

**Evaluation of Large Scale Industrial Development Using Real Options Analysis  
– A Case Study –**

by

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**Evaluation of Large-Scale Industrial Development Using Real-Options Analysis  
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**ABSTRACT**

Recently, real-option analysis has gained attention as an innovative valuation method for complex real estate projects. However, considering its potential, this method has not become as popular as it should have. One major reason may be its complexity, and perhaps, its effectiveness is not yet widely known in the industry.

Accumulating high-quality case studies can help demonstrate the effectiveness of any theory. Case studies can also help standardize the application process, providing guidelines that help people use the model more easily. In addition, it can reveal and provide solutions for various types of properties, and the means to accommodate the specifics of real-world problems met while applying the model.

This case study deals with a large-scale industrial development project, which is suitable for the application of the real-option model. Usually industrial developers obtain large sites and then develop them in a phased manner. This allows them the freedom to choose phase timing and to modify their initial building plans more freely than with other types of property development. This flexibility adds certain amount of value to the land.

We found that, with some modifications, the real-option model is fairly effective when applied to large-scale industrial development. The model facilitates more precise valuations of land by taking into account various options, such as waiting for better timing and selling the vacant land as is. This study also offers a method to analyze the proper timing of each phase's commencement—a useful decision-making tool for the developer.

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## **Chapter 1 Introduction**

### **Background**

Recently, real-option analysis has gained attention as an innovative valuation method for complex real estate projects; however, considering its potential, it has not become as popular as it should be. A major reason could be the complexity of the method; moreover, the effectiveness of this method is not yet widely known in the industry.

By providing a number of case studies that apply the real-option method, it is possible to demonstrate the model's effectiveness as well as point out practical ways to apply the model. Case studies can help standardize an application process, which will in turn facilitate the use of the model. Furthermore, different types of property and development have specific issues that must be dealt with, and each case study can focus on the requirements of a specific property type.

When developing industrial business parks, developers often acquire a large amount of land property, which is then developed in phases. The proper timing for each phase of development is typically based on current market conditions.

This type of large-scale industrial development has relatively short construction periods and requires keeping costs low; therefore, it is a suitable candidate for applying the real-option model.

## **Objectives**

- Investigate the real-option method as an evaluation and decision-making tool for large-scale industrial developments
- Demonstrate the effectiveness of the real-option model in evaluating large-scale industrial developments
- Identify practical problems in applying the real-option model in an actual business situation, and provide solutions.

Our goal is to provide an idea of how the real-option model works in directing an investment strategy, and to provide a useful and understandable tool for making investment decisions in a real industrial-development process.

## Chapter 2 Methodology

### Overview

The method in this case study is based on Geltner and Miller's *Commercial Real Estate Analysis and Investments*, 1<sup>st</sup> edition (2001), and the forthcoming 2<sup>nd</sup> edition (2006). This chapter addresses the fundamentals of the method; specific modifications of the model for the case study will be addressed in the next chapter. This chapter simply provides the essence of the model, to allow the reader to understand the case study. Please refer to the referenced book in order to comprehend the structure of the model.

An *option* is defined as “the right without obligation to obtain something of value upon the payment or giving up of something else of value.” Our objective is to evaluate the value of land for a development project. In this context, the above definition can be restated as “the right without obligation to obtain residual land value based on property developed now, upon giving up land value based on future development.” By choosing the optimal timing to develop their land, landowners can maximize its value. An option to develop a certain site now precludes an option to develop the same site later. The landowner may be better off by developing a given site later, rather than now, but normal DCF (Discounted Cash Flow) methods are not able to evaluate this value. The option model used here, however, can capture the flexibility that landowners may have. Of course, there are other options besides developing or not developing, such as the “switching” option, where the developer chooses to change the use of the land, and the “sellout” option, where the land is sold as is. All these options have value, but we will focus on the “wait” option in this chapter—the option that allows landowners to choose the timing of development.

The fundamental concept of this model is that it can compare the values of the land based on whether it is developed immediately, or if development is postponed.

$$\text{Option value} = \text{Maximum} \left\{ \begin{array}{l} \text{PV of the land developed now,} \\ \text{PV of the land developed in the future} \end{array} \right\}$$

The above equation implies that, if the present value (PV) of immediate development is greater than that of later development, a landowner should commence a project now; otherwise, waiting maximizes the option value.

It is not difficult to calculate the PV of immediate development, because we can simply apply a DCF model. The difficulty is in evaluating the land value after future development because the development has infinite possibilities. The option of future development in this model is regarded as a “call” option, which is an option that can be exercised anytime before it expires. In other words, the landowners can develop their land anytime they want, and this factor, this flexibility, makes calculating the land value based on future development more complex.

We used the binominal model introduced in Geltner and Miller’s book to capture the value of the call option’s flexibility.

Before moving on to the binominal model, we will define the variables used in this study:

## Variables

$PV_t(Exercise)$  = Present value of the land developed at time  $t$

$PV_t(Wait)$  = Present value of the land developed in the future (year 1)

$V_t$  = Value of built property at time  $t$

$$\left( \begin{array}{l} V_{t,up} = \text{Expected value of built property when the value increases at time } t \\ V_{t,down} = \text{Expected value of built property when the value decreases at time } t \\ \%V_{t,up} = V_{t+1,up}/PV_t[V_{t+1}] \\ \%V_{t,down} = V_{t+1,down}/PV_t[V_{t+1}] \end{array} \right)$$

$K_t$  = Construction costs and other costs to develop a property at time  $t$

$C_t$  = Option value (land value) at time  $t$

$$\left( \begin{array}{l} C_{t,up} = \text{Expected option value when option value increases at time } t \\ C_{t,down} = \text{Expected option value when option value decreases at time } t \\ \%C_{t,up} = C_{t+1,up}/PV_t[C_{t+1}] \\ \%C_{t,down} = C_{t+1,down}/PV_t[C_{t+1}] \end{array} \right)$$

$p$  = Probability of option value increasing

$OCC$  = Opportunity cost of capital

$rf$  = Risk free rate

$r_V$  = Expected annual total return on the built property

$y_V$  = Annual net rent income cash yield as a fraction of current building value

$g_V$  = Expected growth rate in built property

$g_K$  = Expected growth rate in construction costs

$$*(1 + r_V) = (1 + y_V)(1 + g_V)$$

$\sigma_V$  = Expected annual volatility of returns of the built property

$y_K$  = Construction cost yield, the difference between the opportunity cost of capital of construction cost cash flows and the expected growth rate in construction costs

$ttb$  = Time to build



**Table 2-1 Inputs for the Study**

Parameters	Value	Remarks
$r_f$	6.00%	5 year treasury note
$r_V$	10.25%	$y_V + 100$ bp
$y_V$	9.25%	Based on the pro-forma created for the project
$g_V$	0.92%	$(1 + r_V) = (1 + y_V)(1 + g_V)$
$g_K$	2.00%	
$\sigma_V$	15.00%	Volatility for the individual project
$y_K$	3.92%	$(1 + r_f) = (1 + y_K)(1 + g_K)$
$ttb$	7–10 months	Based on the pro-forma created for the project

### Binominal Real-Option Model

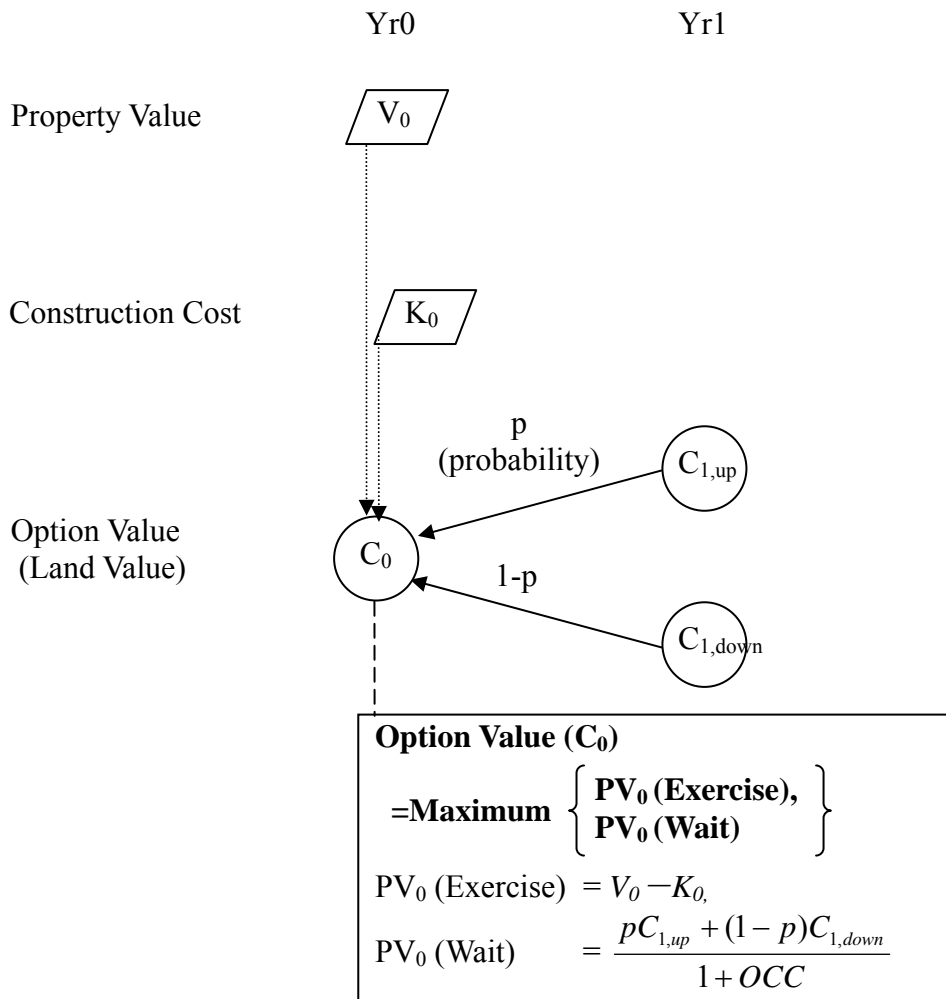
Figure 2-1 shows a simple single-term binominal model. The option value (land value) at time 0 is calculated as

$$\text{Option Value } (C_0) = \text{Maximum } \{PV_0(\text{Exercise}), PV_0(\text{Wait})\}$$

$$PV_0(\text{Exercise}) = V_0 - K_0$$

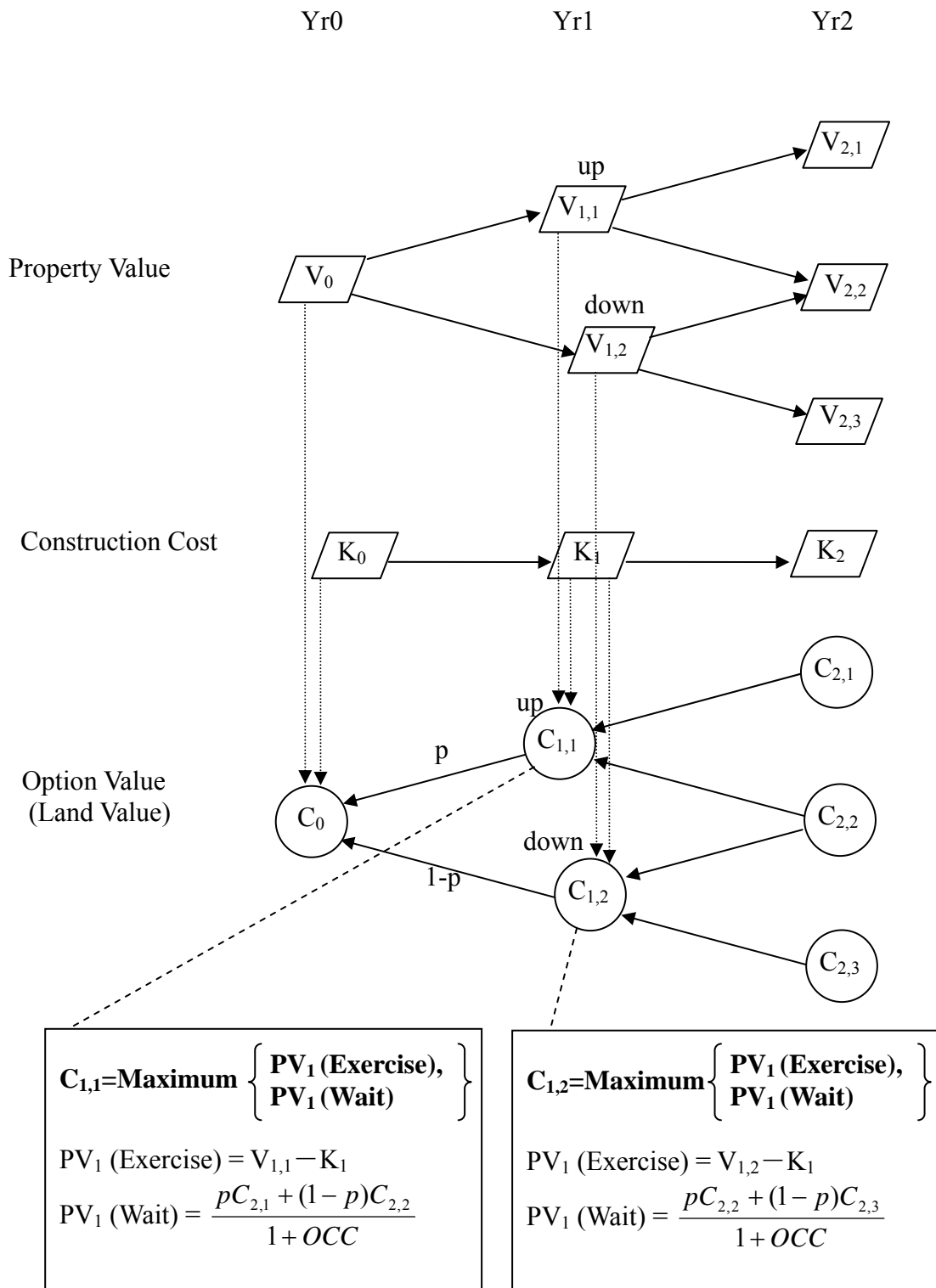
$$PV_0(\text{Wait}) = \frac{\text{Expected Option Value}(\text{yr1})}{1 + OCC} = \frac{pC_{1,up} + (1 - p)C_{1,down}}{1 + OCC}$$

\*Here, construction is assumed to finish instantly. A method for incorporating “time to build” will be stated later.



**Figure 2-1 Simple One-Period Binominal Model**

The essence of the binominal real-option model is included in this simple one-period model. Expanding the unit (triangle) rightward creates a complete model, depending on when the option expires. Figure 2-2 illustrates the model over two periods. Note that the period can be broken into smaller terms, such as quarters or months. In this case, we used terms of a year.



**Figure 2-2 Two- Period Binominal Model**

The following formula represents the value of the option at each node, except for the point at which the option expires. We derive the formula for the wait option and the exercise option described in the following sections.

for all  $t < T$ , ( $T =$  terminal period)

$$\text{Option Value } (C_{t,i}) = \text{Maximum } \{PV_t(\text{Exercise}), PV_t(\text{Wait})\}$$

$$= \text{Maximum} \left\{ v_{t,i} - k_t, \frac{pC_{t+1,i} + (1-p)C_{t+1,i+1}}{1 + OCC} \right\}$$

$$= \text{Maximum} \left\{ V_{t,i}/(1 + y_v)^{ttb} - K_t/(1 + y_k)^{ttb}, \frac{(pC_{t+1,i} + (1-p)C_{t+1,i+1}) - \frac{RPV}{(\%V_{t,up} - \%V_{t,down})} \times (C_{t+1,i} - C_{t+1,i+1})}{(1 + r_f)} \right\}$$

$$= \text{Maximum} \left\{ V_{t,i}/(1 + y_v)^{ttb} - K_t/(1 + y_k)^{ttb}, \frac{(pC_{t+1,i} + (1-p)C_{t+1,i+1}) - \frac{r_v - r_f}{((1 + \sigma_v) - 1/(1 + \sigma_v))} \times (C_{t+1,i} - C_{t+1,i+1})}{(1 + r_f)} \right\}$$

for all  $t = T$ ,

$$\text{Option Value } (C_{t,i}) = PV_t(\text{Exercise})$$

*Numerical Example*

$$V_t = 100, K_t = 90 C_{up} = 8 C_{down} = 0$$

Other parameters follow them in the Table 2-1

*Option Value* ( $C_{t,j}$ ) = Maximum  $\{PV_t(\text{Exercise}), PV_t(\text{Wait})\}$

$$= \text{Maximum} \left\{ V_{t,i}/(1+y_v)^{\text{tb}} - K_t/(1+y_k)^{\text{tb}}, \frac{(pC_{t+1,i} + (1-p)C_{t+1,i+1}) - \frac{r_v - r_f}{((1+\sigma_v) - 1/(1+\sigma_v))} \times (C_{t+1,i} - C_{t+1,i+1})}{(1+r_f)} \right\}$$

$$= \text{Maximum} \left\{ 100/(1+9.25\%)^1 - 90/(1+3.92\%)^1, \frac{(0.8306 \times 8 + 0.1694 \times 0) - \frac{10.25\% - 6\%}{(1.15 - 1/(1.15))} \times (8 - 0)}{(1+6\%)} \right\}$$

$$= \text{Maximum}\{4.93, 5.13\}$$

$$= 5.13$$

If  $PV_t(\text{Exercise}) < PV_t(\text{Wait})$ , then the wait option is better than the exercise option in this example.

## Present Value of the Exercise Option

$$PV_t(\text{Exercise}) = PV_t(\text{Built Property}) - PV_t(\text{Development Costs (excluding land)})$$

$$= v_{t,i} - k_t$$

$$= V_{t,i}/(1 + y_v)^{\text{tb}} - K_t/(1 + y_k)^{\text{tb}}$$

First, calculate the present value of built property at the time for each node using the binominal tree. Starting from  $V_0$ , construct the  $V$  value tree by applying the following formula.

$$V_{t+1,up} = PV_t[V_{t+1}] \times \%V_{t+1,up} = PV_t[V_{t+1}] \times (1 + \sigma_v) = \frac{V_t(1 + g_v)}{(1 + r_v)} \times (1 + \sigma_v) = \frac{V_t}{(1 + y_v)} \times (1 + \sigma_v)$$

$$V_{t+1,down} = PV_t[V_{t+1}] \times \%V_{t+1,down} = PV_t[V_{t+1}] \times \frac{1}{(1 + \sigma_v)} = \frac{V_t(1 + g_v)}{(1 + r_v)} \times \frac{1}{(1 + \sigma_v)} = \frac{V_t}{(1 + y_v)} \times \frac{1}{(1 + \sigma_v)}$$

$$*1 + g_v = (1 + y_v)/(1 + r_v)$$

$$*\%V_{up} = (1 + \sigma_v), \quad \%V_{down} = 1/(1 + \sigma_v)$$

It should be noted that this tree is constructed such that the “down” movements from the upper left and “up” movements from the lower left create the same number. This can reduce the complexity of the model.

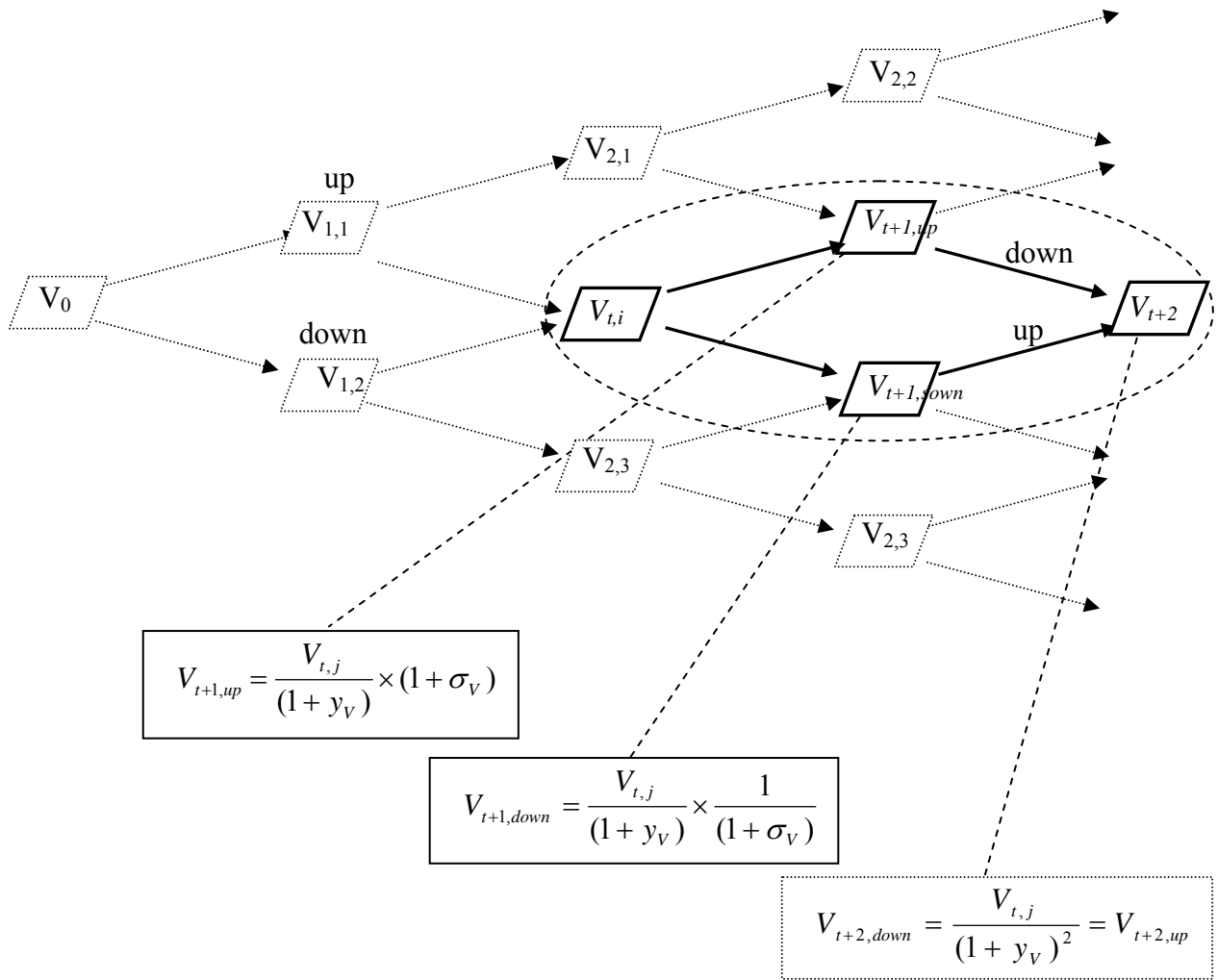
$$V_{t+2,down} = \frac{V_{t,i}}{(1 + y_v)^2} = V_{t+2,up}$$

↑↑

$$V_{t+2,down} = \frac{V_{t+1,up}}{(1 + y_v)} \times \frac{1}{(1 + \sigma_v)} = \frac{\frac{V_{t,i}}{(1 + y_v)} \times (1 + \sigma_v)}{(1 + y_v)} \times \frac{1}{(1 + \sigma_v)} = \frac{V_{t,i}}{(1 + y_v)^2}$$

$$V_{t+2,up} = \frac{V_{t+1,down}}{(1 + y_v)} \times (1 + \sigma_v) = \frac{\frac{V_{t,i}}{(1 + y_v)} \times \frac{1}{(1 + \sigma_v)}}{(1 + y_v)} \times (1 + \sigma_v) = \frac{V_{t,i}}{(1 + y_v)^2}$$

These relations are shown in Figure 2-3.



**Figure 2-3 Property Value Tree**

Because the landowner cannot get these  $V$  values until construction work is completed, the following adjustments are needed to take into account the time to build:

$$PV_t[V_t] = v_t = \frac{\text{Expected } V \text{ at Completion}}{\text{OCC for Built Property}} = \frac{V_t(1 + g_V)^{tb}}{(1 + r_V)^{tb}} = \frac{V_t \left( \frac{(1 + r_V)}{(1 + y_V)} \right)^{tb}}{(1 + r_V)^{tb}} = \frac{V_t}{(1 + y_V)^{tb}}$$

Development costs ( $K$ ), however, are computed simply by applying the development cost (mainly construction cost) growth rate.

$$K_{t+1} = K_t(1 + g_K)$$

The time to build can be taken into account following the same process, using the built-property value.

$$PV_t[K_t] = k_t = \frac{\text{Expected } K \text{ at Completion}}{\text{OCC for Development Cost}} = \frac{K_t(1 + g_K)^{tb}}{(1 + r_f)^{tb}} = \frac{K_t \left( \frac{(1 + r_f)}{(1 + y_K)} \right)^{tb}}{(1 + r_f)^{tb}} = \frac{K_t}{(1 + y_K)^{tb}}$$

Here, we use the “risk-free rate” as an opportunity cost for development, because the negative cash flow (cash outflow) from development costs has almost no correlation with the entire financial market movement (i.e., market portfolio). Also it is notable that the payment of all development costs is assumed to occur at the completion in this model. It is based on the assumption that almost all development costs except for land are usually covered by construction loan. This assumption allows us to think only of *Time 0* (beginning of the construction) and *Time t* (end of construction) in terms of cash flow.

Normally, in order to compute the residual land value from a development project, an estimated *OCC* for the project is applied to the net cash flow accrued during the development process. However, it is often difficult to determine the proper *OCC* for measuring the risk of a



project. This method calculates the present value for the built property separately from the development costs. This method, called the “canonical formula” is introduced and described in detail in Geltener and Miller (Chapter 29).

$$\begin{aligned}
 PV[Land] &= PV[Built Property] - PV[Development Cost] \\
 \frac{V_t - K_t}{(OCC \text{ for Development})^{tb}} &= \frac{V_t}{(OCC \text{ for Built Property})^{tb}} - \frac{K_t}{(OCC \text{ for Development Cost})^{tb}} \\
 \frac{V_t - K_t}{(1 + r_c)^{tb}} &= \frac{V_t}{(1 + r_v)^{tb}} - \frac{K_t}{(1 + r_f)^{tb}}
 \end{aligned}$$

### Present Value of the Wait Option

Discounting the expected option value of two future options (binominal) at each node (one triangle) gives the present value of the wait option. The expected option value is the average of the two options, weighted by their up and down probabilities.

$$\begin{aligned}
 PV_t(\text{Wait}) &= \frac{\text{Expected Option Value (1yr later)}}{1 + OCC} = \frac{pC_{t+1,up} + (1-p)C_{t+1,down}}{1 + OCC} = \frac{pC_{t+1,up} + (1-p)C_{t+1,down}}{1 + r_f + R Pc} \\
 &\quad \Downarrow \\
 (1 + r_f + R Pc)PV_t(\text{Wait}) &= (pC_{t+1,up} + (1-p)C_{t+1,down}) \\
 (1 + r_f)PV_t(\text{Wait}) &= (pC_{t+1,up} + (1-p)C_{t+1,down}) - R PcPV_t(\text{Wait}) \\
 PV_t(\text{Wait}) &= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - R PcPV_t(\text{Wait})}{(1 + r_f)} \dots\dots \textcircled{1}
 \end{aligned}$$

$$\begin{aligned}
RPcPV_t(\text{Wait}) &= \frac{RPc}{(\$C_{t+1,up} - \$C_{t+1,down})} \times (\$C_{t+1,up} - \$C_{t+1,down}) \times PV_t(\text{Wait}) \\
&= \frac{RPc}{(\$C_{t+1,up} - \$C_{t+1,down}) / PV_t(\text{wait})} \times (\$C_{t+1,up} - \$C_{t+1,down}) \\
&= \frac{RPc}{(\%C_{t+1,up} - \%C_{t+1,down})} \times (\$C_{t+1,up} - \$C_{t+1,down}) \\
&= \frac{RPc}{(\%C_{t+1,up} - \%C_{t+1,down})} = \frac{RPv}{(\%V_{t+1,up} - \%V_{t+1,down})} \dots\dots \textcircled{2}
\end{aligned}$$

$$RPcPV_t(\text{Wait}) = \frac{RPv}{(\%V_{t+1,up} - \%V_{t+1,down})} \times (\$C_{t+1,up} - \$C_{t+1,down}) \dots\dots \textcircled{3}$$

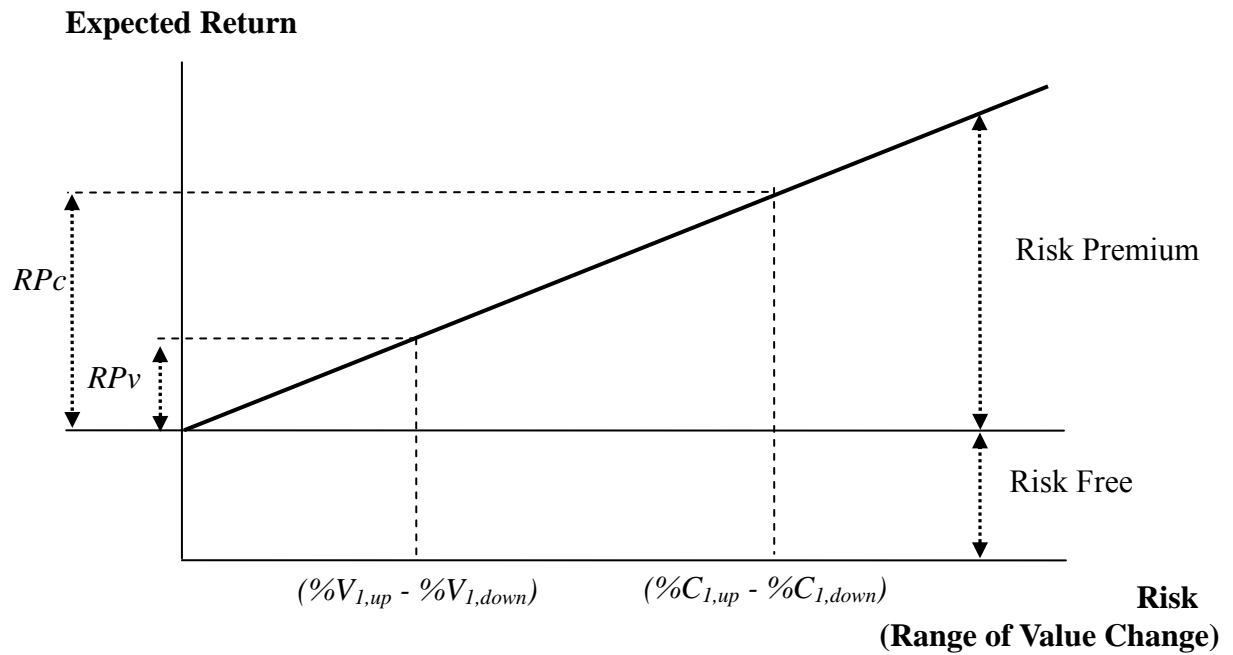
By combining ① & ③, we get

$$\begin{aligned}
PV_t(\text{Wait}) &= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - RPcPV_t(\text{Wait})}{(1+r_f)} \\
&= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - \frac{RPv}{(\%V_{t+1,up} - \%V_{t+1,down})} \times (C_{t+1,up} - C_{t+1,down})}{(1+r_f)} \\
&= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - \frac{r_v - r_f}{((1+\sigma_v) - 1/(1+\sigma_v))} \times (C_{t+1,up} - C_{t+1,down})}{(1+r_f)}
\end{aligned}$$

One of the most important concepts in this model (also, in the entire investment world) is the “price of risk,” i.e., the risk premium per unit of risk. Expected return (risk premium + risk free rate) must be consistent with the risk associated with the investment, regardless of the type of underlying asset. Therefore, the unit of risk must be the same for the built property and the undeveloped land; otherwise, an arbitrage opportunity exists. The preceding formula ② is based on this relationship shown in Figure 2-4. Here, risk, the volatility of value change, can be expressed as the range of expected values.

*Price of Risk for Option (Land) = Price of Risk for Built Property*

$$\frac{RPC}{(\%C_{up} - \%C_{down})} = \frac{RPV}{(\%V_{up} - \%V_{down})}$$



**Figure 2-4 Price of Risk**

The probability ( $p$ ) can be determined to increase the expected  $V$  value one term later to the expected return of built property ( $r_V$ ).

$$p = \frac{1 + r_V - \%V_{1,down}}{\%V_{1,up} - \%V_{1,down}} = \frac{1 + r_V - 1/(1 + \sigma_V)}{(1 + \sigma_V) - 1/(1 + \sigma_V)}$$

↑↑

$$1 + r_V = p\%V_{1,up} + (1 - p)\%V_{1,down}$$

$$p(\%V_{1,up} - \%V_{1,down}) = 1 + r_V - \%V_{1,down}$$

$$\%V_{1,up} = \frac{V_{1,up}}{PV[V_1]} = \frac{V_0(1 + \sigma_V)/(1 + y_V)}{V_0(1 + g_V)/(1 + r_V)} = \frac{V_0(1 + \sigma_V)/(1 + y_V)}{V_0(1 + y_V)} = 1 + \sigma_V$$

$$\%V_{1,down} = \frac{V_{1,down}}{PV[V_1]} = \frac{V_0/(1 + \sigma_V)/(1 + y_V)}{V_0(1 + g_V)/(1 + r_V)} = \frac{V_0/(1 + \sigma_V)/(1 + y_V)}{V_0(1 + y_V)} = 1/(1 + \sigma_V)$$

$$*1 + g_V = (1 + y_V)/(1 + r_V)$$

### Perpetual Option Valuation Model (Samuelson–McKean Formula)

While the binominal model can only evaluate finite-lived options, the Samuelson–McKean formula can capture values with infinite options. Basically, land ownership is perpetual, and an option to develop a particular site of land lasts infinitely. This formula suits the task of evaluating land value with simple singular-phase development without time constraints. Further, it is applicable to the determination of option (land) value of the last phase in a multi-phased project.

$$\text{Option (land) value} = C_0 = (V^* - K_0) \left( \frac{V_0}{V^*} \right)^\eta$$

$$\eta = \left\{ y_V - y_K + \sigma_V^2 / 2 + [(y_V - y_K + \sigma_V^2 / 2)^2 + 2y_K \sigma_V^2]^{1/2} \right\} / \sigma_V^2$$

$$V^* = K_0 \eta / (\eta - 1)$$

\*  $\eta$  = Option elasticity

\*  $V^*$  = Hurdle value of the developed property

## Compound Real Option Model

When a developer purchases a site for development, it is not realistic to delay construction, since the decision to develop the site has normally already been made before the purchase.

Thus far, we have described evaluation of the wait option for simple projects with no phases. The true benefit of the model is in its application to large-scale, multi-phased projects. Such projects require a long time to complete, and the development plan must be continually modified regarding the timing of construction, design, and use, depending on the current market situation. Therefore, these important decisions need to be made not only at the beginning of the project but also periodically as it progresses. In other words, the later phases have a great deal of flexibility to adapt to market conditions, and this flexibility has a huge impact on the land value.

The compound real-option model provides a method of evaluating land value for multi-phased projects. There are two ways to structure the model: simultaneous or sequential. In a simultaneous-option model, the phases are independent of each other, and can start anytime, regardless of the progress of the other phases. In sequential mode, however, subsequent phases cannot start until the current phase finishes. Thus, we call this version the option-on-option model. For this case study, we deal with only the sequential model.

The steps to construct a sequential compound real-option model are

- 1) Construct a binominal option model for each phase. (Normally, the Samuelson–McKean formula is the right choice for the last phase)
- 2) Add an optional value for each subsequent phase only when the timing for exercise of the option is optimal. Normally, the landowners cannot get the value of a subsequent phase until current phase's completion. In order to incorporate this lag into the evaluation, the subsequent option value received by current phase can be computed by

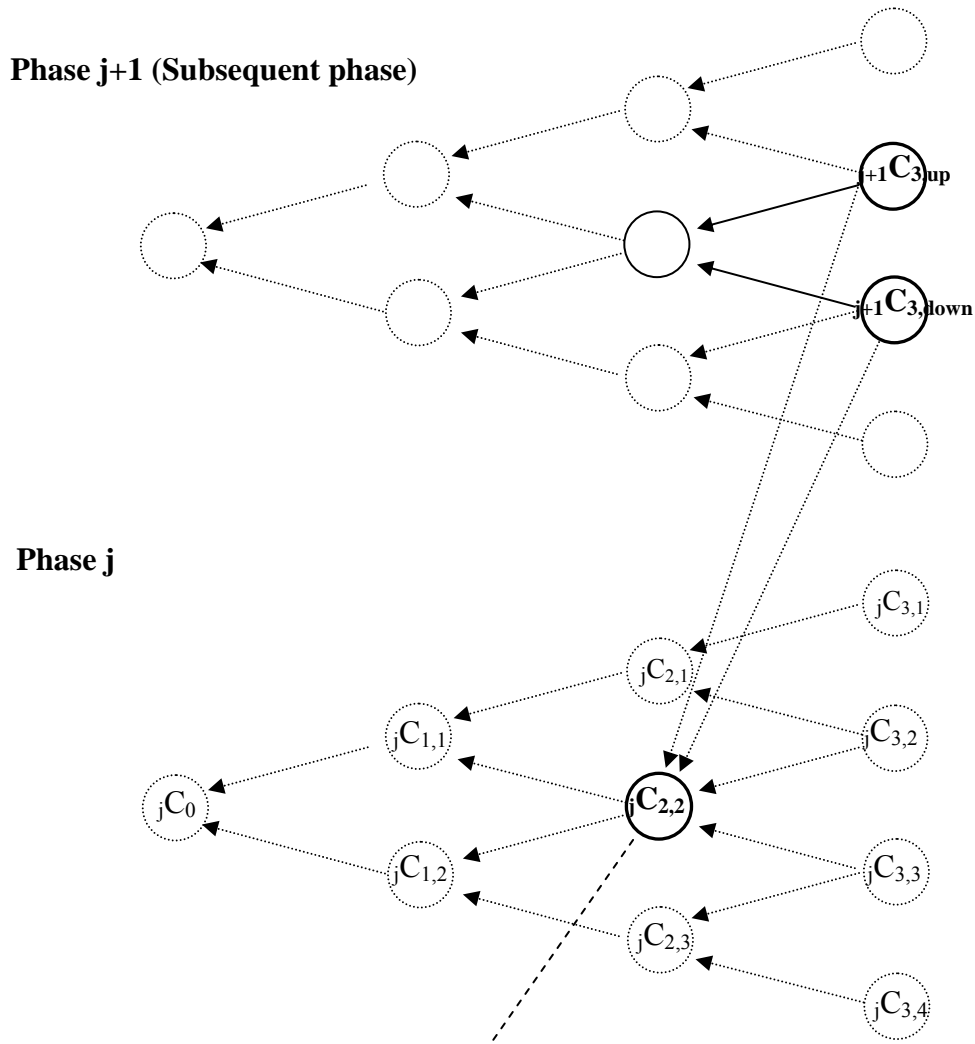
discounting the option value at the time of completion to the value at the time of exercise.

The following formula and Figure 2-5 shows the above procedure (assuming one term lag).

*Option Value* ( ${}_jC_{t,i}$ )

= Maximum { $PV_t$  (*Exercise*) +  $PV_t$  (*Subsequent Phase Option Value*),  $PV_t$  (*Wait*)}

$$\begin{aligned}
 PV_t (\text{Subsequent Phase Option Value}) &= \frac{p_{j+1}C_{t+1,up} + (1-p)_{j+1}C_{t+1,down}}{1 + OCC} \\
 &= \frac{(p_{j+1}C_{t+1,up} + (1-p)_{j+1}C_{t+1,down}) - \frac{RPV}{\left(\frac{\%_{j+1}V_{t+1,up}}{\%_{j+1}V_{t+1,down}} - 1\right)} \times ({}_{j+1}C_{t+1,up} - {}_{j+1}C_{t+1,down})}{(1 + r_f)} \\
 &= \frac{(p_{j+1}C_{t+1,up} + (1-p)_{j+1}C_{t+1,down}) - \frac{r_V - r_f}{((1 + \sigma_v) - 1/(1 + \sigma_v))} \times ({}_{j+1}C_{t+1,up} - {}_{j+1}C_{t+1,down})}{(1 + r_f)}
 \end{aligned}$$



$${}_jC_{2,2} = \text{Maximum} \left\{ \begin{array}{l} \text{PV}_2(\text{Exercise}) + \text{PV}_2(\text{Subsequent Phase Option Value}), \\ \text{PV}_2(\text{Wait}) \end{array} \right\}$$

**PV<sub>2</sub> (Subsequent Phase Option Value)**

$$= \frac{p_{j+1} C_{3,up} + (1-p)_{j+1} C_{3,down}}{1 + OCC}$$

$$= \frac{(p_{j+1} C_{3,up} + (1-p)_{j+1} C_{3,down}) - \frac{r_v - r_f}{((1 + \sigma_v) - 1/(1 + \sigma_v))} \times ({}_{j+1} C_{3,up} - {}_{j+1} C_{3,down})}{(1 + r_f)}$$

\*assuming 1 term lag

Figure 2-5 Option (land) Value -compound option-

## **Chapter 3 Case Study**

### **Data Source**

This case study is based on actual data from a major industrial developer in the US. Some numbers have been modified due to confidentiality issues. We used financial data that was created at the beginning of the project and data created at the beginning of each phase.

### **Project Overview**

#### ➤ Location

This industrial business park is located about 20 miles away from the central business district (CBD) of one of the largest cities in the US. The 164-acre site is ideally suited for a business park because of its easy access to highways and proximity to an airport.

#### ➤ Size

The developer planned to develop nine distribution buildings of total three million square feet in area. The site suits both local and regional distribution facilities, which allowed the developer to adopt the strategy of developing a mixture of different-sized warehouses that would match the needs of both. The planned buildings ranged from 130,000 to 540,000 sf.

#### ➤ Quality and Design

The developer applied high-quality standards to both the design and construction of the buildings and landscape. Establishing organized codes and covenants helped to maintain architectural continuity throughout the business park.

#### ➤ Schedule & Phasing

The developer purchased the site in 1995, and planned to develop the entire site over a period of six years. The project consisted of five phases, with each phase covering the construction of one or two distribution facilities. The infrastructure for the site, including the ingress/egress,



roads, signs, common area landscaping, and utilities, were planned to be constructed with Phases I, II, and IV.

➤ Market Conditions

At the time of the proposal for purchasing the site (1995), the overall vacancy rate was around 5%, which was historically low for this area. The vacancy rate for “Class A” property, such as this one, had fallen to less than 1%.

➤ Valuation Method for the Acquisition Price

The following table shows the costs and profitability summary for Phase I and the total that includes all phases of the project. The first phase site-acquisition price of \$0.82million gives 11% profit margin on costs. The \$5.17 million total site-acquisition cost represents the same price per acreage as paid in Phase I.

**Table 3-1 Costs and Profitability Summary (\$1,000)**

	Phase I	All Phases	
Land Acreage	26	164	
Sight Acquisition	820	5,174	6.3%
Construction	9,406	59,328	72.2%
Soft Costs	2,794	17,623	21.5%
Total Development costs	13,020	82,125	100.0%
NOI	1,378	8,692	
Cap Rate	9.25%		
Projected Sales Price	14,897	93,966	
Selling Expenses	(149)	(940)	
Net Sales Proceeds	14,748	93,026	
Profits	1,458	9,197	
Margin on Costs	11%	11%	

## Valuation Model and Assumptions

The project was divided into five phases, as shown in Table 3-2. We can apply the sequential (compound) real-option model for this case, since a certain percentage of the previous phase's absorption triggers the start of the next phase.

The Phase I option is assumed to expire in a year because the decision to start Phase I had already been made at the time the land was purchased. On the other hand, there are basically no time constraints for exercising the later phases. This could be called the “infinite on infinite” option. Our compound real-option model cannot fully capture an infinite on infinite option value. Thus, 40-year binominal trees approximate the infinite option for the phases between the first and final phase. This is considered to be long enough for the evaluation.

*Table3-2 Valuation Method for Each Phase*

Phase	Building	Option Expiration	Valuation Method
I	A	1 year	Binominal Tree
II	B,I	40 year	//
III	C,F	//	//
IV	D,H	//	//
V	E,G	Infinite	Samuelson–McKean formula

The developer chooses the option that gives the maximum value among “Sellout,” “Exercise,” and “Wait” in each node of the option tree. The following function shows the basic construction of the binominal tree.

### [All Phases Prior to the Last Phase]

For all  $t < T$

$$\begin{aligned} C_t &= \text{Maximum (Sellout Option Value, Exercise Option Value, Wait Option Value)} \\ &\quad - \text{Carrying Costs} \\ &= \text{Maximum (Sellout Option Value, } PV_i[V_{t+1} - K_{t+1}] + PV_i[\text{Subsequent Phase Opt}_{t+1}], \\ &\quad PV_i[C_{t+1}]) - \text{Carrying Costs} \\ C_t &= 0 \text{ (If } C_t < 0) \end{aligned}$$

For  $t = T$

$$\begin{aligned} C_t &= \text{Maximum (Sellout Option Value, Exercise Option Value)} - \text{Carrying Costs} \\ C_t &= 0 \text{ (If } C_t < 0) \end{aligned}$$

### [Last phase]

For all  $t < T$

$$\begin{aligned} C_t &= \text{Maximum (Sellout Option Value, Exercise Option Value, Wait Option Value)} \\ &\quad - \text{Carrying costs} \\ &= \text{Maximum (Sellout Value, } PV_i[V_{t+1} - K_{t+1}], PV_i[C_{t+1}]) \\ C_t &= 0 \text{ (If } C_t < 0) \end{aligned}$$

For  $t = T$  (T=terminal period)

$$\begin{aligned} C_t &= \text{Maximum (Sellout Option Value, Exercise Option Value)} \\ C_t &= 0 \text{ (If } C_t < 0) \end{aligned}$$

#### ➤ Sellout Option Value

Occasionally, the return on a project is optimized by selling the land as is, rather than pursuing industrial development. To value the sellout option value in this case study, considering the surrounding environment, we assumed the development of single-family housing as the fallback alternative use. The basic assumptions for the sellout price valuation are shown in Table 3-3.

**Table 3-3 Assumptions for Sellout Option Valuation**

Total sf	7,143,840	sf	
Lot size	20,000	sf	*Average sigle-family lot size = 16,675 in the US (US Census Bureau)
Efficiency	80%		
Total lots	286		
V <sub>0</sub> /lot	124,200	\$/lot	*Average sigle-family sales price in the area
V <sub>0</sub>	35,491	1,000\$	
Land Value	3,549	1,000\$	
Rezoning Costs	1,775	50%	

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length (T/n) in yrs =	1.0000		
Risk free interest rate (rf) =	6.00%	rf/period=	6.00%
Underlying Asset Total Return (rV) =	10.25%	rV/period =	10.25%
Underlying Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K Growth rate (gK) =		K gro/per=	
Time to build (periods)		V gro/per=	0.92%
Volatility (σ) =	15%	σ /period=	15.00%
V(initial) =		yK/period=	
K(initial) =		"p" real prob=	0.8306
Land Carrying Costs		"u" =	1.1500
Residual Land Ratio (beginning)		"d" =	0.8696

\*Note: All input rates nominal annual rates.

We assumed that the value of the land, based on residential use, would fluctuate in a manner similar to that of the industrial market (More specific assumptions can be made depending on the nature of a specific project, but this simple assumption was applied here). Changing the use can also entail the hurdle of rezoning costs. This includes not only actual rezoning costs, such as legal fees or infrastructure costs, but also the time spent rezoning and the potential degradation of value due to the mix of industrial and residential uses. We used 50% of the land value for rezoning costs as a basic assumption (the sensitivity of land value to the rezoning costs is presented in the subsequent section). The sellout-option value tree is shown in Table 3-4.



➤ Exercise Option Value

As described in Chapter II, the exercise option value consists of the residual land value from exercised property, and the value of options of later phases. The later phase option value cannot be captured until current phase's completion. To incorporate this lag, we added the option value of one year later ( $PV_t[\text{subsequent phase Opt}_{t+1}]$ ). Although the construction schedule for each phase is less than one year in this project (see Table 3-6), we think the one-year lag is reasonable because it takes a few months to prepare for construction after the decision-making process (i.e., 1 year = construction + preparation period).

➤ Wait Option Value

The wait option value is calculated by the following formula, introduced in Chapter II.

$$\begin{aligned}
 PV_t(\text{Wait}) &= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - RPcPV_t(\text{Wait})}{(1+r_f)} \\
 &= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - \frac{RPv}{(\%V_{t+1,up} - \%V_{t+1,down})} \times (C_{t+1,up} - C_{t+1,down})}{(1+r_f)} \\
 &= \frac{(pC_{t+1,up} + (1-p)C_{t+1,down}) - \frac{r_v - r_f}{((1+\sigma_v) - 1/(1+\sigma_v))} \times (C_{t+1,up} - C_{t+1,down})}{(1+r_f)}
 \end{aligned}$$

➤ Carrying Costs

Maintaining vacant land for future development has some carrying costs, such as property tax, insurance, and maintenance. These costs reduce the value of the waiting option. The larger

these costs, the sooner developers must exercise their option and the less valuable the option of waiting.

Property tax for land is usually 1%–2% of the value per year. Our analysis, therefore, uses 2% of the acquisition price as the total carrying cost. Carrying costs naturally decline as the project progresses, because carrying costs refer to land vacant at the time.

During the first four phases, these carrying costs were subtracted from each node, to account for the reality that carrying costs are incurred every year.

In the final phase, based on the Samuelson–McKean formula, carrying costs are roughly the equivalent of an increment to the underlying asset payout rate (yield), the parameter  $y_V$ . This parameter indicates the opportunity cost of *not* exercising the option, in terms of foregone cash flow. Foregoing a positive cash flow is the same as foregoing the elimination of a negative cash flow. Thus, if property taxes (and other carrying costs) are around 2% of the land value annually, and the land value is around 10% of the developed property value, we can add 0.2% (20 basis points) to the yield ( $y_V$ ) value ( $9.45\% = 9.25\% + 0.20\%$ ). This will make the option slightly less valuable, with a tendency to be exercised slightly sooner (lower hurdle benefit/cost ratio).

Accumulated carrying costs can theoretically create a negative option value. However, in reality, a landowner can give up the land for effectively zero cost. Therefore, if the maximum of the three options (sell, exercise, wait) is negative, the option value (abandonment) would equal zero.

➤ Other Assumptions

Table 3-5 shows basic assumptions in this study and Table 3-6 and 3-7 show the calculation of  $V_0$  and  $K_0$  for each building and phase. The project has two types of buildings: “Inventory” and “Build to Suit (BTS).” Inventory buildings have no tenants at the beginning of the construction; thus, the  $V_0$ /SF value for inventory buildings should be lower than that for BTS buildings, reflecting tenant risk (here, we assumed it is 50 basis points lower for BTS buildings in terms of Cap rate).

**Table 3-5 Basic Assumptions**

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length ( $T/n$ ) in yrs =	1.0000		
Riskfree interest rate (rf) =	6.00%	rf/period=	6.00%
Underlying Asset Total Return (rV) =	10.25%	rV/period =	10.25%
Underlying Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K growth rate (gK) =	2.00%	K gro/per=	2.00%
Time to build (periods)	0.58	V gro/per=	0.92%
Volatility ( $\sigma$ ) =	15%	$\sigma$ /period=	15.00%
		yK/period=	3.92%
		"p" real prob=	0.8306
		"u" =	1.1500
Residual Land Ratio (beginning)	100%	"d" =	0.8696
*Note: All input rates nominal annual rates.			



**Table 3-6 V (Built Property Value) and K (Development Costs) Values for Each Building**

Phase	Building	Building Type	Square Footage	Net Acreage	Cap Rate	Start	Completion	Time to build (months)	V <sub>0</sub>	V <sub>0</sub> /SF	Dev. Costs	Dev. Costs/SF
I	A	INV	537,600	26	9.25%	8/95	2/96	7	\$14,748,260	\$ 27.43	\$11,289,989	\$ 21.00
II	B	BTS	400,000	20	8.75%	11/96	5/97	7	\$11,600,460	\$ 29.00	\$ 8,400,289	"
	I	INV	356,000	19	9.25%	11/96	6/97	8	\$ 9,766,333	\$ 27.43	\$ 7,476,258	"
III	C	INV	200,000	10	9.25%	5/98	10/98	5	\$ 5,486,704	\$ 27.43	\$ 4,200,145	"
	F	BTS	400,000	21	8.75%	6/98	1/99	8	\$11,600,460	\$ 29.00	\$ 8,400,289	"
IV	D	INV	130,000	8	9.25%	6/99	2/00	9	\$ 3,566,358	\$ 27.43	\$ 2,730,094	"
	H	BTS	400,000	21	8.75%	6/99	1/00	8	\$11,600,460	\$ 29.00	\$ 8,400,289	"
V	E	INV	480,000	25	9.25%	7/00	3/01	9	\$13,168,089	\$ 27.43	\$10,080,347	"
	G	BTS	255,000	14	8.75%	8/00	5/01	10	\$ 7,395,293	\$ 29.00	\$ 5,355,185	"
	Total		3,158,600	164					\$88,932,415		\$66,332,886	

**Table 3-7 V (Built Property Value) and K (Development Costs) Values for Each Phase**

	Building	Square Footage	Net Acreage	Cumulative (Beginning)	Residual (Beginning)	Start	Completion	Time to build (Weighted avg.)
Phase I	A	537,600	26	-	100%	8/95	2/96	7
Phase II	B,I	756,000	39	16%	84%	11/96	6/97	7.5
Phase III	C,F	600,000	31	40%	60%	5/98	1/99	7.0
Phase IV	D,H	530,000	29	59%	41%	6/99	2/00	8.2
Phase V	E,G	735,000	39	76%	24%	7/00	5/01	9.3
Total		3,158,600	164					

	Building	Square Footage	V <sub>0</sub>	V <sub>0</sub> /SF	Infrastructure	Dev. Costs	K <sub>0</sub>	K <sub>0</sub> /SF	Carrying costs / yr
Phase I	A	537,600	\$ 14,748,260	\$ 27.43	\$ 1,376,000	\$ 11,289,989	\$ 12,665,989	\$ 23.56	\$ 103,481
Phase II	B,I	756,000	\$ 21,366,792	"	\$ 841,000	\$ 15,876,547	\$ 16,717,547	\$ 22.11	\$ 87,075
Phase III	C,F	600,000	\$ 17,087,163	"		\$ 12,600,434	\$ 12,600,434	\$ 21.00	\$ 62,467
Phase IV	D,H	530,000	\$ 15,166,817	"	\$ 841,000	\$ 11,130,384	\$ 11,971,384	\$ 22.59	\$ 42,907
Phase V	E,G	735,000	\$ 20,563,382	"		\$ 15,435,532	\$ 15,435,532	\$ 21.00	\$ 24,608
Total		3,158,600	\$ 88,932,415		\$ 3,058,000	\$ 66,332,886	\$ 69,390,886		\$ 103,481

The value of  $y_V$  is 9.25% based on the pro-forma at the beginning of the project. 10.25% is used as  $r_V$ , reflecting the growth expectation at the time. In addition, we tested the sensitivity of the calculated land value with regard to these parameters (see page 37).

In 1995, using the five-year Treasury note, we assumed 6% as the risk-free rate.

## **Valuation**

Based on these assumptions, we reevaluated the entire project using the real-option model. The calculated site value for this project is \$9.91 million, which is higher than the actual acquisition price of \$5.17 million by \$4.74 million.

The possible reasons for this difference are (1) the market value (acquisition price) assumed lower volatility than our model; (2) the market value did not take into account the option value; or (3) the sellout possibility is ignored.

In addition, the model gives indicators for whether the current market is optimal for starting a phase. As can be seen in Table 3-8, the projected property value at the time justifies the immediate exercise of Phase I.

**Table 3-8 Phase I Valuation Results**

This is a compound call option based on the Phase I assets and the Phase II–V option, expiring at the end of one year from Time 0.

**V tree (net of payout, "ex dividend" values):**

	<b>Year: 1995</b>	<b>1996</b>
Period ("t"):	0	1
Expected Values of V:		14,883
"down" moves ("i"):		
0	14,748	15,524
1		11,739

**K tree (development costs):**

Period ("t"):	0	1
"down" moves ("i"):		
0	12,666	12,919
1		12,919

**Phase Option Value:**

Period ("t"):	0	1
"down" moves ("i"):		
0	9,906	11,893
1		0

**Phase Optimal Exercise:**

Period ("t"):	0	1
"down" moves ("i"):		
0	exer	exer
1		abnd

exer: exercise  
 wait: wait  
 sell: sellout  
 abnd: abandonment

Table 3-9 shows the sensitivity of the calculated site value based on the real-option model.

Our basis assumption of 15% volatility gives \$9.91 million as a site value. If the volatility parameter is 10%, the estimated site value turns out to be \$8.65 million, which is still higher than the actual acquisition price by \$3.48 million. Volatility for industrial property is expected to be

lower than other types of properties because of its relatively shorter delivery lag; therefore, 10% seems more reasonable for this kind of Class A industrial project.

**Table 3-9 Sensitivity Analysis**

[Volatility( $\sigma$ )] vs [Built property cash yield]

Sensitivity Analysis				
		Sigma		
		10%	15%	20%
y <sub>v</sub>	8.25%	\$9,921	\$11,233	\$12,918
	9.25%	\$8,651	\$9,906	\$11,540
	10.25%	\$7,585	\$8,763	\$10,340

[Volatility] vs [Rezoning costs]

Sensitivity Analysis				
		Sigma		
		10%	15%	20%
Rezoning costs /total land value	0%	\$9,144	\$10,381	\$12,052
	50%	\$8,651	\$9,906	\$11,540
	100%	\$8,479	\$9,824	\$11,526

The model can back out the opportunity cost of capital associated with a project that has multiple phases. The following formula calculates the opportunity cost of capital as 29.66% at time 0, i.e., prior to building any phases. The project's implied investment risk is 5.55 times higher than the risk of the built property; in other words, the risk premium for the development project is 5.55 higher than that for built property.

$$PV_0 [\textit{Option Value}] = \frac{\textit{Expected Option Value}}{1 + OCC} = \frac{pC_{1,up} + (1 - p)C_{1,down}}{1 + OCC}$$

$$\Downarrow$$

$$1 + OCC = \frac{\textit{Expected Option Value}}{PV_0 [\textit{Option Value}]} = \frac{pC_{1,up} + (1 - p)C_{1,down}}{C_0} = 1.296$$

$$\frac{\textit{Risk premium for Option (} RP_C \text{)}}{\textit{Risk premium for Option (} RP_V \text{)}} = \frac{29.6\% - 6\%}{10.25\% - 6\%} = 5.55$$

## Review of the Actual Results

### ➤ Actual Project History

We have collected data regarding the actual history of the project, so we can compare our valuation with the up-to-date figures. Since its ground-breaking start in 1995, the project has progressed steadily. Nine buildings have been completed so far, and only one site is left for construction. However, this does not mean the project exactly followed the initial plan and schedule. As shown in Table 3-10, the size of the buildings is largely according to the initial plan, but the construction schedule has changed drastically. This happened because the developer modified the order of the construction to accommodate the needs for buildings of each size.

**Table 3-10 Comparison of Initial Plan and Exercise**

Phase		Phase I		Phase II		Phase III		Phase IV		Phase V	
Building		A	B	I	C	F	D	H	E	G	
<b>Square Footage</b>											
	Initial plan	537,600	400,000	356,000	200,000	400,000	130,000	400,000	480,000	255,000	
	Exercise	537,600	440,000		202,361	485,745	143,017	252,776	502,716	292,800	
<b>Start of Construction</b>											
	Initial Plan	Aug-95	Nov-96	Nov-96	May-98	Jun-98	Jun-99	Jun-99	Jul-00	Aug-00	
	Exercise	Aug-95	Jul-96		Jun-96	Jan-98	Jan-97	Nov-98	Jan-97	Jun-03	
<b>V</b>											
	Initial (V <sub>0</sub> )	\$ 14,748,260	\$ 11,600,460	\$ 9,766,333	\$ 5,486,704	\$ 11,600,460	\$ 3,566,358	\$ 11,600,460	\$ 13,168,089	\$ 7,395,293	
	Exercise (V <sub>t</sub> )	\$ 14,897,232	\$ 12,662,043		\$ 6,235,384	\$ 14,563,021	\$ 4,754,411	\$ 8,651,564	\$ 14,550,793	\$ 10,085,333	
	Initial (V <sub>0</sub> /SF)	\$ 27.4	\$ 29.0	\$ 27.4	\$ 27.4	\$ 29.0	\$ 27.4	\$ 29.0	\$ 27.4	\$ 29.0	
	Exercise (V <sub>t</sub> /SF)	\$ 27.7	\$ 28.8		\$ 30.8	\$ 30.0	\$ 33.2	\$ 34.2	\$ 28.9	\$ 34.4	
<b>K</b>											
	Initial (K <sub>0</sub> )	\$ 12,665,989	\$ 8,845,263	\$ 7,872,284	\$ 4,200,145	\$ 8,400,289	\$ 2,927,849	\$ 9,043,535	\$ 10,080,347	\$ 5,355,185	
	Exercise (K <sub>t</sub> )	\$ 12,665,989	\$ 9,688,729		\$ 4,790,139	\$ 10,747,610	\$ 3,947,854	\$ 7,133,174	\$ 11,058,317	\$ 7,437,707	
	Initial (K <sub>0</sub> /SF)	\$ 23.6	\$ 22.1		\$ 21.0	\$ 21.0	\$ 22.5	\$ 22.6	\$ 21.0	\$ 21.0	
	Exercise (K <sub>t</sub> /SF)	\$ 23.6	\$ 22.0		\$ 23.7	\$ 22.1	\$ 27.6	\$ 28.2	\$ 22.0	\$ 25.4	

\*A part of the site for Building I was sold as land, while the rest of it remains vacant.

We have the proposals for the commencement of building construction, which includes the estimated underlying asset value ( $V_t$ ) and construction costs ( $K_t$ ) at the time of the exercise (they are different from “ex-post numbers,” actual sales price and construction costs).

We calculated the IRR (Internal Rate of Return) for the project based on the projected data at the beginning of the project, hereinafter called “Initial,” and again based on up-to-date projected values, calculated it at the time of the exercise for each building, called “Exercise” (see Table 3-11). “Exercise IRR” is 16.24%, which is higher than “Initial IRR,” 14.99%. This shows that the market turned out to be favorable for the project.

Almost all buildings have been completed and sold. The actual sales prices ended up being mostly higher than the exercise prices stated above. This appears to do as much with the conservative projections used for the proposal as it does with improvement in the market during the construction.



**Table 3-11 Comparison of IRR**

*Initial Plan (\$1,000)*

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	Total
Phase I	(11,290)	14,883									3,593
Phase II		(16,194)	21,760								5,566
Phase III				(13,109)	17,721						4,612
Phase IV					(11,812)	15,874					4,062
Phase V						(16,708)	21,719				5,011
Land	(5,174)										(5,174)
Infrastructure	(1,376)	(841)			(841)						(3,058)
Total	(17,840)	(2,152)	21,760	(13,109)	5,069	(834)	21,719				14,612
IRR	14.99%										

*Exercise (Up-to-date projected value) (\$1,000)*

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	Total
Phase I	(12,666)	15,034									2,368
Phase II		(14,479)	19,070								4,592
Phase III			(15,006)	19,482							4,476
Phase IV				(17,881)	23,427						5,546
Phase V									(16,197)	20,779	4,581
Land	(5,174)										(5,174)
Infrastructure											-
Total	(17,840)	555	4,064	1,601	23,427	-	-	-	(16,197)	20,779	16,388
IRR	16.24%										

\*Infrastructure costs were included in construction costs for each building.

➤ Review of the Exercise Price and Timing

For each phase, developers face the decision of whether to exercise the option or wait.

Making that decision requires understanding if the projected value, based on the current market, justifies moving forward. One of the benefits of the real-option model is that it can give important insights into the optimal timing of construction.

We reviewed the exercise prices when the decision was made. The method is fairly simple.

The option value tree in the previous study starts in 1995. Now, we need to move the starting point to the beginning of each phase that we wish to analyze in this simulation. We can ignore the phases before the targeted phase and calculate the compound option value for all the phases that follow.

For example, if we want to examine Phase IV in 1998, we need to capture the compound real-option value of two phases. This puts the starting point of the trees at 1998; therefore,  $V_0$  ( $=V_{1998}$ ) and  $K_0$  ( $=K_{1998}$ ) should be updated based on current information.

**Table 3-12 V and K Values Projected in 1998 (\$1,000)**

	$V_{1998}$	$K_{1998}$
<b>Phase IV</b>	\$23,215	\$17,881
<b>Phase V</b>	\$20,394	\$15,708

Other parameters, such as the risk free rate or  $y_V$ , could change over time with the market.

Table 3-13 shows the results based on the above values. Our calculations showed that the value of exercising the option in 1999 was higher than the other options, which means that our analysis justified the decision to commence Phase IV. Thus, this procedure could be a helpful tool for decision making in multi-phased projects such as this.

**Table 3-13**

***Phase IV and V Option Value***

Year:	<u>1998</u>	<u>1999</u>	<u>2000</u>	<u>2001</u>	<u>2002</u>	<u>2003</u>
Period ("t"):	0	1	2	3	4	5
"down" moves ("i"):	<i>Phase Option Value:</i>					
0	7,370	8,650	10,072	11,601	13,247	15,020
1		728	1,137	1,756	2,653	3,660
2			140	182	227	274
3				0	0	9
4					0	0
5						0

***Phase Optimal Exercise***

Year:	<u>1998</u>	<u>1999</u>	<u>2000</u>	<u>2001</u>	<u>2002</u>	<u>2003</u>
Period ("t"):	0	1	2	3	4	5
"down" moves ("i"):	<i>Phase Optimal exercise:</i>					
0	exer	exer	exer	exer	exer	exer
1		wait	wait	exer	exer	exer
2			sell	sell	sell	sell
3				abnd	abnd	sell
4					abnd	abnd
5						abnd

## Chapter 4 Conclusions

- We found that the real-option model is fairly effective when applied to a large-scale industrial-development project with multiple phases.
- Recalculated land value based on the real-option model was higher than the actual acquisition price in this case. Possible reasons are (1) implied volatility for the project is lower than our assumption; (2) the market does not fully incorporate the waiting option value; and (3) the possibility of selling the land for another use could be ignored.
- The model requires some modifications for application to specific projects. For example, we incorporated the sellout option value assuming single-family residential use. In addition, we took into account carrying costs, thereby reducing the value of the options.
- The model proves useful in deciding the optimal timing for the exercise of each phase. In this case, our model found that each phase was properly exercised.
- The model can help the design/decision process quantitatively, by providing analysis of alternative phasing schemes.
- Also, the model is useful to evaluate “development right.” A Developer sometimes obtains the right that they can develop the sight anytime until the right expires, instead of purchasing the land itself. The model can capture the value of “development right” just by modifying time of expiration in the model.

**Exhibits**

**Exhibit I-1 Inputs (Phase I)**

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length (T/n) in yrs =	1.0000	$\eta$ =	6.29
Riskfree interest rate (rf) =	6.00%	rf/period=	6.00%
Underl Asset Total Return (rV) =	10.25%	rV/period =	10.25%
Underl Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K growth rate (gK) =	2.00%	K gro/per=	2.00%
Time to build (periods)	0.58	V gro/per=	0.92%
Volatility ( $\sigma$ ) =	15.00%	$\sigma$ /period=	15.00%
V(initial) =	14,748	yK/period=	3.92%
K(initial) =	12,666	"p" real prob=	0.8306
Land Carrying Costs (per yr)	103	"u" =	1.1500
Residual Land Ratio (beginning)	100%	"d" =	0.8696
*Note: All input rates nominal annual rates.		<b>Option Val=</b>	<b>9,906</b>

**Exhibit I-2 V Value Tree (Phase I)**

Year:		1995	1996
Period ("t"):		0	1
Expected Values of V: 14,883			
("i"):			
0		14,748	15,524
1			11,739

**Exhibit I-3 K Value Tree (Phase I)**

Year:		1995	1996
Period ("t"):		0	1
("i"):			
0		12,666	12,919
1			12,919

**Exhibit I-4 Option Value Tree (Phase I)**

Year:		1995	1996
Period ("t"):		0	1
("i"):			
0		9,906	11,893
1			0

**Exhibit I-5 Phase Optimal Exercise (Phase I)**

Year:		1995	1996
Period ("t"):		0	1
("i"):			
0		exer	exer
1			abnd

**Exhibit II-1 Inputs (Phase II)**

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length (T/n) in yrs =	1.0000	$\eta$ =	6.29
Riskfree interest rate (rf) =	6.00%	rf/period=	6.00%
Underl Asset Total Return (rV) =	10.25%	rV/period =	10.25%
Underl Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K growth rate (gK) =	2.00%	K gro/per=	2.00%
Time to build (periods)	0.62	V gro/per=	0.92%
Volatility ( $\sigma$ ) =	15.00%	$\sigma$ /period=	15.00%
V(initial) =	21,367	yK/period=	3.92%
K(initial) =	16,718	"p" real prob=	0.8306
Land Carrying Costs	87	"u" =	1.1500
Residual Land Ratio (beginning)	84%	"d" =	0.8696
*Note: All input rates nominal annual rates.		<b>Option Val=</b>	<b>10,534</b>











**Exhibit III-1 Inputs (Phase III)**

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length (T/n) in yrs =	1.0000	$\eta$ =	6.29
Riskfree interest rate (rf) =	6.00%	rf/period=	6.00%
Underl Asset Total Return (rV) =	11.25%	rV/period =	11.25%
Underl Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K growth rate (gK) =	2.00%	K gro/per=	2.00%
Time to build (periods)	0.58	V gro/per=	1.83%
Volatility ( $\sigma$ ) =	15.00%	$\sigma$ /period=	15.00%
V(initial) =	17,087	yK/period=	3.92%
K(initial) =	12,600	"p" real prob=	0.8663
Land Carrying Costs	62	"u" =	1.1500
Residual Land Ratio (beginning)	60%	"d" =	0.8696
*Note: All input rates nominal annual rates.		<b>Option Val=</b>	<b>8,484</b>











**Exhibit IV-1 Inputs (Phase IV)**

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length (T/n) in yrs =	1.0000	$\eta$ =	6.29
Riskfree interest rate (rf) =	6.00%	rf/period=	6.00%
Underl Asset Total Return (rV) =	11.25%	rV/period =	11.25%
Underl Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K growth rate (gK) =	2.00%	K gro/per=	2.00%
Time to build (periods)	0.69	V gro/per=	1.83%
Volatility ( $\sigma$ ) =	15.00%	$\sigma$ /period=	15.00%
V(initial) =	15,167	yK/period=	3.92%
K(initial) =	11,971	"p" real prob=	0.8663
Land Carrying Costs	43	"u" =	1.1500
Residual Land Ratio (beginning)	41%	"d" =	0.8696
*Note: All input rates nominal annual rates.		<b>Option Val=</b>	<b>5,864</b>









**Exhibit V-1 Inputs (Phase V)**

<b>Enter (input)*:</b>		<b>Resulting (output):</b>	
Period length (T/n) in yrs =	1.0000	$\eta$ =	6.45
Riskfree interest rate (rf) =	6.00%	rf/period=	6.00%
Underl Asset Total Return (rV) =	10.25%	rV/period =	10.25%
Underl Asset Cash yield (yV) =	9.25%	yV/period=	9.25%
K growth rate (gK) =	2.00%	K gro/per=	2.00%
Time to build (periods)	0.78	V gro/per=	0.92%
Volatility ( $\sigma$ ) =	15.00%	$\sigma$ /period=	15.00%
V(initial) =	20,563	yK/period=	3.92%
K(initial) =	15,436	"p" real prob=	0.8306
Land Carrying Costs	25	"u" =	1.1500
Residual Land Ratio (beginning)	24%	"d" =	0.8696
*Note: All input rates nominal annual rates.		<b>Option Val=</b>	<b>3,969</b>











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