end{pmatrix} = \begin{pmatrix} \frac{1}{\sigma} \phi \left( \frac{\lambda + c}{\sigma} \right) (\frac{d\lambda}{dc} + 1) \\ -\frac{1}{\sigma} \phi \left( \frac{\lambda - c}{\sigma} \right) (\frac{d\lambda}{dc} - 1) \\ 0 \end{pmatrix} \]

Rearranging terms and substituting for \( \phi' \) yields

\[ \frac{d\lambda}{dc} = \frac{(c^2 + c\lambda) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c^2 - c\lambda) \phi \left( \frac{\lambda - c}{\sigma} \right)}{(c\lambda + c^2 + \sigma^2) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c\lambda - c^2 - \sigma^2) \phi \left( \frac{\lambda - c}{\sigma} \right)} \]

We similarly differentiate with respect to \( \sigma \):

\[ \frac{c}{\sigma} \phi' \left( \frac{\lambda + c}{\sigma} \right) \left[ \frac{1}{\sigma} \frac{d\lambda}{d\sigma} - \frac{\lambda + c}{\sigma^2} \right] + \frac{c}{\sigma} \phi' \left( \frac{\lambda - c}{\sigma} \right) \left[ \frac{1}{\sigma} \frac{d\lambda}{d\sigma} - \frac{\lambda - c}{\sigma^2} \right] = \phi \left( \frac{\lambda + c}{\sigma} \right) \left[ \frac{1}{\sigma} \frac{d\lambda}{d\sigma} - \frac{\lambda + c}{\sigma^2} \right] \]

\[ -\phi \left( \frac{\lambda - c}{\sigma} \right) \left[ \frac{1}{\sigma} \frac{d\lambda}{d\sigma} - \frac{\lambda - c}{\sigma^2} \right] \]

Rearranging terms yields

\[ \frac{d\lambda}{d\sigma} = \frac{\left[ \frac{c}{\sigma} (\lambda + c)^2 + \sigma \lambda \right] \phi \left( \frac{\lambda + c}{\sigma} \right) + \left[ \frac{c}{\sigma} (\lambda - c)^2 - \sigma \lambda \right] \phi \left( \frac{\lambda - c}{\sigma} \right)}{(c\lambda + c^2 + \sigma^2) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c\lambda - c^2 - \sigma^2) \phi \left( \frac{\lambda - c}{\sigma} \right)} \]
Proof of Theorem 5:

Part (a):

We construct Taylor series for $\Phi$ and $\phi$ around $\lambda$:

\[
\frac{c}{\sigma} \left[ \frac{\lambda + c}{\sigma} \phi \left( \frac{\lambda - c}{\sigma} \right) + \frac{\lambda - c}{\sigma} \phi \left( \frac{\lambda + c}{\sigma} \right) \right] = \frac{2c}{\sigma} \phi \left( \frac{\lambda}{\sigma} \right) + \frac{c^3}{\sigma^3} \phi'' \left( \frac{\lambda}{\sigma} \right) + O \left( c^4 \right)
\]

\[
\Phi \left( \frac{\lambda + c}{\sigma} \right) - \Phi \left( \frac{\lambda - c}{\sigma} \right) = \frac{2c}{\sigma} \phi \left( \frac{\lambda}{\sigma} \right) + \frac{c^3}{3\sigma^3} \phi'' \left( \frac{\lambda}{\sigma} \right) + O \left( c^4 \right)
\]

If $\lambda$ satisfies Equation (\ast) for sufficiently small $c$, it follows that $\phi'' \left( \frac{\lambda}{\sigma} \right) = 0$, or equivalently,

\[
\left[ 1 - \left( \frac{\lambda}{\sigma} \right)^2 \right] \phi \left( \frac{\lambda}{\sigma} \right) = 0 \implies \lambda = \sigma.
\]

Part (b):

\[
\lim_{c \to 0} \frac{d\lambda}{dc} = \lim_{c \to 0} -\frac{(c^2 + c\lambda) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c^2 - c\lambda) \phi \left( \frac{\lambda - c}{\sigma} \right)}{(c\lambda + c^2 + \sigma^2) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c\lambda - c^2 - \sigma^2) \phi \left( \frac{\lambda - c}{\sigma} \right)}
\]

\[
= \lim_{c \to 0} -\frac{\lambda \left[ \phi \left( \frac{\lambda + c}{\sigma} \right) - \phi \left( \frac{\lambda - c}{\sigma} \right) \right]}{\phi \left( \frac{\lambda + c}{\sigma} \right) + \sigma^2 \left[ \phi \left( \frac{\lambda + c}{\sigma} \right) - \phi \left( \frac{\lambda - c}{\sigma} \right) \right]}
\]

\[
= \frac{0}{2\lambda \phi (\sigma) + \sigma^2 \phi' (\sigma)} = 0
\]

Part (c):

\[
\lim_{c \to \infty} \frac{d\lambda}{dc} = \lim_{c \to \infty} -\frac{(c^2 + c\lambda) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c^2 - c\lambda) \phi \left( \frac{\lambda - c}{\sigma} \right)}{(c\lambda + c^2 + \sigma^2) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c\lambda - c^2 - \sigma^2) \phi \left( \frac{\lambda - c}{\sigma} \right)}
\]

\[
= \lim_{c \to \infty} -\frac{(c + \lambda) \phi \left( \frac{\lambda + c}{\sigma} \right) + (c - \lambda)}{\left( \lambda + c + \frac{\sigma^2}{c} \right) \phi \left( \frac{\lambda + c}{\sigma} \right) + \left( \lambda - c - \frac{\sigma^2}{c} \right) \phi \left( \frac{\lambda - c}{\sigma} \right)}
\]

\[
= \lim_{c \to \infty} \frac{(c + \lambda) e^{-\frac{2c\lambda}{\sigma^2}} + (c - \lambda)}{(\lambda + c) e^{-\frac{2c\lambda}{\sigma^2}} + (\lambda - c)}
\]

Note that we cannot have $\lambda \to 0$ as $c \to \infty$; if this were true, then $\frac{2c}{\sigma} \phi \left( \frac{c}{\sigma} \right) \to 1$ as $c \to \infty$,
which is impossible. Hence \( e^{-\frac{2\lambda}{c^2}} \to 0 \) as \( c \to \infty \), so

\[
\lim_{c \to \infty} \frac{d\lambda}{dc} = \frac{\lambda - c}{\lambda - c} = 1.
\]

**Part (d):**

From Lemma 11,

\[
\frac{d\lambda}{dc} = -\frac{(c^2 + c\lambda) \phi\left(\frac{\lambda + c}{\sigma}\right) + (c^2 - c\lambda) \phi\left(\frac{\lambda + c}{\sigma}\right)}{(c\lambda + c^2 + \sigma^2) \phi\left(\frac{\lambda + c}{\sigma}\right) + (c\lambda - c^2 - \sigma^2) \phi\left(\frac{\lambda - c}{\sigma}\right)}
\]

\[
= \frac{c^2 - (c\lambda) \tanh c\lambda}{(c^2 + 1) \tanh c\lambda - (c\lambda)}
\]

First, we consider the case where \( \sigma = 1 \), so that

\[
\frac{d\lambda}{dc} = \frac{c^2 - (c\lambda) \tanh c\lambda}{(c^2 + 1) \tanh c\lambda - (c\lambda)} \quad (1.1)
\]

From part (a), we know that \( \lambda \to 1 \) as \( c \to 0 \). Substituting the power series of \( \tanh \) into (1.1) yields

\[
\frac{d\lambda}{dc} = \frac{c^2(1 - \lambda^2) + \frac{1}{3}c^4\lambda^4 + O(c^6)}{c^2(\lambda - \frac{1}{3}\lambda^3) + O(c^5)}
\]

so that as \( c \to 0 \), \( \frac{d\lambda}{dc} \to 0 \) and \( \frac{d^2\lambda}{dc^2} \to \frac{1}{2} \). Thus \( \lambda \) is increasing in \( c \) when \( c \) is sufficiently close to zero.

Now consider the curve \( S \) defined by the equation

\[ c^2 - (c\lambda) \tanh c\lambda = 0 \quad (1.2) \]

Implicitly differentiating (1.2) yields

\[
\frac{d\lambda}{dc} = \frac{1 - c^2 \text{sech}^2 c\lambda}{\tanh c\lambda + c\lambda \text{sech}^2 c\lambda} > 0 \quad (1.3)
\]

so \( S \) has strictly positive slope. Note that \( S \) contains the point \((c, \lambda) = (0, 1)\), since \( \lambda \to 1 \) as \( c \to 0 \) in equation (1.2). Also, as can be seen in equation (1.3), \( S \) has infinite slope at
Thus, for sufficiently small values of $c$, the solution to (1.1) must lie between the $x$-axis and curve $S$. If the solution to (1.1) intersected $S$ at another point, it must approach $S$ from below, and hence have positive slope at that point. However, the solution to (1.1) must have zero slope at any point at which it intersects $S$. Therefore the solution to (1.1) will always be between the $x$-axis and curve $S$.

In the region between the $x$-axis and curve $S$, the numerator $[c^2 - (c\lambda) \tanh c\lambda]$ in (1.1) will be strictly positive, and the denominator will be positive for all $c, \lambda > 0$. Hence $\frac{dA}{dc}$ is strictly positive, so $\lambda$ is strictly increasing in $c$.

The general case, when $\sigma \neq 1$, now follows easily. Let $\tilde{c} = \frac{c}{\sigma}$, $\tilde{\lambda} = \frac{\lambda}{\sigma}$, and $\tilde{\sigma} = 1$. By the above reasoning, $\frac{dA}{dc}$ is strictly increasing, hence $\frac{dA}{d\tilde{c}}$ is strictly increasing.

**Lemma 12** As $c \to 0$, $g(x) \to x + \frac{3}{2} \frac{\sigma^2 x}{\sigma^2 - x^2}$, and the density function of $x$ approaches

$$\frac{1}{\sqrt{2\pi} \sigma} \left(1 + \frac{3}{2} \frac{\sigma^2 (\sigma^2 + x^2)}{(\sigma^2 - x^2)^2}\right) \exp \left(-\frac{1}{2} x^2 \left(1 + \frac{3}{2} \frac{\sigma^2}{\sigma^2 - x^2}\right)^2\right).$$

**Proof.** We use the Taylor Series expansions for $F^h(x) = \Phi \left(\frac{x}{\sigma}\right)$ and $f^h(x) = \frac{1}{\sigma} \phi \left(\frac{x}{\sigma}\right)$, substituting them into the formula for $g(x)$. This yields

$$g(x) = x + \frac{\sigma}{2} \left(-2c^2 \frac{x}{\sigma} + O(c^2)\right) \to x + \frac{3}{2} \frac{\sigma^2 x}{\sigma^2 - x^2}$$

Now $q_l = h(x)$, so as $c \to 0$,

$$g(x) = x + \frac{\sigma}{2} \left(-2c^2 \frac{x}{\sigma} + O(c^2)\right) \to x + \frac{3}{2} \frac{\sigma^2 x}{\sigma^2 - x^2}$$

Now $q_l = h(x)$, so as $c \to 0$,

$$f^x(x) = f^l(g(x))g'(x)$$

$$\to \phi \left(x + \frac{3}{2} \frac{\sigma^2 x}{\sigma^2 - x^2}\right) \left(1 + \frac{3}{2} \frac{\sigma^2}{\sigma^2 - x^2}\right)$$

$$= \frac{1}{\sqrt{2\pi} \sigma} \left(1 + \frac{3}{2} \frac{\sigma^2 (\sigma^2 + x^2)}{(\sigma^2 - x^2)^2}\right) \exp \left\{-\frac{1}{2} x^2 \left(1 + \frac{3}{2} \frac{\sigma^2}{\sigma^2 - x^2}\right)^2\right\}$$

**Lemma 13** As $c \to 0$, $f^x(x)$ will have a single peak at $x = 0$.

**Proof.** First, assume $\sigma^2 = 1$, so that

$$f^x(x) = \frac{1}{\sqrt{2\pi} \sigma} \left(1 + \frac{3}{2} \frac{(1 + x^2)}{(1 - x^2)^2}\right) \exp \left\{-\frac{1}{2} x^2 \left(1 + \frac{3}{2} \frac{1}{1 - x^2}\right)^2\right\}$$

$$f^x(x) = \frac{1}{\sqrt{2\pi} \sigma} \left(1 + \frac{3}{2} \frac{(1 + x^2)}{(1 - x^2)^2}\right) \exp \left\{-\frac{1}{2} x^2 \left(1 + \frac{3}{2} \frac{1}{1 - x^2}\right)^2\right\}$$

34
and let \( s = \frac{\sigma^2}{x^2} \). Since \( x \) is defined on \([-1, 1]\), \( s \) will be defined on \([1, \infty]\). Substituting \( s \) into the above equation,

\[
\log f^x(x) = -\log \sqrt{2\pi} + \log \left( 1 + \frac{3}{2} s (2s - 1) \right) - \frac{1}{2} \left( 1 - \frac{1}{s} \right) \left( 1 + \frac{3}{2} s \right)^2
\]

Taking derivatives yields

\[
\frac{\partial}{\partial s} \log f^x(x) = \frac{12s - 3}{6s^2 - 3s + 2} - \frac{1}{2} \left[ \frac{1}{s^2} + \left( 3 - \frac{9}{4} \right) + \frac{9}{2} s \right]
\]

and

\[
\frac{\partial^2}{\partial s^2} \log f^x(x) = \frac{6 - \frac{9}{4} + 9(s - 2s^2)}{(6s^2 - 3s + 2)^2} + \left[ \frac{1}{s^3} - \frac{9}{2} \right]
\]

\[
\leq \frac{6 - \frac{9}{4} + 9(-1)}{(6s^2 - 3s + 2)^2} + \left[ 1 - \frac{9}{2} \right], \text{ since } s \geq 1
\]

\[
\leq \frac{6 - \frac{45}{4}}{(6s^2 - 3s + 2)^2} + \left[ 1 - \frac{9}{2} \right]
\]

< 0

Thus \( \frac{\partial}{\partial s} \log f^x(x) \) will be negative everywhere if \( \frac{\partial}{\partial s} \log f^x(x) < 0 \) at \( s = 1 \). Now

\[
\frac{\partial}{\partial s} \log f^x(x) \bigg|_{s=1} = \frac{9}{5} - \frac{25}{8} < 0
\]

which provides the desired result. Also, \( \frac{\partial}{\partial x} > 0 \iff x > 0 \), so \( \frac{\partial}{\partial x} \log f^x(x) < 0 \iff x > 0 \). Thus \( f^x(x) \) is decreasing for \( x > 0 \) and increasing for \( x < 0 \). Therefore \( f^x \) has a single peak at \( x = 0 \) when \( \sigma^2 = 1 \).

Now for \( \sigma^2 \neq 1 \), we normalize the other parameters. Let \( \tilde{c} = \frac{c}{\sigma}, \tilde{x} = \frac{x}{\sigma}, \) and \( \tilde{\sigma} = 1 \). Using the above reasoning, the density function of \( \tilde{x} \) must have a single peak at \( \tilde{x} = 0 \). Since \( x = \sigma \tilde{x}, \) it follows that the density of \( x \) must also have a single peak at \( x = 0 \). \( \blacksquare \)

The above results describe the shape of \( f^x \), the density function of the lower court’s ruling, as \( c \to 0 \). For sufficiently large values of \( \sigma \), \( f^x \) is single-peaked and symmetric around 0, and bounded at \( \pm \sigma \). It follows, and we state without proof, that the variance of \( x \) must be strictly less than \( \sigma \). This means that as \( c \to 0 \), the lower court’s rulings will have lower variance than
the higher court’s, irrespective of the distribution of both courts’ preferences. However, as $c \to 0$, the likelihood that the lower court’s ruling will stand approaches 0. For very small $c$, the second effect dominates, so that a slight increase in $c$ will reduce the variance of the final ruling. This provides intuition for the following result.

**Proof of Theorem 6:**

When $c = 0$, every case will be appealed, so the final ruling will be the appeals court’s preference point. Thus the distribution of the final ruling will be the same as the distribution of the appeals court’s preferences, in this case, a normal distribution with mean 0 and variance $\sigma^2$. Now consider $\varepsilon > 0$. Let $\nu_0 = \sigma^2$ be the variance of the final ruling when $c = 0$, and $\nu_\varepsilon$ be the variance when $c = \varepsilon$. Then the final ruling will be the same in both these cases unless $|x - q_h| \leq \varepsilon$. Thus

$$
\nu_0 - \nu_\varepsilon = \int_{-\infty}^{\infty} \int_{q_h - \varepsilon}^{q_h + \varepsilon} (q_h^2 - x^2) f^x(x) \frac{1}{\sigma} \phi \left( \frac{q_h}{\sigma} \right) dx dq_h
$$

$$
= \int_{-\infty}^{\infty} \frac{1}{\sigma} \phi \left( \frac{q_h}{\sigma} \right) \int_{q_h - \varepsilon}^{q_h + \varepsilon} (q_h^2 - x^2) f^x(x) dx dq_h
$$

Note that the second integral is positive, because

$$
\int_{q_h - \varepsilon}^{q_h + \varepsilon} (q_h^2 - x^2) f^x(x) dx = \int_{0}^{\varepsilon} \left\{ [q_h^2 - (q_h + t)^2] f^x(q_h + t) + [q_h^2 - (q_h - t)^2] f^x(q_h - t) \right\} dt
$$

$$
= \int_{0}^{\varepsilon} \left\{ [-2q_h t - t^2] f^x(q_h + t) dt + [2q_h t - t^2] f^x(q_h - t) \right\} dt
$$

$$
= \int_{0}^{\varepsilon} 2q_h t [f^x(q_h - t) - f^x(q_h + t)] dt + \int_{0}^{\varepsilon} -t^2 [f^x(q_h - t) + f^x(q_h + t)] dt
$$

so it follows that the entire double integral is positive. Therefore $\nu_0 > \nu_\varepsilon$ for small $\varepsilon$.

**Proof of Theorem 7:**
Part (a): Let $\tilde{\lambda} = \frac{\lambda}{\sigma}$ and $\tilde{c} = \frac{c}{\sigma}$. Then 
\[
\tilde{c} \left[ \Phi \left( \tilde{\lambda} + \tilde{c} \right) + \Phi \left( \tilde{\lambda} - \tilde{c} \right) \right] = \Phi \left( \tilde{\lambda} + \tilde{c} \right) - \Phi \left( \tilde{\lambda} - \tilde{c} \right).
\]
Now as $\sigma \to 0$, $\tilde{c} \to \infty$. From Theorem 5, $\frac{\tilde{\lambda}}{\tilde{\lambda} + \tilde{c}} \to 1$ as $\tilde{c} \to \infty$, hence $\frac{\lambda}{c} \to 1$. Thus 
\[
\lim_{\sigma \to 0} \lambda(c, \sigma) = c.
\]

Part (b): As above, let $\tilde{\lambda} = \frac{\lambda}{\sigma}$ and $\tilde{c} = \frac{c}{\sigma}$, so that 
\[
\tilde{c} \left[ \Phi \left( \tilde{\lambda} + \tilde{c} \right) + \Phi \left( \tilde{\lambda} - \tilde{c} \right) \right] = \Phi \left( \tilde{\lambda} + \tilde{c} \right) - \Phi \left( \tilde{\lambda} - \tilde{c} \right).\]
In this case, we have $\tilde{c} \to 0$ as $\sigma \to \infty$. Thus $\tilde{\lambda} \to 1$, so $\frac{\lambda}{c} \to 1$ as $\sigma \to \infty$. Hence 
\[
\lim_{\sigma \to \infty} \frac{dA}{d\sigma} = 1.
\]

Part (c): First, we must show that 
\[
\lim_{\sigma \to 0} \frac{\lambda - c}{\sigma} = -\infty.
\]
Since 
\[
\frac{c}{\sigma} \left[ \phi \left( \frac{\lambda+c}{\sigma} \right) + \phi \left( \frac{\lambda-c}{\sigma} \right) \right] = \Phi \left( \frac{\lambda+c}{\sigma} \right) - \Phi \left( \frac{\lambda-c}{\sigma} \right),
\]
it follows that 
\[
\phi \left( \frac{\lambda-c}{\sigma} \right) < \frac{c}{\sigma}.\]
Thus \(-\frac{1}{2} \left( \frac{\lambda-c}{\sigma} \right)^2 < \log \frac{\sqrt{2\pi}}{c} \implies \left| \frac{\lambda-c}{\sigma} \right| > \sqrt{\log \frac{c^2}{2\pi\sigma^2}}.\]
Note that $\frac{\lambda-c}{\sigma} > \sqrt{\log \frac{c^2}{2\pi\sigma^2}}$ is impossible for large enough $\sigma$, since $\phi()$ will then be convex on $(\frac{\lambda-c}{\sigma}, \frac{\lambda+c}{\sigma})$. Hence, $\frac{\lambda-c}{\sigma} < -\sqrt{\log \frac{c^2}{2\pi\sigma^2}}$. Thus, as $\sigma \to 0$, $\frac{\lambda-c}{\sigma} \to -\infty$.

Now,
\[
\lim_{\sigma \to 0} \frac{d\lambda}{d\sigma} = \lim_{\sigma \to 0} \left[ \frac{c}{\sigma} (\lambda + c)^2 + \sigma \lambda \right] \frac{1}{\phi \left( \frac{\lambda+c}{\sigma} \right)} + \left[ \frac{c}{\sigma} (\lambda - c)^2 - \sigma \lambda \right] \frac{1}{\phi \left( \frac{\lambda-c}{\sigma} \right)}
\]
\[
= \lim_{\sigma \to 0} \left[ \frac{c}{\sigma} (\lambda + c)^2 + \sigma \lambda \right] \frac{e^{-\frac{2c}{\sigma^2}}}{\phi \left( \frac{\lambda+c}{\sigma} \right)} + \left[ \frac{c}{\sigma} (\lambda - c)^2 - \sigma \lambda \right] \frac{e^{-\frac{2c}{\sigma^2}}}{\phi \left( \frac{\lambda-c}{\sigma} \right)}
\]
\[
= \lim_{\sigma \to 0} \frac{c}{\sigma} (\lambda + c)^2 \frac{1}{\phi \left( \frac{\lambda+c}{\sigma} \right)} + \frac{c}{\sigma} (\lambda - c)^2 \frac{1}{\phi \left( \frac{\lambda-c}{\sigma} \right)}
\]
\[
= \lim_{\sigma \to 0} \frac{\lambda - c}{\sigma} = -\infty.
\]

Proof of Theorem 8:

The bounds of rulings will be invariant to $d$ because they are determined by where the denominator of $g(x)$ crosses the $x$-axis. Since $d$ does not appear in the denominator of $g(x)$, it will not affect the bounds of rulings.

To show that the variance of $x$ is decreasing in $d$, note that
\[
g(x) = x + \frac{1}{2} \frac{(c^2+d)}{\sigma} \frac{\phi \left( \frac{x+c}{\sigma} \right) - \phi \left( \frac{x-c}{\sigma} \right)}{\phi \left( \frac{\lambda+c}{\sigma} \right) + \phi \left( \frac{\lambda-c}{\sigma} \right)} - \frac{[\phi \left( \frac{x+c}{\sigma} \right) - \phi \left( \frac{x-c}{\sigma} \right)]}{\phi \left( \frac{\lambda+c}{\sigma} \right) + \phi \left( \frac{\lambda-c}{\sigma} \right)}.
\]
By theorem 2, the fractional part above will be positive when $x > 0$ and negative when $x < 0$. Thus $\frac{\partial g}{\partial x}$ is strictly increasing in $d$, and therefore $\left| \frac{\partial g}{\partial x} \right|$ is strictly decreasing in $d$. Since
\( x = g^{-1}(q_i) \), this means that a larger \( d \) results in lower variance of \( x \).
Chapter 2

Collegial Decision Making in the Courts of Appeals: An Empirical Analysis

This chapter studies the interaction between ideology and collegiality in the U.S. Courts of Appeals. In a collegial court, a ruling issued by a three-judge panel is typically the product of a collaborative effort by all of the judges. Although cases are formally decided by majority vote, it is commonly understood that judges favor unanimity in their rulings, so that judges with different viewpoints will often negotiate to reach a compromise.

Thus, in a collegial court, judges’ observed votes are not necessarily reflective of their true preferences. This presents a significant challenge for any empirical study of judicial decision making. In addition, decisions in different cases are not comparable, and the merits of each case are not readily observable.\(^1\) For instance, if a judge votes in favor of a liberal outcome, there are three possible explanations for the vote: that the judge is liberal, that the judge’s vote was influenced by liberal colleagues on the panel, or that the merits of the case necessitated a liberal outcome. Only by examining many rulings, with repeated interactions over random triples of judges, can we disentangle these effects.

\(^1\) Although judicial opinions are published, and could theoretically be evaluated on some basis, these only reflect ex post justification by the majority; the legal merits and quality of briefing are still unobservable in the data.
The goal of this paper is to analyze how the ideological preferences of judges translate into observed votes. The paper develops a model of panel decision making and uses this model to simultaneously estimate judges' ideological inclinations and the parameters of panel decision making. These parameters can be interpreted as measurements of judges' willingness to compromise, thereby providing measures of an important aspect of judicial collegiality. We then apply this model to a data set of sex discrimination cases.

A second benefit of the model is that by identifying the effects of panel decision making, it can estimate individual judges' ideological preferences based on their observed votes. This is important for any empirical research that seeks to understand the motivations of judges – for instance, how they are influenced by precedent or the possibility of appellate review, or how their backgrounds affect their philosophies.

A willingness to compromise is one indication that the judges in a court maintain a norm of collegiality. Judges who view their duty to interpret the law as a common objective – rather than a means of achieving political goals – will strive to reach consensus in their opinions. While some dissent can be healthy, too much public disagreement among judges can undermine the prestige of the judiciary and the legitimacy of the courts.

A collegial environment may also have a positive effect on judicial rulings. Judges who are open to opposing viewpoints will be more able to recognize the weaknesses of their own positions and will therefore be able to produce opinions that can better withstand scrutiny. An opinion that can garner the approval of all three judges on a panel is less likely to be controversial than an opinion that produced by a 2-judge majority. The deliberative process may lead to greater moderation: the minority judge can negotiate for the removal or revision of controversial passages as a condition for joining the opinion. On the other hand, it is also conceivable that compromise would result in narrow rulings that dispose of the case at hand without establishing a workable precedent. Over time, such opinions could diminish the coherence of case law.

There has been recent interest in panel decision making in the academic literature, partly as a result of debate between empirical scholars who have emphasized the ideological component of judging, e.g., Sunstein, Schkade, and Ellman (2004), Revesz (1997), Tiller and Cross (1998, 1999a, 1999b), and judges themselves, e.g., Edwards (2003), Wald (1999), who downplay the importance of ideology and emphasize collegiality in decision making. Previous empirical
studies, however, have relied on variables such as race, gender, and party affiliation, as proxies for judicial ideology. For example, Sunstein, Schkade, and Ellman (2004) use the party of the appointing president to examine ideological voting and “panel effects” across various areas of law, and Tiller and Cross (1998) and Revesz (1997) do the same for challenges to EPA regulations in the D.C. Circuit. Farhang and Wawro (2004) use race, gender, and Poole common space scores (McCarty and Poole 1995) of appointing presidents and home-state senators to study panel decision making in employment discrimination cases, while Peresie (2005) adds additional variables for judges’ prior experience to study the effect of female judges on panel rulings in sex discrimination cases.

These variables are valid predictors under many circumstances, and have been used to identify the presence of “panel effects,” when judges’ votes are affected by the other judges on a panel. (Sunstein, et. al. 2004) However, the imprecision and limited variation of these proxy variables make them poor candidates for developing a richer model of panel decision making or determining the impact of intrapanel interactions on case outcomes. For example, several papers have observed that unified panels are more likely to issue extreme rulings than mixed panels, and have attributed this effect to the moderating effect of the minority judge on a mixed panel. (Sunstein, et. al. 2004, Tiller and Cross 1998) However, these results are also consistent with unobserved heterogeneity among judges appointed by presidents of the same party. In the presence of unobserved heterogeneity, the median judge on a panel of three Republicans should be more conservative, in expectation, than the median judge on a panel with two Republicans and one Democrat. Without a richer model, it is impossible to distinguish the moderating effects of a minority judge from the effects of within-party heterogeneity.

Instead of relying on proxy variables, this paper uses a structural model to estimate judges’ ideological inclinations, or “attitudes” (Segal and Spaeth 2002), in a one-dimensional spectrum based on their actual votes. The model uses a judicial negotiation game with flexible parameters to predict observed votes based on judicial inclinations, and then estimates both the parameters of the game and the inclinations of individual judges using maximum likelihood estimation. The estimation procedure controls for the incomparability of cases by treating the merits of each case as a random effect and exploiting the exogenous variation resulting from the random assignment of judges to panels. By eliminating the problem of proxy error, the model can
provide more precise estimates of judicial ideology and better explore the dynamics of panel decision making.

The spatial estimates derived from this model are significant because they can enable empirical research on judicial decision making in situations where subtle influences on judicial decision making might otherwise be obscured by "panel effects" and the imprecision of proxy variables. This is especially important in situations where proxy variables are poor predictors\(^2\) and where there is insufficient variation in these variables\(^3\). Moreover, by providing numerical estimates of the parameters of panel decision making, this model can analyze the impact of collegial decision making on case outcomes and has the potential to compare collegiality across judicial circuits, time periods, and areas of law.

Another advantage of the structural approach is that it can generate out-of-sample predictions. For example, it could predict the effect on case outcomes in a particular circuit if a liberal judge were replaced by a conservative, or after several more years of Bush appointees. It could also potentially evaluate the impact of court reorganization plans, such as proposals to split the Ninth Circuit.

The primary assumptions underlying the model are that judges have preferences over ideology, and that they prefer unanimous rulings. These assumptions are relatively uncontroversial. While there has been much debate about the importance of ideology in judicial decision making, few would deny that it plays some role. Several empirical studies have found judicial ideology to be a significant factor in case outcomes. (Sunstein, Schkade, and Ellman 2004, Revesz 1997, Tiller and Cross 1998) Even several federal judges, who have argued against the importance of ideology in judicial decision making, concede that it is still a factor in some cases. (Edwards 2003, Wald 1999)

It is also widely accepted that judges strive to reach unanimity in their decisions. A dissenting opinion weakens the legitimacy of the panel's ruling, and frequent dissents can diminish the authority of the court. Dissenting opinions require significant effort on the part of the minority

\(^2\) For example, Sisk & Heise (2005) find that party affiliation has no predictive value for judges' votes in religious freedom cases, although they find that other demographic variables are significant. In state courts, foreign courts, and earlier periods in history, judicial appointments may be less partisan, and these variables may be poor indicators.

\(^3\) This is especially true in some state courts, where one party has dominated appointments. For example, in most southern states for the century after Reconstruction, there would be little or no variation among judges in race, gender, or party of appointment.
judge, and are also potentially costly to the majority, by increasing the likelihood that the ruling will be overturned by a higher court or a non-judicial entity. (Daughety and Reinganum 2002) The “cost” of dissent also reflects the “danger of crying wolf too often”: issuing too many dissents may diminish their signaling value. (Ginsburg 1990) These intuitions have also been confirmed by judges themselves, who have described consensus as a goal of panel deliberation. (Edwards 2003, Coffin 1994)

The paper is organized as follows. Section I constructs a model of negotiation within appellate panels, where judges have ideological preferences and prefer unanimity. This model allows judges to vote against their ideological inclination when they derive a greater benefit from consensus. Section II discusses the technique for estimating the parameters of the model. Section III describes the data set. Section IV discusses the estimation results for the judges and for the parameters governing the dynamics of panel decision making. Section V uses the model and the estimated parameters to make out-of-sample predictions. The first part of Section V estimates how often a unanimous decision results from actual agreement among the judges, as opposed to judicial compromise, and how often the majority position dominates when compromise occurs. The second part compares the distribution of outcomes when cases are decided by single judges and three-judge panels, to provide a sense of how panel decision making moderates rulings.

2.1 Model

We model the panel deliberation process as a two-stage game, where judges have ideological preferences on a one-dimensional spectrum and favor unanimous rulings. The merits of each case can be represented as a point \( \eta \) on the real line, and the position of each judge participating in the case will be represented by a point \( a_i \in \mathbb{R} \). Each judge may choose a ruling \( v_i \in \{ P, D \} \), representing a vote in favor of the plaintiff or defendant, respectively.\(^4\)

We represent the ideological component of the judge’s utility as \((-1)^{I_P} (a_i - \eta)\), where \( I_P \) is an indicator function such that \( I_P = 1 \) when \( v_i = P \). Thus, when \( a_i > \eta \), the judge will prefer to choose \( v_i = P \), and when \( a_i < 0 \), the judge will choose \( v_i = D \). We can think of

\(^4\)In cases at the appellate level, the term “plaintiff” will always denote the original plaintiff in the case.
a judges with greater \( a_i \) as being more liberal, in the sense of being more sympathetic to sex discrimination plaintiffs, and judges with lower \( a_i \) as being more conservative. Cases with greater \( \eta \) are stronger cases for the defendant while cases with lower \( \eta \) are stronger cases for the plaintiff.

We model the preference for consensus by imposing a cost \( c_m \) on each of the majority judges and a cost \( c_d \) on the dissenting judge when there is a dissent. Each judge's total utility is therefore

\[
U_i = (-1)^{I_p}(a_i - \eta) - I_m c_m - I_d c_d
\]

where \( I_m \) and \( I_d \) are indicator variables for judge \( i \) being in the majority and the dissent, respectively. When cases are unanimous, \( I_m = I_d = 0 \).

The judicial deliberation process is modeled as a full-information two-stage game. In the first stage, the judges take an intial vote. If there is disagreement, the minority judge may choose to change his vote in the second stage.\(^5\)

When \( \eta < \min\{a_i\} \) or \( \max\{a_i\} < \eta \), the decision will be unanimous: all of the judges will have the same preference, and none of them will have an incentive to switch sides. The following results focus on the case where there is ex ante disagreement. Let \( a_d \) denote the judge in the minority position, \( a_{m1} \) denote the median judge, and \( a_{m2} \) denote the other judge in the majority, so that \( a_d < \eta < a_{m1} < a_{m2} \) or \( a_d > \eta > a_{m1} > a_{m2} \).

The following propositions will describe the unique subgame perfect equilibrium.

Proposition 14 When there is disagreement in the first stage, the minority judge will switch sides in the second stage if and only if \( |a_d - \eta| < \frac{\eta}{2} \). When this condition is satisfied, the majority judges will vote their true preferences in stage 1.

Proof. The minority judge's utility from switching will be \(-|a_d - \eta|\), and the utility from

\(^5\)We model deliberation as a two-stage game and use subgame perfection as the equilibrium concept in order to avoid the multiplicity of equilibria that could obtain in a one-stage game. When the cost of dissent is sufficiently high for both the majority and the dissent, then either unanimous outcome would be a Nash equilibrium. If all three judges voted in favor of the minority's position, then there would be no profitable deviation for the majority judges, since the cost of dissent would outweigh the ideological gain. This would also be true in a cooperative framework.

The choice of a two-stage game and the assumption that \( c_m \leq c_d \) eliminates the multiplicity of equilibria, thus allowing straightforward estimation of the parameters. When \( c_m \) is close to 0, most of the indeterminate cases in a one-stage game result in a unanimous ruling in favor of the majority's position. As \( c_m \) increases, a greater proportion of these cases result in unanimous rulings in favor of the minority's position.}
not switching will be $|a_d - \eta| - c_d$. Hence the minority judge will switch sides if and only if $|a_d - \eta| < \frac{\eta}{2}$. Since there will be unanimity in the final stage, the majority judges get utility $|a_{m,i} - \eta|$ in the final stage if they vote their true preferences, so there is no profitable deviation for them. 

**Proposition 15** The judges in the majority will vote against their true preferences if and only if $|a_d - \eta| > \frac{\eta}{2}$ and $|a_{m1} - \eta| < \frac{\eta}{2}$ and $|a_{m2} - \eta| < \frac{\eta}{2}$.

**Proof.** First, suppose that $|a_d - \eta| > \frac{\eta}{2}$ and $|a_{m1} - \eta| < \frac{\eta}{2}$ and $|a_{m2} - \eta| < \frac{\eta}{2}$. Then by proposition 14, the minority judge will always vote his true preferences. If both majority judges vote sincerely, the median judge will have utility $|a_{m1} - \eta| - c_m$, but if the median judge votes against her ideological preference, then the other majority judge will switch sides in the final stage, and the median judge will have utility $-|a_{m1} - \eta|$. Thus, there will be a unanimous opinion in favor of the minority judge’s preference when these conditions are satisfied.

If any of these conditions fail, then the majority judges will vote their ideological preferences. If $|a_d - \eta| < \frac{\eta}{2}$, then proposition 14 shows that the minority judge will not dissent, and the majority will not switch sides. When $|a_{m1} - \eta| > \frac{\eta}{2}$, the median judge would prefer a non-unanimous opinion to switching sides. When $|a_{m2} - \eta| > \frac{\eta}{2}$, the more extreme majority judge would dissent if the median judge switched sides; there will be a dissenting opinion either way. Thus, the median judge will vote her true preferences. 

**Proposition 16** The judges in the majority will vote true preferences and the minority judge will dissent if and only if $|a_d - \eta| > \frac{\eta}{2}$ and either $|a_{m1} - \eta| > \frac{\eta}{2}$ or $|a_{m2} - \eta| > \frac{\eta}{2}$.

**Proof.** If $|a_d - \eta| > \frac{\eta}{2}$ and $|a_{m2} - \eta| > \frac{\eta}{2}$, then these two judges have an irreconcilable disagreement, and one of them will dissent in either case. Since the median judge will incur the cost $c_m$ in either case, she will maximize her utility by voting in favor of her ideological preference.

If $|a_d - \eta| > \frac{\eta}{2}$ and $|a_{m1} - \eta| > \frac{\eta}{2}$, then the median judge gets utility $|a_{m1} - \eta| - c_m$ from voting in favor of her ideological preference, and a maximum utility of $-|a_{m1} - \eta|$ if she joins the minority judge. Thus voting her true preference will always be the dominant strategy. 

45
These propositions describe the outcome of judicial deliberations. Proposition 14 provides conditions for a “collegial concurrence” (Sunstein et. al. 2004): when the judge in the minority position does not feel too strongly, he will join the majority opinion. Proposition 15 demonstrates that it is possible, under certain conditions, to have “minority rule.” In these circumstances, a minority judge with strong ideological preferences will be able to induce a relatively indifferent majority to switch sides. Proposition 16 provides conditions for when the judges will not reach unanimity.

The following table summarizes the results of these propositions for judges with $a_i < a_j < a_k$:

<table>
<thead>
<tr>
<th>Conditions for $a_i, a_j, a_k, \eta$</th>
<th>Outcome: $(v_i, v_j, v_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i &lt; a_j &lt; a_k &lt; \eta$</td>
<td>$(P, P, P)$</td>
</tr>
<tr>
<td>$\eta &lt; a_i &lt; a_j &lt; a_k$</td>
<td>$(D, D, D)$</td>
</tr>
<tr>
<td>$a_i &lt; a_j &lt; \eta &lt; a_k &lt; \eta + c_d$</td>
<td>$(P, P, P)$</td>
</tr>
<tr>
<td>$\eta - c_d &lt; a_i &lt; \eta &lt; a_j &lt; a_k$</td>
<td>$(D, D, D)$</td>
</tr>
<tr>
<td>$a_i &lt; \eta - c_d$ and $a_j &lt; \eta - c_m$ and $a_k &gt; \eta + c_d$</td>
<td>$(P, P, D)$</td>
</tr>
<tr>
<td>$a_i &lt; \eta - c_d$ and $a_j &gt; \eta + c_j$ and $a_k &gt; \eta + c_d$</td>
<td>$(P, D, D)$</td>
</tr>
<tr>
<td>$\eta - c_d &lt; a_i$ and $\eta - c_m &lt; a_j &lt; \eta$ and $a_k &gt; \eta + c_d$</td>
<td>$(D, D, D)$</td>
</tr>
<tr>
<td>$a_i &lt; \eta - c_d$ and $-\eta &lt; a_j &lt; \eta + c_m$ and $a_k &lt; \eta + c_m$</td>
<td>$(P, P, P)$</td>
</tr>
</tbody>
</table>

The following corollaries provide the conditions for independent voting and majority rule in panels.

**Corollary 17** When $c_d = c_m = 0$, judges will vote purely on an ideological basis, and will not be influenced by the other judges on the panel.

**Proof.** Propositions 14 and 15 provide conditions for when judges will change votes in order to reach consensus. When $c_d = c_m = 0$, these conditions cannot occur. Since judges will never switch sides, their votes will be determined solely by their ideological preferences.

**Corollary 18** When $c_m = 0$, all case outcomes will be determined by majority rule.
Proof. By proposition 15, the majority will never vote against their true preferences, since
$$|a_{m1} - \eta| \geq \frac{\epsilon_2}{2} = 0.$$ Hence the majority's ideological preferences will determine the case outcome. □

Corollary 17 shows that when $c_d = c_m = 0$, there will be no panel effects in the data: each judge's vote will be independent of the characteristics of the other judges on the panel. All cases will be decided by majority vote, and the proportion of unanimous rulings observed in the data will reflect the actual degree of agreement among the judges.

When $c_m = 0$ and $c_d > 0$, corollary 18 shows that the minority judge may still switch sides, and we will therefore observe panel effects in the data, but case outcomes will be the same as under independent voting. In this case, the proportion of unanimous rulings will overstate the level of actual agreement among the judges.

2.2 Estimation

In this section, we describe how to estimate the parameters of the above model discussed in section I. Consider a data set with $n$ judges and $T$ cases. Each case will be decided by a panel of three judges, selected at random. We represent each judge by a parameter $\alpha_i$ representing judge $i$'s position on an ideological spectrum, where a greater $\alpha_i$ corresponds to a more liberal judge.

Let $S_t = \{\alpha_1, \alpha_2, \alpha_3\}$ be the set of judges who participate in case $t$. We assume that each judge's preference in case $t$ is of the form $a_{it} = \alpha_i + \epsilon_{it}$, where $\alpha_i$ is a judge fixed effect, and $\epsilon_{it}$ are iid $N(0,1)$. This error term accounts for the fact that $\alpha_i$ is not a perfect predictor of each judge's approach to each case. Since the merits of the case $\eta_t$ are not observable, we treat it as a random effect, with $\eta_t \sim N(0, \sigma^2)$, where $\sigma$ is a parameter to be estimated. This accounts for the fact that the judges' attitudes toward a particular case may be highly correlated; in many cases, liberals and conservatives will agree on the merits. The parameter $\sigma$ represents the magnitude of the correlated component of the judges' preferences: a larger $\sigma$ (keeping all other parameters constant) means that there will be more "easy cases" and consequently more ex ante agreement.
Let $y_{it}$ be judge $i$'s preference in case $t$. Then

$$\Pr(y_{it} = P) = \Pr(\alpha_i + \epsilon_{it} > \eta_t) = \Phi(\alpha - \eta_t)$$

We will estimate the parameters using maximum likelihood estimation. To derive the likelihood for a unanimous ruling, recall that such a ruling can occur three ways: from ex ante agreement, the minority switching sides, or the majority switching sides. The probability of a unanimous "P" vote following from ex ante unanimity is

$$\Phi(\alpha_1 - \eta_t)\Phi(\alpha_2 - \eta_t)\Phi(\alpha_3 - \eta_t)$$

By proposition 14, the probability of arriving at a unanimous "P" ruling after the minority judge switches sides is

$$\sum_{j,k \in S_t \setminus \{i\}} \sum_{i=1}^3 \left[ \Phi \left( \alpha_i - \eta_t + \frac{C_d}{2} \right) - \Phi(\alpha_i - \eta_t) \right] \Phi(\alpha_j - \eta_t)\Phi(\alpha_k - \eta_t)$$

We can derive the probability of the majority switching sides to reach a unanimous "P" vote using proposition 15:

$$\sum_{\substack{i=1 \\ j,k \neq k \neq i}}^3 \Phi \left( \alpha_i - \eta_t - \frac{C_d}{2} \right) \Psi(\alpha_j, \alpha_k)$$

where

$$\Psi(\alpha_j, \alpha_k) = \left[ \Phi \left( \alpha_j - \eta_t + \frac{C_d}{2} \right) - \Phi(\alpha_j - \eta_t) \right] \left[ \Phi \left( \alpha_k - \eta_t + \frac{C_m}{2} \right) - \Phi(\alpha_k - \eta_t) \right]$$

$$+ \left[ \Phi \left( \alpha_k - \eta_t + \frac{C_m}{2} \right) - \Phi(\alpha_k - \eta_t) \right] \left[ \Phi \left( \alpha_j - \eta_t + \frac{C_d}{2} \right) - \Phi(\alpha_j - \eta_t) \right]$$

$$- \left[ \Phi \left( \alpha_k - \eta_t + \frac{C_m}{2} \right) - \Phi(\alpha_k - \eta_t) \right] \left[ \Phi \left( \alpha_k - \eta_t + \frac{C_m}{2} \right) - \Phi(\alpha_k - \eta_t) \right]$$
Hence

\[ L((P, P, P) \mid \eta_t) = \]
\[ \sum_{i=1}^{3} \Phi \left( \alpha_i - \eta_t + \frac{c_d}{2} \right) \Phi(\alpha_j - \eta_t) \Phi(\alpha_k - \eta_t) \]
\[ + \sum_{i,j,k} \Phi \left( \alpha_i - \eta_t - \frac{c_d}{2} \right) \Psi(\alpha_j, \alpha_k) \]
\[ -2\Phi(\alpha_1 - \eta_t)\Phi(\alpha_2 - \eta_t)\Phi(\alpha_3 - \eta_t) \]

And similarly,

\[ L((D, D, D) \mid \eta_t) = \]
\[ \sum_{i=1}^{3} \Phi \left( -\alpha_i + \eta_t + \frac{c_d}{2} \right) \Phi(-\alpha_j + \eta_t)\Phi(-\alpha_k + \eta_t) \]
\[ + \sum_{i=1}^{3} \Phi \left( -\alpha_i + \eta_t - \frac{c_d}{2} \right) \Psi(-\alpha_j, -\alpha_k) \]
\[ -2\Phi(-\alpha_1 + \eta_t)\Phi(-\alpha_2 + \eta_t)\Phi(-\alpha_3 + \eta_t) \]

To derive the likelihood function in the case where one judge dissents, we use the conditions given in proposition 16:

\[ L((P, P, P) \mid \eta_t) = \Phi \left( -\alpha_3 + \eta_t - \frac{c_d}{2} \right) \left[ \Phi(\alpha_1 - \eta_t)\Phi(\alpha_2 - \eta_t) - \Psi(\alpha_1, \alpha_2) \right] \]

Since \( \eta_t \) is unobserved, we must integrate over \( \eta_t \) to get the unconditional likelihood:

\[ L((P, P, P)) = \int L((P, P, P) \mid \eta) \phi \left( \frac{\eta}{\sigma} \right) d\eta \]

which can be estimated using Gauss-Hermite quadrature. The unconditional likelihoods for the other cases can be derived in a similar fashion.

The parameters to be estimated are the judge fixed effects \( \alpha_i, c_d, c_m, \) and \( \sigma \). The maximum of the log-likelihood function is estimated using Newton’s method. Standard errors are derived
from the inverse Hessian matrix. All optimization routines were coded in Matlab.

2.3 Data

The data consists of 1080 sex discrimination and sexual harassment cases decided by three-judge panels in the U.S. Courts of Appeals between 1995 and 2002. The data set was taken from a larger data set collected and made available by the Chicago Judges Project of the University of Chicago Law School.\(^6\)

Because the model assumes that judicial preferences can be mapped onto a one-dimensional spectrum, the data is limited to a single area of law. Sex discrimination cases were chosen for several reasons. First, the laws governing sex discrimination cases have substantial ambiguities, thus allowing judges substantial discretion in interpreting the law and providing for a role for judicial ideology. Second, judges’ attitudes toward sex discrimination law are plausibly one-dimensional; although there are numerous subtleties in different areas of the law, it is believable that judges’ interpretations would be influenced by their attitudes toward workplace discrimination, and whether they think the laws should be applied broadly or narrowly. Finally, sex discrimination law is “moderately ideological,” in the sense that there are likely to be ideological differences among judges, but not so large as to overwhelm collegial effects.

For each case, the data set provides the identities and votes of the three judges. A case is coded as “P” if the court provides any relief to the plaintiff; otherwise, it is coded as a “D.”\(^7\) Plaintiffs won 41% of the cases, and 42% of the judges’ individual votes. 92% of the cases were decided unanimously, and about 3% of judges’ votes are dissenting opinions.

There are 438 judges who appear at least once in the data, and the most appearances for a single judge is 52. Because of the time period involved, the data are dominated by appointees of Presidents Carter, Reagan, Bush I, and Clinton. The differences between the Democratic and Republican appointees is immediately evident: Carter and Clinton appointees ruled in favor of

---

\(^6\)The cases were gathered using a Lexis search of the terms “sex! discrimination” and “sex! harassment.” (Sunstein et. al. 2004) One case was removed from the data because it was decided by only two judges after the death of the third judge.

\(^7\)Note that this method of coding overstates the degree of unanimity in the data. Some cases may be coded as (1,1,1) even though a judge may have dissented on some portion of the ruling.
plaintiffs 53% of the time, versus 35% of the time for Reagan and Bush I appointees. Figure 1 shows the number of votes in each direction, where judges are grouped by appointing president.

An examination of the dissenting votes provides an even starker contrast between Democratic and Republican appointees. Carter and Clinton appointees combined for 36 dissenting opinions, of which 34 favored plaintiffs. On the other hand, Reagan and Bush I appointees combined for 34 dissents, of which only 7 favored plaintiffs. Figure 2 shows the number of dissents in each direction, by president.

Of the 438 judges who appear in the data, 210 judges participated in at least 5 cases; the rest are grouped together by appointing president. There are 10 such judge groups, corresponding to presidents Eisenhower through George W. Bush. Table 4 in the Appendix provides summary statistics for the individual judges and groups.

Note that most judges voted in favor of plaintiffs between 30% and 60% of the time, but
Dissenting Votes, by Appointing President

Figure 2-2:
there are also outliers. On the conservative side, Judge Demoss supported the plaintiff in none of the 13 cases in which he participated, with 3 dissents, while on the liberal side, Judge Pregerson supported the plaintiff all nine cases in which he participated, with one pro-plaintiff dissent.

2.4 Results

2.4.1 Judge Preferences

The results for each judge and judge-group are shown in Table 4 in the Appendix. For each judge, we report the fixed effect $\alpha_i$ and $p_i$, the unconditional probability that a judge would vote in favor of the plaintiff in a case drawn at random, where

$$p_i = \int \Phi(\alpha_i + \eta) \phi\left(\frac{\eta}{\sigma}\right) d\eta$$

We report standard errors and 90% confidence intervals for the $p_i$'s. Table 1 summarizes the distribution of the fixed effects and unconditional probabilities for the individual judges.\footnote{This table only includes those judges who appear at least 10 times in the data. The judge-group fixed effects are excluded.}

53
Table 1:

<table>
<thead>
<tr>
<th></th>
<th>α_i</th>
<th>P_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05 quantile</td>
<td>-16.01</td>
<td>9%</td>
</tr>
<tr>
<td>0.25 quantile</td>
<td>-2.54</td>
<td>33%</td>
</tr>
<tr>
<td>median</td>
<td>-0.49</td>
<td>44%</td>
</tr>
<tr>
<td>0.75 quantile</td>
<td>1.94</td>
<td>66%</td>
</tr>
<tr>
<td>0.95 quantile</td>
<td>15.33</td>
<td>90%</td>
</tr>
<tr>
<td>standard deviation</td>
<td>2.00</td>
<td>15%</td>
</tr>
<tr>
<td>median Republican</td>
<td>-1.88</td>
<td>34%</td>
</tr>
<tr>
<td>median Democrat</td>
<td>1.73</td>
<td>66%</td>
</tr>
</tbody>
</table>

This table provides distributional information on the estimated judge fixed effects α_i and unconditional probabilities p_i of issuing a pro-plaintiff ruling. These values only include the 115 judges who appear at least 10 times in the data.

The histogram in Figure 3 shows the dispersion of p_i’s for the individual judges estimated in the model. Note that there are two sharp peaks in the histogram, while there are relatively few judges in the center.⁹

Figure 4 provides a histogram comparing Democratic and Republican appointees. This histogram explains the bimodality in the ideological distribution of judges. The peak probability for Republican appointees is in the range of 30-35%, which the peak for Democratic appointees is 60-70%. Note that while there are significant differences between the two groups, there is also substantial within-group heterogeneity. As Table 1 indicates, the median probability for Republican appointees is 34%, versus 66% for Democratic appointees. The standard deviation of probabilities among judges within each party is about 19%.

These results have some implications for empirical research that relies on political party as a proxy for ideology. First, differences between Democratic and Republican appointees are

⁹ Also note that the extreme outliers participated in relatively few cases in the data, and that these estimates may be less precise.
Probability of Pro-Plaintiff Vote for Individual Judges

Figure 2-3:
Probability of Pro-Plaintiff Vote for Individual Judges
(by Party of Appointment)

Figure 2-4:
significant, and party is therefore a valid predictor in sex discrimination cases. Second, there is substantial within-party heterogeneity. A likelihood ratio test on the restriction that judges of the same party vote alike is clearly rejected.10

Thus the likelihood of success for a plaintiff should vary by the number of judges of each party, even when judges vote independently. This suggests that use of party variables alone, as proxies for ideology, would not be sufficient to identify the determinants of panel decision making.

Figure 5 compares estimates for judges by gender. Because there are a small number of female judges who are estimated individually, the differences are subtle. The mean probability for female judges is 56%, compared with 46% for male judges.

Note that estimated probabilities for judges are strongly correlated with the judges’ voting

10 The test statistic has a p-value of $10^{-13}$. 
records, there are some substantial deviations. Since dissenting opinions provide strong evidence of ideological differentiation from other judges on the panel, dissents will have an impact on a judge’s estimate. For example, Judges Rovner, Dennis, and Lipez are estimated to be much more liberal than their voting records would suggest, because they each wrote multiple pro-plaintiff dissents. Dissenting opinions by other judges on the panel also impact a judge’s estimate.

2.4.2 Costs of Disagreement

Table II provides estimates of $c_d$, $c_m$, and $\sigma$:

<table>
<thead>
<tr>
<th></th>
<th>estimate</th>
<th>standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_d$</td>
<td>6.28</td>
<td>0.94</td>
</tr>
<tr>
<td>$c_m$</td>
<td>2.16</td>
<td>0.55</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>16.02</td>
<td>15.54</td>
</tr>
</tbody>
</table>

This table provides estimates of the “costs” of dissent, where $c_d$ cost incurred by the dissenter and $c_m$ is the cost incurred by the majority. The variable $\sigma$ is the variance of the random effect associated with each case. This represents magnitude of the importance of the legal merits of the case, compared to the ideological component.

The estimates of $c_d$ and $c_m$ represent the “cost” of disagreement. The higher these costs, the more judges will be willing to compromise in order to achieve unanimity. Note that both $c_d$ and $c_m$ are positive and significant. Proposition 14 shows that outvoted judges are willing to “travel” an distance of $\frac{\sigma_d}{2} = 3.14$ in order to achieve a unanimous opinion. On the ideological spectrum, this distance is a bit less than the distance between the median Republican and the median Democrat (3.61). It is slightly greater than the distance from the median judge to either the 0.25 quantile or the 0.75 quantile.

The parameter $c_m$ is smaller but also statistically significant. A likelihood ratio test rejects
the restriction $c_m = 0$. Recall that $c_m = 0$ corresponds to the case where the majority never compromises, so that we can reject the possibility that outcomes are decided purely on the basis of majority preferences.

We can also test for the presence of “panel effects.” The restriction $c_d = c_m = 0$ corresponds to the case where judges have no preference for unanimity, and therefore vote purely on an ideological basis. A likelihood ratio test on this restriction clearly rejects, so that we can reject the hypothesis that judges vote independently. This confirms the conclusion made by Sunstein, Schkade, and Ellman (2004) on the same data set using party data.

2.5 Simulations

One benefit of a structural model is that it lends itself to simulations and out-of-sample predictions. In this section, we will explore a few applications. First, we will estimate the frequency of compromise. We will use Monte Carlo simulation to estimate the proportion of unanimous decisions that were the result of a compromise by one or more of the judges. Second, we will estimate how appellate rulings would be different if they were decided by a single judge rather than a three-judge panel. By simulating these results for the actual panels in the data, and also for individual judges, we can get a sense of how panel decision making increases predictability in rulings.

2.5.1 Frequency of Compromise

To understand the magnitude of “panel effects” we estimate the probability that one or two judges would have switched votes in order to achieve unanimity. We estimate these probabilities using Monte Carlo simulation, generating random values for $\eta_t$ and $\varepsilon_{it}, \varepsilon_{jt}, \varepsilon_{kt}$ for the actual cases that occur in the data and including only those values that result in the observed vote. Using these same parameters, we compare the judges’ actual preferences to their observed votes. The results are presented in the following table:

---

11The test statistic has a $p$-value on the order of $10^{-6}$.
12As we discuss in Section V, however, this effect is small in magnitude.
13The test statistic has a $p$-value on the order of $10^{-61}$. 

59
Table 3:

<table>
<thead>
<tr>
<th></th>
<th>Percent of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>ex ante unanimity</td>
<td>81.6%</td>
</tr>
<tr>
<td>one judge switched</td>
<td>9.3%</td>
</tr>
<tr>
<td>two judges switched</td>
<td>1.4%</td>
</tr>
<tr>
<td>dissent</td>
<td>7.7%</td>
</tr>
</tbody>
</table>

This table provides simulated estimates of the process by which panels achieved unanimous rulings. These estimates are obtained by simulating judges’ preferences in the data using the estimated parameters, conditional on these simulated preferences resulting in the observed outcomes.

Several observations can be made from these results. First, there was ex ante unanimity in a large majority of the cases. In 82% of cases, the judges’ individual preferences all would have led to the same outcome. This suggests that judicial ideology only plays a role in a relatively small proportion of the cases.

Second, the judges are able to reach unanimity in about 60% of the cases in which there was ex ante disagreement. Thus there would be more than twice as many dissenting opinions if judges’ votes were truly independent. Finally, the percentage of cases in which two judges switch sides is tiny and insignificant. In most of these cases, the majority’s position prevailed. Although we have estimated some amount of minority influence, this result suggests that majority rule determines the outcome most of the time, at least in the context of sex discrimination cases.

Third, the minority’s position prevailed only in about 1.4% of cases. Note that this frequency, though small, is statistically significant. However, since we only observe the outcomes in the data, this may understate the influence of the minority judge. The minority judge may have had a stronger influence on the content of the opinion, even if the outcome was unchanged.

These results also suggest that the outcome would only change in a small proportion of cases if all appeals were decided by a single judge. Suppose that one judge were chosen from the panel at random to decide the case. The probability that this judge would have ruled the
same way as the panel is $81.6\% + \frac{2}{3}(9.3\%) + \frac{1}{3}(1.4\%) + \frac{2}{3}(7.7\%) = 93\%$. Thus, only in 7% of cases would the outcome be different if decided by a single judge on the panel.

### 2.5.2 Individual Judge vs. Three-Judge Panels

In this section, we estimate the probability of success for the plaintiff for various levels of case difficulty, based on the panel compositions that were actually observed in the data. This value is the probability of a ruling favoring a plaintiff, using the parameters estimated in Section IV, conditional on a given $\eta$. We can also estimate the probability of each outcome if cases were decided by a single judge. For simulated single-judge rulings, we weight the likelihood of a judge being assigned by the number of times that judge occurred in the data. Figure 6 shows the probability of the plaintiff winning the case, conditional on $\eta$. 

![Case Outcome, Conditional on Legal Merits](image)
The horizontal axis of Figure 6 measures the quantile of \( \eta \), the random effect, which represents the legal merits of the case. The point 0 on the x-axis represents the weakest case, and the point 1 represents the strongest case. Note that the slope is increasing very sharply in the center of the distribution. Thus there is only a small proportion of cases in the middle of the distribution for which the outcome is truly unpredictable. On the other hand, there is only a small proportion (about 10% of cases at either extreme) for which the outcome is completely certain.\(^{14}\)

The dashed line in Figure 6 shows the probability of outcomes if the cases were decided by a single judge chosen at random. Note that the outcomes are a bit less predictable under this assumption. These results confirm the intuition that a panel of three judges will rule more moderately and more predictably than a single judge.

### 2.6 Conclusion

Using a model to predict how judges' ideological inclinations will translate into observed votes, this paper estimated judges' inclinations as well as the parameters governing panel decision making in sex discrimination cases. The benefits of this method are that it can estimate how both judicial ideology and panel collegiality affect votes and case outcomes. At least in the context of sex discrimination cases, ideology and collegiality are clearly significant factors.

Although the results in this paper confirm the presence of panel effects, they cannot support the claim that these effects have an impact on case outcomes. Rather, the panel effects seem to result primarily from "collegial concurrences," when an outvoted judge joins the majority opinion.

This conclusion is relevant to a proposal by Tiller and Cross (1999) to eliminate random assignment in favor of a system that ensures diversity within each panel. If a judge in the ideological minority exercises influence over the case outcome, such a proposal could result in more moderate and uniform judicial rulings. On the other hand, if a "collegial concurrence" has no impact on the case outcome, such a proposal would have little benefit.

One important caveat: the data only reveal case outcomes, and not the content of the

\(^{14}\)Note that these results may be sensitive to the distributional assumption of the random effect.
opinions. The model in this paper therefore cannot capture how panel deliberations affect opinion. Even if minority judges do not affect outcomes, in a directional sense, they may have an impact on the degree of relief provided or the content of the judicial opinion. Since all of the judges must sign on to a unanimous opinion, it is highly likely that judges do exert influence over the content when they choose to join an opinion.

The methodology used in this paper lends itself easily to other studies of judicial decision making. This model could be used to compare the effects of collegiality in areas of law that are more highly contested, as well as areas of law that are viewed as relatively nonideological. This method could also be used to compare panel decision making by judicial circuit, to test whether judges are less collegial in larger and more geographically dispersed circuits.
Chapter 3

Asymmetric Liability and Incentives for Innovation

Firms’ decisions to invest in product safety are influenced by both market and legal incentives. A firm will save on liability costs if a product causes fewer accidents, and consumers will also pay a premium for a safer product, but the size of this premium will depend on how well the consumers are compensated for injuries under tort law. This paper explores the interaction between product markets and product liability, and develops some insights on how the liability system affects safety innovation.

Case studies and academic commentary have identified opposing effects of product liability on innovation. Clearly, product liability may encourage firms to develop safer products in order to reduce liability costs. On the other hand, liability may also lead firms to decide against developing or releasing new products. (Viscusi 1991) National surveys have cited both effects on firms’ innovation decisions (e.g., Conference Board 1987, 1988). Viscusi and Moore (1993) find empirical evidence that liability costs have a nonmonotonic effect on R & D intensity: low levels of liability costs spur innovation, but high levels can have a stifling effect.

This paper develops a model that considers the distortions caused when liability does not apply equally to all substitutes of a given product. If the alternative to purchasing a product is purchasing no product, substituting the consumer’s own labor supply, or buying from a judgment-proof seller, then the product will be subject to stricter liability than its alternatives.
This may also be the case when a preexisting product has well-known risks that are not perceived as design defects, but a new product with unfamiliar risks is subject to a higher standard. In this case, the product will be competing against alternatives that have lower liability costs (but also offer less protection) to consumers.

This “asymmetric liability” does not necessarily deter innovation. In fact, when the liability system is efficiently administered, it may provide stronger incentives for the development of safer products. When liability is efficient, higher liability costs translate directly into greater benefits for consumers, and also provides greater differentiation between products. When liability is inefficient, however, in the sense that liability costs exceed benefits to consumers, then goods outside the liability system will be at an advantage, and asymmetric liability may indeed deter innovation.

There are few other papers that consider the link between liability and innovation. Viscusi and Moore (1993) provide empirical evidence that liability costs have a nonmonotonic effect on innovation, and also develop a theoretical model that explains this conclusion. They consider innovation and safety as separate attributes, where consumers will pay a premium for “product novelty,” and consider firms’ joint investment decisions.

Daughety and Reinganum (2006) develop a rich model of competition among oligopolists under product liability law. Like this paper, they explicitly model differentiation along a safety attribute, treat safety investments as fixed costs, and consider incomplete compensation in the liability system. Their model takes substitutability between products to be exogenous, and shows that the degree of substitutability has a nonmonotonic effect on the provision of safety by firms.

The structure of the paper is as follows. Section 1 describes the model and analyzes the simple case where consumers are homogeneous. Section 2 extends the model to allow consumers to be heterogeneous in their level of expected damages. Section 3 concludes.

3.1 Homogeneous consumers

We consider products that are differentiated only along a safety dimension. Let \( r_i \) denote the probability of an accident occurring for product \( i \). Assume that there is an outside option
(product 0) with risk $r_0$ of causing injury to the consumer. We assume at first that there is no producer liability associated with this option. This would occur when the outside option consists of buying no product or a "do it yourself" option. This would also be the case if the outside option is a product supplied by small, judgment-proof firms. We will subsequently consider the case where there is equal liability for the new product and the outside option.

A single firm has the opportunity to develop a new product with risk $r_1$, where $r_1 \leq r_0$. We assume that the firm incurs a fixed cost $I(r_1)$ to develop this product, where $I(r_0) = 0, I(0) = \infty, I' < 0, I'' > 0$. These assumptions mean that there are no fixed costs for producing a product with the same risk level as the outside option, safer products are costlier to develop, and it is impossible to develop a product that is completely safe.

We model the tort system as imposing strict liability, but with inefficient transfer. When an accident occurs, the firm must pay damages $\alpha \theta$, but the consumer receives compensation of $\beta \theta$, where $(\alpha, \beta)$ are parameters representing the efficiency of the legal system. A perfectly efficient tort system would have $\alpha = \beta = 1$, so that all injured consumers would be perfectly compensated. Similarly, the case of $\alpha = \beta = 0$ represents a system of no liability. In general, we will have $\alpha > \beta$, since some costs will be incurred by both sides in litigating the claim. We will also assume that $\beta < 1$, so that consumers are always willing to pay some premium for a safer product.

In order to simplify the analysis, we make the following assumptions:

- the products are heterogeneous only in the safety dimension
- the safer product can be produced at the same marginal cost $c$ as the generic product, so that safety innovations only take the form of fixed costs
- the likelihood of an accident depends solely on the risk level $r_i$ of the product (i.e., there is no role for consumer precautions)
- the benefit from consuming the goods is sufficiently large to ensure that all consumers will choose one of the options

We consider the case in which there is no consumer heterogeneity, so that all consumers suffer damages $\gamma$ in the event of an accident.
3.1.1 Asymmetric Liability

First, we assume that there is no liability for the outside option. In this case, a consumer who chooses the outside good will have utility

\[ U_0(\theta) = \bar{U} - \gamma r_0 - c \]

and a consumer who chooses the new product will have utility

\[ U_1(\theta) = \bar{U} - \gamma (1 - \beta) r_1 - p_1 \]

where \( \bar{U} \) denotes the benefit of consuming the good and \( p_1 \) is the price of product 1. Thus all consumers will favor the new product \(^1\) if

\[ p_1 \leq c + \gamma [r_0 - (1 - \beta) r_1] \]

and the firm’s profit will be

\[ \pi_1 = p_1 - c - \alpha \gamma r_1 \]

\[ = \gamma [r_0 - (1 + \alpha - \beta) r_1] - I(r_1) \]

If the firm chooses to enter the market, the first order condition with respect to \( r_1 \) must be satisfied:

\[ I'(r_1^*) = -\gamma (1 + \alpha - \beta) \quad (3.1) \]

and the firm will enter if

\[ \gamma [r_0 - (1 + \alpha - \beta) r_1^*] - I(r_1^*) \geq 0 \]

A necessary condition for the above is that

\[ r_1^* \leq \frac{r_0}{1 + \alpha - \beta} \]

\(^1\)Here we must assume that when consumers are indifferent, they will choose the new product.
An immediate consequence of the above is that small quality improvements will not be profitable in a liability system with inefficiencies (i.e., $\alpha > \beta$).

Now consider the choice that a social planner would make. If it is optimal to develop a safer product, it will allocate the new product to all consumers. Total social welfare will be

$$\bar{U} - \gamma r_1 - I(r_1)$$

which leads to the first order condition

$$I'(r_{SW}) = -\gamma$$

Note that a new product will be developed as long as $I'(r_0) \geq -\gamma$. However, in some cases, the firm will not have the incentive to develop a safer product, even though it would be socially optimal to do so.

When $\alpha > \beta$, it follows that $I'(r*) < I'(r_{SW})$ and since $I'$ is increasing, $r^* < r_{SW}$. This means that the firm will overinvest in product safety relative to the social optimum.

These results are summarized in the following proposition:

**Proposition 19** When there is no liability for the outside option, the firm will develop a safer product if there exists $r_1$ satisfying $\gamma [r_0 - (1 + \alpha - \beta) r_1] \geq I(r_1)$. If the firm does develop a safer product, it will overinvest in safety relative to the social optimum if $\alpha > \beta$.

### 3.1.2 Symmetric Liability

Now, suppose that there is liability for the outside option. In this case, it could be a product that is competitively supplied.

In this case, we now have

$$U_0(\theta) = \bar{U} - \gamma(1 - \beta)r_0 - p_0$$
$$U_1(\theta) = \bar{U} - \gamma(1 - \beta)r_1 - p_1$$
so that consumers favor the new product if

\[ p_1 \leq p_0 + \gamma (1 - \beta) (r_0 - r_1) \]

Here, we have

\[ p_0 = c + \alpha \gamma r_0 \]

due to the fact that the providers of the competitive good also have liability costs. The firm’s profit will now be

\[ \pi_1 = p_1 - c - \alpha \gamma r_1 = \gamma (1 + \alpha - \beta) (r_0 - r_1) - I(r_1) \]

Note that we have the same first order condition for \( r_1 \) as in the case of no liability for the outside option, \( I'(r_1^*) = -\gamma (1 + \alpha - \beta) \), but total profits are strictly lower.

**Proposition 20** When there is liability for the outside option, the firm will develop a safer product if there exists \( r_1 \) satisfying \( \gamma (1 + \alpha - \beta) (r_0 - r_1) \geq I(r_1) \). This condition is stronger than in the case of no liability for the outside option. If the firm does develop a safer product, it will develop a product of the same risk level as in the case of no liability for the outside option, but the firm’s profit will be lower.

When the liability system is efficient (\( \alpha = \beta \)), then the investment, pricing, and profits will be the same under symmetric and asymmetric liability. This is true because the difference in liability costs is perfectly offset by consumer’s willingness to pay. When liability is inefficient, then the innovative product is at a disadvantage, and must lower its price in order to compete with the outside good, which results in lower profit. Although the first order condition is the same under both scenarios, the lower profit under asymmetric liability means that in some cases, the firm will be unwilling to incur the fixed-cost investment in product safety.

One result that follows in both cases is that the firm will always choose the optimal investment when there is no liability (\( \alpha = \beta = 0 \)). In this simple model, firms will choose the optimal investment in the absence of liability, but will never choose the social optimum when there is inefficient liability.
3.2 Heterogeneous consumers

3.2.1 Asymmetric Liability

In many instances, liability cannot be imposed on the outside option. This will be the case when the outside option consists of purchasing no good or substituting the consumer’s own labor supply, rather than purchasing a good from a firm. It could also be the case when the outside good is supplied competitively by judgement-proof firms.

In this case consumers have utility

\[ U_0(\theta) = \bar{U} - \theta r_0 - p_0 \]
\[ U_1(\theta) = \bar{U} - \theta (1 - \beta) r_1 - p_1 \]

where \( \theta \sim U[\theta_0, \theta_1] \).

For the following analysis, we take \( r_1 \) to be fixed; given the initial investment decision, we solve for the optimal pricing decision. Because the outside good is supplied competitively and not subject to liability, its price will be equal to marginal cost, i.e., \( p_0 = c \). Consumer \( \tilde{\theta} \) will be indifferent between the two choices when \( U_0(\bar{\theta}) = U_1(\bar{\theta}) \), or equivalently,

\[ \tilde{\theta} = \frac{p_1 - c}{r_0 - (1 - \beta) r_1} \]

If \( \tilde{\theta} \) is interior to \([\theta_0, \theta_1] \), the firm’s profits will be

\[ \pi_1 = \left[ p_1 - c - \alpha r_1 \left( \frac{\tilde{\theta} + \theta_1}{2} \right) \right] \left[ \frac{\theta_1 - \tilde{\theta}}{\theta_1 - \theta_0} \right] - I(r_1) \]

where \( \alpha r_1 \left( \frac{\tilde{\theta} + \theta_1}{2} \right) \) represents the average liability cost per consumer for firm 1. Substituting \( p_1 - c = [r_0 - (1 - \beta) r_1] \tilde{\theta} \) gives \( \pi_1 \) as a function of \( \tilde{\theta} \):

\[ \pi_1 = \left[ (r_0 - \left( 1 + \frac{\alpha}{2} - \beta \right) r_1) \tilde{\theta} - \alpha r_1 \theta_1 \right] \left[ \frac{\theta_1 - \tilde{\theta}}{\theta_1 - \theta_0} \right] - I(r_1) \]
which yields the first order condition

\[ \tilde{\theta}^* = \frac{\theta_1}{2} \frac{r_0 - (1 - \beta) r_1}{r_0 - \left(1 + \frac{\alpha}{2} - \beta\right) r_1} \]

Hence, when \( \tilde{\theta} \) is interior to \([\theta_0, \theta_1]\), the firm’s profit in the asymmetric liability case is

\[ \pi_{AL} = \frac{\theta_1^2 \left[r_0 - (1 + \alpha - \beta) r_1\right]^2}{4 (\theta_1 - \theta_0) \left[r_0 - \left(1 + \frac{\alpha}{2} - \beta\right) r_1\right]} - I(r_1) \] (3.3)

It can easily be shown that an interior solution obtains if and only if \( r_1 \leq \frac{r_0}{1 + \alpha - \beta} \); otherwise, \( \pi_{AL} = 0 \).

### 3.2.2 Symmetric Liability

When the outside good is subject to the same liability regime as the innovative good, we have

\[ U_0(\theta) = \bar{U} - \theta (1 - \beta) r_0 - p_0 \]
\[ U_1(\theta) = \bar{U} - \theta (1 - \beta) r_1 - p_1 \]

Let \( \tilde{\theta} \) denote the indifferent consumer. Then

\[ \tilde{\theta} = \frac{p_1 - p_0}{(1 - \beta) (r_0 - r_1)} \]

and

\[ \pi_1 = \left[p_1 - c - \alpha r_1 \left(\frac{\tilde{\theta} + \theta_1}{2}\right)\right] \left[\frac{\theta_1 - \tilde{\theta}}{\theta_1 - \theta_0}\right] - I(r_1) \]

\[ = \left[(1 + \frac{\alpha}{2} - \beta) (r_0 - r_1) \frac{\tilde{\theta} - \alpha}{2} (r_0 \theta_0 - r_1 \theta_1)\right] \left[\frac{\theta_1 - \tilde{\theta}}{\theta_1 - \theta_0}\right] - I(r_1) \]

The first order condition for \( \tilde{\theta} \) is

\[ \tilde{\theta}^* = \frac{\theta_1}{2} - \frac{\alpha}{4} \frac{r_0 \theta_0 - r_1 \theta_1}{\left(1 + \frac{\alpha}{2} - \beta\right) (r_0 - r_1)} \]
and hence when \( \tilde{\theta} \) is interior to \( [\theta_0, \theta_1] \),

\[
\pi_{SL} = \frac{(1 + \frac{\alpha}{2} - \beta) (r_0 - r_1)}{\theta_1 - \theta_0} \left[ \frac{\theta_1}{2} + \frac{\alpha}{4} \frac{r_0 \theta_0 - r_1 \theta_1}{(1 + \frac{\alpha}{2} - \beta) (r_0 - r_1)} \right]^2 - I(r_1) \tag{3.4}
\]

### 3.2.3 Analysis

First, we consider the profitability of small innovations. In many instances, marginal improvements in safety will be unprofitable due to the adverse selection effect: the small improvement will provide only a small increase in value to consumers, but will have a large effect on the firm’s liability costs by attracting the most injury-sensitive consumers.

In the case of asymmetric liability, demand for the new good will be positive whenever

\[
\frac{\theta_1}{2} \frac{r_0 - (1 - \beta) r_1}{r_0 - (1 + \frac{\alpha}{2} - \beta) r_1} < \theta_1
\]

or equivalently,

\[
\frac{r_1}{r_0} < \frac{1}{1 + \alpha - \beta} \tag{3.5}
\]

Thus when liability is costly to administer, i.e., \( \alpha > \beta \), very small innovations will be unprofitable. Note that (3.5) is not sufficient to ensure that innovation will occur; profits must also be sufficient to cover the fixed investment costs.

\[
\tilde{\theta}^* = \frac{\theta_1}{2} \frac{r_0}{r_0 - \frac{1}{2} r_1} < \theta_1
\]

which holds if \( r_1 < r_0 \). Thus, ignoring fixed costs, any safety innovation can generate positive profits.

When liability is symmetric, demand will be positive when

\[
\frac{\theta_1}{2} - \frac{\alpha}{4} \frac{r_0 \theta_0 - r_1 \theta_1}{(1 + \frac{\alpha}{2} - \beta) (r_0 - r_1)} < \theta_1
\]

or equivalently, when

\[
\frac{r_1}{r_0} < 1 - \left( \frac{\alpha}{1 + \alpha - \beta} \right) \left( \frac{\theta_1 - \theta_0}{2 \theta_1} \right) \tag{3.6}
\]

Note that the right hand side of both (3.5) and (3.6) is decreasing in \( \alpha \). In both of these
circumstances, a liability regime that imposes high liability costs on firms will make it harder for firms to profit from "small" safety innovations, and hence may deter the development of safer products.

The right hand side of (3.5) is *increasing* in $\beta$, which shows that when consumers value liability protection, it is easier for a new product to compete with a liability-proof outside good. However, when the outside good is also subject to liability, then an increase in $\beta$ will make it harder for a new product to be profitable, since (3.6) is *decreasing* in $\beta$. When the tort system efficiently transfers damages from the firm to the consumer, the combination of adverse selection and minimal differentiation will make it harder for the new product to be profitable.

For example, consider the case where the liability system is efficient, so that $\alpha = 1$ and $\beta = 1$.\(^2\) Under asymmetric liability, demand for the new good will always be positive, as seen in (3.5). However, demand will only be positive under symmetric liability if

$$\frac{r_1}{r_0} < \frac{\theta_1 + \theta_0}{2\theta_1}$$

This illustrated the case where a firm is *more* likely to invest in product safety when the alternatives are outside the liability system. The profitability of various investment levels (not including fixed costs) are illustrated for both cases in Figure 1.

When liability becomes inefficient, so that $\alpha > \beta$, then it will always be harder to profit under asymmetric liability. Since an inefficient tort system imposes costs on firms that do not result in benefits to consumers, any option outside the tort system will be at a relative advantage. The effect of inefficient liability is illustrated in Figure 2, which assumes $\alpha = 1.3$ and $\beta = 0.7$. Compared to Figure 1, the profitability of the two regimes is reversed.

We formalize the above argument in the following proposition:

**Proposition 21** Let $\bar{\tau}_{SL}$ and $\bar{\tau}_{AL}$ denote the highest risk levels of products that can be profitably

\(^2\)When $\beta = 1$, consumers will be indifferent between the products when the prices are equal. In order to simplify the analysis, we assume that indifferent consumers will always choose the new product.
Figure 1: \(\alpha = 1, \beta = 1, \theta_0 = 1, \theta_1 = 3\)

Figure 3-1:
Figure 2: $\alpha = 1.3, \beta = 0.7, \theta_0 = 1, \theta_1 = 3$

Figure 3-2:
sold under symmetric and asymmetric liability, respectively. Then

\[ \bar{r}_{SL} = 1 - \left( \frac{\alpha}{1 + \alpha - \beta} \right) \left( \frac{\theta_1 - \theta_0}{2\theta_1} \right) \]

\[ \bar{r}_{AL} = \frac{1}{1 + \alpha - \beta} \]

and

\[ \bar{r}_{SL} > \bar{r}_{AL} \text{ if and only if } \frac{\theta_0}{\theta_1} > 1 - 2(\alpha - \beta) \]

Several results follow immediately. First, when \( \alpha = \beta \) (no inefficiency of transfer), then sufficiently small safety innovations will not be profitable under symmetric liability but may be under asymmetric liability.

This also includes the case where there is no liability, i.e., \( \alpha = \beta = 0 \). Thus is some instances, firms will be deterred from introducing welfare-enhancing safety innovations under a liability system that they would have introduced in the absence of liability. This is true even if liability is efficient. Moreover, in the no liability case, the firm will underinvest in safety relative to the social optimum, because the firm does not internalize all of the benefit of the investment. At least when the optimal investment is small, the firm will therefore underinvest even more under a liability regime. This result stands in contrast with the traditional analysis of strict liability that only considers marginal costs. When safety precautions only take the form of marginal cost, then firms will always invest optimally in accident avoidance under a regime of strict liability. (Brown 1973)

Finally, when \( \alpha - \beta > \frac{1}{2} \), then \( \bar{r}_{SL} > \bar{r}_{AL} \). This means that there are some small safety improvements that could be profitable under symmetric liability but not under asymmetric liability. When the inefficiency of compensation is severe enough, the unequal application of liability law can be a strong impediment to the development of new products.

### 3.3 Conclusion

The simple model presented in this paper highlights several distortions inherent in product liability law. When there is any inefficiency in the administration of liability and safety innovations take the form of fixed, rather than marginal costs, firms may be deterred from
making socially optimal safety improvements but may also overinvest in safety.

When the tort system does not treat all risks equally – when there is a substitute for a good that is not subject to liability – then investment decisions may be distorted. In particular, when asymmetric liability is combined with inefficient transfer of damages, then firms incentives for developing safer products will be reduced. This effect is most acute when the optimal innovation is small.

This paper uses a simple model with a competitive good and a single innovative product to develop these points. An important question for further work would be to understand how disparate liability and inefficiency of transfer affect firm’s incentives in a more complex market environment.
Bibliography


79


