

PRICING POLICIES IN THE LINEN SUPPLY INDUSTRY

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ABSTRACT

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Linen supplies use a number of different types of pricing systems. Some of these systems charge customers a single price for each piece of linen soiled. Other systems charge customers in two parts: Customers pay a price per piece of linen soiled, and also a price for the right to rent linen at those piece rate prices.

As long as a linen supply has some degree of monopoly power, it can earn larger profits by using a two-part pricing system rather than charging a single price for the linen it provides. What form of two part pricing system is best for a linen supply to use depends on the market in which the linen supply operates. For markets which include a sizeable segment of large, very price sensitive customers, it will be best for the linen supply to charge customers a right to rent linen price in the form of a delivery fee which is the same for all customers. For markets in which large customers are not more price sensitive than smaller customers, a linen supply should charge a right to rent linen price in the form of an inventory fee. This inventory fee is charged for each piece of linen the linen supply holds to service each customer and hence, larger customers will be charged larger right to rent linen prices.

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## CHAPTER I

### THE LINEN SUPPLY INDUSTRY

#### A. What is a Linen Supply?

A linen supply provides customers with a variety of clean linen by way of a regular system of pick up and delivery. A linen supply, unlike a laundry, owns most of the linens it washes and in effect leases these linens to its customers. Deliveries are made as frequently as once a day and as infrequently as once a month. Typically, the linen supply plant operates on a five day cycle. Each day, each delivery driver runs a different part of his route; that is, each day of the week the driver delivers clean and picks up dirty linen from a different string of customers. The dirty linen picked up by the driver on the first day of the cycle is counted and sorted by workers inside the linen plant on the second day of the cycle. On the third day the linen is washed and dried in huge machines made expressly for industrial laundries. On the fourth day, the linen is pressed, folded and bundled into packages designated for particular customers. Again, large specialized equipment is normally used to process the linen. On the final day of the cycle the packaged linen is separated by route and stacked in huge baskets, which the delivery men wheel to their trucks for loading. Thus, on the next morning, the cycle is ready to begin again.

There are over four hundred types of linens which a linen

supply might offer its customers, although typically a full-service linen supply will provide only seventy-five to a hundred different types and colors of linen. Exactly what products are offered varies, of course, among linen supplies, but the following table does provide a general picture of the product lines carried by a linen supply plant.

<u>General Product Type</u>	<u># of Products</u>	<u>Colors Available</u>	<u>Typical Customers</u>
Uniforms	10	5	Heavy industrial companies; restaurants; automobile-related companies
Flat towels	10	1	Restaurants; doctors' offices; gyms; country clubs; garages; bars
Bed linens	3	1	Motels; hotels
Table cloths	6	4	Restaurants
Napkins	2	3	Restaurants
Aprons	3	2	Restaurants; bakeries; butcher shops; print shops
Mops	3	1	Various
Entryway Latex Mats	2	1	Various
Continuous towels	2	2	Public restrooms in restaurants, bars, gyms, offices
Oven mitts	2	1	Restaurants
Patient gowns	2	1	Doctors' offices
Paper towels	1	1	Various
Toilet paper	1	1	Various

The percent of total revenues associated with each general product type varies so widely among linen supplies as to make it impossible to associate a typical percent of total revenue figure with each general product type. Nevertheless, we may observe that for nearly all linen supplies, uniform rentals are a major source of revenue. In fact, some linen supplies specialize in providing uniform service. For many, continuous towels are a very important product, and yet some linen supplies do not carry continuous towels at all. For a few, bed linens are a major revenue producer, although many linen supplies have tried to keep their volume of bed linens relatively low due to the rather high shrinkage associated with these items.

A linen supply's costs, other than capital costs, are of four basic types: processing costs, which are those costs directly associated with washing, pressing or dry cleaning linens; inventory costs, or the costs of maintaining an inventory of linen sufficient to meet customer needs; delivery costs, which are those costs associated with maintaining the linen company's fleet of trucks and staffing them with delivery drivers; and finally, general and administrative costs. Although there is considerable variation among linen supplies, the following distribution of these costs would be fairly common in the industry:

<u>Type of Cost</u>	<u>% of Total Cost</u>
Production	25%
Textile	30%
Delivery	15%
General & Administrative	30%

The key person in a linen supply plant is the plant manager. It is his job to coordinate the sales force, delivery department and production department. It is his responsibility to set up prices for each of the linen supply's products, although in practice he may share this responsibility with his sales manager and delivery department manager.

Most plant managers are college educated, although very few hold advanced degrees in business or economics. Typically, the plant manager has worked for a linen supply for a number of years and has spent time in both the the production department and in the delivery department. He is generally well-acquainted with the market conditions in the area served by his plant, and in fact may have considerable knowledge about which of his customers are sensitive to the price of the products his plant provides and which are more concerned with a product's quality and appearance.

#### B. Industry Structure

Large companies in the linen supply industry generally consist of chains of linen supplies spread over wide geographical areas with plants in many smaller cities as well as in large metropolitan areas. The five largest such linen supply companies in the United States are:

1. National Service Industries. This company owns approximately 40 linen supply plants in the United States. It is publicly traded and has an approximate

(1979) market value of \$203,000,000.

2. Steiner Corporation. Steiner owns thirty linen supplies in the United States and an additional thirty plants outside the United States, making it the largest company in the industry world-wide. It is privately held.
3. American Linen Supply Co.. This company operates twenty-seven plants in the United States. It also is privately held.
4. F. W. Means. Means owns 28 plants in the United States. It is publicly traded and has an approximate market value of \$28,000,000.
5. Workwear. This company owns 19 plants in the United States. It is publicly traded and has an approximate market value of \$17,500,000.

It would be a mistake to think that any of these large linen supply companies have, on the level of the firm as a whole, any monopoly power. The market for linen supply services is essentially local. Thus, particular plants belonging to these large linen supply companies may have some monopoly power, while others may operate in very competitive markets.

Nearly all cities in the United States are serviced by at least one linen supply. Large cities such as New York or Chicago may be serviced by as many as fifteen or twenty large

linen supplies and a good number of smaller ones. In some cases, plants located as far as seventy-five miles away, will send a delivery driver into such metropolitan centers. Competition in the big cities is generally rather fierce, and hence managers are left with little discretion as to where to set prices. In general, no one linen supply is able to secure more than a fairly small market share.

In smaller cities competition among linen supplies is much less severe. This is particularly true in small western cities such as Casper, Wyoming (population: 40,000) or Grand Junction, Colorado (population: 25,000) which are so isolated from other cities as to prevent linen supplies located elsewhere from invading the local market. In such cities, there is unlikely to be more than three linen supplies, and hence a large linen supply would enjoy a near monopoly position. In Casper, Wyoming, for example, one linen supply plant controls 60% of the market for linen supply services, with the remaining 40% divided among two or three smaller linen supplies.

The monopoly positions of these small city linen supplies are protected by three fairly formidable barriers to entry:

1. There is a lot to know in order to run a linen supply. There are more and less efficient ways to lay out a plant and to organize a sales force and delivery department. Certain mixes of soap, starch and hot water are much harder on textiles than others.

Finally, certain laundry equipment is more efficient than others, and in fact some of a large linen supply's equipment must be specially designed and built to order.

2. Building a large linen supply requires a considerable amount of capital--\$1,000,000 at least.
3. One could start up a small linen supply for considerably less than a million dollars, of course, but there are significant economies of scale associated with a large linen supply plant. The large industrial type machines are much more efficient than smaller washers and dryers. More importantly, many of a linen supply's customers demand a large variety of products. Without special equipment and a large investment in textiles, a smaller linen supply cannot process the variety of products required by these customers and hence, cannot efficiently compete for the business of these customers.

#### C. Industry Pricing Policies

There are four pricing systems which have been used in linen supplies. Very commonly, one linen supply may use more than one pricing system. Most linen supplies, for example, do not use the same pricing system for uniform rentals as they use for flat goods--towels, bed linens, tablecloths,

etc. Many linen supplies, in fact, use different pricing systems for the same type of product, depending on the size and type of customer. For all types of linen supply products, volume discounts are common, which for very large customers may be as high as 20% off list prices. The four most commonly used pricing systems are as follows:

1. Piece-Rate System. Plants using this pricing system charge customers a fee for each item of linen soiled by the customer. The fee varies according to the type of linen used. Thus, for example, a restaurant which used 100 napkins and 25 tablecloths during a particular billing period would be charged as follows:

<u>Quantity</u>	<u>Product</u>		<u>Piece Rate Price</u>		<u>Total</u>
100	Napkins (white, regular)	@	\$ .08	=	\$8.00
25	Tablecloths (white, 64 x 64)	@	.43	=	<u>10.75</u>
			<u>Amount Billed:</u>		<u>\$18.75</u>

The piece-rate system was formerly the standard pricing system in the industry. However, in the late 50's as managers became more concerned with increased costs in the industry, they determined that there was a critical customer size: For customers generating revenues less than this critical size of customer, the costs of billing, keeping records and delivering to that customer exceeded the revenue brought in by that customer.



It was simply not profitable to rent two aprons a month @ 35¢ each to a hamburger stand or a Mom-and-Pop grocery store. Hence, plant managers began to change from the piece rate system to the pricing systems discussed below.

2. Piece-Rate with a Minimum. The simplest solution to the problem of critical customer size was to augment the piece rate system with a minimum charge for small customers. Under this system, customers are billed the greater of a minimum monthly charge, which is the same for all customers, or what the customer would be charged under a pure Piece-Rate system. For example, a linen supply may charge 25¢ for each Turkish towel and 16¢ for each massage towel soiled by the customer. Should the linen supply have a minimum charge of \$8 per month and should a particular customer soil only 20 Turkish towels and 10 massage towels, the customer would be charged the \$8 minimum rather than the \$6.60 he would pay on a pure Piece-Rate basis.

<u>Quantity</u>	<u>Product</u>		<u>Piece-Rate Price</u>		<u>Total</u>
20	Turkish	@	\$ .25	=	\$5.00
10	Massage	@	.16	=	<u>1.60</u>
					\$6.60
				<u>Minimum</u>	\$8.00
				<u>Amount Billed</u>	<u>\$8.00</u>

The Piece-Rate with a Minimum system is very common in linen supplies. Typically, the minimum monthly charge is set by the plant manager at what he believes to be the critical size of customer revenue for his plant. That is, the minimum monthly charge is set at that point for which the revenue which would be obtained from a certain size of customer is just equal to the cost of providing linen service to that customer.<sup>1</sup>

3. Flat Rate. The Flat Rate system is another simple solution to the problem of critical customer size. Under this system, the customer is charged the same fixed amount each billing period. This amount is based on the volume of linen the customer is expected to use rather than the volume of linen he actually does use each period. If a potential customer is not expected to generate revenues of the critical size, his business is simply not accepted.

Suppose, for example, that a bakery contracts with a linen supply to provide it with 10 white aprons per week and agrees to be billed on a flat-rate basis. The bakery might during one month soil 40 aprons and be billed \$14 for that month. The following month, due to vacations or illness, the bakery may use only 35 aprons. Nonetheless, its linen bill for the month would again be \$14.-

The flat rate system is very common among linen supplies, largely because it greatly simplifies bookkeeping and profit planning for the linen supply. Also, as linen suppliers

are quick to point out, it assures customers that their linen bills will be exactly as budgeted.

#### 4. Inventory Charge Plus Laundering Fee

Some linen supplies charge customers according to the amount of linen inventory that the linen supply must purchase and maintain in order to service that customer. This inventory charge is augmented by a relatively small laundering fee which the linen supply charges customers to pick up soiled linen, process it and return the linen clean to the customer. Thus, the linen supply, in effect, rents to the customer an inventory of linen and charges the customer for maintaining that linen in a clean and usable state.

There are standard rules of thumb in the industry to determine the quantity of linen that should be held to service a particular customer. Basically, these rules relate the appropriate level of inventory to hold to the quantity of linen the customer soils per month, since, obviously, the greater the quantity of linen a customer soils, the larger will be the inventory it is appropriate to hold for that customer.

To see how the Inventory Charge Plus Laundering Fee pricing system works, let us consider as an example an automotive repair shop employing ten mechanics each of whom normally uses three coveralls per week. The repair shop receives its delivery every week and thus, for every coverall soiled by the customer per week the linen supply believes it must maintain

an inventory of three coveralls. The monthly inventory charge for this customer might be calculated as follows.

<u># of Coveralls Expected Soiled x Per Week</u>		<u>Inventory Multiple</u>	x	<u>Weekly Inventory Charge per Coverall</u>	=	<u>Weekly Inventory Charge</u>
30		3		\$.39		\$35.10

		<u>Weekly Inventory Charge</u>		<u>Monthly Inventory Charge</u>
4	x	\$35.10	=	\$140.40

Suppose that each week during the month the automotive repair shop soiled only 28 coveralls. Then the repair shop would have soiled 112 coveralls during the month, and would be charged a laundering fee as follows:

<u># of Coveralls Soiled</u>	x	<u>Laundering Fee Per Coverall</u>	=	<u>Laundering Fee</u>
112		\$1.10		\$123.20

The total amount billed the repair shop would then be:

<u>Inventory Charge</u>	+	<u>Laundering Fee</u>	=	<u>Amount Billed</u>
\$140.40		\$123.20		\$263.60

The Inventory Charge plus Laundering Fee pricing system is relatively new to the linen supply industry. It was introduced in Portland, Oregon in the 1960's. At present, the system has gained relatively little acceptance. Those plants which do use it, normally employ it only for large uniform accounts, seasonal flat linen accounts or accounts which require the linen

supply to purchase linen it would not ordinarily buy.

D. Cost-Plus Pricing

Whichever pricing system is used, the linen supply plant manager must at some point choose a particular price to charge for each type of linen his plant provides. In very competitive markets, the price is largely determined for the plant manager by prevailing market conditions: The manager can only charge as much as his competitors do. In less competitive markets, however, managers have considerable discretion in setting prices and it is important to understand how this decision is typically made in the linen supply industry.

When a plant manager does have discretion in setting prices, by far the most common way for a manager to select a particular level of prices is for the manager to choose some target rate of return and to set prices so as to achieve some target rate of return. The type of rate of return used may be return on inventory investment, return on the book value of average total assets, or return on the book value of average equity. The target rate of return selected, of course, depends on particular market conditions.

An industry journal provides the following example of how to set piece-rate prices to achieve a target return on inventory investment:<sup>2</sup>

If you find from cost accounting that an item costs approximately 17 cents, for example (i.e., full cost-production, distribution and administrative costs included), and you estimate that you get an average of 17 servings per year and the net purchase cost of the item is a dollar then--

- o Yearly return required is  $30\% \times \$1.00 = 30\text{¢}$
- o Profit needed per serving is  $\frac{30}{17 \text{ servings}} = 1.8\text{¢}$
- o Price that should be charged to yield a 30% return on textile investment is 17¢ (your cost) + 1.8¢ (profit required per serving) = .19¢ (rounded off)

This may not be the price you wish to charge because of your market but this information can be extremely valuable.

The important fact to notice about rate of return methods for setting prices, is that higher rates of return do not necessarily mean higher profits for the linen supply. A linen supply earning a relatively high rate of return may do a smaller volume of business than it would if its target rate of return were lower and hence earn smaller total profit than it would at a lower rate of return. Consider for example two linen supplies, A and B, which operate in the same market and have identical costs. Both A and B carry Turkish towels which, as in the example above, cost an average 17¢ to process and \$1.00 to purchase. Each linen supply expects to get 17 servings per year, or in other words, expects to process the towel 17 times before it wears out, is stolen or lost.

Plant A has a target return of 30% per year and hence charges 19 cents per towel on a piece-rate basis. (Calculations are identical to those in the example.) At 19 cents per towel, the linen supply does a volume of 22,000 Turkish towels over the year and earns a profit of \$400 during the year.

<u>Volume</u>		<u>Price</u>	=	<u>Total Revenue</u>
20,000	x	19¢	=	\$3,800
<u>Volume</u>		<u>Average Cost</u>	=	<u>Total Cost</u>
20,000	x	17¢	=	\$3,400
				\$3,800
				- 3,400
				<hr/>
Profit:				\$ 400

Plant B has a target return of 15% per year. The price it charges is calculated as follows:

- o Yearly return required is  $15\% \times \$1.00 = .15¢$
- o Profit needed per serving is  $\frac{.15}{17 \text{ servings}} = .09$
- o Price that should be charged to yield a 15% return on inventory investment is  $.17 + .09 = \$ .18$ , rounded off

At 18 cents per towel, Plant B is able to steal quite a few customers from Plant A and hence does a volume of 50,000 Turkish towels during the year. It earns a profit of \$500, as opposed to Plant A's profit of \$400.

<u>Volume</u>		<u>Price</u>	=	<u>Total Revenue</u>
50,000	x	18¢	=	\$9,000
<u>Volume</u>		<u>Average Cost</u>	=	\$8,500
50,000	x	17¢	=	\$8,500
				\$9,000
				- 8,000
				<hr/>
				\$ 500

The rate of return methods of setting price are, of course, versions of cost-plus pricing: The linen supply charges so as to insure a profit margin--expressed as a percentage rate of return --over its total costs of providing linen service. As we have just seen, these cost-plus pricing methods need not lead a linen supply to maximize profits. Why then, it is reasonable to ask, are rate of return methods for setting prices so common in the industry?

The answer, it seems to me, is that cost-plus pricing is very easy to use. The linen supply manager, after all, must set prices for each of the seventy-five to a hundred different types and colors of products his plant provides, as well as attend to the hundreds of other aspects of running the linen supply. Any methods of setting prices which require market surveys and detailed data analysis, however theoretically acceptable those methods may be, are entirely useless to the plant manager. He simply does not have the time, nor the



resources to use them. Unless some more sophisticated method for setting price levels can be made easy and quick to use, the linen supply manager is better off using his cost-plus pricing tempered by his judgement of what the market will bear.

We shall return to this theme in Chapter V where I shall present what I believe to be a workable and sophisticated method for setting prices at the appropriate levels. In Chapter II and Chapter III, however, I shall take up the topic of which sort of pricing system--Piece Rate, Piece-Rate with a Minimum, Flat-Rate, or Inventory Plus Laundering Fee--is best for linen supplies to use.

## CHAPTER ONE

## FOOTNOTES

<sup>1</sup>It has been my experience that "the cost of providing linen service" is intended by most managers to mean the average total cost rather than the marginal cost.

<sup>2</sup>Laurason, Jim. "How to Calculate and Use Return on Textile Investment," Linen Supply News, November 1976, p. 40.

## CHAPTER II

### ONE PART PRICING SYSTEMS

In this chapter, I shall begin to present economic analyses of the various pricing systems used by linen supplies. Specifically, this chapter has four sections. In Section (A) I shall introduce several important economic concepts and in Section (B), use those concepts to define the maximum profit that can be earned in the short run<sup>1</sup> by a given firm operating in a given market. As we shall see, these maximum profits may be obtained under a discriminating one-part pricing system. In Section (C), I shall examine how profits may be maximized under a non-discriminating one-part pricing system and finally in Section (D), I shall discuss to what extent the Piece Rate pricing system may be explained in terms of a non-discriminating one-part pricing system.

#### A. Economic Concepts

Generally speaking, there are just two elements which determine the maximum short term profit that can be earned by a firm: what customers are willing to pay for the firm's output and what it costs the firm to produce that output.

What customers are willing to pay for a firm's output varies, of course, from firm to firm depending on what the firm is selling, the tastes of its customers, the incomes of those customers, the availability of substitutes for

the firm's output and a myriad of other factors. Nonetheless, for particular firms operating at a specific time in a particular market, there is a unique relationship between the price the firm charges for its output and the amount of output the firm can sell. This relationship between price and the amount of output a firm can sell is called the firm's demand curve. More formally, a demand curve is the relation between the price charged per-unit of output  $P$  and the total quantity produced  $Q$ , such that the pair  $(P_i, Q_i)$  is on a firm's demand curve if and only if the maximum price the firm can charge for its  $Q_i^{\text{th}}$ , or last, unit of output is  $P_i$ .

Demand curves may have various shapes. In a purely competitive market, the demand curve faced by a particular firm will be a flat, horizontal line. This shape reflects the fact that at prices above the going market price, the firm will sell no output. In a monopolistic market, the demand curve faced by the monopolistic firm will have a negative slope; that is, the demand curve will slope downward to the right. This shape of demand curve reflects the fact that the amount of output the monopolistic firm can sell will vary depending on the price it charges for its output: at high prices the firm's customers will purchase fewer units of output than they would purchase at low prices. In a very few markets, the demand curve faced by a firm may have a portion which is positively sloped reflecting the fact

that for a certain range of prices customers value the firm's output more highly the higher the price is.

For a given firm, the total cost that firm incurs depends only on the level of output the firm produces. Normally, the greater the output the firm produces the higher will be its total costs. For some firms, as volume increases total costs will increase at a constant rate. For other firms, as volume increases total costs will increase at varying rates. Put another way, for some firms the cost of producing an additional unit of output remains the same as the level of total output increases, while for other firms, the cost of producing an additional unit of output varies as the level of output increases. The cost of an additional unit of output, or equivalently, the rate of change of a firm's total costs, is called the marginal cost of production.

There is an interesting interpretation of a firm's marginal cost function. Since any firm will produce its  $Q^{\text{th}}$  unit of output only if the price it can receive for that unit of output is greater than or at least equal to its cost of producing that unit, the minimum price which the firm must receive for it to produce its  $Q^{\text{th}}$  unit of output is the marginal cost of producing the  $Q^{\text{th}}$  unit. Thus, for example, a firm will produce its  $100^{\text{th}}$  unit of output only if the price it will receive for that  $100^{\text{th}}$  unit is greater than or equal to the firm's cost of producing the 100th unit--

i.e., the marginal cost function of the firms evaluated at the 100<sup>th</sup> unit.

Thus, it is apparent that a firm's marginal cost function places a lower bound on the price at which the firm would be willing to sell each additional unit. More specifically, if  $P_i$  is the price charged per-unit, and the total quantity produced is  $Q_i$ , then if the pair  $(P_i, Q_i)$  is on a firm's marginal cost curve, then the firm will be willing to produce the  $Q_i^{\text{th}}$  unit of output only if the price it receives for that unit is greater than or equal to  $P_i$ .

B. The Maximum Profit Achievable by a Given Firm Operating in a Given Market

We have now identified two important ways in which price and the quantity of output a firm produces are related. The demand curve facing a firm gives, for each level of output the firm might produce, the maximum price per unit of output at which the firm could sell all of its output. A firm's marginal cost function gives, for each level of output the firm might produce, the cost of producing one additional unit of output.

With this background, the rule for a given firm to maximize profits by operating in a given market is very simple:

Rule For Maximizing Profit Under a Discriminating One-Part Pricing System

First, charge each customer the maximum amount he is willing to pay for each unit of output he purchases.

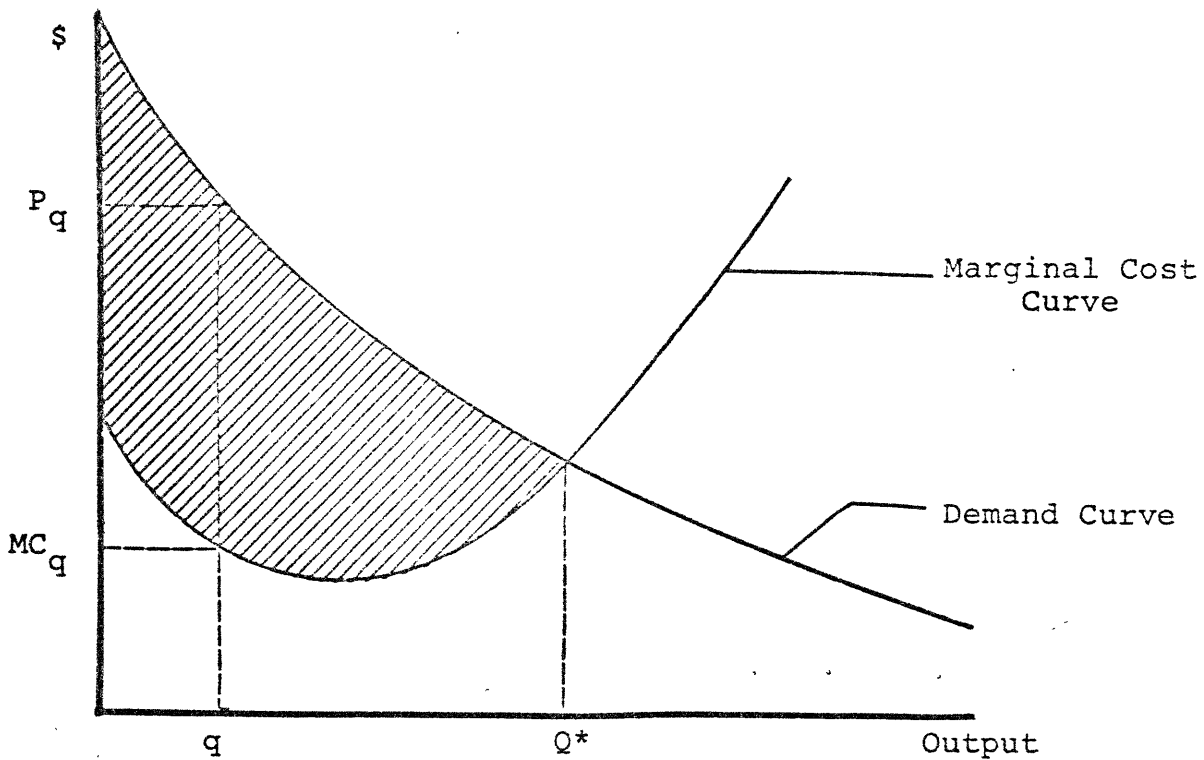
Second, continue to produce and sell output until the marginal cost of production, i.e., the cost of producing one additional unit of output, is equal to the maximum price that customers are willing to pay for that unit of output.

This rule defines a discriminating, one-part pricing system. The system is discriminating because it charges different customers different prices for each unit of output they purchase, and hence the system discriminates among customers. The system is a one-part pricing system, because, although the price charged for different units of output may vary, each customer is charged a single price for each unit he purchases.

To see how this rule works, we shall consider an example. Let us suppose that a certain firm has a monopoly in a given market and hence is faced with a downward sloping demand curve. Also, we shall suppose that the firm's total costs increase at varying rates as volume increases and hence, the firm will have a varying marginal cost curve.

Demand and marginal cost functions satisfying these

descriptions are depicted on the following graph.



Demand and Marginal Cost Curves for the Firm

If the firm charges each customer the maximum amount he is willing to pay for each unit of output, then the firm will not charge all customers the same price nor even the same customers the same price for different units of output. Rather, the firm will negotiate the price for each unit of output purchased by each customer. Thus, customers who are willing to pay a large amount for a unit of output will be charged a large amount. Other customers, who are very nearly satiated with the firm's output, would be charged much less



for additional units of output.

More specifically, we might think of the firm as producing its first unit of output and selling that unit in the market for the highest price any customer would be willing to pay for it. What that price would be is determined by finding that point on the demand curve where the quantity sold is one unit and observing the maximum price the firm can charge to sell that one unit. The firm then brings its second unit of output to the market and sells that unit for the highest price any customer is willing to pay. What that price would be is again determined from the demand curve: the firm locates that point on the demand curve where the quantity sold is two units and charges for the second unit of output the maximum price at which it can sell the second unit. The firm continues this process for the third, fourth and following units of output.

In effect, by charging each customer the maximum amount he is willing to pay for each unit of output, the firm lowers the price it charges for each subsequent unit of output. Furthermore, since the firm's demand curve given for each quantity of output the maximum price the firm can charge to sell its last unit of output, the optimal price to charge for each unit of output is determined from the demand curve facing the firm.

If the firm continues to produce and sell units until the cost of producing an additional unit is equal to the maximum

price that some customer would be willing to pay for that unit of output, then the firm will produce and sell exactly  $Q^*$  units of output. The marginal cost of producing the  $Q^{*th}$  unit of output is equal to the amount the firm would earn by selling that unit in the market. Should the firm produce more than  $Q^*$  units, the cost of producing those extra units will exceed the amount that the firm can receive by selling those units and hence the firm would lose money. Should the firm produce less than  $Q^*$  units, the price some customer is willing to pay for an additional unit is greater than what it would cost the firm to produce that unit. Hence the firm could increase its profit by producing and selling additional units. In fact, only by producing and selling  $Q^*$  units, will the firm achieve maximum profit.

The profit earned by the firm on each unit it sells is equal to the difference between the price it can charge for that unit and the cost of producing that unit. In terms of the above graph, the profit the firm earns on its  $q^{th}$  unit of output is represented by the distance between the demand curve evaluated at  $q$  and the marginal cost curve evaluated at  $q$ . The total profit earned by the firm is equal to the sum of the profits it earns on each unit it sells. Thus, the total profit earned by the firm is equal to the entire area between the demand curve facing the firm and its marginal cost curve. This is the area shaded in the above graph.

This profit is the maximum profit that the firm could earn by selling its output in the market. As is apparent from studying the graph, there is no way to increase the firm's profits without either shifting the demand curve outward--thereby defining a different market for the firm's products, or shifting the marginal cost curve inward--supposing thereby, that a different firm with different costs now produces for the market. Thus, a discriminating, one-part pricing system achieves the maximum profits possible for a given firm operating in a given market.

Despite the profitability of the system, there are two important reasons why a firm might choose not to use a discriminating one-part pricing system:

First, for a firm, such as a linen supply, which deals with a large number of customers, many of whom demand large quantities of output, it would be entirely impractical to negotiate with each customer the maximum price he is willing to pay for each unit of output. The transactions cost of performing such negotiations far outweigh the benefits which the firm can expect to receive from price discrimination.

Second, price discrimination among customers, which is unjustified by differences in costs incurred to sell to different customers, is illegal. Price discrimination is explicitly forbidden by Section 1 (a) of the Robinson-Patman Act (15 U.S.C. Sections 13 and 13a):

That it shall be unlawful for any person engaged in commerce, in the course of such commerce, either directly or indirectly, to discriminate in price between different purchasers of commodities of like grade and quality, where either or any of the purchases involved in such discrimination are in commerce, where such commodities are sold for use, consumption, or resale within the United States or any Territory thereof or the District of Columbia or any insular possession or other place under the jurisdiction of the United States, and where the effect of such discrimination may be substantially to lessen competition or tend to create a monopoly in any line of commerce, or to injure, destroy, or prevent competition with any person who either grants or knowingly receives the benefit of such discrimination, or with customers of either of them: Provided, that nothing herein contained shall prevent differentials which make only due allowance for differences in the cost of manufacture, sale, or delivery resulting from the differing methods or quantities in which such commodities are to such purchasers sold or delivered.

Because of these reasons a firm is likely not to wish to discriminate in price among its customers. Nevertheless, the discriminating one-part pricing system will continue to interest us in the following analyses as a benchmark from which to judge the effectiveness of other pricing systems.

### C. Maximizing Profit Under a Non-Discriminating One-Part Pricing System

In this section, I shall discuss how the optimal pricing strategy of a firm would change if the firm chose to impose on itself the restriction that it will charge all customers the same, single price for all units of output they purchase. At first, I shall consider the effect that such a restriction will have on a monopolistic firm. Then, once we have understood this case, I shall consider the effect this restriction has for firms operating in perfectly competitive markets.

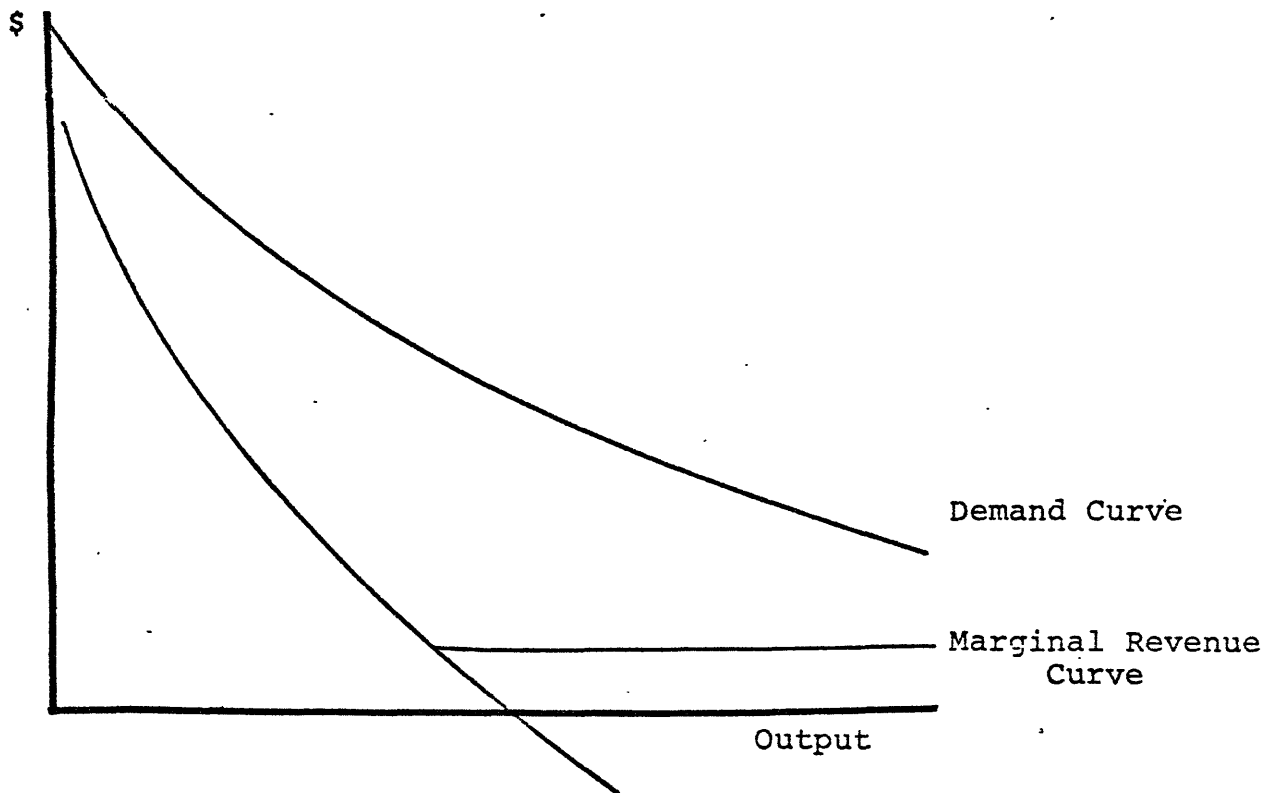
Recall that the demand curve facing a monopolistic firm slopes downward as the quantity of output sold by the firm increases. This negative slope of the monopolistic firm's demand curve simply reflects the fact that the firm can sell additional units of its output only at a lower price. Of course, if the firm is restricted to charge the same price for all units of output it sells, then if the firm does sell an additional unit of output, it must lower the price on all units of output it sells. Similarly, if the firm were to sell one fewer units, it would be able to raise the price it charges for all units. Thus, when operating under the restriction that it must charge the same, single price for all units of output it sells, the maximum price the firm can charge for its output will always be equal to the maximum price any of the firm's customers would be willing to pay for the last unit produced by the firm.

It is easy to see that when this restriction is imposed the decision of whether or not to produce an additional unit of output becomes much more important to the firm: If the firm is allowed to discriminate in price among its customers, then the firm will always increase its revenues--although not always its profits--by selling additional units. When the firm is constrained to charge the same price for all units of output, total revenue becomes subject to two opposing influences: since by selling an additional unit, the firm is

increasing the quantity of output it sells, total revenue will tend to increase. However, since to sell the additional unit, the firm must lower the price it charges for all units, total revenue will tend to decrease.

We could imagine using the demand curve facing the firm to calculate both the positive and negative impacts selling one additional unit of output would have on the total revenues of the firm. The net change in total revenue of these impacts need not, of course, be always the same for all levels of total output which the firm might supply to the market. If we were to calculate the change in total revenue which would result from selling one additional unit for all levels of output which the firm might currently be producing, then we would be able to define a function which relates the level of output and the change in total revenue which would result from producing and selling one additional unit. This function is called the marginal revenue function of the firm.

A graph of a typical monopolistic firm's marginal revenue curve would look something like the following. (see page 35).



### Demand and Marginal Revenue Curves

Notice that the marginal revenue curve lies always below the firm's demand curve. This is due to the fact that at any level of output, the firm can sell one additional unit only by charging a price lower than the price it currently receives. Notice also that the marginal revenue can be negative. This will occur when the increase in total revenue due to an additional unit sold is outweighed by the decrease in total revenue due to lowering the price of all output sufficiently, to sell one additional unit.

A firm's marginal revenue function is conceptually very similar to another function we have already discussed:

the firm's marginal cost of production function. As you remember, the firm's marginal cost of production function gives the cost of producing one additional unit of output for each level of output at which the firm may be operating. The firm's marginal revenue function gives the change in total revenue which would result from selling an additional unit of output for all levels of output which the firm might currently be producing.

With these two concepts as background we may now state the rule for maximizing profit when the constraint is imposed that all customers must be charged the same price for each unit of output they purchase:

Rule For Maximizing Profit Under a Non-Discriminating One-Part Pricing System

Continue to produce and sell output until the increase in total revenue resulting from the sale of an additional unit of output is equal to the cost of producing that unit of output.

Put another way the rule is this:

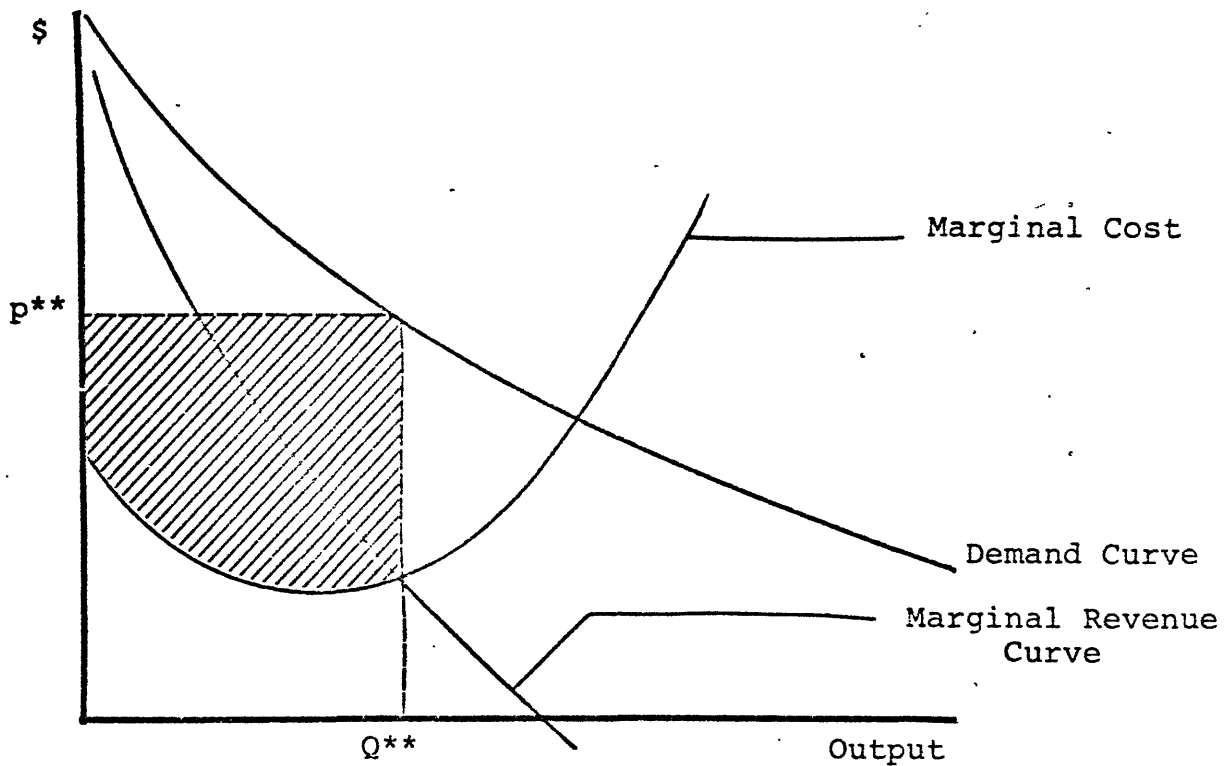
Continue to produce and sell output until marginal revenue becomes equal to marginal cost.

This rule defines the optimal use of a non-discriminating, one-part pricing system. The system is non-discriminating since all customers are charged the same price for each unit of output they purchase. The system is a one-part system because



customers are charged only one price for the output they buy.

To see how this rule works, and to observe how the profits it generates differ from those obtained under a discriminating one-part pricing system, we shall again consider the case of a monopolistic firm with varying marginal costs. The demand and marginal cost functions of our former example are repeated in the following graph; in addition, however, the firm's marginal revenue curve is now included.



Demand, Marginal Revenue and Marginal Cost Curves

If the firm follows the rule given above, then it will produce  $Q^{**}$  units of output since for that quantity of output marginal cost is equal to marginal revenue. If the firm

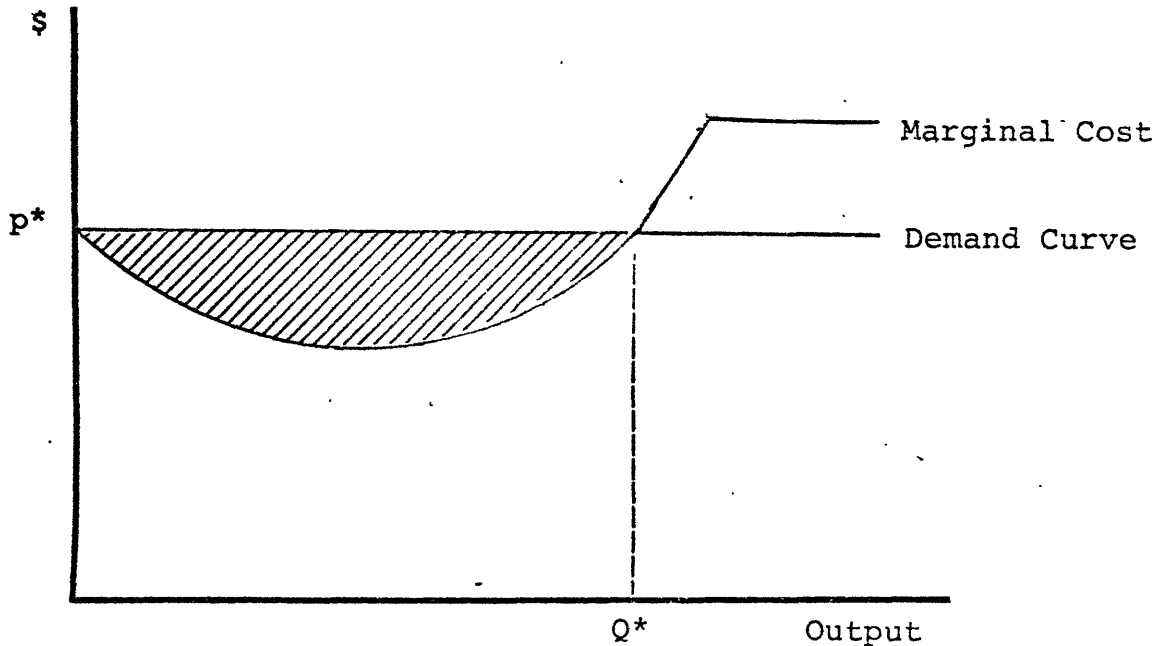
produces more than  $Q^{**}$  units, then the cost of producing an additional unit of output will exceed the increase in total revenue that will be received if that unit is sold. Hence, the firm will lose money if it produces more than  $Q^{**}$  units. If the firm were to produce less than  $Q^{**}$  units, then the cost of producing an additional unit will be less than the increase in total revenue that the firm would receive by selling the additional unit. Thus, the firm would make money by producing and selling the additional unit.

By referring to the demand curve facing the firm, we see that at the optimal output level  $Q^{**}$ , the maximum price that the firm can charge for its output and be sure of selling it all is  $P^{**}$ . The profit which would be earned by the firm is equal to the total revenue it receives from selling  $Q^{**}$  units;  $P^{**} \cdot Q^{**}$ , minus the total cost of producing  $Q^{**}$  units, which may be written as  $\int_0^{Q^{**}} MC(Q) dQ$  where  $MC$  is the firm's marginal cost function. This amount of profit is represented by the shaded area in the above graph.

One important fact to notice about the non-discriminating one-part pricing system is that it is less profitable for a monopolistic firm to use, than the discriminating one-part pricing system. Under the discriminating one-part system, the profits earned by the firm were equal to the entire area between the demand curve facing the firm and the firm's marginal cost curve. Under the non-discriminating one-part system however, the firm's profits were equal to only a part of that

area, and thus the monopolistic firm will earn less under a non-discriminating one-part pricing system.

Let us now briefly consider what impact imposing the restriction that all customers must be charged the same single price for all units of output they purchase, has for firms operating in a perfectly competitive market. The demand curve facing a firm in a perfectly competitive market is a flat horizontal line reflecting the fact that the maximum price the firm can charge for its output is the going market rate. We shall suppose that one firm operating in such a competitive market has the demand and marginal cost curves as given in the following graph.



Demand and Marginal Cost Curves

Let us suppose the firm in our example wishes to use a discriminating one-part pricing system to sell output to its customers. Following the rule for the optimal use of this system, the firm would, first, charge each customer the maximum amount he is willing to pay for each unit he purchases, and second continue to produce and sell output until the cost of producing an additional unit of output is equal to the maximum price customers are willing to pay for that unit. The second part of this rule would lead the firm to produce  $Q^*$  units since the marginal cost of producing the  $Q^{\text{th}}$  unit is just equal to the price the firm can sell it for in the market. However, if the firm follows the first part of this rule and charges each customer the maximum amount he is willing to pay, the firm will end up always charging the same price,  $P^*$ . Since the market is perfectly competitive, no customer will be willing to pay more than market price  $P^*$ , and since the firm can always sell its output for  $P^*$ , it will never charge less than  $P^*$ .

Thus, due to the nature of competitive markets, the discriminating one part pricing system collapses into the non-discriminating system: The optimal pricing strategy in a competitive market is to voluntarily submit to the restriction that all customers be charged the same single price for all units of output they purchase. Thus, in competitive markets, there is no advantage to using a discriminating rather than a non-discriminating one-part pricing system.

D. The Piece-Rate Pricing System

As you recall, under the Piece-Rate pricing system, linen supplies charge a single price for each uniform, towel, tablecloth, etc., which a customer soils. Aside from volume discounts, each customer is charged the same piece-rate as every other customer and each customer pays the same amount for each piece of linen he soils whether it is his tenth or his fiftieth.

Thus, the Piece-Rate pricing system is a close approximation to the non-discriminating one-part pricing system we have just discussed. The good, or what I have called the unit of output corresponds to a rental of a piece of linen of a given type. The price-per-unit is, of course, just the piece-rate charged for a particular type of linen. Thus, presumably the way to maximize profits under a Piece-Rate system, will be to follow the rule for maximizing profits under a non-discriminating one-part pricing system: Continue to sell output until the marginal cost of producing an additional unit becomes equal to the marginal revenue received from selling it.

In fact, however, the Piece-Rate pricing system is not identical with the non-discriminating one part pricing system we have been discussing. There are three important differences each of which will require some refinements in our analysis of the non-discriminating one-part pricing system, if that system is to be used properly to explain the Piece-Rate system.

First, the analysis of the non-discriminating one-part pricing system takes no account of critical customer size. Yet, linen supplies which do use the Piece-Rate system often find that some customers choose to receive such small amounts of linen that the revenues they provide is less than the costs of providing service to those customers. The issue of critical customer size is, of course, entirely different from the issue of the linen supply's marginal costs: Some customers are so small that, at whatever level of total volume the plant is operating and thus wherever the plant is on its marginal cost function, it still will cost the linen supply more to deliver to those customers than it would receive in revenue.

Unfortunately, I do not see any way of amending the analysis so as to explicitly recognize the issue of critical customer size. Thus, I am led to propose the rather ad hoc measure of redefining the demand curves facing linen supplies so as to exclude the quantities demanded by customers who demand less than the critical customer size. In effect, by this proposal I have assumed that linen supplies simply refuse to provide service to potential customers whose linen requirements are less than the minimum customer size.

The second way in which the Piece-Rate pricing system differs from the non-discriminating one-part pricing system we have discussed is the fact that it is common practice for linen supplies to offer volume discounts to large customers. To the extent that these large customers are more price sensitive

than smaller customers, this practice can be seen as a form of price discrimination. Thus, when volume discounts are offered to large price sensitive customers, the Piece-Rate system could be seen as a version of a discriminating one-part pricing system. If so, then one cannot help wondering about the legality of granting such volume discounts. On the other hand, if large customers are not more price sensitive than smaller customers, the practice of granting volume discounts is, in economic terms, inexplicable: A firm which adopts this practice, in effect, is willing to accept less profit than it would earn if it charged all customers the same piece rate per unit.

Finally, my analysis of the non-discriminating one-part pricing system assumed that the firm produced only one product and hence that all the firm's costs could be traced in one way or another to the product the firm is attempting to price. A linen supply however provides its customers with many different products and unfortunately for my analysis, many of the costs a linen supply incurs are joint-costs which cannot be precisely traced to any particular product. Thus, how a linen supply plant manager should interpret the rule, "Continue to sell output until the marginal cost of producing an additional unit becomes equal to the marginal revenue received from selling it," is not at all clear.

There are, I think, reasonable ways to resolve the problem that joint costs in linen supplies raises for my

analysis, but their presentation is rather lengthy and complex. Hence, I will defer a discussion of these solutions until Chapter V in which I shall present my recommendation for how linen supplies should go about setting prices.

The problems of critical customer size, volume discounts and joint costs are not unique to the Piece-Rate system. Rather, elements of these same problems can be found in all of the pricing systems commonly used by linen supplies. Since, with the exception of the problem of joint costs, what little I have to say concerning the solution of these problems I have said in this section, I shall not re-say that little bit in future chapters. Nonetheless, in reading my analysis of other pricing systems, you should keep in mind that these problems do exist, and thus, they introduce discrepancies between the real-world pricing systems we shall discuss and their idealized economic counterparts.



## FOOTNOTES

## CHAPTER TWO

<sup>1</sup>I have chosen to speak of short term profit rather than long term profit in order to avoid a discussion of whether firms would be better off in the long term--i.e. over the entire life of the firm if they forego optimizing short term profit in order to keep competition at a fairly low level. Briefly, the point is that if firms make too much money in the short term, other firms are likely to enter their markets, thereby increasing competition and decreasing profits. The trade-off between short and long term profit is an extremely important issue which, however, is outside the scope of this thesis.

## CHAPTER III.

### TWO-PART PRICING SYSTEMS

A two-part pricing system charges customers a per-unit price for each unit purchased plus a lump sum fee for the right to buy goods at that per unit price. Two-part pricing systems are commonly used by a number of industries such as the office equipment leasing industry, the short-term car rental industry, amusement parks, private clubs and, as we shall see, the linen supply industry. In each of these industries, customers are charged for each unit of good they consume--hours of computer time, document copies made, miles driven, amusement rides, or rounds of golf played--plus a charge that must be paid in order to consume these goods--a monthly equipment rental, a daily car rental charge, an entrance fee to an amusement park, or an entrance fee to a club.

In this chapter, I shall present the economic theory of two-part pricing systems. Specifically, in Part I, I shall discuss why a firm might wish to use a two-part pricing system. In Part II, I shall discuss uniform two-part pricing systems, that is two part pricing systems in which the right-to-buy tariff charged is the same for all customers, and I shall use this analysis to explain linen supplies' Piece Rate With a Minimum pricing system. Finally in Part III, I shall consider variable two part pricing systems, or two-part systems in

which the right-to-buy tariff is varied depending on the number of units the customer wishes to purchase. This type of pricing system will be used to explain the Inventory Charge Plus Laundering Fee and the Flat Rate pricing systems.

#### Part I: General Analysis of Two-Part Pricing Systems

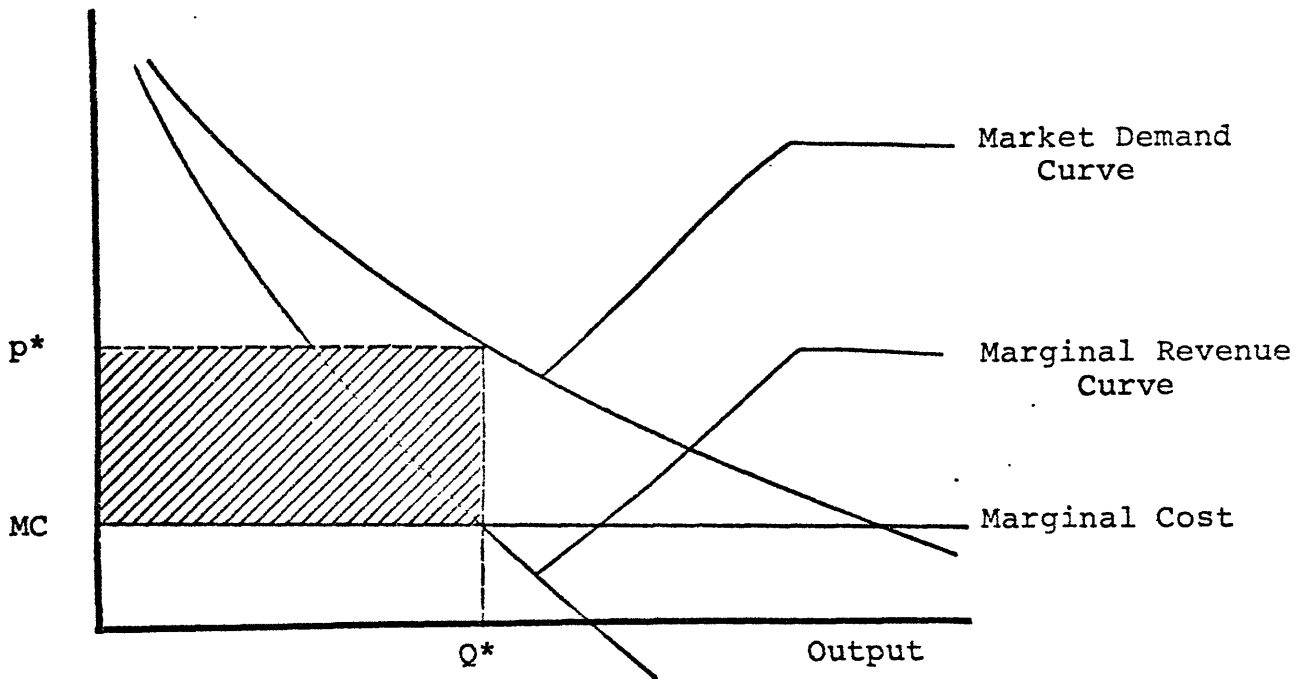
The demand curve faced by a firm records the highest price that any customer would be willing to pay for the last unit of output produced by the firm, as the total quantity of output available for purchase increases. Alternatively, if the firm is constrained to charge all customers the same price for each unit of output they purchase, the demand curve may be thought of as recording the maximum uniform price that the firm can charge per unit of output and still sell all it produces.

The demand curve facing a firm may be analyzed into the demand curves of the firm's individual customers. The demand curve of an individual customer gives the maximum price he would be willing to pay for the last unit of output he purchases, or alternatively, if the customer is charged the same price for all units he purchases, the demand curve of an individual customer records the quantity of the firm's output that that customer would purchase at various prices per-unit of output. Obviously, for any price the firm charges, the quantity of output that the aggregate of the firm's customers would purchase must be equal to sum of the quantities that

individual customers are willing to purchase. Hence, the demand curve facing the firm, which I shall henceforward call the market demand curve, is equal to the sum of the demand curves of the firm's individual customers. Thus, if a firm sold its output only to one customer, then the market demand curve would be identical with the demand curves of the firm's one customer. If a firm sold its output only to two customers then the market demand curve would be equal to the sum of the demand curves of both customers.

We shall now consider why a monopolistic firm might wish to charge its customers using a two-part pricing system. Later on, we shall take up the question of how a two-part pricing system might function in a perfectly competitive market.

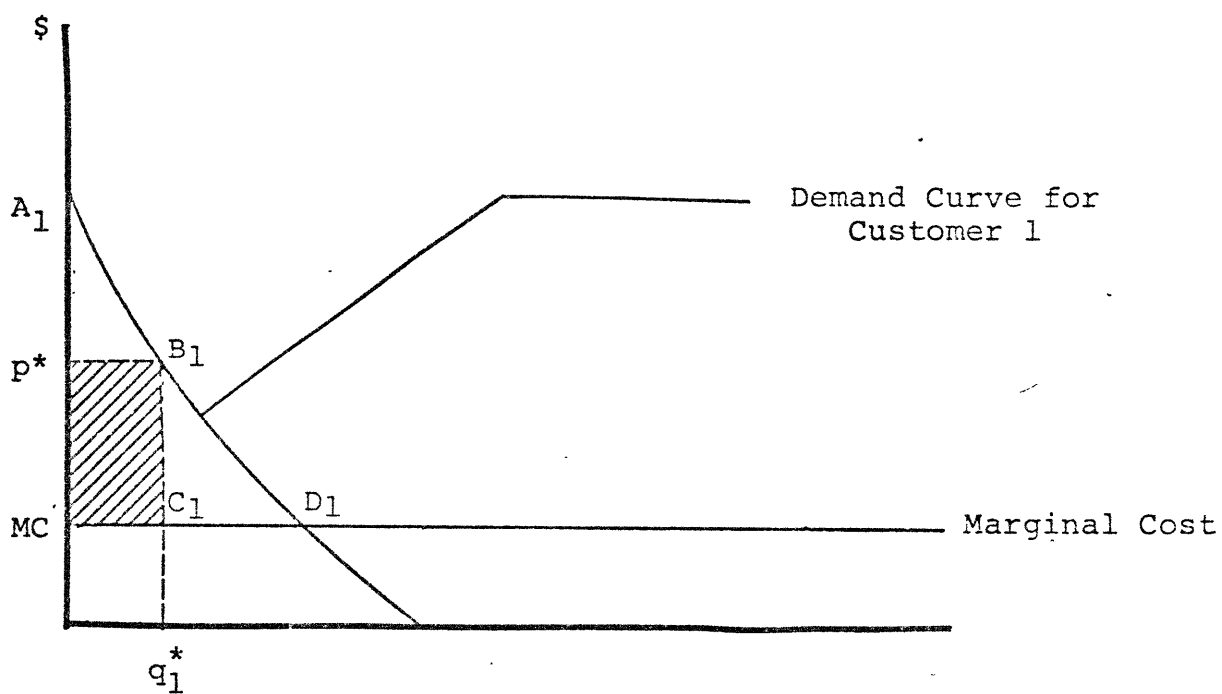
Let us take as an example a monopolistic firm which has constant marginal costs. For simplicity we shall assume that the firm operates in a market of only two customers and that the firm has decided to charge both customers the same price for each unit of output they purchase. Following the appropriate rule given in Chapter II, such a firm will continue to produce and sell output until the marginal cost of producing an additional unit of output is equal to the marginal revenue which would be received from selling an additional unit. This is depicted in the following graph:



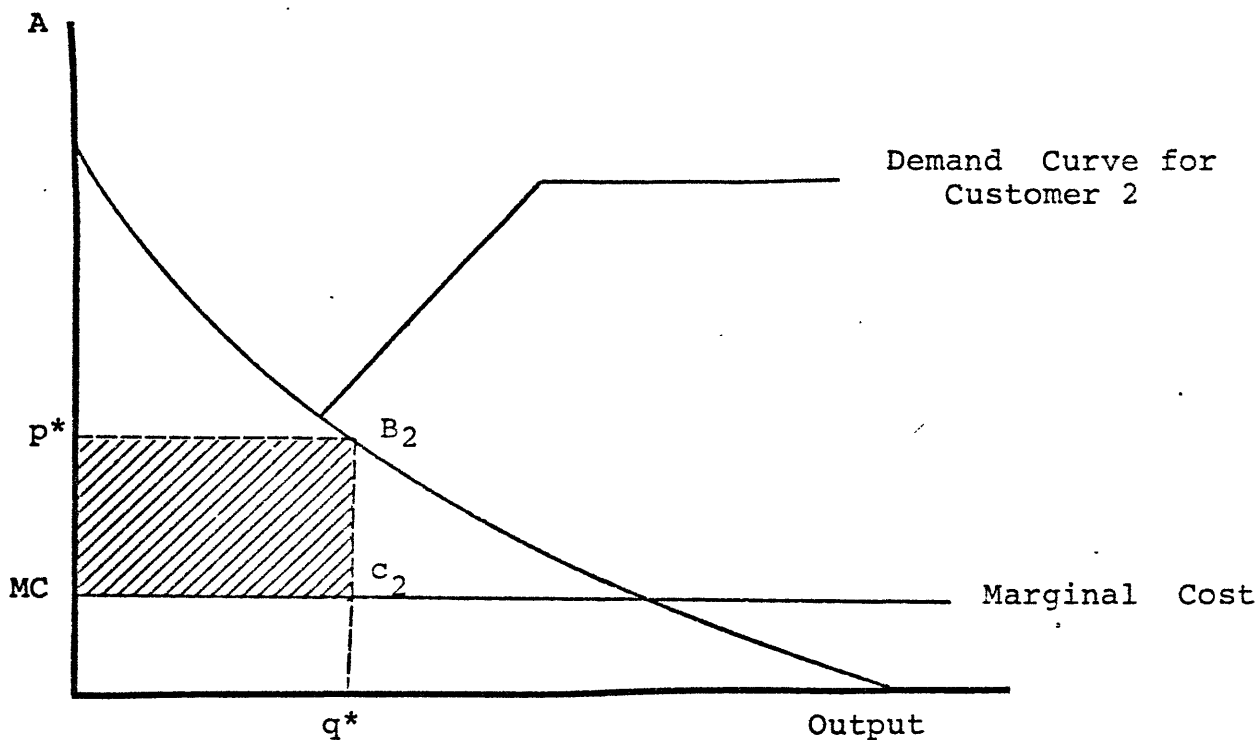
Profit Maximization for a Monopolistic Firm with  
Constant Marginal Costs

The monopolistic firm will produce exactly  $Q^*$  units since at that level of output, the marginal cost of producing one additional unit of output is just equal to the marginal revenue received from selling that additional unit. If only  $Q^*$  units of output are available for purchase, then the firm will be able to sell each of those units for a price of  $P^*$  as determined from the market demand curve. The profit earned by the firm will be equal to the total revenue it receives,  $P^* \cdot Q^*$ , minus the cost it incurred in producing those units  $MC \cdot Q^*$ . This profit,  $P^* \cdot Q^* - MC \cdot Q^*$ , is represented by the shaded area in the above graph.

We can distinguish the profits that would be earned by the firm from selling to each of its two customers by replacing the market demand curve in the above graph with the demand curves of the individual customers. This is done in the following two graphs.



Profit Earned by Selling to Customer 1 at a  
Price of  $P^*$



Profit Earned by Selling to Customer 2 at a  
Price of  $P^*$

By setting the price for a unit of output at  $P^*$ , the firm earns a profit of  $P^* \cdot q_1^* - MC \cdot q_1^*$  from selling units of output to Customer 1. This amount of profit is represented by the shaded area in the first graph. Similarly, at a price of  $P^*$ , the firm earns a profit of  $P^* \cdot q_2^* - MC \cdot q_2^*$  from selling units of output to Customer 2 and this amount of profit is represented in the second graph by the shaded area. Since  $q_1^* + q_2^* = Q_1^*$ , the sum of the profits earned by selling to Customer 1 and Customer 2 is equal to  $P^* Q^* - MC Q^*$ , the profit

we determined the firm would earn by considering only the market demand curve.

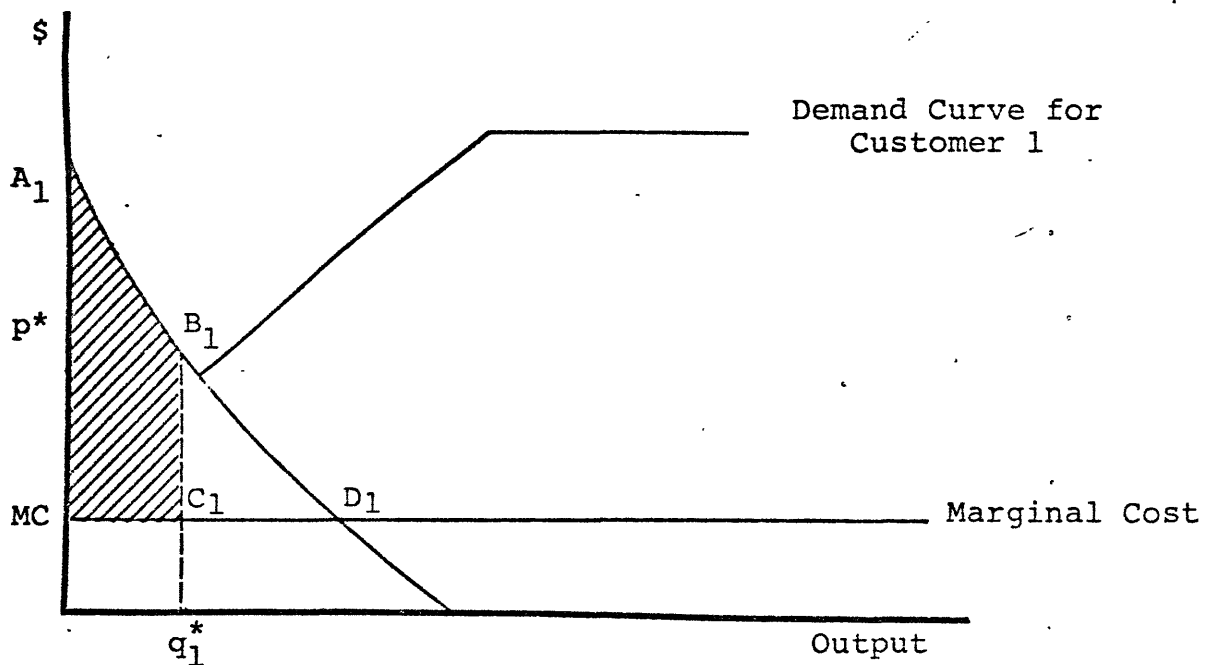
Let us now look more closely at the profit earned by selling to Customer 1. Although he does pay the maximum amount he is willing to pay for his  $q_1^{*th}$  unit, Customer 1 is able to purchase his first  $q_1^* - 1$  units at less than the maximum amount he would be willing to pay. This surplus of value achieved by Customer 1 under a non-discriminating one-part pricing system is called Customer 1's consumer surplus, and in the above graph is defined by the triangular area  $P^* A_1 - B_1$ , or equivalently by  $\int_{P^*}^{\infty} \psi_1(P) dP$ , the area beneath Customer 1's demand curve  $\psi_1(P)$  and above the price line  $P = P^*$ .

We have already considered one pricing system the firm might use to force Consumer 1 to surrender his consumer surplus. That system, called the discriminating one-part pricing system, required that the firm discriminate in the price it charged customers for each unit of output purchased. When adapted to insure that the firm receives the maximum amount that Customer 1 would be willing to pay for the  $q_1^*$  units he purchases, the rule is the following:

- (1) Charge Customer 1 the maximum amount he is willing to pay for each unit of output he purchases.
- (2) Sell exactly  $q_1^*$  units to customer 1.<sup>1</sup>



The profit that would be earned by the firm should it follow this pricing rule in selling its output to Customer 1 is given by the shaded area in the following graph. Notice that the increase in profit that the firm earns by adopting this pricing policy, the area  $P^*A_1B_1$ , is exactly equal to the consumer surplus enjoyed by Customer 1 under a non-discriminating one-part pricing system.

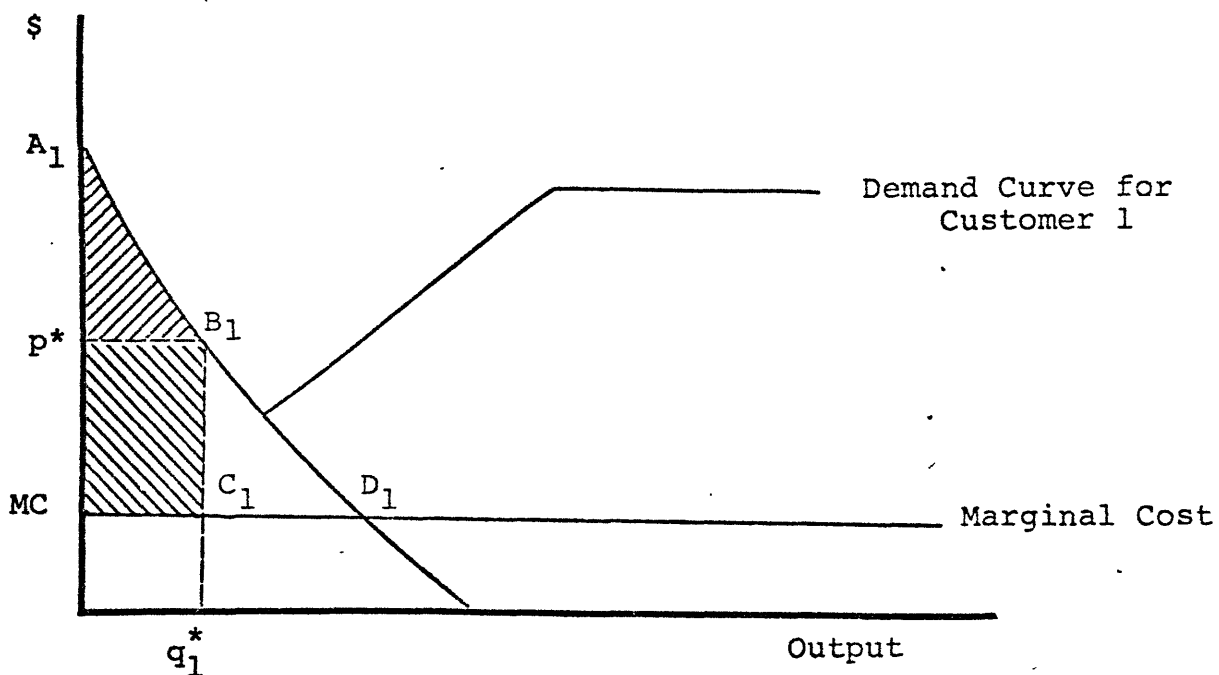


Profit Earned by Selling  $q_1^*$  Units to Customer 1  
Using Price Discrimination

Obviously, the firm would like to receive the greater profit that price discrimination will gain for it, but as we have seen, there are legal constraints and high transactions cost

which effectively disallow the firm from practising price discrimination. Fortunately for the firm in our example, there is another way of setting prices, two-part pricing, which would be just as effective as price discrimination in forcing Customer 1 to surrender the consumer surplus he enjoys by purchasing  $q_1^*$  units at a price of  $P^*$ . Fortunately also, two-part pricing is a much easier system to use and infinitely more legal!

A two-part pricing system, as the name implies, would charge Customer 1 in two parts: Before Customer 1 were allowed to purchase any of the firm's output, he would be sold the right to buy the firm's output. Only after Customer 1 has paid this right-to-buy tariff will he be permitted to pay a per-unit price for each unit of output he purchases. If, as in our example, it is assumed that Customer 1 will purchase only  $q_1^*$  units of output, then the per-unit price will be set at  $P^*$  and the right-to-buy tariff will be set equal to the consumer surplus that Customer 1 would enjoy if he were allowed to purchase  $q_1^*$  units at a per-unit price of  $P^*$ . The profit that the firm would earn by selling  $q_1^*$  units of output to Customer 1 is given in the following graph.



Profit Earned by Selling  $q_1^*$  Units to Customer 1

Using a Two-Part Tariff

In this graph, we can distinguish the profit earned by the firm from charging the right-to-buy tariff and those profits earned by charging a price per unit purchased by Customer 1: The area  $P^*-B_1-C_1-MC$ , shaded by negatively sloped lines, is the profit earned by selling  $q_1^*$  units to Customer 1 at a price of  $P^*$  each. This profit is, of course, just the profit that the firm would have received had the firm charged Consumer 1 a non-discriminating, one-part price of  $P^*$ ; that is, the rectangle  $P^*-B_1-C_1-MC$  is equal to the firm's revenues from unit sales,  $P^*.q_1^*$  minus the marginal

costs of producing those units,  $MC - q_1^*$ . The area  $P^* - B_1 - A_1$  shaded by positively sloped lines, is the profit the firm earns by charging Customer 1 a right-to-buy tariff. Notice that when a two-part pricing system is used, the total profit earned from both per unit sales and the right-to-buy tariff is exactly equal to the profit earned by price discrimination, which is, in turn, the maximum amount of profit which can be earned by the firm on sales of  $q_1^*$  units to Customer 1.

Let us now examine how the two-part pricing system may be extended to Customer 2. As you recall, the demand curve of Customer 2 lay always outside and to the left of Customer 1's demand curve. We may infer from this fact not only that Customer 2 is a larger customer than Customer 1--at each price per unit, Customer 2 is willing to purchase more units than Customer 1--but also that Customer 2 has a larger consumer surplus than Customer 1--for each unit of output he purchases, Customer 2 is willing to pay a higher price than Customer 1. How then should a two-part pricing system handle customers with different sized consumer surpluses?

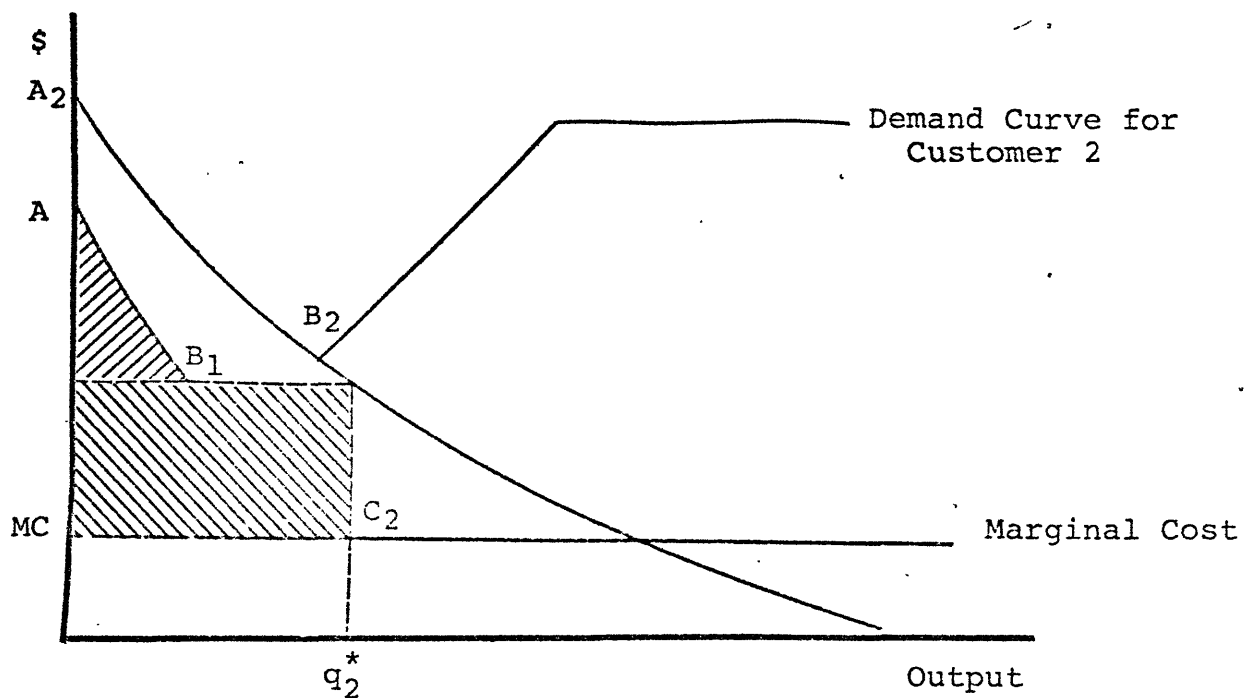
There are basically two alternatives:

First, the firm could attempt to charge customers with larger consumer surpluses a larger right to buy tariff. Should the firm choose this alternative, we shall say that the firm has adopted a variable two-part pricing system, since the right-to-buy tariff charged varies among customers. The advantages

of a variable two-part pricing system are obvious: By charging a larger right to buy tariffs to customers with larger consumer surpluses, the firm extracts more consumer surplus from its customers and hence increases its total profits. However, this method is fraught with legal danger. Since charging a customer a right-to-buy tariff is essentially selling that customer the right to purchase output from the firm, a firm which charges customers different right-to-buy tariffs is engaging in a subtle form of price discrimination among its customers. Price discrimination which is unjustified on the basis of different costs involved in selling to different customers is, as we have seen, forbidden by the Robinson-Patman Act. Hence, firms which do attempt to charge different customers different right-to-buy tariffs must be extremely cautious. In Part III of this chapter, we shall examine two systems for charging different customers different right-to-buy tariffs which seem to be permitted by the Robinson-Patman Act.

The second alternative is extremely simple. The firm could charge customers with large consumer surpluses the same right-to-buy tariff as it charges its customer with the smallest consumer surplus. In effect, each customer is charged a right-to-buy tariff equal to the smallest consumer surplus belonging to any of the firm's customers. In this second case, we will say that the firm has adopted a uniform two-part tariff.

If this second alternative were adopted by the firm in our example, then Customer 2 would be charged a right-to-buy tariff equal to that charged Customer 1. As was shown earlier, the proper right-to-buy tariff to charge Customer 1 when the per-unit charge is  $P^*$  is equal to the consumer surplus Consumer 1 would enjoy if he were allowed to purchase output at a non-discriminating one-part price of  $P^*$ . Thus, the profit that would be earned by the firm by selling output to Customer 2 is depicted in the following graph.



Profit Earned by Selling  $q_2^*$  Units to Customer 2

Using a Two-Part Tariff

As for Customer 1, we can distinguish the profit earned by the firm from charging the right to buy tariff and those profits earned by charging a price per unit purchased by Customer 2. The area  $P^*-B_2-C_2-MC$ , shaded by negatively sloped lines is the profit earned by selling  $q_2^*$  units to Customer 2 at a price of  $P^*$  each. This profit is, of course, just the profit that the firm would have received had the firm charged Consumer 2 a non-discriminating one-part price of  $P^*$ . That is, the rectangle  $P^*-B_2-C_2-MC$  is equal to the firm's revenues from unit sales,  $P^* q_2^*$  minus the marginal costs of producing those units  $MC \cdot q_2^*$ . The area  $P^*-B_1-A_1$ , shaded by positively sloped lines, is the profit the firm earns by charging Customer 2 a right-to-buy tariff equal to that charged Customer 1. Notice, however, that because Customer 2 has a larger consumer surplus than Customer 1, Customer 2 still retains some unexploited consumer surplus. He would be willing to pay an amount greater than he is being charged under our two-part pricing system to receive  $q_2^*$  units. Hence, a two-part pricing system which is constrained to charge each each customer the same right to buy tariff is a less than perfectly profitable pricing system when used in a market of two or more customers with consumer surpluses of unequal sizes.

We have now calculated the profits that the firm would earn by selling its output by way of a fixed tariff, two-part pricing system for both Customer 1 and Customer 2. By adding

these profits we may calculate the total profit that our firm would earn.

The profit the firm earns through its charge of a right-to-buy tariff is the same for Customer 1 and Customer 2. In each case, the customer is charged the amount defined by the area  $P^* - B_1 - A_1$  or, equivalently,  $\int_{P^*}^{\infty} \Psi_1(P) dP$ , the consumer surplus of Customer 1. Thus, the firm's total profit due to its charge of a right-to-buy tariff is equal to  $2 \int_{P^*}^{\infty} \Psi_1(P) dP$ .

The profit the firm earns through its sales of output at a per unit price of  $P^*$  differs, of course, between Customer 1 and Customer 2. Customer 1 is willing to purchase only  $q_1^*$  units of output at a price of  $P^*$  and hence generates revenues from those sales of  $P^* \cdot q_1^*$ . Since it cost the firm  $MC \cdot q_1^*$  to produce those units, the profit earned from unit sales to Customer 1 is  $P^* q_1^* - MC q_1^*$ . Customer 2, however, is willing to purchase  $q_2^*$  units at a price of  $P^*$  and hence sales to Customer 2 result in revenues of  $P^* q_2^*$ . The cost of producing  $q_2^*$  units is  $MC q_2^*$  and hence the profit earned from unit sales to Customer 2 is  $P^* q_2^* - MC q_2^*$ . Total profit from unit sales is then  $P^* (q_1^* + q_2^*) - MC (q_1^* + q_2^*)$ .

Total profit for the firm in our example therefore would be equal to its profit on charging right to buy tariffs plus its profit on unit sales:-

$$\Pi = 2 \int_{P^*}^{\infty} \Psi_1(P) dP + P^* (q_1^* + q_2^*) - MC (q_1^* + q_2^*)$$

Notice however that the firm's profit on unit sales,



$P^*(q_1^*+q_2^*) - MC(q_1^*+q_2^*)$ , is exactly the profit that the firm would earn by charging both customers the same price for each unit of its output as was shown earlier in this chapter. Hence, the profit earned by the firm in our example using a uniform two-part pricing system exceeds that which would be earned by the firm if it used a non-discriminating one-part tariff. The amount that was gained by using a two-part pricing system in our example was equal to  $2 \int_{p^*}^{\infty} \Psi_1(P) dP$ --the number of customers times the smallest consumer surplus of any of the firm's customers.

Let us now briefly consider how a firm which wishes to use a two-part pricing system would fare in a perfectly competitive market.

In a competitive market, the maximum per unit price the firm can charge for its output is the going market rate. However, since no customer would be willing to pay more than the market price per unit for any unit of output he purchases, no producer can capture the consumer surplus of any customer. Thus, if the firm attempts to charge any right-to-buy tariff the firm will drive all customers out of the market for its products and into the market for its competitor's products. In effect, the maximum right-to-buy tariff the firm can charge is zero. At a zero right-to-buy tariff, of course, a two-part pricing system is really no different than a non-discriminating one-part pricing system. Thus, in

perfectly competitive markets, a two-part pricing system has no scope to operate. It can earn profits no greater than a simple non-discriminating one-part system.

## Part II: Uniform Two-Part Tariffs

### A. Optimal Use

We have seen that a monopolistic firm can achieve higher profit by using a two-part pricing system than it could by using a non-discriminating one-part pricing system. That demonstration was accomplished by setting the per-unit price charged under the two-part tariff equal to  $P^*$  the optimal price to charge under a non-discriminating one-part tariff, thus ensuring that the profit earned from the per-unit sales under the two-part tariff would be exactly equal to the total profit earned under a one-part tariff. Then, we simply charged both customers a right-to-buy tariff equal to the smallest consumer surplus of any of the firm's customers.

Although these gains may indeed seem substantial, it is possible that the firm in our example could earn still greater profits under a two-part pricing system. These additional gains would come from two sources. First, the firm might find that there is a more profitable combination of a per-unit price and a right-to-buy tariff than the one chosen in our example. Second, the firm might choose to set its per unit price or its right-to-buy tariff at a high enough level to drive the smaller of the two customers out of the market.

In this section I shall present rules to resolve each of these issues.<sup>2</sup> In company with the presentation of each rule, I shall discuss a modification of the example given in Part I (A) which will demonstrate the validity of the rule by showing the increase in profit that a firm would earn by following the rule.

Rule 1: Optimal Combinations of Per-Unit and Right to Buy Prices.

Under a two-part pricing system, a firm earns profit both from its sales of individual units of output and from charging a right to buy tariff. We might therefore write the profit earned by a firm serving  $N$  customers who demand in aggregate  $X$  units as

$$\pi(N) = N \cdot T + X \cdot P - TC(X)$$

where  $T$  is the right-to-buy tariff,  $P$  is the price per unit and  $TC(X)$  is the total cost of producing  $X$  units of output. As we have seen, the largest right-to-buy tariff that can be charged without driving customers out of the market is equal to the smallest consumer surplus of any of the firm's customers. The largest right-to-buy tariff that can be charged is therefore equal to the area between the demand curve of the smallest customer and the price set by the firm. Hence, we may write the largest right-to-buy tariff the firm can charge as a function of the per-unit price charged for output.

$$T^* = \int_p^{\infty} \Psi(P) dp$$

Substituting the expression for  $T$  in the above equation, we

find that profits are a function of just one parameter  $P$ . We may therefore find the price  $P^*$  which maximizes profit by taking the derivative of the above equation, setting it equal to zero and solving for  $P^*$ . Doing so, we find,

$$P^* = \frac{MC}{1 + \frac{1-NS_1}{E}}$$

where  $S_1$  is the market share demanded by the smallest consumer in the market and  $E$  is the price elasticity of the market demand curve.<sup>3</sup> (Details of this calculation are given in Appendix I of this Chapter).

These calculations give rise to the following rule for selecting the optimal combination of a right-to-buy tariff and a per-unit price for a firm operating in a given market:

Optimal Price Combination Rule

In a market of  $N$  customers, who in aggregate would be willing to purchase  $X$  units of output, a firm should charge a right to buy tariff  $T^*$  equal to,

$$\int_{P^*}^{\infty} \psi_1(P) dP$$

and a per unit price  $P^*$  equal to

$$\frac{MC}{1 + \left( \frac{1-NS_1}{E} \right)}$$

Where:

$\Psi_1$  = the demand curve of the customer with the smallest consumer surplus. Hence,  $\int_{p^*}^{\infty} \Psi_1 (P) dP$  is equal to the smallest consumer surplus of any of the  $N$  consumers in the market.

MC = Marginal Cost of Producing the  $x^{\text{th}}$  unit of output

E = The elasticity of the Market Demand Curve. That is, E is equal to the percentage change in quantity of output demanded due to a 1 per cent change in the per unit price of output.

$S_1$  = Market Share of the customer having the smallest consumer surplus.

We shall now consider a modification of the example given in Part I (A) which will demonstrate the merit of this rule.

Suppose that the two customers in the market described in Part I have identical demand curves. At all prices the firm charges per unit of its output each customer would be willing to purchase the same quantity of output as the other customer.

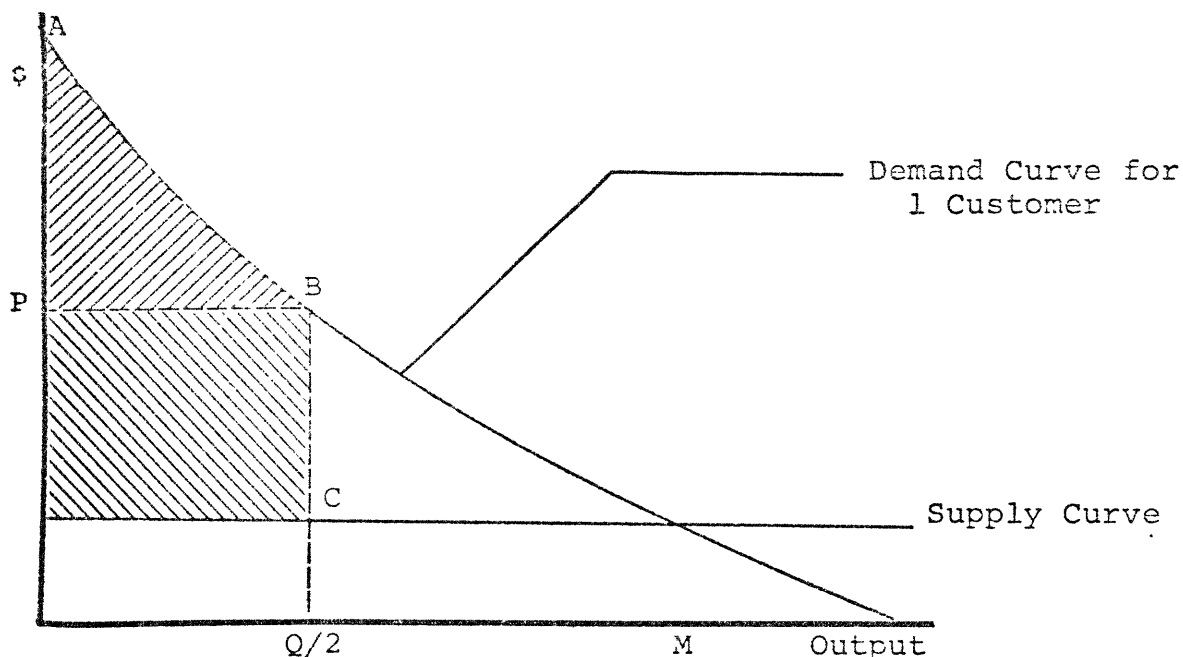
Let us now compare the method of selecting per unit and right to buy tariffs used in Part I, with the rule just developed. Following the reasoning in Part I, the firm would produce that quantity of output such that the marginal cost of producing an additional unit of output is exactly equal to the

marginal revenue which the firm would receive from its per unit sales. The maximum uniform price which the firm could charge per unit of output is then determined from the market demand curve. In addition to this per unit price, the firm also charges each customer a right-to-buy tariff equal to the consumer surplus enjoyed by each of the firm's identical customers at the given per unit price.

If the total quantity that the firm chooses to sell is  $Q$  units and the maximum uniform price the firm can charge for those  $Q$  units is  $P$ , then the maximum right-to-buy tariff the firm can charge is  $\int_p^{\infty} \Psi(P) dP$ , the consumer surplus of one of the firm's identical customers. Thus, the total profits the firm would earn are given by the equation:

$$\pi = 2 \cdot \int_p^{\infty} \Psi(P) dP + P \cdot Q - TC(Q)$$

The profit that the firm earns on each of its two customers is given in the following graph, in which the area shaded by positively sloped lines represents the firm's profits from charging a right to buy tariff of  $\int_p^{\infty} \Psi(P) dP$  and the area shaded by negatively sloped lines represent the profits due to charging a price  $P$  per unit of output.



Profit Earned from One of Two Identical Customers

Suppose now that the firm in our example were to follow the rule presented above for determining the optimal combination of per-unit and right-to-buy tariffs. If so, then the firm will charge a right to buy tariff  $T^*$  equal to  $\int_p^\infty \psi(p) dp$ , the smallest consumer surplus of any of the firm's customers and a per unit price of  $P^* = \frac{MC}{1 + \frac{(1-NS_1)}{E}}$ . Notice, however, that since in our example all consumers demand exactly the same amount of output.  $S_1$  the market share demanded by the customer with the smallest consumer surplus is equal to  $1/N$ . Hence, the term  $\left(\frac{1-NS_1}{E}\right)$  becomes equal to zero and hence the optimal per-unit price  $P^*$  is set equal to MC, the marginal cost of producing one additional unit of output. By establishing a new price  $P^*$  equal to marginal cost, the firm in effect

agrees to sell each unit of output for the amount it costs the firm to make that unit. Hence, the firm will make no profit on its per-unit sales to either customer.

However, by lowering the per-unit price charged, the firm has increased the maximum right-to-buy tariff the firm can charge. When the per-unit price is set equal to marginal cost, the maximum right to buy tariff the firm can charge is equal to the consumer surplus enjoyed by either of the firm's customers at a per unit price equal to marginal cost. The amount of this consumer surplus is  $\int_{MC}^{\infty} \Psi(P) dp$  or equivalently the area between each customer's demand curve and the firm's marginal cost curve. Thus, the firm's total profits may be expressed by the following equation:

$$\pi = 2 \cdot \int_{MC}^{\infty} \Psi(P) dP + P^* \cdot Q - TC(Q)$$

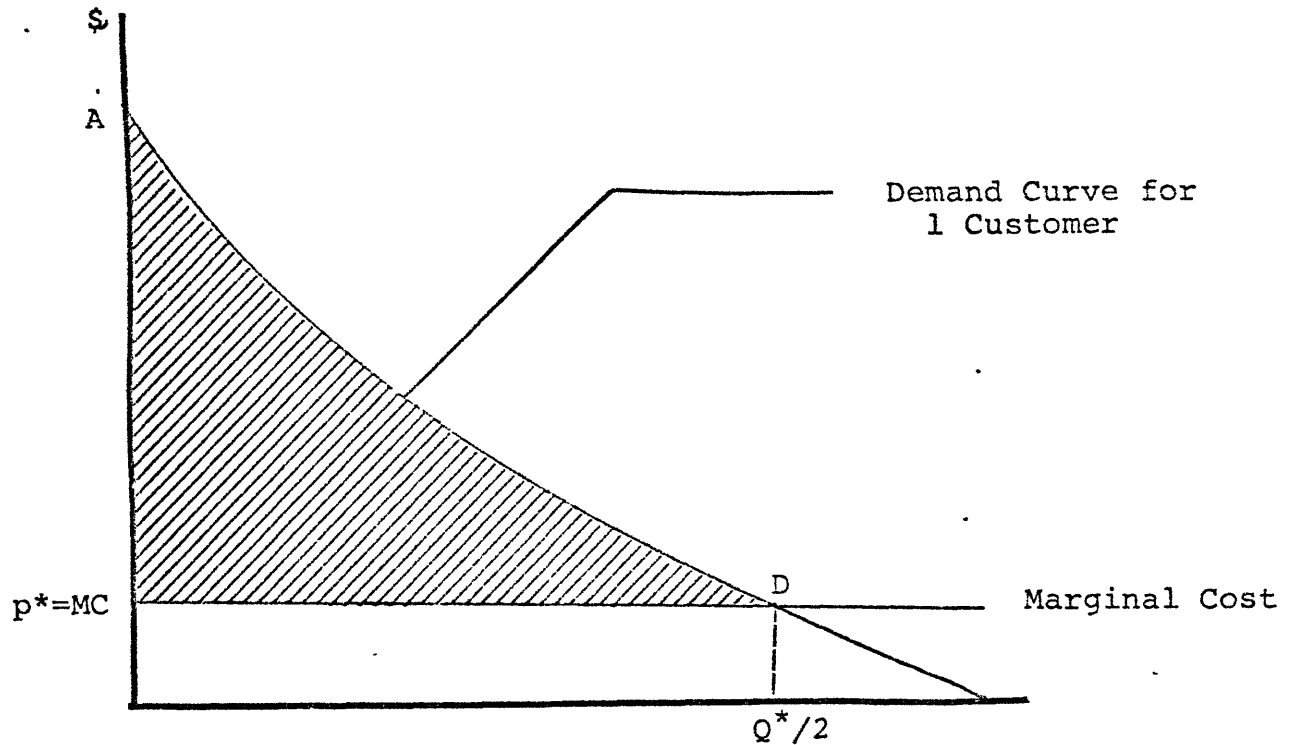
Or, since the firm's marginal costs are constant and since  $P^*$  is equal to the firm's marginal cost

$$= 2 \cdot \int_{MC}^{\infty} \Psi(P) dP + MC \cdot Q - MC \cdot Q$$

$$= 2 \int_{MC}^{\infty} \Psi(P) dP$$

The profit that the firm earns on each of its two customers is given in the following graph.





Profit Earned from One of Two Identical Customers

Under a Uniform Two-Part Tariff.

As is apparent from this graph, the profits earned by the firm increase when the firm follows the rule for selecting the optimal combination of per unit and right-to-buy tariffs. In a way, this result should not be surprising. At first, we selected the optimal per-unit price  $P$  to charge according to the rule for maximizing profit under a non-discriminating one-part pricing system. However, when the firm is permitted to charge in two parts for its output, there is no guarantee that the optimal per-unit price under a two-part pricing system will be the same as the per-unit price under a one-part system.

Notice that in this example, the amount of profit earned by the firm from its dealings with each customer is exactly equal to the area between that customer's demand curve and the firm's marginal cost curve. Thus, in this particular example, the uniform two part pricing system was able to earn the same amount of profit that the firm could earn by using a discriminating one-part tariff. This profit, as we have seen, is the maximum profit that the firm could earn by operating in that given market.

Rule 2: Selecting the Proper Customer Mix

The idea on which the rule for selecting the proper customer mix is based is really very simple. The maximum right-to-buy tariff that the firm can charge its customers is  $\int_p^{\infty} \Psi(P) dP$ , the smallest consumer surplus of any of the firm's customers. If one customer or one group of customers have very small consumer surpluses relative to those of the firm's other customers, then the firm may earn higher total profits, by ignoring those customers with small consumer surpluses. The firm would do this by charging a right-to-buy tariff which exceeds the smallest of its customers' consumer surpluses, thus effectively driving the customer with the smallest consumer surplus from the market. If such a strategy is to be profitable, the amount of profit the firm foregoes by driving some of its customers from the market must be more than made up by the increase in profit the firm earns by charging the customers remaining in the market a higher right-to-buy tariff.

The rule for selecting the optimal customer mix involves two steps.

First, the firm should assume that all customers are to remain in the market. The firm will use the rule for determining an optimal combination of per-unit and right-to-buy tariffs to calculate the optimal prices to charge if all customers remain in the market. Using these optimal prices, the firm will calculate the profits it expects to earn, according to the formula:

$$\pi(N) = N \cdot T_1^* + P_1^* \cdot X - TC(X_N)$$

Where

$N$  = Number of customers

$T_1^*$  = Right-to-buy tariff

$P_1^*$  = Per-unit price

$X_N$  = Total Quantity Purchased by  $N$  Customers

$TC(X_N)$  = Total cost of producing  $X$  units

Second, the firm should assume that the customer with the smallest consumer surplus is to be driven from the market. Again the firm will use the rule for determining an optimal combination of per-unit and right-to-buy tariffs, this time however for a market which excludes the customer with the smallest consumer surplus. Using the new optimal prices, the firm will again calculate the profits it expects to earn:

$$\pi(N-1) = (N-1) T_2^* + P_2^* \cdot X_{N-1} - TC(X_{N-1})$$

If the profit earned is greater when the customer with the

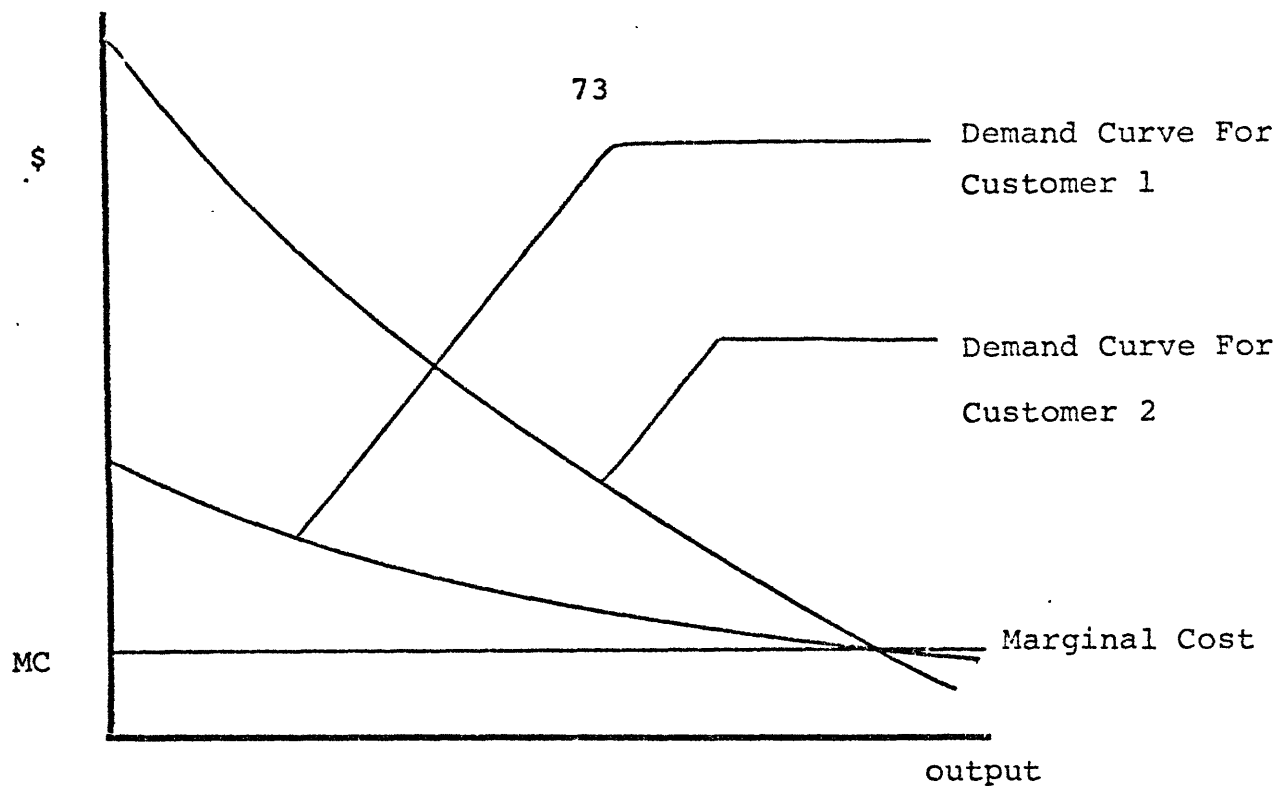
smallest consumer surplus remains in the market, then the optimal number of customers for the firm to deal with is  $N$ . If the profit earned is greater when the customer with the smallest consumer surplus is excluded from the market, then the firm should consider excluding the customer with the second smallest consumer surplus, and so on. The optimal mix of customers for the firm to deal with will be those customers remaining when total profits cannot be increased by excluding customers from the market.

More succinctly put, the rule for selecting the optimal customer mix is this:

Optimal Customer-Mix Rule.

Continue to exclude customers from the market, in the order of which remaining customer has the smallest consumer surplus, until anticipated profits begin to decline.

To see how this rule works, let us consider again the two customer market of Part I of this chapter. We shall assume that the firm has constant marginal costs. We shall also suppose that Customer 1 is much more price sensitive than Customer 2, although when the per-unit price is set equal to marginal cost, both Customer 1 and Customer 2 will choose to purchase the same quantity of output. Specifically, we shall assume that when price is set equal to marginal cost, the consumer surplus of Customer 2 is five times that of Customer 1. This market is depicted in the following graph.



### Market of Two Customers

If the firm were to sell its output to both Customer 1 and Customer 2, then it would set the right-to-buy tariff equal to the consumer surplus of Customer 1,  $\int_{P^*}^{\infty} \Psi_1(P) dP$ , and charge the per-unit price  $P^*$  calculated from the formula:

$$P^* = \frac{MC}{1 + \frac{1 - NS_1}{E}}$$

In our example, the  $P^*$  which satisfies this formula is  $P^* = MC$ , as can be easily verified by substituting 2 for  $N$ , the number of customers and  $1/2$  for  $S_1$ , the market share demanded by the customer with the smallest consumer surplus at a per-unit price of  $P^*$ . Thus, since  $P^*$  is equal to marginal cost, and since marginal costs are constant, the total profit of the firm is given by the following equations:

$$\pi(2) = N \cdot T^* + P^* \cdot X - TC(X)$$

$$\begin{aligned}
&= N \cdot T^* + MC \cdot X - MC \cdot X \\
&= N \cdot T^* \\
&= 2 \cdot \int_{MC}^{\infty} \Psi_1(P) dP
\end{aligned}$$

Let us now consider how the firm's profits would change if Customer 1, the customer with the smallest consumer surplus were excluded from the market.

If the firm were to sell its output only to Customer 2, it would again calculate the optimal per-unit price from the formula:

$$P^* = \frac{MC}{1 + \frac{1 - NS_1}{E}}$$

But since, in this case both  $N$ , the number of customers, and  $S_1$ , the market share demanded by the smallest customer are equal to 1, the optimal price  $P^*$  will again be equal to marginal cost. The total profit earned by the firm will then be derived entirely from the right-to-buy tariff charged Customer 2:

$$\begin{aligned}
\pi(1) &= \int_{MC}^{\infty} \Psi_2(P) dP + MC(X_2) - MC(X_2) \\
&= \int_{MC}^{\infty} \Psi_2(P) dP
\end{aligned}$$

Or, since at a per-unit price equal to marginal cost the consumer surplus of Customer 2 was assumed to be five times that of Customer 1:

$$\pi(1) = 5 \int_{MC}^{\infty} \Psi_1(P) dP$$

Thus, the profits earned by the firm when Customer 1 is excluded from the market are greater than when both customers are allowed to remain in the market. In fact, the firm in our example can increase its profits by 150% by driving Customer 1 from the market.

#### B. Uniform Two-Part Pricing Systems for Linen Supplies

No customer of a linen supply ever sees an item on his monthly bill:

Right to Buy Charge . . . . . \$10

Yet, there are ways that linen supplies can and do charge their customers under a uniform two-part pricing system. The trick, of course, is to disguise the right-to-buy charge in a form which seems fair or at least palatable to the customer.

One way in which linen supplies might implement a uniform two-part pricing system would be to charge its customers a fixed delivery charge for each type of linen they receive plus a laundering fee for each piece of linen soiled. The delivery charge for a particular type of linen would be charged monthly and would be the same for all customers receiving that item of linen. The laundering fee charged a customer would also be different for each type of linen the customer uses, although all customers would again be charged the same laundering fee for the same type of linen.

This pricing system, which I shall call the Delivery Charge Plus Laundering Fee system, is clearly a uniform

two-part pricing system. Since nearly all customers of a linen supply receive their linen by way of the linen supply's delivery system, the delivery charge corresponds to the right-to-buy tariff of a uniform two-part pricing system: Customers can use a linen supply's products, and hence pay a per-unit charge for each piece of linen they soil only if they receive clean linen to start with. But since the way customers receive clean linen is through the linen supply's delivery system, customers can acquire the right to use a linen supply's products, and pay a per-unit charge for each piece soiled, only if they pay the delivery charges for each type of linen they use. Hence, the delivery charge associated with a given product is, in effect, a charge for the right to rent that product at a given per-unit charge. This per-unit charge is, of course, the laundering fee charged by the linen supply.

The Delivery Charge Plus Laundering Fee pricing system, to my knowledge, is not currently used by any linen supply to price any of its products. However, as we shall see in the following chapter, linen supplies operating in some markets would do well to adopt it.

One of the most common pricing systems used by linen supply companies is the Piece-Rate With a Minimum system. Under a Piece-Rate With a Minimum system, as you recall, the customer is charged a per-unit price for each piece of linen he soils subject to the requirement that he must purchase a minimum dollar volume from the linen supply per month. This

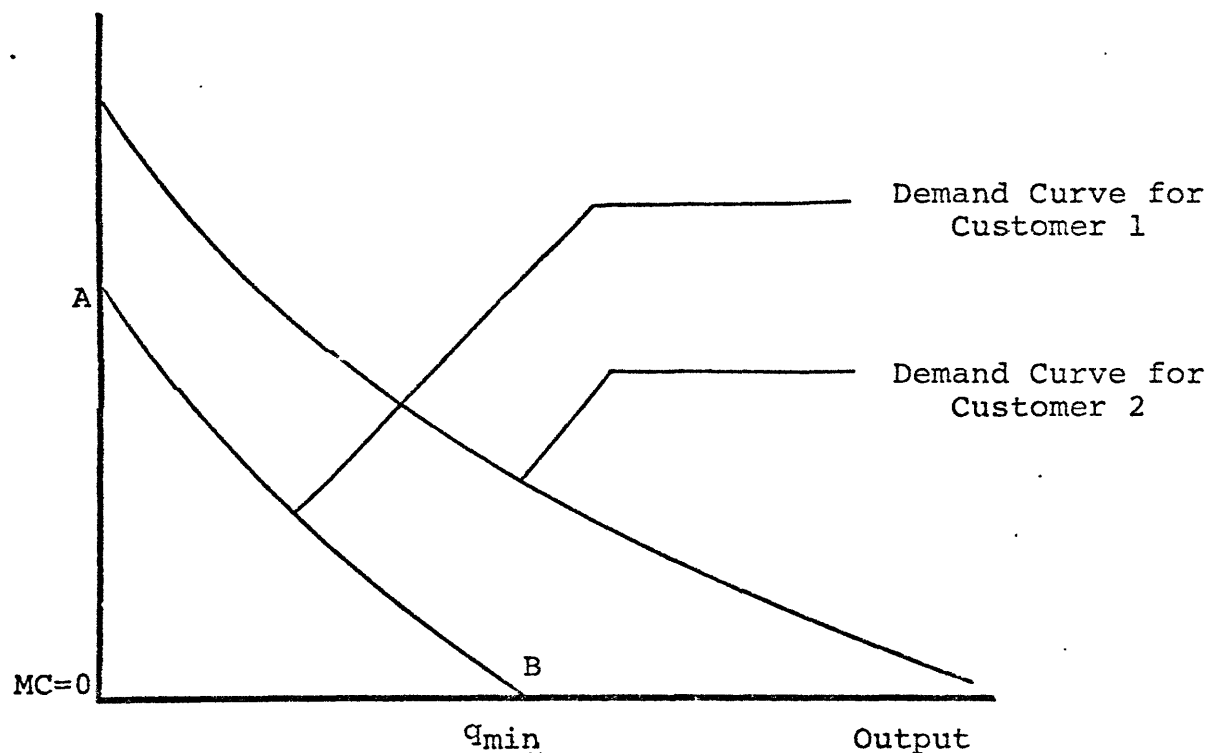


pricing system was invented by the linen supply industry as a solution to the problem of critical customer size. That is, since some customers may demand such small amounts of linen at given prices that it is not profitable for the linen supply to accept the business of such customers, linen supplies needed a way to drive those customers from the market. By establishing a monthly minimum for small customers, the linen supply either forces its small customers to cover the linen supply's cost of providing them linen service or drives those customers from the market.

Interestingly enough, the Piece-Rate with a Minimum pricing system turns out to be a very subtle variety of uniform two-part pricing system. Although the correspondence is not exact, we may think of the required monthly minimum as the right-to-buy tariff and the linen supply's various piece-rate charges as the per-unit price charged under a two-part pricing system. The minimum monthly charge is like a right-to-buy tariff in that, if a customer wishes to rent any amount of linen at all, he must pay the monthly minimum. The monthly minimum charge, however, is different from the right-to-buy tariff of the uniform two part pricing system we have been considering in two respects: First, by paying the required minimum, a customer acquires the right to receive not just one but all of the products a linen supply provides. Second, by paying the fee, a customer is entitled to receive linen, at no extra charge, up to the value of the monthly minimum fee. Similarly, the piece-rate

price under a Piece-Rate with a Minimum pricing system is different from the per-unit price of a uniform two-part tariff in that it becomes effective only after the customer exceeds his monthly minimum. Until that point, piece rate prices are used merely to calculate how much of his minimum a customer has used.

To see more exactly how the Piece-Rate with a Minimum pricing system might be construed as a sort of uniform two-part tariff, let us consider a market of two customers who are served by a monopolistic linen supply with zero marginal costs. To simplify matters, we shall assume that the linen supply does not grant volume discounts and that the linen supply provides customers with only one type of linen. Thus, in our example, we shall not be concerned with the problem of minimum customer size, nor with the problem of volume discounts, nor with the problem of joint costs in linen supplies. Also, since the linen supply provides only one type of product, we shall have the advantage of being able to refer to the required minimum in terms of dollars or in units of linen soiled per month.



#### Individual Demand Curves in a Market of Two Customers

Suppose for a moment that the linen supply were to ignore Customer 2 and concentrate its efforts on fully exploiting the smaller customer, Customer 1. To do this, the linen supply might use a uniform two-part tariff of the sort we have been discussing. It would set the per-unit price it charges for linen equal to zero and charge Customer 1 a right-to-buy tariff equal to the area  $\int_0^{\infty} \Psi_1(P)dP$  or, equivalently, the area  $O-A-B$  in the above graph. By following such a strategy, the linen supply rents Customer 1 exactly as many units of linen as is profitable,  $q_{min}$  units in the above graph, and charges Customer 1 the maximum amount he is willing to pay to receive those  $q_{min}$  units.

The linen supply, however, could have accomplished the same end by charging Customer 1 a simple piece-rate price of

$P^*$ , where  $P^* = \frac{\int_0^{\infty} \psi_1(P) dP}{q_{\min}}$  and requiring that he purchase a minimum of  $q_{\min}$  units. Under these conditions, Customer 1 would just be willing to purchase  $q_{\min}$  units since the total amount Customer 1 would have to pay to receive a total of  $q_{\min}$  units is exactly to Customer 1's consumer surplus:

$$\begin{aligned} P^* \cdot q_{\min} &= \frac{\int_0^{\infty} \psi_1(P) dP}{q_{\min}} \cdot q_{\min} \\ &= \int_0^{\infty} \psi_1(P) dP \end{aligned}$$

If the same price and minimum were quoted to Customer 2, then one of two things could happen.

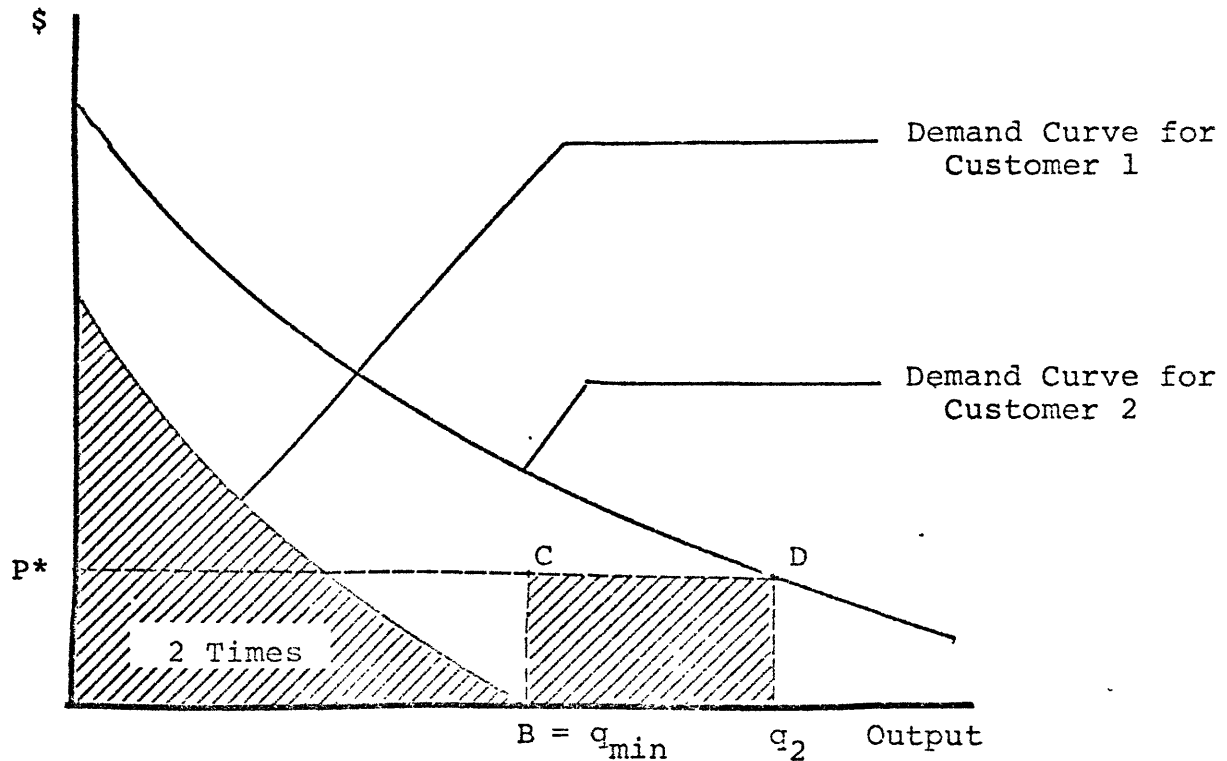
First, if in the absence of a minimum at a price of  $P^*$ , Customer 2 would prefer to purchase less than  $q_{\min}$  units, then Customer 2 would purchase only the minimum number of units required. We know that he would purchase at least the minimum since his demand curve lies everywhere above that of Customer 1, and thus he would be willing to pay more for  $q_{\min}$  units than would Customer 1. How this would occur is depicted in the following graph.



The linen supply's profits in this case are equal to twice the piece-rate price times the monthly minimum; these profits are represented by twice the shaded area in the above graph.

Under this alternative, the pricing strategy we derived for fully exploiting Customer 1, remains the best strategy the linen supply could follow when dealing with both customers: If the linen supply were to increase the piece-rate price, then it would have to decrease its minimum, or lose the business of Customer 1. Yet, if it does decrease its minimum, then both customers will demand less and the firm's profits will fall. If the linen supply were to decrease its price, neither customer would respond at least initially, by demanding more linen and hence its profits would decline.

The second alternative which might occur is that at a piece-rate price of  $P^* = \frac{\int_0^{\infty} \psi_1(P) dP}{q_{\min}}$ , Customer 2 would prefer to purchase more than the required minimum of  $q_{\min}$ . This event is depicted in the following graph.



Profit Earned Under a Piece-Rate with a Minimum Pricing System

In this case, at a piece-rate price of  $P^*$  the amount that Customer 2 would choose to purchase in the absence of a minimum is equal to  $q_2$  which is greater than the proposed minimum  $q_{\min}$ . In effect, the minimum is not binding on Customer 2: at a price of  $P^*$ , Customer 2 will purchase  $q_2$  units independently of whether or not a minimum of  $q_{\min}$  is enforced by the linen supply. The profit earned by the linen supply in this case is given by the shaded area in the above graph. Notice that the linen supply derives profit from two sources: it receives profit equal to twice the consumer surplus of Customer 1 by charging both customers a piece-rate of  $P^*$  and requiring them to purchase

at least  $q_{\min}$  units at that price. Also, it receives profit equal to the area  $q_{\min}-q_2-D-C$  from its sales to Customer 2 in excess of the required minimum.

Under this second alternative, it is no longer clear that the pricing strategy we derived for fully exploiting Customer 1 remains the best strategy for the linen supply to follow. Since the minimum requirement of  $q_{\min}$  pieces of linen is not binding on Customer 2 at a piece-rate price of  $P^*$ , the linen supply might be able to increase the profit it earns from Customer 2 by adjusting the piece rate-price it charges. However, since any change in the piece-rate price or the minimum will decrease the profits earned from Customer 1, the linen supply must balance the gains that may be had from Customer 2 through changing the piece-rate, with the losses it will suffer from Customer 1.

Thus, we might state the problem faced by the linen supply in our example as follows:

Choose a piece-rate price  $P$ , and a monthly minimum  $M$  so as to maximize total profit as given by the equation:

$$\pi = 2 \cdot P \cdot M + P \cdot (\phi_2(P))$$

(Where  $\phi_2(P)$  is the quantity of linen demanded by the larger customer in excess of the required minimum.)

Subject to:

$$(1) \quad M \leq q_{\min}$$

The monthly minimum must be less than the maximum amount customer 1 is willing to purchase.



$$(2) \quad P \leq \frac{\int_0^{\infty} f_1(P) dP}{M}$$

The piece-rate price and the monthly minimum must be set so as to allow Customer 1 to remain in the market.

Having solved this constrained maximization problem, the linen supply will then wish to compare the profit earned under this solution with the profit that could be earned by excluding the smaller customer from the market and using the Piece-Rate with a Minimum Pricing System to fully exploit the larger customer.

The rather complex formulation of the linen supply's profit maximization problem under a Piece-Rate With a Minimum pricing system, even in such a simple situation as the one we have been considering, is a hint of the complexities that would ensue were we to extend the analyses to more realistic situations. When a linen supply has more than two customers, or provides more than a single product, profit maximization under the Piece-Rate with a Minimum becomes unfathomable. Fortunately, as we shall see in the following chapter, other pricing systems whose analyses are much easier, can be shown to produce profits which are at least as great as those of the Piece-Rate With a Minimum system.

### Part III. Variable Two-Part Pricing Systems

Under a variable two-part tariff, a firm charges its customers two prices: First, the firm charges its customers a right-to-buy tariff which varies in size from customer to

customer. Second, the firm charges a per-unit price for each unit of its output the customer purchases, and this per-unit price is the same for all of the firm's customers.

Nearly all variable two-part pricing systems have one fatal flaw: they are not legal. Recall that Sections 1 and 3 of the Robinson-Patman Act provide:

That it shall be unlawful for any person engaged in commerce, in the course of such commerce, either directly or indirectly, to discriminate in price between different purchasers of commodities of like grade and quality, where either or any of the purchases involved in such discrimination are in commerce, where such commodities are sold for use, consumption, or resale within the United States or any Territory thereof or the District of Columbia or any insular possession or other place under the jurisdiction of the United States, and where the effect of such discrimination may be substantially to lessen competition or tend to create a monopoly in any line of commerce, or to injure, destroy, or prevent competition with any person who either grants or knowingly receives the benefit of such discrimination, or with customers of either of them: Provided, that nothing herein contained shall prevent differentials which make only due allowance for differences in the cost of manufacture, sale, or delivery resulting from the differing methods for quantities in which such commodities are to such purchasers sold or delivered.

Although the application of the Robinson-Patman Act to variable two-part tariffs has not been tested, it would seem that, barring very special circumstances, variable two-part tariffs will not be allowed by the courts. Firms would seem to be required by the phrase, "Provided that nothing herein contained shall prevent differentials which make only allowance for differences in . . . cost . . ." (emphasis added) to quote its most costly customers a price equal to the price it

charges its less costly customers plus the amount of the extra cost the firm incurs in selling to its more costly customers. When this restriction is applied, a variable two-part tariff will not be significantly different from a uniform two-part tariff, since the right to buy tariff charged different customers can vary only slightly. Thus, the profits achieved under a variable two-part system will be approximately equal to the profits achieved under a uniform two-part system.

The linen supply industry, however, may provide an exception to this general proscription of variable two-part tariffs. From the customer's point of view, the good which a linen supply provides is clean linens delivered regularly to him. Yet, arguably, a linen supply is really providing two goods: It is renting customers an inventory of linen, and it is washing those linens and returning them to customers. If linen supplies only rented customers linen inventories, they would be a very specialized sort of leasing company. If linen supplies only washed and returned linens to its customers, then they would not be linen supplies but laundries. Thus, a linen supply would seem to have a right to charge customers a price for each of the goods it provides. If so, then as we shall see linen supplies can arrange the prices for the two goods it provides in such a way as to establish a variable two-part tariff.

A. Inventory Charge Plus Laundering Fee as a Variable Two Part Tariff

Under the Inventory Charge Plus Laundering Fee pricing system, the linen supply charges its customers in two parts. First, the linen supply charges its customers a fee according to the amount of linen inventory that the linen supply must purchase and maintain in order to service each customer. The amount of this monthly inventory charge varies among customers depending on the quantity and types of linen each customer uses. Second, the linen supply charges customers a laundering fee for each piece of linen that it picks up soiled from the customer, washes and returns clean to the customer. This laundering fee is, of course, the same for all customers although the fee will differ depending on the type of linen involved.

The Inventory Charge Plus Laundering Fee pricing system is a sort of variable two-part tariff. The laundering fee charged for a particular type of linen corresponds to the per-unit of output price, and the inventory charge for a particular customer corresponds to right-to-buy tariff of a variable two-part pricing system. Since normally, a customer agrees that only the linen supply will have the right to wash the linen it delivers to the customer, by paying his monthly inventory fee, the customer really only acquires the right to receive clean linen in exchange for paying the appropriate laundering fee. Thus, the inventory fee is a right-to-buy tariff. Since the inventory fee charged a particular customer depends on the

amount of linen inventory the linen supply maintains on his behalf, and since different customers require different amounts of inventory, the inventory fee charged customers will vary from customer to customer. Thus, the inventory fee is a variable right-to-buy tariff.

We may represent the profit that a linen supply would earn on a given type of linen under an Inventory Charge Plus Laundering Fee pricing system by the following equation.

$$\pi = T \cdot \sum_{i=1}^N I(x_i) + P \cdot X_N - TC(X_N)$$

Where:

$T$  = Inventory fee per unit of inventory

$N$  = Number of customers

$x_i$  = Quantity of the given type of linen received by the  $i^{\text{th}}$  customer

$I(x_i)$  = Quantity of inventory the linen supply must hold to provide the  $i^{\text{th}}$  customer with  $x_i$  units of linen

$P$  = Laundering fee

$X_N$  = Total quantity of the given type of linen soiled by  $N$  customers.

$TC(X_N)$  = Total cost of providing customers with those  $X_N$  units of linen.

What this equation says is, in effect, that the profits of the linen supply are equal to the revenues it receives from charging an inventory fee plus the revenues it receives from charging a laundering fee minus its total costs. The profit maximization

problem faced by a linen supply wishing to use this system is to select a laundering fee  $P$ , a per-unit of inventory charge  $T$ , and a mix of customers  $N$  so as to maximize the profits given in the above equation.

We may simplify the profit maximization problem of the linen supply by establishing two rules very similar to the rules set out above for maximizing a firm's profit under a uniform two-part pricing system. That is, first we shall define a rule to determine the optimal combination of inventory fee and laundering fee, assuming a given number of customers are allowed to remain in the market. Second, we shall define a rule for the firm to follow in order to select an optimal mix of customers.

Rule 1: Defining the Optimal Combination of Inventory Fee and Laundering Fee

Following the analysis for profit maximization under a uniform two-part pricing system, I shall begin the analysis of the optimal mix of inventory fee and laundering fees by considering what the greatest right-to-buy tariff is that the linen supply could charge if the laundering fee is fixed and no customers are to be driven from the market.

For a given laundering fee  $P$ , and a given number of customers  $N$ , the maximum right-to-buy tariff the linen supply can charge is equal to the smallest consumer surplus of any of the linen supply's customers. That is, since a customer's consumer surplus is equal to the maximum amount that that customer would

be willing to pay for the right to receive linen at a laundering fee of  $P$ , the largest amount that the linen supply can charge for that right, without driving customers from the market, is the smallest consumer surplus that any customer would enjoy at a laundering fee price of  $P$ . However, under an Inventory Charge Plus Laundering Fee system, the right-to-buy tariff is the inventory charge, and the inventory charge a customer is required to pay depends on the amount of inventory the linen supply must hold to service that customer. Thus, the maximum inventory fee the linen supply can charge per-unit of inventory is the smallest amount per-unit of inventory that any customer would be willing to pay for the right to receive linen at a laundering fee of  $P$ . Equivalently, the maximum inventory fee  $T^*$  the linen supply can charge is the smallest ratio of consumer surplus to inventory of all customers the linen supply serves:

$$T^* = \text{Min} \left( \frac{\int_P^\infty \Psi_1(P) dP}{I(x_1)}, \dots, \frac{\int_P^\infty \Psi_N(P) dP}{I(x_N)} \right)$$

Or, in abbreviated form,

$$T^* = \text{Min}_{i=1 \text{ to } N} \left( \frac{\int_P^\infty \Psi_i(P) dP}{I(x_i)} \right)$$

Thus, if Customer  $J$  is the customer with the smallest ratio of consumer surplus to inventory, Customer  $J$  will be required to pay a total inventory charge exactly equal to his consumer surplus:

$$\begin{aligned}
 \text{Inventory Charge} &= T^* \cdot I(x_J) \\
 \text{for Customer J} & \\
 &= \frac{\int_p^\infty \Psi_J(P) dP \cdot I(x_J)}{I(x_J)} \\
 &= \int_p^\infty \Psi_J(P) dP
 \end{aligned}$$

Customer J need not always be the customer with the smallest consumer surplus. In fact, a customer could have a very large consumer surplus and still have the smallest ratio of consumer surplus to inventory, if the inventory the linen supply held on his behalf were very large.

When this expression for  $T^*$  is substituted into the equation describing a linen supply's profit under the Inventory Charge Plus Laundering Fee pricing system, where the number of customers  $N$  is assumed to be fixed, the linen supply's profits are expressed as a function of one variable, the laundering fee  $P$ .

$$\pi = \text{Min}_{i=1 \text{ to } N} \left( \frac{\int_p^\infty \Psi_i(P) dP}{I(x_i)} \right) \cdot \left( \sum_{i=1}^N I(x_i) \right) + P \cdot X_N - TC(X_N)$$

Where the terms in this equation have the following interpretations:

$$\text{Min}_{i=1 \text{ to } N} \left( \frac{\int_p^\infty \Psi_i(P) dP}{I(x_i)} \right) = \text{Largest per unit of inventory fee the linen supply can charge without driving any of its potential customers from the market}$$



$\sum_{i=1}^N I(x_i)$  = Total inventory of linen (of the relevant type) that the linen supply must hold to service all N customers

$X_N$  = Total amount of linen soiled by customers during one month

P = The laundering fee price

$T_C(X_N)$  = Total cost of providing N customers with X pieces of linen during the month

To find that value of P for which the linen supply's profits are maximized, we take the derivative of this equation and set it equal to zero.

$$\frac{d\pi}{dP} = 0 = X_N + P \cdot \frac{dX_N}{dP} + \underset{\text{to } N}{\text{Min}}_{i=1} \left( \frac{\int_P^{\infty} \psi_J(P) dP}{I(x_i)} \right) \cdot \frac{d \left( \sum_{i=1}^N I(x_i) \right)}{dP}$$

$$\frac{+ d \underset{\text{to } N}{\text{Min}}_{i=1} \left( \frac{\int_P^{\infty} \psi_i(P) dP}{I(x_i)} \right) \cdot \left( \sum_{i=1}^N I(x_i) \right) - \frac{d(TC)}{dP}}{dP}$$

The laundering fee price P which satisfies this equation, we call it  $P^*$ , is the optimal laundering fee price that the linen supply can charge. Of course, without knowing more about the demand curves of linen supply's customers and the nature of the linen supply's inventory-stocking policy, we cannot solve for  $P^*$ . Let us therefore rely on current practice in the linen supply industry and some common sense about customer behavior to supply this missing bit of information.

Specifically, I propose to make the following two assumptions:

First, the inventory a linen supply holds to service customers, is determined by multiplying some constant by the amount of linen those customers are expected to soil per month. Thus, for a given customer  $K$ , the amount of inventory the linen supply will hold to service customer  $K$  is:

$$I(x_K) = C \cdot x_K$$

That this assumption is reasonable is evident from the fact that Steiner Corporation, a company which owns several linen supplies, recommends that the initial stocking of a customer's inventory of flat linen be calculated in just the fashion I have described.<sup>4</sup>

Second, we shall also assume that customers respond to relative changes in price rather than absolute changes in price. Thus, when deciding on his response to a change in the laundering fee price, a customer will consider the percentage increase or decrease in price. A price change from 25¢ to 50¢ (a doubling in price) will elicit the same percentage decrease in the quantity of linen received as would a price increase from \$1.00 to \$2.00 (also a doubling in price). Economists believe that, in general, this assumption better describes people's behavior than the alternative assumption that customers respond to absolute changes in price: an increase in price from 25¢ to 50¢, (a doubling in price), elicits the same response from customers as would an increase from \$2.00 to \$2.25 (a 13% increase in price).

In order to incorporate this assumption into the analysis, we shall take the demand curve of Customer J, the customer with the smallest consumer surplus per unit of inventory remaining in the market, to be of the form:

$$x_J = a_J P^{b_J}$$

where  $x_J$  is the number of linen pieces Customer J soils per month,  $a_J$  is same constant,  $P$  is the laundering fee and  $b_J$  is Customer J's constant price elasticity, and where  $b_J$  is assumed to be less than negative one.<sup>5</sup>

When these two assumptions hold, the optimal laundering fee price  $P^*$  is given by the following equation:

$$P^* = \frac{MC \cdot E}{1 + E - \left(\frac{E+1}{b_J+1}\right)}$$

Where:

MC = Marginal Cost

E = Elasticity of market demand curve

$b_J$  = Price elasticity of customer with the smallest consumer surplus per unit of inventory.

( $b < -1$ ).

(Calculations are included in Appendix II of this chapter).

Once the linen supply has determined the optimal laundering fee price  $P^*$ , it can substitute  $P^*$  into the equation for  $T^*$  to determine the optimal per-unit of inventory fee to charge:

$$T^* = \frac{\int_{P^*}^{\infty} a_J P^{b_J} dP}{I(x_J)}$$

We may summarize these results by way of the following rule for the optimal combination of inventory fee and laundering fee:

Optimal Price Combination Rule

In a market of  $N$  customers, who demand in aggregate  $X_N$  units of linen, and in which the following conditions hold:

- (1) The amount of inventory held on behalf of a customer receiving delivery at a given frequency, is determined by multiplying a constant by the quantity of linen the customer is expected to soil each month (different constants may be used for different delivery frequencies);
- (2) The customer with the smallest consumer surplus per unit of inventory has a demand curve of the form

$$x_J = a_J P^{b_J}, \quad b < -1$$

where  $x_J$  is the quantity of linen received by the customer,  $a_J$  is a constant,  $P$  is the laundering fee and  $b_J$  is that customer's constant price elasticity;

then, the linen supply should charge an inventory fee  $T^*$  equal to,

$$\frac{\int_{P^*}^{\infty} a_J P^{b_J} dP}{I(x_J)}$$

and a laundering fee equal to:

$$\frac{MC \cdot E}{1+E - \frac{E+1}{b_J+1}}$$

where:

$a_J P^{b_J}$  = Demand curve of the customer with the smallest consumer surplus per-unit of inventory of any customer remaining in the market

$I(x_J)$  = Inventory held to service Customer J =  $C \cdot a_J P^{b_J}$

MC = Marginal Cost

E = Price elasticity of market demand curve

$b_J$  = Price elasticity of Customer J

### Rule 2: Selecting the Optimal Customer Mix

Again following the analysis for profit maximization under a uniform two-part pricing system, the linen supply will wish to compare the profit it would earn by serving all customers in the market with the profit it would earn by adjusting the prices it charges so as to exclude the customer with the smallest consumer surplus per-unit of inventory from the market. The linen supply would thus use the above rule to calculate the optimal inventory fee and laundering fee when all N customers remain in the market, and substitute those prices into the profit equation for a linen supply using an Inventory Charge Plus Laundering Fee system:

$$\pi = T^* \cdot \sum_{i=1}^N I(x_i) + (P^* \cdot X_N) - TC$$

Where:

$\sum_{i=1}^N I(x_i)$  = Total quantity of inventory the linen supply must hold to provide service to all N customers.

$X_N$  = Total quantity of linen soiled by N customers  
 TC = Total Cost

The linen supply will then apply the above rule to determine the profit it would earn by serving all customers but the customer with the smallest consumer surplus per-unit of inventory.

Again, it will substitute those values into the profit equation:

$$\pi = T^* \sum_{i=1}^{N-1} I(x_i) + P \cdot X_{N-1} = TC$$

The linen supply would continue this application of the optimal price combination rule for successively smaller groups of customers. The optimal group of customers for the linen supply to serve will be that group for which the linen supply's expected profits are the greatest.

Optimal Customer-Mix Rule:

Continue to exclude customers from the market in the order of which remaining customer has the smallest consumer surplus per-unit of inventory, until anticipated profits begin to decline.

The complexity of these two rules definitely limits their usefulness to a linen supply manager who prefers to set prices by way of back-of-the envelope calculations. However, as we shall see in Chapter V, if the manager is willing to use a computer program to assist him in setting prices, these rules for the optimal use of the Inventory Charge Plus Laundering Fee pricing system may be quite useful. Computers

after all, are much better suited to tedious and complicated calculations than are ordinary mortals--especially time-pressed linen supply managers.

B. The Flat Rate Pricing System as a Variable Two-Part Tariff

Under the Flat Rate pricing system customers are charged according to the amount of linen the customer is expected to soil rather than according to the amount he actually does soil each month. Thus, for example, if a particular customer is expected to use 25 Turkish towels each month, then under the Flat Rate system his monthly bill for Turkish towels will be the same for months in which he actually uses 20 towels as for months in which he uses 25 towels.

The Flat Rate pricing system may be seen as a very special sort of variable two-part tariff. Recall that under a variable two-part tariff, customers are charged two prices: a per-unit price charged customers for each unit of output they receive, and a right-to-buy tariff charged for the right to purchase output at the per-unit price. Under a variable two-part tariff, of course, different customers are charged different right-to-buy tariffs, although all customers are charged the same per-unit price. Under the special case in which the per-unit price is set equal to zero, a variable two-part tariff collapses into a pricing system in which different customers are charged different amounts for the right to receive units at no cost to the customer per-unit. This special case of a

variable two-part pricing system, I believe, describes very well the Flat Rate pricing system.

Under the Flat Rate system, the amount billed a customer each month remains constant, although the amount of linen he actually receives may vary from month to month. Thus, since the amount billed under the flat rate system is not directly tied to the amount of linen a customer actually uses, it would not be proper to regard the flat rate charged a customer as a price charged per-unit of linen. Rather, it is more reasonable to think of the flat rate charged a particular customer as a right-to-buy tariff. By paying his monthly flat rate, a customer acquires the right to receive linen at no additional cost. In effect, a customer pays a lump sum each month for the right to receive linen at a per-unit of linen cost equal to zero. Furthermore, since the flat rate charged a customer depends on the amount of linen he is expected to use each month, and since different customers are expected to use different amounts of linen per month, different customers will be charged different right-to-receive linen tariffs. Thus, the Flat Rate pricing system is a sort of variable two-part tariff--albeit a variable two-part tariff of a very special variety.

Under a Flat-Rate system, the profits a linen supply earns are equal to the sum of the right-to-buy tariffs, or flat rates it charges customers, less the total cost of providing linen service to those customers. We have seen that the maximum right-to-buy tariff that can be charged a particular customer



is the amount of that customer's consumer surplus. Thus, when the laundering fee price is set equal to zero, the largest flat rate the linen supply can charge its  $i^{\text{th}}$  customer is the amount of his consumer surplus, which when expressed mathematically is the following:

$$\int_0^{\infty} \Psi_i(P) dP$$

Of course, if the linen supply is to steer clear of price-discrimination charges, then the flat rates it charges each customer must be clearly explicable in terms of the quantities each customer is expected to receive each month. Specifically, it would seem reasonable to require that the linen supply charge customers the same flat rate per-unit of linen that the customer is expected to use:

$$\frac{\text{Flat Rate Billed Customer I}}{\text{Quantity of Linen Customer I Expects to Receive}} = \frac{\text{Flat Rate Billed Customer J}}{\text{Quantity of Linen Customer J Expects to Receive}}$$

There is, of course, no guarantee that when this restriction is imposed, the linen supply will be able to charge both Customer I and Customer J the amount of their consumer surpluses. In fact, both customers will be charged the maximum amount they would be willing to pay for the right to receive linen at a zero price per-unit only if,

$$\frac{\int_0^{\infty} \Psi_I(P) dP}{\text{Quantity of Linen Customer I Expects to Receive}} = \frac{\int_0^{\infty} \Psi_J(P) dP}{\text{Quantity of Linen Customer J Expects to Receive}}$$

But this will be true only in very rare circumstances. More commonly the linen supply will not be able to charge both customers the amount of their consumer surpluses. If not, and if the linen supply does not drive either customer from the market, then the best the linen supply can do is to charge one of its customers a flat rate equal to his consumer surplus and all other customers a flat rate less than their consumer surpluses. The customer who will be charged a flat rate equal to his consumer surplus is the customer for which  $\frac{\int_0^{\infty} \psi(P) dP}{\text{Quantity of Linen that Customer Expects to Receive}}$  is the smallest. For example,

Quantity of Linen  
that Customer Expects  
to Receive

in a market of two customers, if

$$\frac{\int_0^{\infty} \psi_1(P) dP}{q_1} < \frac{\int_0^{\infty} \psi_2(P) dP}{q_2}$$

where  $q_1$  and  $q_2$  are the quantities of linen that Customer 1 and Customer 2 are respectively expected to receive, then Customer 1 would be charged a flat rate equal to his consumer surplus:

$$\begin{array}{l} \text{Flat Rate Billed} \\ \text{Customer 1} \end{array} = \int_0^{\infty} \psi_1(P) dP$$

However, due to our requirement that the linen supply charge customers the same flat rate per-unit of linen that the customer is expected to use, it must be the case that:

$$\frac{\text{Flat Rate Billed Customer 2}}{q_1} = \frac{\text{Flat Rate Billed Customer 1}}{q_2} = \frac{\int_0^{\infty} \psi_1(P) dP}{q_1}$$

Hence the flat rate billed Customer 2 is given by the following:

$$\frac{\text{Flat Rate Billed Customer 2}}{q_2} = \frac{\int_0^{\infty} \psi_2(P) dP}{q_1} \cdot q_2$$

The flat rate billed Customer 2 is, however, less than the maximum amount he would be willing to pay for the right to receive linen at a zero price per-unit:

$$\begin{aligned} \text{Consumer Surplus of Customer 2} &= \int_0^{\infty} \psi_2(P) dP \\ &= \frac{\int_0^{\infty} \psi_2(P) dP}{q_2} \cdot q_2 \end{aligned}$$

And since by assumption

$$\frac{\int_0^P \psi_2(P) dP}{q_2} < \frac{\int_0^{\infty} \psi_1(P) dP}{q_1}$$

$$\text{Consumer Surplus of Customer 2} > \frac{\int_0^{\infty} \psi_1(P) dP}{q_1} \cdot q_2$$

$$\text{Consumer Surplus of Customer 2} > \text{Flat Rate Billed Customer 2}$$

This method of setting flat rates provides the linen supply with as much revenue as possible under a flat rate system in which both customers are allowed to remain in the market. The flat rate charged Customer 1 cannot be increased without charging him more than his consumer surplus; more than he would be willing to pay for the right to receive linen at a zero price per-unit. Similarly, the flat rate billed Customer 2 cannot be increased without violating the condition that both customers be charged the same flat rate per-unit of linen that the customer is expected to use.

The profit the linen supply would expect to earn is equal to the sum of the flat rates charged both customers less the expected total cost of providing linen to both customers:

$$E(\pi) = \int_0^{\infty} \psi_1(P) dP + \left( \frac{\int_0^{\infty} \psi_1(P) dP}{q_1} \cdot q_2 \right) - TC(q_1+q_2)$$

$$= \frac{\int_0^{\infty} \psi_1(P) dP}{q_1} \cdot (q_1+q_2) - TC(q_1+q_2)$$

Because the revenue the linen supply would receive when all customers are allowed to remain in the market is maximized by the procedure for setting flat rates, the only way that the linen supply can affect the amount of profit it expects to receive is by driving customers from the market. Thus, in our example, the linen supply will wish to calculate the expected profit it would earn by driving Customer 1 from the market and concentrating its efforts solely on supplying linens to Customer 2.

When generalized to a market of more than two customers, the arguments of this section yield the following rule for maximizing profit under a Flat Rate pricing system.

Rule for Maximizing Profit Under a Flat Rate Pricing System:

For a market of N customers, and for a linen supply which is constrained to set flat rates such that, for any two customers, I and J:

<u>Flat Rate Billed</u> <u>Customer I</u>	=	<u>Flat Rate Billed</u> <u>Customer J</u>
Quantity of Linen Customer I Expects to Soil		Quantity of Linen Customer J Expects to Soil

the linen supply will maximize its profits by following two steps:

First, the linen supply should identify which of its  $N$  potential customers has the smallest consumer surplus per unit of linen the customer is expected to soil per month. That is, the linen supply should determine which of its customers is the customer  $K$  such that:

$$\frac{\int_0^{\infty} \Psi_K(P) dP}{q_K} = \text{Min}_{i=1 \text{ to } N} \frac{\int_0^{\infty} \Psi_i(P) dP}{q_i}$$

Where  $q_K$  is the quantity of linen Customer  $K$  is expected to soil each month and  $q_i$  is the quantity of linen the  $i^{\text{th}}$  customer is expected to soil each month. The Customer  $K$  will be charged a flat rate equal to his consumer surplus,  $\int_0^{\infty} \Psi_K(P) dP$ . The flat rates charged all other customers are then determined by the flat rate charged Customer  $K$  and the requirement that for any customer  $J$ :

$$\frac{\text{Flat Rate Billed Customer K}}{q_K} = \frac{\text{Flat Rate Billed Customer J}}{q_J}$$

The linen supply should then calculate the profit it would expect to earn by dealing with all  $N$  customers according to the formula:

$$E(\pi) = \frac{\int_0^{\infty} \Psi_K(P) dP}{q_K} \cdot \sum_{i=1}^N q_i - TC \left( \sum_{i=1}^N q_i \right)$$

Second, the linen supply should determine what expected

profits would be if it excluded Customer K, the customer with the smallest consumer surplus per unit of linen the customer is expected to soil per month. The linen supply would do this by identifying the customer that, next to Customer K, has the smallest consumer surplus per unit of linen expected to be soiled. This customer would be charged a flat rate equal to his consumer surplus, and all other customers would be charged flat rates consistent with the requirement that all customers be charged the same flat rate per unit of linen expected soiled. The flat rate calculated for Customer K will exceed the amount of his consumer surplus, and thus drive him from the market. The linen supply would then calculate the profit it would expect to earn by excluding Customer K. If the linen supply's expected profits decline when Customer K is excluded, then expected profits are at a maximum when all N customers remain in the market. If the linen supply's expected profits increase when Customer K is excluded, then the linen supply will wish to determine whether it can increase its expected profits still more by excluding other customers from the market. The linen supply's expected profits are at a maximum when the result of excluding one additional customer from the market is to decrease rather than increase expected profits.

In the following chapter, Chapter IV, I shall discuss which of the four pricing systems discussed in this chapter-- Delivery Charge Plus Laundering Fee, Piece-Rate With a Minimum, Inventory Charge Plus Laundering Fee, and Flat Rate--would be

most profitable for linen supplies to use. The arguments of that chapter will show that the linen supply industry could make larger profits by abandoning its more traditional pricing systems and adopting other systems which to date have not received general acceptance in the industry.

## FOOTNOTES

## CHAPTER THREE

<sup>1</sup>The rule for maximizing profits under a discriminating one-part pricing system would actually require the firm to continue to sell output to Customer 1 as long as the price he is willing to pay is greater than marginal cost. I have altered the rule here only for the purpose of the argument I am constructing in order that Customer 1 will purchase only  $q_1^*$  units.

<sup>2</sup>The analysis I shall present is borrowed from an article by Walter Y. Oi, "A Disneyland Dilemma: Two-Part Tariffs For a Mickey Mouse Monopoly," Quarterly Journal of Economics, Vol. 85, (February 1971), 77.

<sup>3</sup>The price elasticity of a demand curve is defined as the percentage change in quantity demanded that results from a one percent change in price. Thus, if the per-unit price were raised 1% and the quantity demanded declined by 1% in response, the demand curve would have an elasticity of -1. More precisely, the price elasticity of a demand curve is:

$$E = \frac{P}{X} \cdot \frac{dX}{dP}$$

where X is the quantity of output demanded.

<sup>4</sup>Steiner Corporation's Standard Operating Procedures Manual recommends:

In stocking flat work on the first delivery for a new account, install 125% of their estimated usage, based upon frequency of delivery. This applies to deliveries every 4 weeks, E.O.W., weekly, semiweekly, or daily.

Thus, for flat work, inventory is determined by the equation:

$$\text{Inventory} = 1.25 \cdot \text{Amount Delivered}$$

<sup>5</sup>If a customer has a demand curve of the form  $X = aP^b$ , and if b is not less than negative one, then at any finite price,  $P_1$ , that customer would have an infinitely large consumer surplus. That is, if  $b = -1$ , then the consumer surplus the customer would enjoy is the following:

$$\text{Consumer Surplus} = \int_{P_1}^{\infty} aP^{-1} dP$$



$$\begin{aligned}
&= \lim_{x \rightarrow \infty} \int_{P_1}^x a P^{-1} dP \\
&= \lim_{x \rightarrow \infty} (a \ln X - a \ln P_1)
\end{aligned}$$

But since, as  $X$  takes on larger and larger values, the logarithm of  $X$  increases indefinitely,

$$\lim_{x \rightarrow \infty} (a \ln X - a \ln P_1) = \infty$$

If  $b > -1$ , we encounter a similar problem.

$$\begin{aligned}
\text{Consumer Surplus} &= \int_{P_1}^{\infty} a P^b dP \\
&= \lim_{x \rightarrow \infty} \int_{P_1}^x a P^b dP \\
&= \lim_{x \rightarrow \infty} \left( \frac{a x^{b+1}}{b+1} - \frac{a P_1^{b+1}}{b+1} \right)
\end{aligned}$$

But if  $b > -1$ , then  $b+1$  will be positive. Hence, as  $X$  takes on larger and larger values,  $X^{b+1}$  will increase indefinitely:

$$\lim_{x \rightarrow \infty} \left( \frac{a x^{b+1}}{b+1} - \frac{a P_1^{b+1}}{b+1} \right) = \infty$$

Of course, the customer with smallest consumer surplus per unit of inventory cannot have an infinitely large consumer surplus, since that would imply his consumer surplus per unit of inventory is also infinite. Thus, I have restricted the price elasticity of the consumer with the smallest consumer surplus to be less than negative one.

Appendix IDerivation of the Optimal Combination of Right-to-Buy and Per-Unit Prices Under a Uniform Two-Part Pricing System

The optimal combination of prices rule was developed by Walter Y. Oi in his article, "A Disneyland Dilemma: Two-Part Tariffs For a Mickey Mouse Monopoly" (Quarterly Journal of Economics, Vol. 85, February 1971, pp. 77-94). His derivation of the rule is the following:

Suppose initially that the monopoly establishes a feasible tariff (consisting of a price  $P$  and lump sum tax  $T$ ) that insures that all  $N$  consumers remain in the market for his product. The profits from this feasible tariff are given by

$$(A.1) \quad \pi = XP + NT - C(X), \quad \left[ X = \sum_{j=1}^N x_j \right]$$

Let  $\psi_j(P)$  describe the constant utility demand curve of the  $j$ th consumer,  $T_j^*$ , is a function of the price  $P$ :

$$(A.2) \quad T_j^* = \int_p^{\infty} \psi_j(P) dP.$$

For any price  $P$ , profits can be increased by setting the lump-sum tax  $T$  equal to the smallest of the  $N$  consumer surpluses that is assigned to the first consumer; that is,  $T = T_1^*$ . The demand for rides by the smallest consumer is thus determined by his constant utility demand,  $x_1 = \psi_1(P)$ . Since the remaining  $N-1$  consumers still enjoy some consumer surplus, their demands for rides depend on the price  $P$  and net incomes ( $M_j - T$ ):

$$(A.3) \quad x_j = D_j(P, M_j - T), \quad [j=2, 3, \dots, N].$$

[I]t is assumed that all  $N$  consumers must be kept in the market. Consequently, the tax  $T$  must be adjusted whenever the price  $P$  is varied in order to keep the smallest consumer in the market. The requisite adjustment is  $(dT/dP) = -x_1$ . In this manner, total profits, equation (A.1), can be reduced to a function of only one parameter, the price per ride  $P$ . Setting  $(d\pi/dP)$  equal

to zero, we get the equilibrium condition for an optimum price  $P$  given a market of  $N$  consumers:

$$(A.4) \quad c' = P \left[ 1 + \left( \frac{1 - NS_1}{E} \right) \right]$$

where  $E$  is the "total" price elasticity of the market demand for rides;

$$(A.5) \quad E = \left( \frac{P}{X} \right) \left[ \sum_{j=1}^N \left( \frac{dx_j}{dP} \right) + x_1 \sum_{j=2}^N \left( \frac{dx_j}{dM_j} \right) \right] = \varepsilon + s_1 \sum_{j=2}^N \lambda_j \mu_j$$

where  $s_1 = x_1/X$  is the smallest consumer's share of the market demand,  $\lambda_j = x_j P/M_j$  represents the budget share devoted to variable outlays for rides, and  $\mu_j$  is the income elasticity of demand for rides.† The optimum price per ride is thus set to satisfy equation (A.4); by substituting this optimum price in equation (A.2) for the smallest consumer ( $j=1$ ), we get the optimum lump sum tax  $T=T_1^*$ . This procedure could conceptually be repeated for any number of consumers,  $n$ , thereby obtaining an optimum tariff and constrained maximum monopoly profits  $\pi(n)$  for a market of  $n$  consumers.

---

†In differentiating profits  $\pi$  with respect to  $P$ , it must be remembered that the market demand  $X$  is a function of  $P$  and  $T$  via equation (A.3). Hence, we have

$$\frac{d\pi}{dP} = X + P \left[ \frac{dX}{dP} + \left( \frac{dX}{dT} \right) \left( \frac{dT}{dP} \right) \right] + N \left( \frac{dT}{dP} \right) - c' \left[ \frac{dX}{dP} + \left( \frac{dX}{dT} \right) \left( \frac{dT}{dP} \right) \right]$$

Recall that  $(dT/dP) = -x_1$  and that  $(dx_j/dT) = -(dx_j/dM_j)$ . Thus, we get

$$\left( \frac{dX}{dT} \right) \left( \frac{dT}{dP} \right) = x_1 \sum_{j=2}^N \left( \frac{dx_j}{dM_j} \right), \quad \frac{dX}{dP} = \sum_{j=1}^N \left( \frac{dx_j}{dP} \right)$$

Collecting terms, we obtain

$$\frac{d\pi}{dP} = (X - x_1 N) + (P - c') \left[ \sum_{j=1}^N \left( \frac{dx_j}{dP} \right) + x_1 \sum_{j=2}^N \left( \frac{dx_j}{dM_j} \right) \right]$$

The "total" price elasticity of the market demand  $E$  is defined as follows:

(Continuation of Footnote †

$$E = \left(\frac{P}{X}\right) \left[ \sum \left(\frac{dx_j}{dP}\right) + x_1 \left(\frac{dx_1}{dM_1}\right) \right]$$

The "total" price elasticity thus incorporates the induced change in demand for rides resulting from the requisite adjustment in T when P is used. If we substitute, we get

$$\frac{d\pi}{dP} = X(1-s_1N) + P - c' \left[ \frac{-XE}{P} \right]$$

Setting  $d\pi/dP$  equal to zero thus yields equation (A.4). In the expression for the "total" price elasticity, the term  $\epsilon$  represents the price elasticity unadjusted for the induced effect of changes in T. More precisely,

$$\epsilon = \left(\frac{P}{X}\right) \sum \left(\frac{dx_j}{dP}\right)$$

Appendix II:Derivation of Optimal Price Combination Rule for the Inventory  
Charge Plus Laundering Fee System

Profits from a particular product are given by the formula:

$$= T \cdot \sum_{i=1}^N I(X_i) + P \cdot X - TC$$

Where:

T=Inventory fee

$\sum_{i=1}^N I(X_i)$ =Total inventory held to service all customers

P=Laundering fee

X=Total quantity of linen soiled

TC=Total cost

These profits will be maximized when:

$$0 = \frac{d(T \sum_{i=1}^N I(X_i))}{dP} + \frac{d(P \cdot X)}{dP} - \frac{d(TC)}{dP}$$

To simplify the equation, we shall consider each of the three component terms in this equation separately.

$$(A) \quad \frac{d(T \sum_{i=1}^N I(x_i))}{dP}$$

Suppose that the amount of inventory the linen supply holds to service any customer is determined by multiplying a constant by the quantity of linen that customer soils per month. If that inventory multiple is C, then the total inventory of the relevant type of linen the linen supply holds

to service all customers may be expressed as follows:

$$\sum_{i=1}^N I(x_i) = C \cdot X$$

where  $X$  is the total quantity of linen soiled by all customers.

Suppose that the customer with the smallest consumer surplus per unit of inventory, Customer  $J$ , has the constant elasticity demand curve:

$$x_j = a_j p^{b_j}$$

Where:

$x_j$  = Quantity of linen soiled at a laundering fee of  $p$

$a_j$  = A constant

$p$  = Laundering fee

$b_j$  = Price elasticity

Thus, the largest inventory fee  $T$  the linen supply can charge without driving customers from the market is the consumer surplus of Customer  $J$  divided by the inventory held on behalf of Customer  $J$ .

$$T = \frac{\int_p^{\infty} a_j p^{b_j} dp}{I(X_j)}$$

$$= \frac{-p}{(b_j+1)C}, \text{ for } b < -1. \text{ (See footnote 6)}$$

Upon substituting these values in for  $T$  and  $\sum_{i=1}^N (x_i)$  we find:

$$\frac{d\left(\sum_{i=1}^N I(x_i)\right)}{dP} = \frac{d\left(\frac{-P}{b_j+1} \cdot X \cdot C\right)}{dP}$$

$$= \frac{-P}{(b_j+1)} \cdot \frac{dX}{dP} - \frac{1}{(b_j+1)} \cdot X$$

(B)  $\frac{d(P \cdot X)}{dP}$

The analysis of this term is relatively simple:

$$\frac{d(P \cdot X)}{dP} = X \cdot \frac{dP}{dP} + P \cdot \frac{dX}{dP}$$

$$= X + P \cdot \frac{dX}{dP}$$

(c)  $\frac{d(TC)}{dP}$

The analysis of this term is also relatively simple:

$$\frac{d(TC)}{dP} = \frac{d(TC)}{dX} \cdot \frac{d(X)}{dP}$$

$$= MC \cdot \frac{d(X)}{dP}$$

Combining the analyzed forms of these three terms, we find that profits will be maximized only if:

$$0 = \frac{-P}{(b_j+1)} \cdot \frac{dX}{dP} - \frac{X}{(b_j+1)} + \frac{P}{dP} \cdot \frac{dX}{dP} - MC \cdot \frac{dX}{dP}$$

Multiplying through by  $P/X$  we obtain:

$$0 = \frac{-P}{(b_j+1)} \cdot \frac{P}{X} \cdot \frac{dX}{dP} - \frac{P}{(b_j+1)} + P \cdot \frac{P}{X} \cdot \frac{dX}{dP} - MC \cdot \frac{X}{P} \cdot \frac{dX}{dP}$$

Or, since  $\frac{P}{X} \frac{dX}{dP}$  is equal to  $E$ , the price elasticity of the market demand curve:

$$0 = \frac{-P}{(b_J+1)} \cdot E = \frac{P}{(b_J+1)} + P \cdot E - MC \cdot E$$

Rearranging terms, we find:

$$P = \frac{MC \cdot E}{1+E - \left(\frac{E+1}{b_J+1}\right)}$$



## CHAPTER IV

### WHICH SORT OF TWO-PART PRICING SYSTEM IS BEST?

We saw in Chapter III that so long as a firm has some degree of monopoly power, it can earn greater profit under a two-part pricing system than it can under a non-discriminatory one-part tariff. In competitive markets, however, we saw that no advantage can be gained from using a two-part rather than a one-part pricing system. Also in Chapter III, we noticed that there are four types of two-part tariffs that linen supplies might use:

- o Piece-Rate With a Minimum
- o Flat-Rate
- o Delivery Charge Plus Laundering Fee
- o Inventory Charge Plus Laundering Fee

In this chapter, we shall compare these four types of pricing systems to determine which will provide monopolistic linen supplies with the greatest profits. Since a linen supply's profits are equal to the revenues it receives from customers less its total costs of providing linen service, we shall first wish to investigate how a linen supply's total costs may be affected by the choice of a pricing system. Having understood how total costs can be expected to differ under different pricing systems, we shall then be able to compare the profitability of the various pricing systems.

(a) Total Costs Under Various Pricing Systems

The total costs a linen supply incurs in providing its customers with linen service are commonly divided into four groups:

- (1) Processing Costs. These costs include the wages paid workers inside the plant, depreciation of plant equipment, and the soap, detergent, electricity, and fuel required to process linen.
- (2) Inventory Costs. These are the costs of purchasing and maintaining an inventory of linen adequate to provide linen service to customers.
- (3) Delivery Costs. These include the wages of delivery drivers, fuel and maintenance of delivery trucks.
- (4) General and Administrative Costs. These costs include the wages of office workers, depreciation of office machinery, depreciation of the linen supply plant building, and general overhead.

Depending on the type of pricing system a linen supply uses, there will be different pressures on each of these cost categories. That is, barring some corrective action by the linen supply, there will be a tendency for the linen supply's total costs to vary depending on the type of pricing system selected.

Many processing costs are fixed costs. Due to the great durability of the equipment used in linen supplies, the depreciation of machinery is not closely tied to the quantity of linen processed by those machines. Although a plant manager is generally free to add or to lay-off workers inside the plant, he generally has little control over the hours worked by the plant workers. Most linen supplies do not operate near capacity and hence are not likely to work overtime. On the other hand, due to union contracts and to standard practice in the industry, plant workers are in general guaranteed a minimum forty-hour week. Thus, the processing costs that will vary with the quantity of linen processed are to some extent labor costs, and the relatively insignificant costs of soap, detergent, electricity, fuel and plastic wrap to package the linen bundles.

We may expect the quantity of linen processed, and hence some part of processing costs, to vary under different pricing systems, according to three factors: whether a monthly minimum is required, the optimal laundering fee charged, and the optimal right to buy tariff charged.

When a monthly minimum is required under a Piece-Rate With a Minimum pricing system, customers are billed the greater of the monthly minimum or what their linen bill would be under a pure Piece-Rate system. At the piece-rate prices the linen supply charges, some customers might prefer to receive less than the required minimum. However,

due to imposition of the minimum, these customers can expect to be billed the amount of the minimum whether or not the quantity of linen they actually use is large enough to result in their being charged the minimum under a pure Piece-Rate system. Since there is no difference in cost to these customers between receiving the minimum amount of linen and the amount they would prefer to receive in absence of a required minimum, these small customers are likely to increase the amount of linen they receive up to the required monthly minimum. Thus, the effect of requiring a monthly minimum is to increase the quantity of linen demanded by small customers and hence to increase the linen supply's processing costs.

Similarly, the right-to-buy tariff charged under a two part pricing system may also have an effect on the total quantity of linen soiled and hence, on the processing costs of a linen supply. The right-to-buy tariff requires customers to pay a part of their consumer surpluses for the right to receive linen at a given laundering fee. Under different pricing systems, this right-to-buy tariff may be set at different levels and hence different customers may be driven from the market for the linen supply's products. Thus, the total quantity of linen demanded may vary under different pricing systems if a different mix of customers is allowed to remain in the market under different pricing systems.

It is important to notice, however, that as long as the linen supply provides service to the same group of customers under different pricing systems, total costs will not depend on the right-to-buy tariff charged by different pricing systems. Since a right-to-buy tariff charges customers part of their consumer surplus--part of what they would be willing to pay for the right to receive linen at a given laundering fee--the right-to-buy tariff will affect only the mix of customers which are to remain in the market, not the quantity of linen those customers will soil.

Inventory costs are entirely variable costs, since they will vary only according to the quantity of linen soiled. The larger the quantity of linen soiled, the larger will be the inventory of linen that the linen supply must hold to provide its customers with linen service. Thus, if a monthly minimum is required, the linen supply can expect its inventory costs to be greater than they otherwise would be. Similarly, the lower the laundering fee, the greater will be the linen supply's inventory costs. Again, if the linen supply deals with the same group of customers, the right-to-buy tariff charged under different pricing systems will not affect the quantity of linen soiled and hence will not affect the linen supply's inventory costs.

Inventory costs however, are affected by other factors than the quantity of linen soiled. For example, some customers are very concerned that they might run out of linen before their next delivery.

Thus, these customers have a tendency to stockpile linen in amounts sufficient to allay their concerns. Of course, this hoarding effect tends to increase the linen supply's inventory costs: In addition to the appropriate amount of linen required to provide a given customer with linen service, the linen supply must also purchase a buffer supply of linen for the customer to stockpile.

Under all pricing systems but the Inventory Charge Plus Laundering Fee system, customers who do stockpile linen are not charged any differently than customers who do not stockpile linen. Under the Inventory Charge Plus Laundering Fee pricing system, customers must pay an inventory fee on all linen the linen supply holds on their behalf whether they use that inventory fully or not. Under all other pricing systems, it costs customers nothing to keep a larger than necessary inventory of linen. Thus, all other things being equal, the linen supply's inventory costs will be smaller under an Inventory Charge Plus Laundering Fee system than under any other pricing system.

Linen supply plant managers, however, are rarely content to allow all other things to remain equal. Plant managers are aware of the effects of customer stockpiling and when they decide to use pricing systems other than the Inventory Charge Plus Laundering Fee system, they usually supplement these pricing systems with some sort of inventory control program. Typically, under such programs, the dollar drivers

are required to observe the inventory of linen each customer keeps and report any excessive inventories to the plant manager. The plant manager will then discuss the problem with the offending customers and in most cases, succeeds in reducing that customer's inventory.

Although no data exists on the effectiveness of these inventory control programs, the general feeling in the industry is that when other pricing systems are supplemented with an inventory control program, the inventory cost advantage of the Inventory Charge Plus Laundering Fee pricing system is not significant.

The Delivery Costs a linen supply incurs are fixed costs. Most delivery drivers are not required to keep regular hours and hence are not normally paid overtime wages. Within a fairly broad range of the quantity of linen soiled and the number of customers served, the linen supply will neither add delivery drivers nor lay drivers off. This practise is due not only to requirements of union contracts, but also because the costs of adding or deleting a route are quite substantial: When the decision is made to change the number of routes a linen supply runs, normally the linen supply must replan its entire delivery system. All stops of all routes for each day of the five-day cycle must be reallocated over the new number of routes. This reallocation is usually done not by computer but manually, and hence is a very time consuming and expensive

undertaking.

Of all the costs included under general and administrative costs, the only cost which might conceivably vary among pricing systems is bookkeeping cost. Some pricing systems involve more detailed record-keeping and more complicated calculations than other systems. Although for linen supplies with manual billing systems these cost differences may be considerable, most linen supplies do not have manual but computerized billing systems. Hence for these linen supplies, even the most complicated pricing system will increase general and administrative costs by only a negligible amount.

In sum, the major differences in cost among the four types of two-part pricing systems are due to whether or not a minimum is required by the pricing system and to differences in the optimal laundering fees and right-to-buy charges under the various pricing systems. The imposition of a monthly minimum will tend to increase the quantity of linen soiled by small customers and hence will increase both processing and inventory costs. Differences in the optimal laundering fee and right-to-buy charges under various pricing systems will also lead to differences in the quantity of linen soiled and thus to differences in processing costs and inventory costs.

B. Delivery Charge Plus Laundering Fee vs. Piece-Rate With a Minimum

As you recall, the Delivery Charge Plus Laundering Fee



is a uniform two-part pricing system. Under the Delivery Charge Plus Laundering Fee system, a linen supply chooses a laundering fee to charge for each piece of each type of linen the customer soils, and a delivery fee to charge customers for each type of linen they receive. By its choice of a laundering fee and delivery fee, a linen supply implicitly chooses the number of customers it serves. According to the analysis presented in Chapter III, the laundering fee corresponds to the per-unit price and the delivery fee to the right-to-buy tariff charged under a uniform two-part pricing system.

The Piece-Rate With a Minimum pricing system is a special sort of uniform two-part pricing system. Under the Piece-Rate With a Minimum pricing system, the right-to-buy tariff is the required monthly minimum. The monthly minimum is different from most right-to-buy tariffs in two respects. First, by paying the minimum, a customer acquires the right to receive not just one, but all of the products of the linen supply. Second, by paying the minimum, the customer is entitled to receive a quantity of linen up to the amount of the minimum at no additional charge. The piece-rate charged under the system corresponds to the per-unit of output charge of a two-part tariff. The piece-rate is also somewhat different from most per-unit prices in that it becomes effective only after a customer has soiled a quantity of

linen equal in value to the amount of the minimum. Until that point, piece-rate prices are merely used to calculate how much of his minimum the customer has used.

I shall argue that, in practise, linen supplies will never wish to use the Piece-Rate With a Minimum pricing system. My argument will have two parts: First, I shall argue that, in general, a linen supply will be better off charging a right-to-buy tariff on a product by product basis, as under the Delivery Charge Plus Laundering Fee system, rather than charging one right-to-buy tariff which applies to all products, as under the Piece-Rate With a Minimum system. Second, I shall argue that even when the monthly minimum is assumed to apply to only one product, the Delivery Charge Plus Laundering Fee system can provide larger profits than the Piece-Rate With a Minimum system in all but the most rare circumstances.

What is at stake in the question of whether a linen supply should set one overall right-to-buy tariff or a different right-to-buy tariff for each of its products? To answer this question, let us consider a linen supply which serves  $N$  customers but which provides only two products, towels and tablecloths, to those customers.

If the linen supply were to set its right-to-buy tariffs on a product by product basis, then the maximum right-to-buy tariff it can charge for each product is the amount of the smallest consumer surplus belonging to any customer in the final market for each product.

If linen supply were to set a right-to-buy tariff on an overall basis, then it would charge a single fee in exchange for the right to receive both of the linen supply's products. The maximum right-to-buy tariff the linen supply could charge would, of course, be the smallest amount that any of its customers would be willing to pay for that right. This, in turn, is equal to the smallest total amount any customer would be willing to pay for the right to receive towels plus what he would be willing to pay for the right to receive tablecloths. Thus, the maximum overall right-to-buy tariff the linen supply could charge is equal to the smallest sum of consumer surpluses that any one customer would enjoy from purchasing both of the linen supply's products.

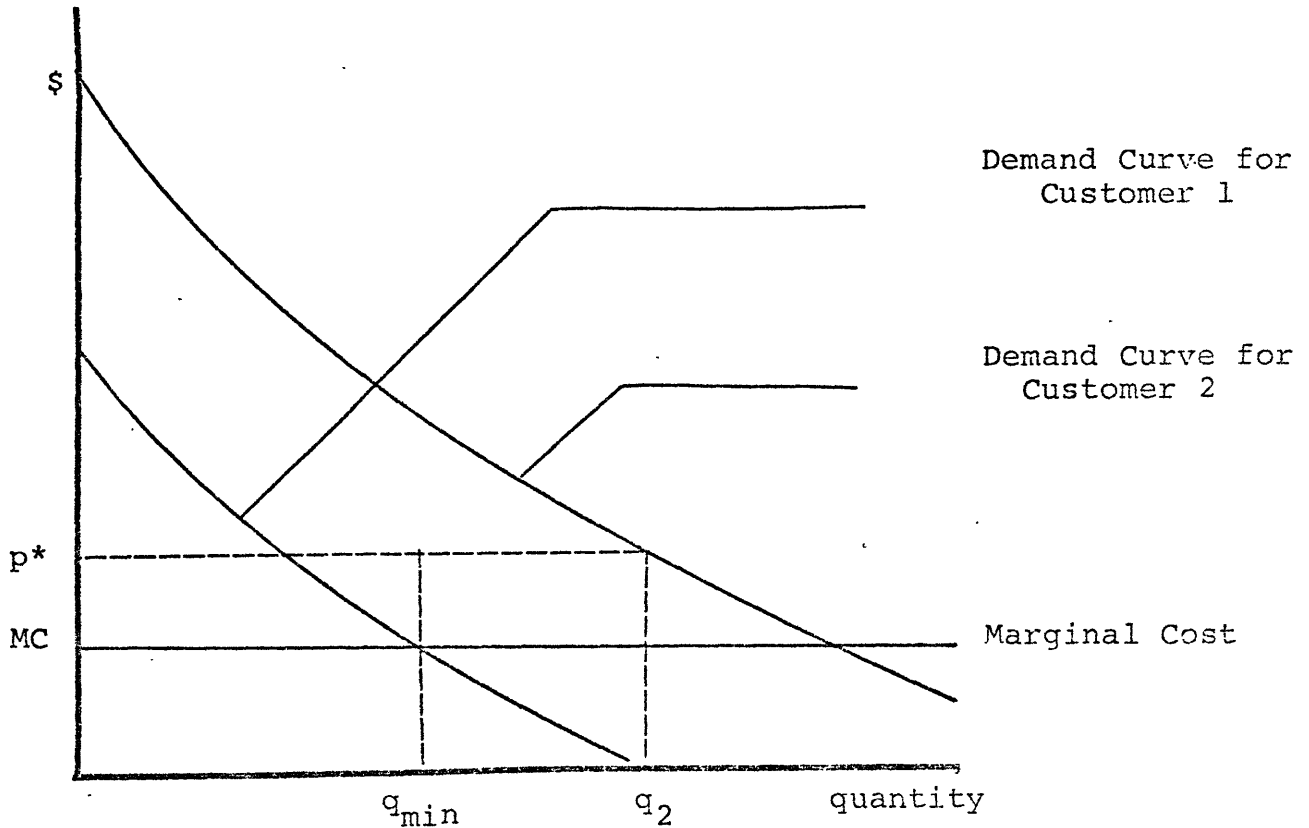
If the customer with the smallest consumer surplus for towels wishes to receive only towels then the linen supply can charge an overall right-to-buy tariff no larger than the amount of consumer surplus of that customer, without driving him from the market. If this customer were to receive only towels, and if no customers are to be driven from the market, then those customers who receive both towels and tablecloths would pay an overall right-to-buy tariff equal to the right-to-buy tariff for towels that they would pay if right-to-buy tariffs were charged on a product by product basis. Thus, by charging an overall right-to-buy tariff, the linen supply would forego the revenues it could receive from charging customers who receive both towels and tablecloths a

right-to-buy charge for tablecloths. Thus, the total revenues the linen supply would collect from charging an overall right-to-buy tariff will tend to be smaller than the total revenues the linen supply would receive from charging a different right-to-buy tariff for each product.

This example shows in the case of a two product linen supply the problem that a full-service linen supply would encounter by charging an overall right-to-buy tariff. A full-service linen supply normally offers seventy-five to a hundred different types of products, ranging from stove mitts and kitchen aprons to barber towels and doctors' coats. No one customer will ever wish to receive all the linen supply's products. In fact, small customers usually receive only one or two of the products available to them. Thus, the maximum overall right-to-buy tariff the linen supply could charge would be limited to what small customers would be willing to pay for the one or two products they are interested in receiving. As in the case of the two-product linen supply, a full-service linen supply would do better to divide its total market into distinct product markets and set different right-to-buy tariffs for each product market, rather than to set a single overall right-to-buy tariff.

Let us now turn to the second part of my argument in which I shall show that even if separate minimums were established for each of the linen supply's products, a linen supply will normally prefer to use a Delivery Charge Plus

Laundrying Fee system rather than a Piece-Rate With a Minimum system. To make matters simple, we shall consider an example in which a linen supply with constant marginal costs offers one product to only two customers. Such a market is depicted in the following graph.



Market of Two Customers

Suppose that the linen supply in this example were to set prices under the Piece-Rate With a Minimum system.

In this market, the optimal minimum to require is  $q_{\min}$ , the maximum number of units it is profitable to rent to the smaller customer, Customer 1. The optimal total amount to bill Customer 1 is the maximum amount he would be willing

to pay to receive those  $q_{\min}$  limits of linen, which in turn is equal to the area under Customer 1's demand curve between the price axis and the quantity  $q_{\min}$ . Thus, the optimal piece-rate to charge Customer 1 is the maximum amount he would be willing to pay per-unit:<sup>1</sup>

$$P^* = \frac{\int_{MC}^{\infty} \psi_1(P) dP + MC \cdot q_{\min}}{q_{\min}}$$

When a piece-rate price of  $P^*$  is charged, and a minimum of  $q_{\min}$  units of linen is required, the profits the linen supply earns from serving the smaller customer is equal to the maximum total amount Customer 1 is willing to pay for  $q_{\min}$  units less the total cost of providing Customer 1 with those units:

$$\text{Profit from Customer 1} = P^* \cdot q_{\min} - \text{total cost}$$

Or, since marginal cost is constant:

$$\begin{aligned} \text{Profit from Customer 1} &= P^* \cdot q_{\min} - MC \cdot q_{\min} \\ &= (P^* - MC) q_{\min} \end{aligned}$$

Or, upon substituting in the above value for  $P^*$ , we find:

$$\text{Profit from Customer 1} = \int_{MC}^{\infty} \psi_1(P) dP$$

The larger customer in this market will demand  $q_2$  units at a piece-rate price of  $P^*$ . As I have depicted matters,  $q_2$  is greater than the required minimum of  $q_{\min}$  units, and

hence, the imposition of a monthly minimum will not affect the quantity of linen soiled by Customer 2.<sup>2</sup> The profit the linen supply earns from renting linen to Customer 2 is equal to the total amount charged Customer 2 less the total cost of providing linen services to Customer 2:

$$\begin{aligned}
 \text{Profit from} \\
 \text{Customer 2} &= P^* \cdot q_2 - \text{total cost} \\
 &= (P^* - MC)q_2 \\
 &= (P^* - MC)q_{\min} + (P^* - MC)(q_2 - q_{\min}) \\
 &= \int_{MC}^{\infty} \Psi_1(P) dP + (P^* - MC)(q_2 - q_{\min})
 \end{aligned}$$

The total profit the linen supply earns from serving both customers is equal to the profit it earns from Customer 1 plus the profit it earns from Customer 2:

$$\begin{aligned}
 \text{Total Profit} &= (P^* - MC)q_{\min} + (P^* - MC)q_2 \\
 &= (P^* - MC)(q_{\min} + q_2)
 \end{aligned}$$

Or, since by our choice of a value for  $P^*$  we have insured that,

$$(P^* - MC)q_{\min} = \int_{MC}^{\infty} \Psi_1(P) dP$$

we may write the total profit of the linen supply in the following form:

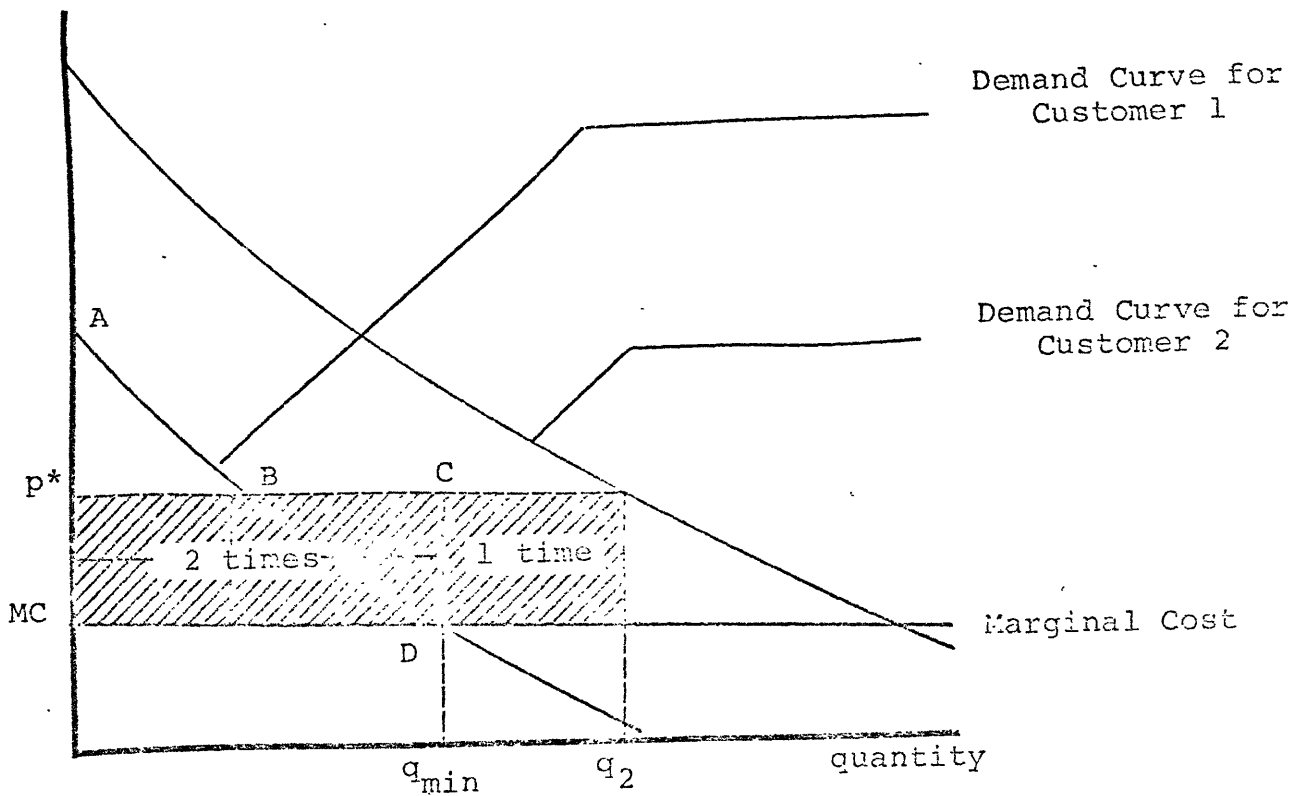
$$\text{Total Profit} = 2 \int_{MC}^{\infty} \Psi_1(P) dP + (P^* - MC)(q_2 - q_{\min})$$

Each of these two ways of writing the total profit the linen supply earns has a different graphic interpretation. It will be important in the following analysis to recognize these different graphic interpretations and to note that the profits they express are identical.

When the linen supply's total profits written in the form,

$$\pi = (P^* - MC) \cdot (q_{\min} + q_2)$$

then these profits correspond to the shaded area in the following graph.



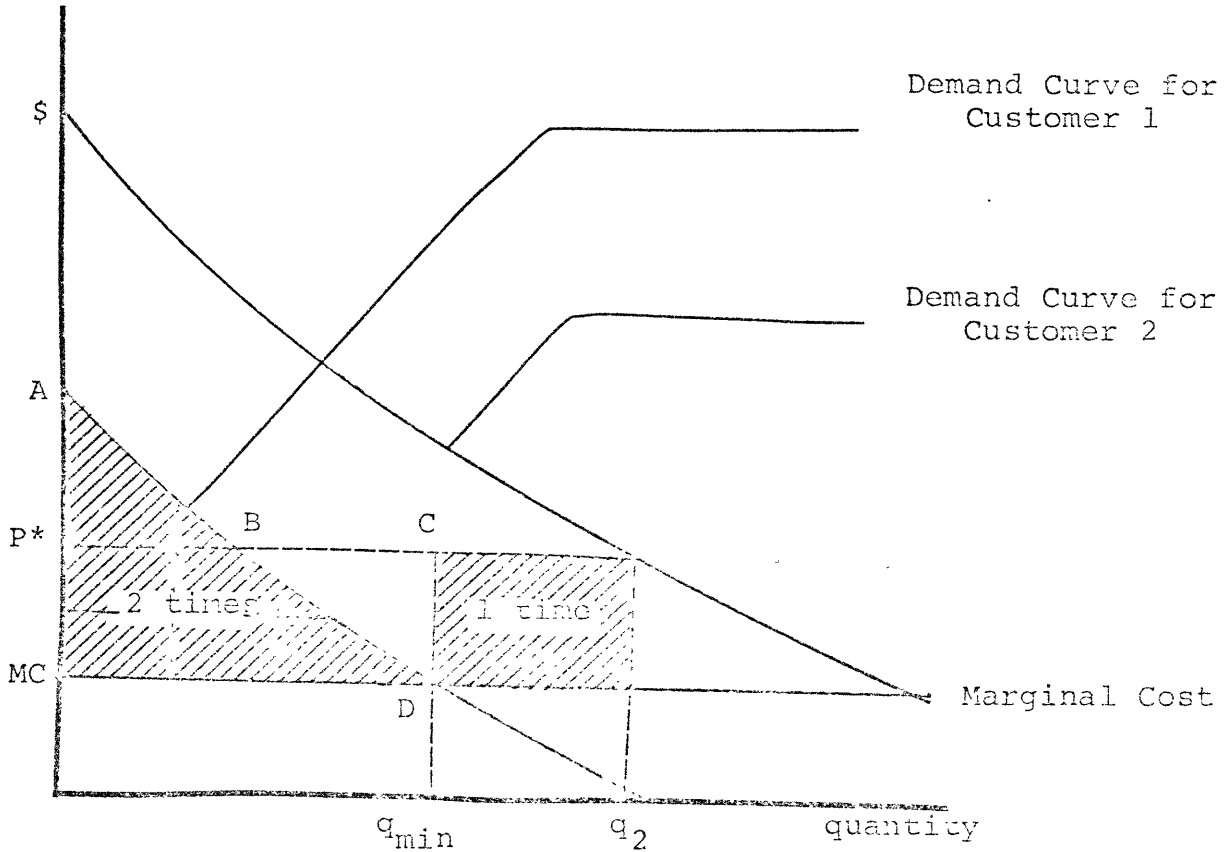
Profit Under a Piece-Rate With a Minimum System

Alternatively, since  $(p^* - MC)q_{\min} = \int_{MC}^{\infty} \psi_1(p) dp$ , the linen supply's total profits may be expressed in the following form:

$$\pi = 2 \int_{MC}^{\infty} \psi_1(p) dp + (p^* - MC)(q_2 - q_{\min})$$



In this form, the total profits of the linen supply are represented by the shaded area in the following graph.



Profit Under a Piece-Rate With a Minimum Pricing System

The only difference between these two graphs is that in the former, the linen supply's profits include the triangle B-C-D, while in the latter graph, the triangle  $P^*-B-A$  is substituted for triangle B-C-D. Since, however, we express profits the total amount of profit must be the same, the triangle  $P^*-B-A$  must be exactly equal in area to the triangle B-C-D.

Let us now consider the profit that the linen supply could earn under a Delivery Charge Plus Laundering Fee system. In particular, we shall consider the version of the Delivery Charge Plus Laundering Fee system in which the delivery fee and laundering fee are given the following assignments:

$$\text{Delivery Charge } D = \int_{P^*}^{\infty} \psi_1(P) dP$$

$$\text{Laundering Fee } P = P^*$$

Thus, the laundering fee charged will be equal to the optimal piece-rate price  $P^*$ , and the delivery fee charged both customers will be the consumer surplus that the smaller customer would enjoy at a laundering fee of  $P^*$ .

Suppose that in absence of a required minimum, the smaller customer would prefer to receive  $q_1$  units at a laundering fee of  $P^*$ . Clearly,  $q_1$  will be less than  $q_{\min}$  since the smaller customer would be willing to receive  $q_{\min}$  units of linen in absence of a required minimum only at a per-unit price equal to marginal cost. The profits the linen supply earns from serving the smaller customer, Customer 1, are equal to the revenues it receives from charging Customer 1 a delivery fee plus the revenues the linen supply receives from charging a laundering fee minus the total cost of providing Customer 1 with  $q_1$  units of linen:

$$\begin{aligned} \text{Profits from} &= D + P^* \cdot q_1 - \text{Total Cost} \\ \text{Customer 1} & \end{aligned}$$

$$= \int_{P^*}^{\infty} \psi_1(P) dP + P^* \cdot q_1 - MC \cdot q_1$$

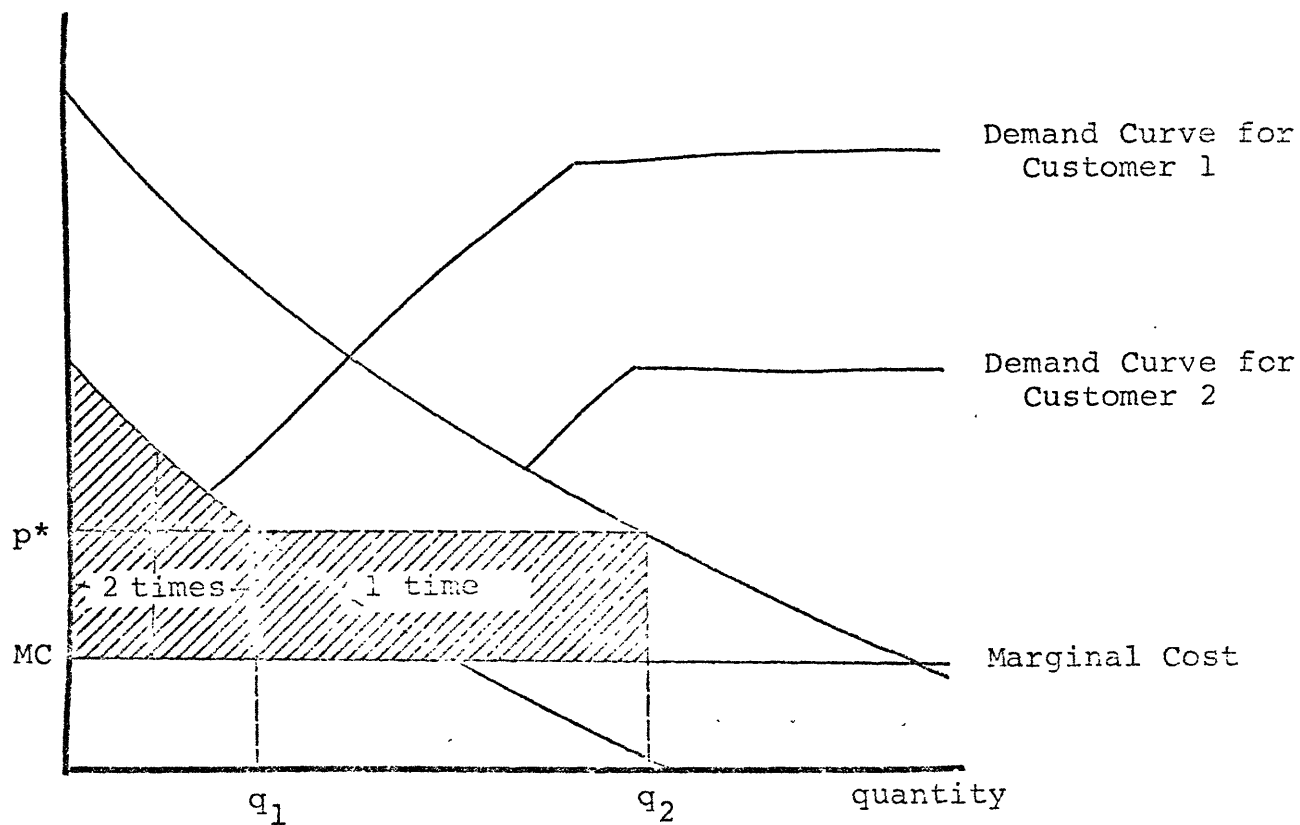
At a laundering fee of  $P^*$ , Customer 2 will be willing to receive  $q_2$  units of linen. Thus, the profits the linen supply earns from serving Customer 2 are equal to the revenue it receives from charging Customer 2 a delivery fee plus the revenue the linen supply receives from charging a laundering fee minus the total cost of providing Customer 2 with  $q_2$  units of linen:

$$\begin{aligned} \text{Profits from Customer 2} &= D + P^* \cdot q_2 - \text{Total Cost} \\ &= \int_{P^*}^{\infty} \Psi_2(P) dP + \left[ P^* \cdot q_2 \right] - MC \cdot q_2 \end{aligned}$$

The total profit the linen supply would earn from serving both customers is equal to the profit it earns from Customer 1 plus the profit it earns from Customer 2:

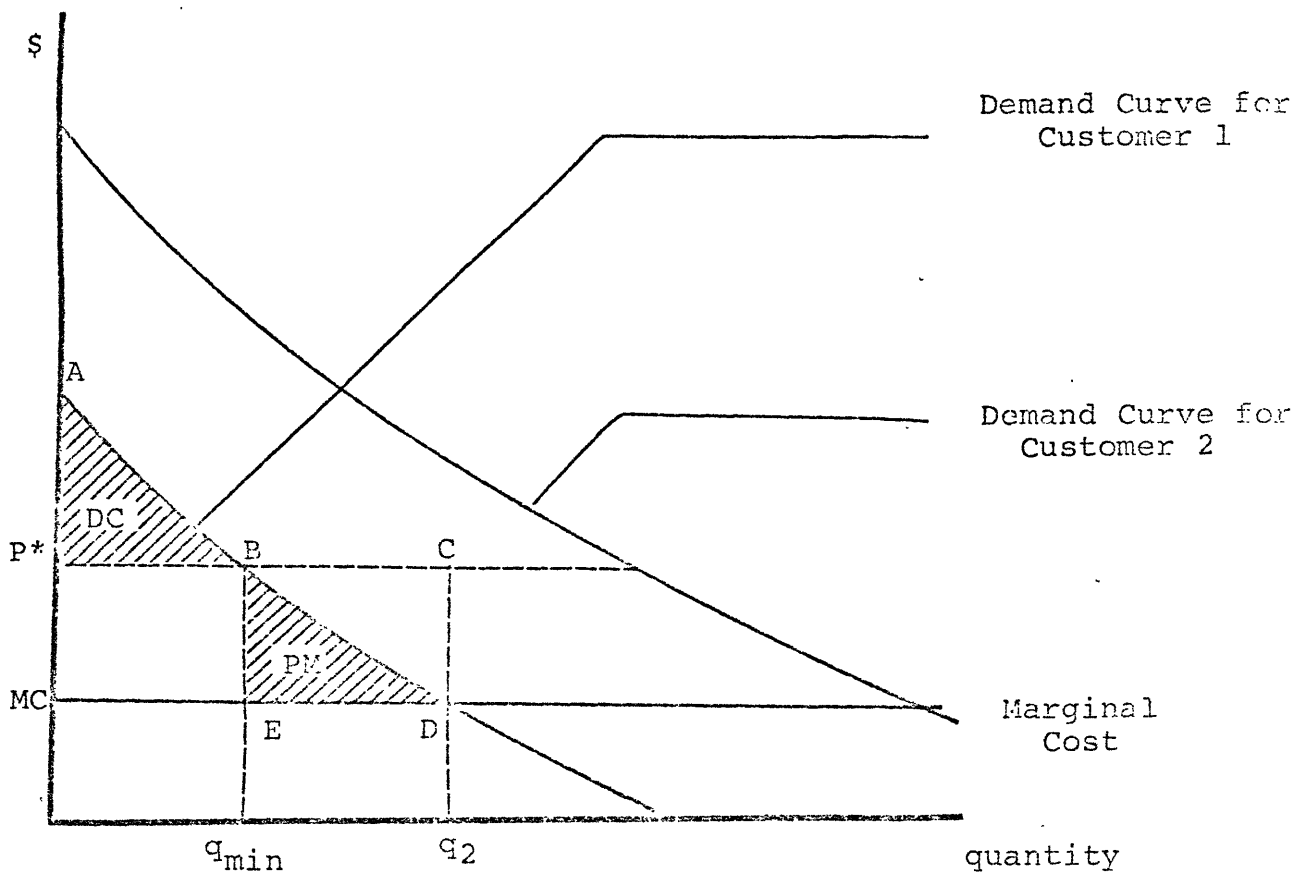
$$\text{Total Profit} = 2 \int_{P^*}^{\infty} \Psi_1(P) dP + (P^* - MC)(q_1 + q_2)$$

In graphic form, these profits are equal to the shaded area in the following diagram.



Total Profit Under a Delivery Charge Plus Laundering  
Fee System

We shall now compare the profits earned under both systems. To do so, it will be most convenient to consider the following graph which eliminates the profits earned under both systems and thus only depicts profits which are earned by one system but not the other.



Differences in Profits Between the Piece-Rate With a  
Minimum and the Delivery Charge Plus Laundering Fee  
Pricing Systems

In this graph, the area labeled DC represents profits earned by the Delivery Charge Plus Laundering Fee system which are not earned by the Piece-Rate With a Minimum system. The area labeled PM are those profits earned by the Piece Rate With a Minimum system but not earned by the Delivery Charge Plus Laundering Fee System. Thus, the question of which pricing system will produce larger profits can be answered by determining whether the area DC is greater or less

than the area PM.

Notice that the area DC is identical to the triangle P\*-B-A. In discussing the Piece Rate With a Minimum system, we observed that the triangle P\*-B-A was equal in area to the triangle B-C-D. Therefore, if the smaller customer, Customer 1, had a straight line demand curve, then the area B-C-D would be equal to the area PM since both would be equal to half the area of the rectangle, B-C-D-E. Hence, the area DC would be equal to the area PM and thus, the Delivery Charge Plus Laundering Fee system would earn profits exactly equal to the maximum profits achievable under a Piece Rate With a Minimum pricing system.

As we have seen, however, it is rather unlikely that a customer will have a straight line demand curve. In the much more common circumstance, customers demand curves will be concave (like the inside of a bowl), reflecting the fact that customers will be willing to pay more for an additional piece of linen when they currently receive few pieces of linen than when they currently receive a large quantity of linen and hence are nearly satiated. If the demand curve of Customer 1 is concave, then the area PM will be less than the triangular area B-C-D. Thus, since the area DC is equal to B-C-D, the area DC will be greater than the area PM. The Delivery Charge Plus Laundering Fee system, therefore, will provide the linen supply with greater profits than the maximum

profits achievable under a Piece Rate With a Minimum system.

When this argument is generalized to a market of more than two customers, the complexity of the argument increases substantially, but the basic result of the argument does not change: As long as at least half of the linen supply's customers would be willing to receive more than the required minimum number of linen pieces at the given piece-rate price, the Delivery Charge Plus Laundering Fee system will always be able to produce profits at least equal to the maximum profits achievable under the Piece-Rate With a Minimum system.<sup>3</sup>

It is quite reasonable to expect this condition to be satisfied in nearly all markets in which linen supplies operate. Recall that the optimal minimum to require can be no greater than the maximum amount the customer with the smallest consumer surplus would receive at a piece-rate equal to marginal cost. Of course, for customers with identical price elasticities, the smaller a customer's consumer surplus, the smaller will be the quantity he demands at all piece-rate prices, including the piece-rate equal to marginal cost. Therefore, since different price elasticities seem to be randomly distributed among various sizes of customers, one would expect the customer with the smallest consumer surplus to be a fairly small customer. Hence, the optimal minimum to require will nearly always be less than what half the linen supply's customers would be willing to receive at the optimal piece-rate price.<sup>4</sup>

In sum, there are two reasons to believe that linen supplies will prefer to use the Delivery Charge Plus Laundering Fee pricing system rather than the Piece-Rate With a Minimum system. First, the Piece-Rate With a Minimum System charges an overall right-to-buy tariff rather than different right-to-buy tariffs for each product. In general, since most customers receive only a few of the linen supply's products, the maximum overall right-to-buy tariff the linen supply can charge will be less than the sum of the right-to-buy tariffs it charges on a product-by-product basis. Thus, the linen supply's profits will tend to be smaller under the Piece-Rate With a Minimum system than under a system such as the Delivery Charge Plus Laundering Fee system, which charges different right-to-buy tariffs for each product. Second, even if separate minimums were established for each of the linen supply's products, the Delivery Charge Plus Laundering Fee system will earn greater profits than the Piece Rate With a Minimum in nearly all cases.

C. Inventory Charge Plus Laundering Fee vs. Flat Rate

The Flat Rate pricing system is a special sort of variable two-part pricing system in which the per-unit of output price is set equal to zero. The Inventory Charge Plus Laundering Fee pricing system is also a sort of variable two-part tariff, but under this system the per-unit of output price need not be set equal to zero. Thus, it is not



surprising that in nearly all cases the Inventory Charge Plus Laundering Fee system will provide a linen supply with greater profit than it would earn under a Flat Rate system. In fact, only in the very special case in which the optimal per-unit price (laundering fee) of an Inventory Charge Plus Laundering Fee system is exactly equal to zero will the profits under a Flat Rate system be as large as those a linen supply would earn under an Inventory Charge Plus Laundering Fee system.

As you recall, under the Flat Rate system, the only variable of choice open to the linen supply is  $N$ , the number of customers it chooses to serve. Once the linen supply has determined the optimal number of customers to serve,  $N^*$ , the setting of flat rates for its customers is an entirely mechanical procedure. The linen supply must first identify which of its  $N^*$  customers has the smallest consumer surplus per unit of linen the customer is expected to soil each month. That is, the linen supply should determine which of its customers is the Customer  $K$  such that:

$$\frac{\int_0^{\infty} \psi_k(P) dP}{q_k} = \text{Min}_{i=1 \text{ to } N} \left( \frac{\int_0^{\infty} \psi_i(P) dP}{q_i} \right)$$

where  $q_k$  is the quantity of linen Customer  $K$  is expected to soil each month and  $q_i$  is the quantity of linen the  $i^{\text{th}}$  customer is expected to soil each month.

The customer  $K$  is charged a flat rate equal to his

consumer surplus,  $\frac{\int_0^{\infty} \psi_K(P) dP}{q_K}$ . The flat rates charged Customer K and the requirement imposed in order to insure compliance with the Robinson-Patman Act, that for any customer j:

$$\frac{\text{Flat Rate Billed Customer K}}{q_K} = \frac{\text{Flat Rate Billed Customer j}}{q_j}$$

The linen supply would then expect to earn the profits given in the following equation:

$$\pi = \frac{\int_0^{\infty} \psi_K(P) dP}{q_K} \sum_{i=1}^{N^*} q_i - TC_F$$

where  $\sum_{i=1}^{NA} q_i$  is the total quantity of linen expected to be soiled and  $TC_F$  are the linen supply's total costs under the flat rate system.

As you recall, under the Inventory Charge Plus Laundering system, a linen supply chooses a laundering fee to charge customers for each unit of linen the customer soils and an inventory fee to charge for each unit of linen the linen supply must purchase and maintain to service the customer. By its choice of a laundering fee and inventory fee, a linen supply implicitly chooses the number of customers it serves. According to the analysis presented in Chapter III, the laundering fee corresponds to the per-unit price, and the inventory fee to the right-to-buy tariff charged under a

two-part pricing system.

In order to show that an Inventory Charge Plus Laundering Fee pricing system is better for a linen supply to use than a Flat Rate system, it is sufficient to show that there is some, not necessarily optimal, assignment of values to the inventory price  $T$  and the laundering fee  $P$  which will produce profit equal to the profit that a linen supply would expect to earn from the optimal use of the Flat Rate system. If there is some such assignment, then when the Inventory Charge Plus Laundering Fee system is used to its best advantage, it can be relied on to produce profits at least as large as those produced under a Flat Rate system. The profit that a linen supply earns under an Inventory Charge Plus Laundering Fee pricing system generally is given by the following equation:

$$\pi = T \cdot \sum_{i=1}^N I(X_i) + P \cdot X - TC_I$$

where:

$T$  = Inventory fee per-unit of linen

$N$  = Number of customers

$I(X_i)$  = Amount of inventory the linen supply must hold to service the  $i^{\text{th}}$  customer assuming weekly delivery.

Thus,  $\sum_{i=1}^N I(X_i)$  is the total inventory held by the linen supply.

$P$  = Laundering fee per-unit of linen

$X$  = Total quantity of linen soiled per month

$TC_I$  = Total cost under the Inventory Charge Plus  
Laundering Fee system.

Consider now the following assignment for the inventory fee and laundering fee under an Inventory Charge Plus Laundering Fee pricing system.

Laundering Fee  $P = 0$

$$\text{Inventory Fee } T \text{ Per-Unit of Inventory} = \frac{\int_0^{\infty} \Psi_K(P) dP}{C \cdot q_K}$$

where  $C$  is the inventory multiple the linen supply uses to calculate the optimal amount of inventory to hold, given the quantity of linen a customer soils per-month.

Upon substituting these values in for  $T$  and  $P$  in the equation for the linen supply's expected profit, we find:

$$\pi = \frac{\int_0^{\infty} \Psi_K(P) dP}{C \cdot q_K} \cdot \sum_{i=1}^N I(X_i) + O \cdot X - TC_I$$

Or, since the inventory a linen supply holds to service a given customer is determined by multiplying an inventory multiple  $C$  by the quantity of linen a customer is expected to soil each month, we may write the linen supply's profit in the following form:

$$\begin{aligned} \pi &= \frac{\int_0^{\infty} \Psi_K(P) dP}{C \cdot q_K} \cdot \sum_{i=1}^N C \cdot q_i - TC_I \\ &= \frac{\int_0^{\infty} \Psi_K(P) dP}{q_K} \cdot \sum_{i=1}^N q_i - TC_I \end{aligned}$$

Thus, this assignment of values to the prices charged under an Inventory Charge Plus Laundering Fee system results in each customer being charged the same right-to-buy tariff and the same per-unit price that they would be charged under a Flat-Rate system: Each customer is charged a laundering fee equal to zero and each customer ends up paying a right-to-buy tariff equal to  $\frac{\int_0^{\infty} \psi_K(P) dP}{q_K}$  times the amount of linen he expects to soil each month. Hence, this version of the Inventory Charge Plus Laundering Fee system will drive no customers from the market, and will generate revenues identical to those under the Flat-Rate system. Furthermore, since for systems not requiring a monthly minimum, the only difference in total costs will be due to different right-to-buy and per-unit prices, the linen supply's total costs should be the same under this version of the Inventory Charge Plus Laundering Fee system as under the Flat Rate system. Thus, this version of the Inventory Charge Plus Laundering Fee system will produce expected profits exactly equal to the maximum profits the linen supply could expect to earn under an optimal use of the flat Rate system.

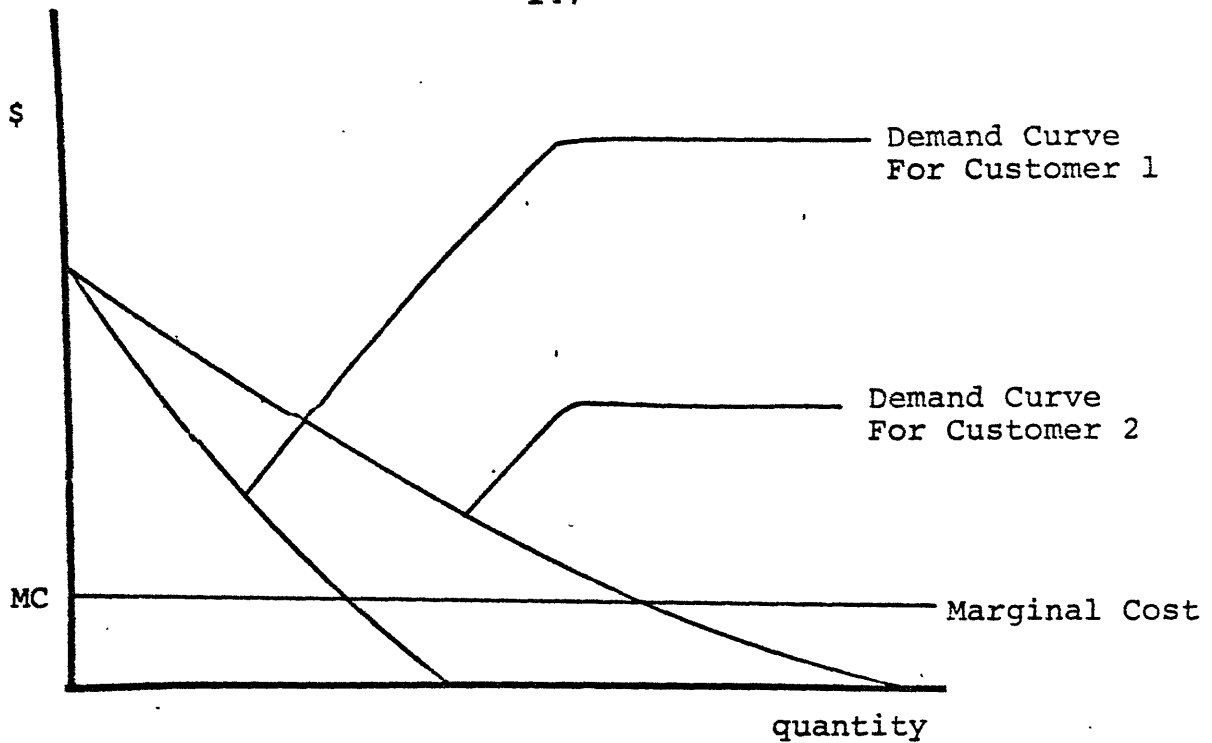
Since the assignment of values to the inventory fee and laundering fee under this version will be an optimal selection of values for the inventory fee and laundering fee only in very special circumstances, we may expect that in most situations an Inventory Charge Plus Laundering Fee

pricing system will produce profits not just equal to, but greater than the linen supply would earn under the Flat-Rate system.

D. Inventory Charge Plus Laundering Fee vs. Delivery Charge Plus Laundering Fee

Neither the Inventory Charge Plus Laundering Fee pricing system nor the Delivery Charge Plus Laundering Fee pricing system is best under all circumstances. In some markets, it will be more profitable to charge customers a variable right-to-buy tariff in the form of an inventory price per-unit of inventory. In other markets, it will be better for a linen supply to charge customers a uniform right-to-buy tariff by charging the same delivery fee to all customers for the right to receive a given product.

An example of a market in which a linen supply would do better to use an Inventory Charge Plus Laundering Fee system rather than a Delivery Charge Plus Laundering Fee system is given in the following graph. This market has the special feature that of the two customers on the market, Customer 2 demands twice as much linen as Customer 1 at all laundering fees the linen supply might set.



Demand and Marginal Cost Curves for a Market  
of Two Customers

If we represent the demand curve of Customer 1 as  $\psi_1(P)$ , then since Customer 2 demands twice as much as Customer 1 at any laundering fee price, we may write the demand curve of Customer 2 as either  $\psi_2(P)$  or  $2\psi_1(P)$ .

Let us suppose that the optimal prices for the linen supply to charge under a Delivery Charge Plus Laundering Fee system are the following:

$$\text{Optimal Laundering Fee} = P^*$$

$$\text{Optimal Delivery Fee} = D^* = \int_{P^*}^{\infty} \psi_1(P) dP$$

The profit that a monopolistic linen supply earns by using a Delivery Charge Plus Laundering Fee pricing system is given by the following equation:

$$\pi = N \cdot D + P \cdot X - TC_D$$

where

N = Number of customers

D = Delivery charge

P = Laundering fee

X = Total quantity of linen soiled

$TC_D$  = Total cost of providing linen under a Delivery Charge Plus Laundering Fee system.

Thus, the maximum profit that the linen supply in our example could earn under the Delivery Charge Plus Laundering Fee system is given by substituting these values for D and P into the equation for the linen supply's profits.

$$\pi = 2 \cdot \int_{p^*}^{\infty} \psi_1(P) dP + p^* \cdot X - TC_p$$

We shall now consider what profit the linen supply would earn in this market under an Inventory Charge Plus Laundering Fee system. Let us suppose that the linen supply sets the laundering fee under an Inventory Charge Plus Laundering Fee system equal to  $P^*$ , the optimal laundering fee under the Delivery Charge Plus Laundering Fee system. If the laundering fee charged by the linen supply is  $P^*$ , then the maximum inventory fee  $T^*$  that the linen supply can charge is equal to the smallest consumer surplus per-unit of inventory of either customer:

$$T^* = \text{Min} \left( \frac{\int_{p^*}^{\infty} \psi_1(P) dP}{C \cdot X_1}, \frac{\int_{p^*}^{\infty} \psi_2(P) dP}{C \cdot X_2} \right)$$

However, since at all laundering fee prices Customer 2 demands twice the amount of linen demanded by Customer 1, the consumer



surplus of Customer 2 will be twice that of Customer 1.

$$\begin{aligned} \text{Consumer Surplus} &= \int_{p^*}^{\infty} \Psi_2(P) dP \\ \text{of Customer 2} &= \int_{p^*}^{\infty} 2\Psi_1(P) dP \\ &= 2 \int_{p^*}^{\infty} \Psi_1(P) dP \end{aligned}$$

Also, Customer 2 will require twice the amount of inventory that Customer 1 requires:

$$C \cdot x_2 = 2 \cdot C \cdot x_1$$

Thus, both customers will have the same consumer surplus per unit of inventory:

$$\begin{aligned} T^* &= \frac{\int_{p^*}^{\infty} \Psi_1(P) dP}{C \cdot x_1} = \frac{2 \int_{p^*}^{\infty} \Psi_1(P) dP}{2 C x_1} \\ &= \frac{\int_{p^*}^{\infty} \Psi_2(P) dP}{C \cdot x_2} \end{aligned}$$

The profit that a linen supply earns from using an Inventory Charge Plus Laundering Fee system is given by the following equation.

$$\pi = T \cdot \sum_{i=1}^N C \cdot x_i + P \cdot X - TC_I$$

where

$T$  = Inventory Fee  
 $\sum_{i=1}^N C \cdot x_i$  = Total inventory the linen supply holds to service all customers

$P$  = Laundering fee

$X$  = Total quantity of linen soiled

$TC_I$  = Total cost under an Inventory Charge Plus Laundering Fee system

Substituting in values for  $T$ ,  $P$  and  $\sum_{i=1}^N C \cdot x_i$ , we obtain:

$$\begin{aligned} \pi &= \frac{\int_{p^*}^{\infty} \Psi_1(P) dP}{C \cdot x_1} \cdot C \cdot x_1 + \frac{\int_{p^*}^{\infty} \Psi_2(P) dP}{C \cdot x_2} \cdot C \cdot x_2 \\ &+ P^* \cdot X - TC_I \\ &= \int_{p^*}^{\infty} \Psi_1(P) dP + \int_{p^*}^{\infty} \Psi_2(P) dP + P^* \cdot X - TC_I \end{aligned}$$

Or since the consumer surplus of Customer 2 is twice that of Customer 1,

$$\pi = 3 \cdot \int_{p^*}^{\infty} \Psi_1(P) dP + P^* \cdot X - TC_I$$

We have seen that the major differences in cost that arise between two different pricing systems is due to the difference in the quantity of linen soiled under the two systems. This difference in the quantity of linen soiled will arise either because one system requires a minimum, or because one system selects a laundering fee less than the other, or because one system sets its right-to-buy tariff higher than the other and hence drives more customers from the market. In the present case, no difference in total cost can be expected to arise from any of these sources: No minimums are required by either system. We have not allowed the Inventory Charge Plus Laundering Fee system to drive either customer from the market. Also, we have set the laundering fee under the Inventory Charge Plus Laundering Fee system equal to  $P^*$ , the optimal laundering fee under the Delivery Charge Plus Laundering Fee system and thereby insured that the quantity of linen soiled will be the same under both systems. Hence, the linen supply's total costs will be the same under both systems.

If so, then it is apparent that in this market, the Inventory Charge Plus Laundering Fee system will earn larger profits than the Delivery Charge Plus Laundering Fee system. Both pricing systems secured revenues equal to  $P^* \cdot X$  from the charge of a laundering fee. The Delivery Charge Plus Laundering Fee system secured  $2 \cdot \int_{P^*}^{\infty} \psi_1(P) dP$  from its charge of a right-to-buy tariff. Yet, the revenues received under the Inventory Charge Plus Laundering Fee system from its charge of a right-to-buy tariff were  $3 \cdot \int_{P^*}^{\infty} \psi_1(P) dP$ . Thus, in this market, the profits earned under the Inventory Charge Plus Laundering Fee system exceed those that the linen supply would earn under the Delivery Charge Plus Laundering Fee system.

The derivation of this result depended on the rather strong assumption that at each laundering fee price, both customers differed in the quantity of linen demanded by a certain constant multiple. However, in order for the Inventory Charge Plus Laundering Fee pricing system to be superior to the Delivery Charge Plus Laundering Fee system, this particular assumption need not hold. In fact, it is not difficult to show that a monopolistic linen supply will prefer to use the Inventory Charge Plus Laundering Fee system so long as the amount of inventory the linen supply holds to service the customer with the smallest consumer surplus per-unit of inventory, is less than the average amount of inventory held per customer. That is, in a monopolistic market of  $N$  customers, where  $\psi_i(P)$  is the  $i^{\text{th}}$  customer's demand curve,  $I(X_i)$  is the amount of inventory held for the  $i^{\text{th}}$  customer

and where  $P^*$  is the optimal laundering fee under the Delivery Charge Plus Laundering Fee system, if Customer J is the customer such that,

$$\frac{\int_{P^*}^{\infty} \Psi_J(P) dP}{I(X_J)} = \text{Min}_{i=1 \text{ to } N} \frac{\int_{P^*}^{\infty} \Psi_i(P) dP}{I(X_i)}$$

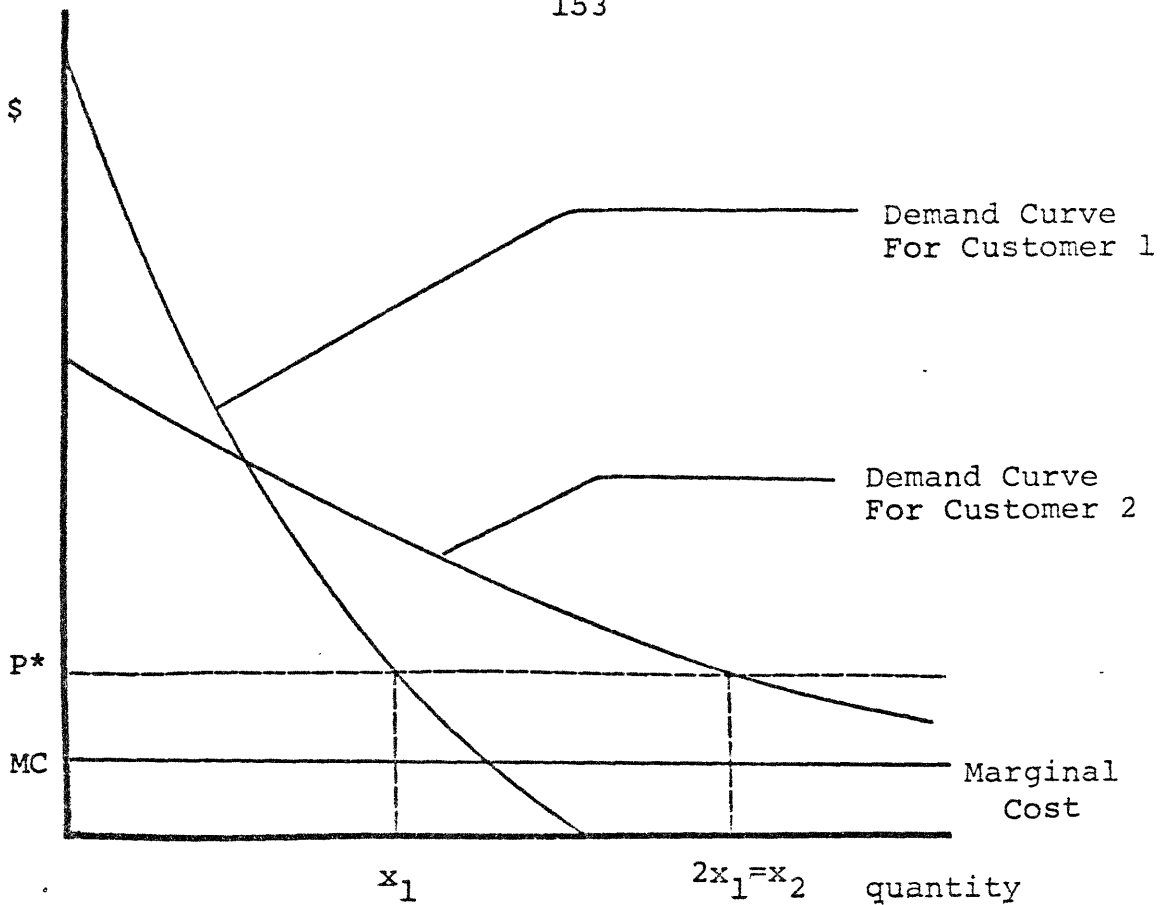
and if

$$I(X_J) \leq \frac{\sum_{i=1}^N I(X_i)}{N}$$

then the linen supply will earn greater profit under an Inventory Charge Plus Laundering Fee system than it would earn under a Delivery Charge Plus Laundering Fee system.<sup>6</sup>

As we shall now see, however, in some markets a linen supply would do better to use a Delivery Charge Plus Laundering Fee system rather than an Inventory Charge Plus Laundering Fee system.

Consider the market of two customers given in the following graph. This market has two special features: First, the area between the demand curve of Customer 2 and the price line  $P^*$  is exactly equal to the corresponding area for Customer 1, and thus at a laundering fee price of  $P^*$  Customer 1 and Customer 2 have identical consumer surpluses. Second at a laundering fee price of  $P^*$ , Customer 2 will choose to receive exactly twice the amount of linen that Customer 1 receives.



Demand and Marginal Cost Curves For a Market of Two Customers

To make things simple, let us suppose that under an Inventory Charge Plus Laundering Fee pricing system, the optimal laundering fee price to charge is  $P^*$ . The optimal inventory price to charge per unit of inventory is then the smallest consumer surplus per-unit of inventory that either customer would enjoy at a laundering fee price of  $P^*$ :

$$T^* = \text{Min} \left( \frac{\int_{P^*}^{\infty} \psi_1(P) dP}{C \cdot x_1} \dots, \frac{\int_{P^*}^{\infty} \psi_2(P) dP}{C \cdot x_2} \right)$$

(where the numerator in each fraction is the consumer surplus of each customer and the denominator is the amount of inventory

that the linen supply must hold to service each customer). Since by assumption, both customers have the same consumer surplus at a laundering fee price of  $P^*$ , then

$$\int_{P^*}^{\infty} \Psi_1(P) dP = \int_{P^*}^{\infty} \Psi_2(P) dP$$

Furthermore, we have assumed that at a laundering fee price of  $P^*$ , Customer 2 will demand twice the amount of linen that Customer 1 demands. Hence, since the amount of inventory the linen supply holds for a given customer is determined by multiplying the quantity of linen demanded by that customer by some constant, the amount of linen the linen supply holds for Customer 2 must be twice that it holds for Customer 1.

Thus, we find that Customer 2 has smaller per unit of inventory than does Customer 1:

$$\frac{\int_{P^*}^{\infty} \Psi_2(P) dP}{C \cdot 2} = \frac{1}{2} \frac{\int_{P^*}^{\infty} \Psi_1(P) dP}{C \cdot 1}$$

The optimal inventory price to charge per unit of inventory is therefore Customer 2's consumer surplus per-unit of inventory or, equivalently half Customer 1's consumer surplus per-unit of inventory.

The maximum profit that a linen supply could earn in this market by using the Inventory Charge Plus Laundering Fee pricing system is given in the following equation:

$$\pi = T^* \cdot \sum_{i=1}^2 C \cdot x_i + P^* \cdot X - TC$$

and, since

$$T^* = \frac{\int_{P^*}^{\infty} \Psi_2(P) dP}{C \cdot X_2} = \frac{\frac{1}{2} \int_{P^*}^{\infty} \Psi_1(P) dP}{C \cdot X_1}$$

$$\pi = \int_{P^*}^{\infty} \Psi_1(P) dP + \int_{P^*}^{\infty} \Psi_2(P) dP + P^* \cdot X - TC_1$$

And since the consumer surplus of Customer 1 at a laundering fee price of  $P^*$  is equal to that of Customer 2:

$$\pi = 1.5 \int_{P^*}^{\infty} \Psi_1(P) dP + P^* \cdot X - TC_I$$

Let us now consider the profit that this same linen supply would earn under a Delivery Charge Plus Laundering Fee Pricing system. Again it will be reasonable to expect that the linen supply's total cost will not vary substantially between the two systems. We shall not allow the Delivery Charge Plus Laundering Fee system to set a right-to-buy tariff that will drive either customer from the market. Also, we shall set the laundering fee under the Delivery Charge

Plus Laundering Fee pricing system equal to  $P^*$ , the optimal laundering fee under the Inventory Charge Plus Laundering Fee system, and thereby insure that the quantity of linen soiled will be the same under both systems. Hence, the linen supply's total costs as well as the revenues the linen supply receives from its charge of a laundering fee will be the same under both systems. The only difference in profit that will result under the two systems will be due to differences in the right-to-buy tariffs charged.

If the laundering fee charged by the linen supply is  $P^*$ , then the maximum delivery fee that the linen supply can charge, without driving either customer from the market, is the amount of the smallest consumer surplus of any customer. However, since we have assumed that at a laundering fee price of  $P^*$ , Customer 1 and Customer 2 have the same amount of consumer surplus, the maximum delivery fee  $D^*$  that the linen supply can charge will be equal to the consumer surpluses of both customers:

$$D^* = \int_{P^*}^{\infty} \Psi_1(P) dP = \int_{P^*}^{\infty} \Psi_2(P) dP$$

The profit the linen supply would earn under a Delivery Charge Plus Laundering Fee pricing system is given by the following equation:

$$\pi = N \cdot D + P \cdot X - TC_D$$

When,  $N$  the number of customers is replaced by 2, the number of customers in our example;  $D$ , the delivery charge is



replaced by  $\int_{P^*}^{\infty} \psi_1(P) dP$ , the consumer surplus of Customer 1; and  $P$  is replaced by  $P^*$  the given laundering fee price, this equation becomes the following:

$$\pi = 2 \cdot \int_{P^*}^{\infty} \psi_1(P) dP + P^* \cdot X - TC_D(X)$$

Comparing these profits with the profit the linen supply would earn under an Inventory Charge Plus Laundering Fee Pricing system, we find that in this special sort of market, the Delivery Charge Plus Laundering Fee will provide greater profits than the Inventory Charge Plus Laundering Fee system. This result, of course, contrasts with the results of our earlier example in which we found the Inventory Charge Plus Laundering Fee system to be more profitable than the Delivery Charge Plus Laundering Fee system. Thus, the question of whether it is more profitable to use a variable two-part tariff, such as the Inventory Charge Plus Laundering Fee system, or a uniform two-part tariff such as the Delivery Charge Plus Laundering Fee system, cannot be decided a priori: whether one system is better than the other depends on the market in which the linen supply is operating.

The example I used to demonstrate that under certain circumstances a linen supply would prefer to use a Delivery Charge Plus Laundering Fee system, relied of course on some rather far-fetched assumptions. We assumed that at a laundering fee price of  $P^*$

both customers had identical consumer surpluses, and that at a laundering fee price of  $P^*$  one customer would choose to soil twice the amount soiled by the other. Of course, these particular assumptions need not hold in order for a Delivery Charge Plus Laundering Fee pricing system to produce greater Profits than an Inventory Charge Plus Laundering Fee system. In fact, it can be shown that a monopolistic linen supply will prefer to use a Delivery Charge Plus Laundering Fee pricing system so long as the amount of inventory the linen supply must hold to service the customer with the smallest consumer surplus is greater than the average inventory of linen it holds per customer. That is, in a monopolistic market of  $N$  customers, where  $\psi_K(P)$  is the  $i^{\text{th}}$  customer's demand curve,  $I(X_i)$  is the amount of inventory held for the  $i^{\text{th}}$  customer, and where  $P^*$  is the optimal laundering fee price under the Inventory Charge Plus Laundering Fee system, if Customer  $K$  is the customer such that,

$$\int_{P^*}^{\infty} \psi_K(P) dP = \text{Min}_{i=1 \text{ to } N} \left( \int_{P^*}^{\infty} \psi_i(P) dP \right)$$

and if

$$I(X_K) > \sum_{i=1}^N \frac{I(X_i)}{N}$$

then the linen supply will earn greater profit under a Delivery Charge Plus Laundering Fee pricing system than it would under an Inventory Charge Plus Laundering Fee system.<sup>7</sup>

We have seen in this section two sufficient conditions

for determining whether the Inventory Charge Plus Laundering Fee system is more profitable than the Delivery Charge Plus Laundering Fee system:

- (1) If Customer J is the customer with the smallest consumer surplus per unit of inventory and if the linen supply holds less than the average amount of inventory for Customer J, then the Inventory Charge Plus Laundering Fee system is more profitable.
- (2) If Customer K is the customer with the smallest consumer surplus and if the linen supply holds more than the average quantity of linen to service Customer K, then the Delivery Fee plus Laundering Charge system will be more profitable.

If neither of these sufficient conditions is fulfilled, then to my knowledge, there is no way to determine which of these two pricing systems is preferable without performing the computations required to calculate the maximum profit achievable under an Inventory Charge Plus Laundering Fee system and the maximum profit achievable under a Delivery Charge Plus Laundering Fee system and comparing the two.

## FOOTNOTES

## CHAPTER IV

<sup>1</sup>This formula is somewhat different from the formula for the optimal piece-rate price presented in Chapter III. This difference is due to the fact that in Chapter III, we assumed for simplicity that the linen supply had constant marginal costs which were always equal to zero. However in both cases the basic idea behind the formula is the same: Charge the smaller customer the maximum amount he would be willing to pay for the maximum number of units it is profitable to rent to him. In both cases, the formulae give that amount in terms of the area under the customer's demand curve, between the price axis and the minimum required quantity of linen.

<sup>2</sup>I have chosen to consider this type of market, rather than the market in which Customer 2 would prefer to receive less than the required minimum of  $q_{min}$  units, because in this latter case, it is trivially easy to show that the Delivery Charge Plus Laundering Fee system can exactly duplicate the profits earned under the Piece-Rate With a Minimum system: If Customer 2 would prefer to receive less than the required  $q_{min}$  units at a piece-rate of  $P^*$  in the absence of a minimum, then when a minimum of  $q_{min}$  units is enforced, Customer 2 will choose to receive no more than  $q_{min}$  and will thus pay a total amount equal to  $P^* \cdot q_{min}$ . (We know that Customer 2 would be willing to pay this amount because  $P^* \cdot q_{min}$  exhausts the consumer surplus of Customer 1 and the consumer surplus of Customer 2 is always larger than that of Customer 1.) Thus, the total profit that the linen supply would earn in this case, would be equal to the profit it earns from serving Customer 1 plus the profit it earns from serving Customer 2:

$$\begin{aligned} \text{Total Profit} &= (P^* - MC)(q_{min} + q_{min}) \\ &= 2(P^* - MC) \cdot q_{min} \end{aligned}$$

Or substituting for  $P^*$  the appropriate value:

$$\text{Total Profit} = 2 \cdot \int_{MC}^{\infty} \psi_1(P) dP$$

Consider now the Delivery Charge Plus Laundering Fee system in which:

$$\text{Delivery Fee} = \int_{MC}^{\infty} \psi_1(P) dP$$

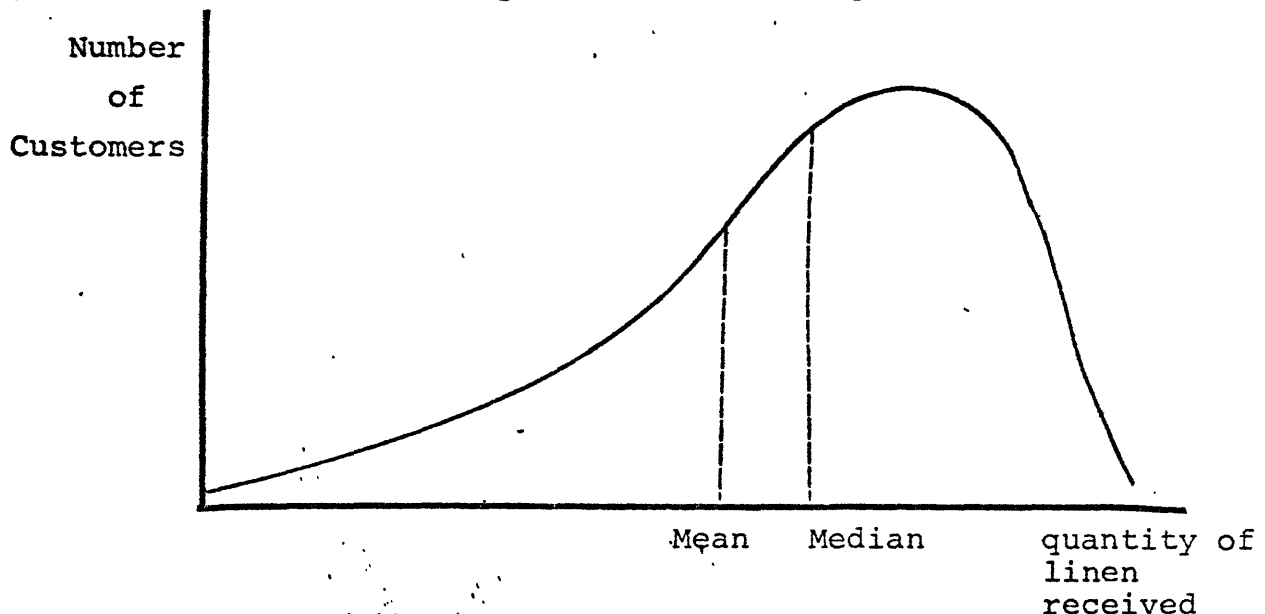
$$\text{Laundering Fee} = MC$$

Since marginal costs are constant, the revenues the linen supply receives from charging a laundering fee equal to marginal cost will exactly equal the linen supply's cost of providing each unit of linen rented. All profits will be derived from charging delivery fees to both customers, and thus, total profits will be exactly equal to the maximum profits that could be earned by the Piece-Rate With a Minimum pricing system:

$$\text{Total Profits} = 2 \cdot \int_{MC}^{\infty} \psi_1(P) dP_1$$

<sup>3</sup>The Piece-Rate With a Minimum earns greater profit than the Delivery Charge Plus Laundering Fee system on each customer who would prefer to receive less than the required minimum at the optimal piece-rate price. The Delivery Charge Plus Laundering Fee system earns greater profit than the Piece-Rate With a Minimum system on each customer who prefers to receive more than the required minimum at the given piece rate. Thus, so long as the number of customers who prefer to receive more than the required minimum is greater than the number of customers who would prefer to receive less than the required minimum, the Delivery Charge Plus Laundering Fee system will be better than the Piece Rate With a Minimum.

<sup>4</sup>A further observation will amplify this point. Typically, there is much more variety in size among a linen supply's smaller customers than among its larger customers. That is, the frequency distribution of the size of a linen supply's customers is most commonly skewed to the right:



Frequency Distribution of Customers

Thus, a linen supply's smallest customers are normally much smaller than the median customer size. Thus, since one is most likely to find the customer with the smallest consumer surplus among the linen supply's smallest customers, it is extremely unlikely that less than half of the linen supply's customers will choose to receive more than the required minimum.

<sup>5</sup>To show that, under these circumstances, the Inventory Charge Plus Laundering Fee system is preferable to the Delivery Charge Plus Laundering Fee system, it is sufficient to show that the Inventory Charge Plus Laundering Fee system can produce profits greater than the profits achieved under an optimal choice of number of customers served,  $N^*$ , laundering fee  $P^*$  and delivery fee  $D^*$  of a Delivery Charge Plus Laundering Fee system.

To show this, we shall require the Inventory Charge Plus Laundering Fee to serve the same group of  $N^*$  customers and to charge the same laundering fee price  $P^*$ . Thus, since neither system requires a minimum we shall expect total costs under both systems to be the same. Also, since the laundering fee charged is the same under both systems, the revenues received from charging the same group of customers the same laundering fee will be the same. Thus, any difference in profits between the two systems will occur as a result of the right to buy tariffs charged.

If Customer K is the customer with the smallest consumer surplus of any customer remaining in the market, then the Delivery Charge Plus Laundering Fee pricing system will charge a right-to-buy tariff equal to his consumer surplus:

$$D^* = \int_{P^*}^{\infty} \Psi_K(P) dP$$

Total revenues from charging this delivery fee to all customers will be the delivery fee times the number of customers.

$$R_D = N^* D^*$$

Suppose that of the  $N^*$  customers in the market Customer J is the customer with the smallest consumer surplus per-unit of inventory, and that the inventory the linen supply holds to service Customer J is less than the average amount of inventory held per customer:

$$I(X_J) < \frac{\sum_{i=1}^{N^*} I(X_i)}{N^*}$$

where  $I(X_i)$  is the amount of inventory held on behalf of the  $i$ th customer. Thus, the inventory fee charged by an Inventory Charge Plus Laundering Fee system will be the consumer surplus of Customer  $J$  divided by the inventory held for Customer  $J$ :

$$T^* = \frac{\int_{p^*}^{\infty} \Psi_K(P) dP}{I(X_K)}$$

The revenues received will be equal to the inventory fee  $T^*$  times the total amount of inventory the linen supply holds to service all customers:

$$R_I = T^* \sum_{i=1}^{N^*} I(X_i),$$

Clearly, since Customer  $K$  is the customer with the smallest consumer surplus, his consumer surplus will be less or equal to that of Customer  $J$ :

$$\int_{p^*}^{\infty} \Psi_K(P) dP \leq \int_{p^*}^{\infty} \Psi_J(P) dP$$

Multiplying through by  $\sum_{i=1}^{N^*} I(X_i)/I(X_J)$

we obtain:

$$\sum_{i=1}^{N^*} I(X_i) \frac{\int_{p^*}^{\infty} \Psi_K(P) dP}{I(X_J)} \leq \sum_{i=1}^{N^*} I(X_i) \frac{\int_{p^*}^{\infty} \Psi_J(P) dP}{I(X_J)}$$

But since by assumption,  $I(X_J) < \frac{\sum_{i=1}^{N^*} I(X_i)}{N^*}$ ,

$$\sum_{i=1}^{N^*} I(X_i) \frac{\int_{p^*}^{\infty} \Psi_K(P) dP}{\frac{\sum_{i=1}^{N^*} I(X_i)}{N^*}} \leq \sum_{i=1}^{N^*} I(X_i) \frac{\int_{p^*}^{\infty} \Psi_J(P) dP}{I(X_J)}$$

Simplifying, we obtain:

$$N^* \int_{p^*}^{\infty} \Psi_K(P) dP \leq \frac{\sum_{i=1}^{N^*} I(X_i) \int_{p^*}^{\infty} \Psi_J(P) dP}{I(X_J)}$$

$$N^* \cdot D^* \leq T^* \cdot \sum_{i=1}^{N^*} I(X_i)$$

Thus, the revenues produced by an Inventory Charge Plus Laundering Fee system's charge of a right-to-buy tariff are greater than or equal to those of a Delivery Charge Plus Laundering Fee system. Since differences in profits were shown to rely only on differences in the revenues from right-to-buy tariffs charged, the Inventory Charge Plus Laundering Fee system will provide profits at least as great as the maximum profits achievable under a Delivery Charge Plus Laundering Fee system in this market.

<sup>6</sup>To show that, under these circumstances, the Delivery Charge Plus Laundering Fee system is preferable to the Inventory Charge Plus Laundering Fee system, it is sufficient to show that the Delivery Charge Plus Laundering Fee system can produce profits greater than the profits achieved under an optimal choice of number of customers  $N^*$ , laundering fee  $P^*$  and inventory fee  $T^*$  of an Inventory Charge Plus Laundering Fee system.

To show this, we shall require the Delivery Charge Plus Laundering Fee system to serve the same group of  $N^*$  customers and to charge the same laundering fee  $P^*$  as the Inventory Charge Plus Laundering Fee system. Thus, since neither system requires a minimum, we shall expect both total costs under both systems to be the same. Also, since the laundering fee is the same under both systems, the revenues received from charging the same group of customers the same laundering fee will be the same. Thus, any difference in profits between the two systems will occur as a result of the right to buy tariff charged.

If Customer J is the customer with the smallest consumer surplus per-unit of inventory of any customer remaining in the market, then the Inventory Charge Plus Laundering Fee system will charge a right-to-buy price equal to Customer J's consumer surplus divided by the amount of inventory required to service Customer J:

$$T^* = \frac{\int_{p^*}^{\infty} \Psi_J(P) dP}{I(X_J)}$$



Total revenues from charging this inventory fee to all customers will be equal to the inventory fee  $T^*$  times the total amount of inventory the linen supply holds to service all customers:

$$R_I = T^* \cdot \sum_{i=1}^{N^*} I(X_i)$$

Suppose that of the  $N^*$  customers in the market, Customer K is the customer with the smallest consumer surplus, and that the inventory the linen supply holds to service Customer K is greater than the average amount of inventory held per-customer:

$$I(X_K) < \frac{\sum_{i=1}^{N^*} I(X_i)}{N^*}$$

The optimal delivery fee to charge will be the consumer surplus of Customer K:

$$D^* = \int_{p^*}^{\infty} \psi_K(P) dP$$

Revenues received from charging all customers this delivery charge will be equal to the delivery fee  $D^*$  times the number of customers:

$$R_D = N^* \cdot D^*$$

Clearly, since Customer J is the customer with the smallest consumer surplus per-unit of inventory, the consumer surplus per-unit of inventory for Customer K will be greater than or equal to that of Customer J:

$$\frac{\int_{p^*}^{\infty} \psi_K(P) dP}{I(X_K)} \geq \frac{\int_{p^*}^{\infty} \psi_J(P) dP}{I(X_J)}$$

Multiplying through by  $\sum_{i=1}^{N^*} I(X_i)$  we obtain:

$$\sum_{i=1}^{N^*} I(X_i) \frac{\int_{p^*}^{\infty} \psi_K(P) dP}{I(X_K)} \geq \frac{\int_{p^*}^{\infty} \psi_J(P) dP}{I(X_J)} \cdot \sum_{i=1}^{N^*} I(X_i)$$

But since by assumption,  $I(X_K) > \frac{\sum_{i=1}^{N^*} I(X_i)}{N^*}$

$$\frac{\sum_{i=1}^N I(X_i) \int_{p^x}^{\infty} \psi_K(P) dP}{\frac{\sum_{i=1}^{N^*} I(X_i)}{N^*}} \geq \frac{\int_{p^*}^{\infty} \psi_J(P) dP}{I(X_J)} \cdot \sum_{i=1}^{N^*} I(X_i)$$

Simplifying, we obtain:

$$N^* \int_{p^*}^{\infty} \psi_K(P) dP \geq \frac{\int_{p^*}^{\infty} \psi_J(P) dP}{I(X_J)} \sum_{i=1}^{N^*} I(X_i)$$

$$N^* \cdot D^* \geq T^* \sum_{i=1}^{N^*} I(X_i)$$

Thus, the revenues received from charging a delivery fee under the Delivery Charge Plus Laundering Fee system are greater than or equal to the revenues received from charging an inventory fee under the Inventory Charge Plus Laundering Fee system. Since differences in profits were shown to rely only on differences in the revenues received from right-to-buy tariffs charged, the Delivery Charge Plus Laundering Fee system will provide profits at least as large as the maximum profits achievable in this market under an Inventory Charge Plus Laundering Fee system.

## CHAPTER V

### APPLICATIONS

In the last chapter we saw that the two most preferable pricing systems for a linen supply to use are the Inventory Charge Plus Laundering Fee system and the Delivery Charge Plus Laundering Fee system. Which of these two systems is most preferable depends on the particular markets in which the linen supply operates. In some markets, a linen supply will earn greater profits under a Delivery Charge Plus Laundering Fee system than under an Inventory Charge Plus Laundering Fee system. In other markets an Inventory Charge Plus Laundering Fee system will produce larger profits than a Delivery Charge Plus Laundering Fee system.

In Chapter III, we developed rules for the optimal use of both of these pricing systems. As you recall, however, in order to make optimal use of either the Delivery Charge Plus Laundering Fee system or the Inventory Charge Plus Laundering Fee system, a linen supply manager must know a tremendous amount about the customers his plant serves. In particular, the optimal use of both these pricing systems requires that the plant manager know the demand curve of each of its potential customers. Linen supply plant managers are, however, not omniscient. Even the most informed plant manager would never know the demand curves of each of his potential

customers, and thus unless a way can be found to simplify the information requirements of two-part pricing systems, the theoretical groundwork we have laid thus far will be of little practical use in the linen supply industry.

Consequently, in this chapter I plan to develop in rough outline a computer program to help plant managers decide which pricing system to use and at what level to set prices. Basically the idea will be to treat the market for a particular item of linen as being composed of a relatively small number of customer types. The program will require managers to estimate the demand curve of each customer type and to estimate what percent of the total market for that item of linen is best described by that customer type. The manager will also be required to estimate his plant's marginal cost of providing that item of linen. Given this estimate of the marginal cost function and the simplified description of the market for that item of linen, the program will calculate the following for both the Inventory Charge Plus Laundering Fee system and the Delivery Charge Plus Laundering Fee system:

- (1) whether it is worthwhile to deal with all customer types;
- (2) the optimal right-to-buy tariff to charge (inventory fee or delivery charge);
- (3) the optimal per-unit of output price (laundering fee) to charge;

- (4) the profit the linen supply would expect to earn from dealing with the optimal mix of customer types and charging the optimal right-to-buy tariffs and laundering fees.

By comparing the expected profit under both systems, the manager can determine whether an Inventory Charge Plus Laundering Fee system is preferable to a Delivery Charge Plus Laundering Fee system, or vice-versa. Furthermore, by altering the estimates he inputs to the program, the manager will be able to determine whether the dominance of one pricing system over the other is sensitive to the accuracy of his estimates.

The computer program I envision to accomplish this task can best be explained in four parts:

- (a) Estimating the marginal cost function;
- (b) Analysis of the market;
- (c) Calculations by the program;
- (d) Possible extensions of the program.

We shall now consider each of these four parts.

#### A. Estimating the Linen Supply's Marginal Cost Function

A firm's marginal cost is the increase in the firm's total costs that would result from producing one more unit of output. A marginal cost function depicts how total costs will change as volume changes. A linen supply, however, provides its customers with many different types of products, and many of the costs a linen supply incurs in providing those products

are joint costs, that is, costs such as general overhead or the cost of operating a piece of equipment which process several products, which cannot be assigned with certainty to any particular product.

Typically, cost accountants will allocate joint costs to a particular product on the basis of what percent that product constituted of the total products which together were responsible for incurring the joint cost. That is, cost accountants will generally allocate joint costs in three steps:

First, the total amount of joint costs are determined.

Second, the percent of all joint-cost products which a particular type of product constituted is determined. This is simply done by dividing the number of products of the particular type by the total number of products which were together responsible for incurring the joint costs.

Third, the joint costs allocated to products of a particular type is calculated by multiplying the total amount of joint costs by the percent of all joint-cost products that the particular type of product constituted.

Thus, for example, if 5% of the total pounds of linen washed were Turkish towels, a linen supply's cost accountant might

assign 5% of the total cost of washing all products to Turkish towels.

This procedure seems reasonable enough for allocating a linen supply's joint costs to the products it provides and hence, I shall propose a method for estimating the marginal cost of a particular item of linen which incorporates this procedure. More exactly, the method I propose will require the computer program to make an assumption about how the mix of products provided by a linen supply will change as total plant volume changes. The program will then ask the plant manager to estimate how his plant's total costs will change as total plant volume changes. Finally, the program will use the cost accountants' procedure to allocate the change in total cost to the changes in volume of particular products, and from this derive the marginal cost functions for particular products.

To see how this method for estimating the marginal cost functions of particular products will work, we shall consider an example. Let us suppose that a linen supply currently provides  $N$  Turkish towels per month. If the linen supply knew how its mix of products would change as the total volume of linen rented changes, then the linen supply could determine what level of total volume it would have to reach in order for one additional Turkish towel to be rented. Let us denote the linen supply's current total volume as  $V_0$  and the total volume it would have to reach in order to rent an additional

Turkish towel  $V_1$ . If the linen supply also knew how its total costs would change as total volume changes, then it would be able to determine the total costs it would incur if its volume were to increase to  $V_1$ . We shall call the linen supply's current total costs  $TC_0$  and denote by  $TC_1$  the linen supply's total costs at a volume of  $V_1$ . Following the procedure for allocating joint costs, we find that the linen supply's current cost of providing  $N$  Turkish towels is the following:

$$\begin{array}{l} \text{Current Cost} \\ \text{of } N \text{ Turkish} \\ \text{Towels} \end{array} = \frac{N}{V_0} \cdot TC_0$$

Again, following the procedure for allocating joint costs, we find that the cost of providing  $N+1$  Turkish towels would be:

$$\begin{array}{l} \text{Cost of } N+1 \\ \text{Turkish} \\ \text{Towels} \end{array} = \frac{N+1}{V_1} \cdot TC_1$$

Subtracting the linen supply's cost of providing  $N$  Turkish towels from the cost of providing  $N+1$  Turkish towels we get the increase in total cost associated with renting one additional Turkish towel, or equivalently the marginal cost of Turkish towels:

$$\begin{array}{l} \text{Marginal Cost} \\ \text{of Turkish} \\ \text{Towels} \end{array} = \frac{N}{V_0} \cdot TC_0 - \frac{N+1}{V_1} \cdot TC_1$$

This method for estimating the marginal cost of a particular item of linen requires that we know both how the mix of products provided by a linen supply will change as



total volume changes, and how total costs will change as total plant volume changes. We may, I believe, rely on plant managers to provide this latter sort of information. That is, the computer program could ask managers to estimate how total costs would change as total plant volume changes, by asking the plant manager such questions as the following:

If the total poundage washed pressed and delivered by your plant were to increase 10% by what percent would total costs increase?

Once the plant manager has answered such questions for a number of different percentage changes in plant volume, the program would be able to estimate the general trend of how total costs will change as volume changes and use that general trend to estimate, for any particular level of volume, what total costs are likely to be.

The second type of information my method for estimating marginal costs will require is more elusive. There is, after all, no causal relation running from changes in plant volume to changes in product mix, as there is running from changes in total cost. Rather, an increase in plant volume could result in any of an infinite number of different product mixes.

In this situation, the best that can be done is to make some fairly plausible assumption about how a linen supply's product mix will change as total plant volume changes. Two

alternatives seem reasonable:

First, we might assume that a linen supply will always provide the optimal, that is, the most profitable, mix of products possible for a given market of customers. To incorporate this assumption into the computer program would require that the program be designed to solve for the optimal mix of products for the linen supply to rent at all levels of total volume at which the plant might operate. Basically, such a program would collect data on the current capacity of each department in the plant and the costs of changing the capacity of each department. For each level of total volume at which the plant might operate, the program would then allocate each unit of capacity in each department to that product whose marginal revenue is the greatest. The resulting mix of products will be the optimal mix of products the linen supply could provide.

This version of our computer program has two disadvantages. First, the complexity of designing such a program is overwhelming. It would take years longer and thousands of dollars more to build than any linen supply company would be willing to devote to building it. Second, under a program of this type, the prices of all products would be set simultaneously. Thus, a manager could not effectively use the program to reconsider his pricing of a particular item of linen. Also, the manager would not be effectively able to test the sensitivity of this program's results to the descriptions of the

he provides.

Second, we might assume that the product mix a linen supply currently provides will remain constant as total volume changes. That is, according to this assumption, if the total volume of the linen supply were to increase 10%, then the quantity of Turkish towels would also increase 10% as would the quantity of tablecloths, uniforms and so on.

There are two factors which recommend the second assumption. First, a computer program making this assumption will be relatively easy and inexpensive to build. Second, although the mix of products may vary widely among different linen supplies, the mix of products provided by a single linen supply will normally change rather slowly over time. Since the mix of products for a given linen supply does not seem to be very volatile, it is reasonable to expect that, at different levels of total volume the resulting mix of products will not be far different from the current mix of products.

When this assumption is incorporated into the computer program, we may summarize the procedures the program will follow to estimate the marginal cost of a particular item of linen by way of the following four steps.

First, the program will ask the plant manager to specify his current total volume and current total costs, and to estimate how total costs will change as total volume changes.

Second, the program will ask the plant manager to specify

his current product mix. That is, the manager will be asked what percent of his plant's current total volume is accounted for by the product he is attempting to price.

Third, the program will solve for that level of total plant volume which would result in one additional unit of the relevant type of linen being rented. The program will then estimate the total costs the linen supply would incur at that level of total volume.

Finally, the program will calculate what part of total costs should be assigned to the product for both the current level of total volume, and that volume at which one additional unit is rented. The difference between these two shares of total costs is the marginal cost of the product he is attempting to price.

#### B. Analysis of the Market

One fundamental difficulty that a linen supply manager would have in using the theoretical analyses of two-part pricing systems presented in Chapter III, is that the optimal use of these pricing systems require that the manager know the demand curves of each of his potential customers. In this section I shall present a method for simplifying the informational requirements of these systems. Basically, the idea will be to treat the market for a particular item of linen as being composed of a relatively small number of customer types. The program will require plant managers to estimate

the size of the total market, the demand curve of each customer type, and the percent of the total market for that item of linen best described by each customer type.

All simplifications pay a price in the accuracy of the information they convey. To avoid any confusion, I would like now to state explicitly what price my system of customer types pays in terms of the accuracy of the results the program will generate: My analysis of the market into customer types assumes that all customers have constant elasticity demand curves of the form,

$$x = aP^b$$

where  $x$  is the quantity of the relevant type of linen received per month,  $a$  is some parameter varying from customer to customer,  $P$  is the laundering fee charged and  $b$  is the customer's price elasticity. We have seen this form of demand curve before, in Chapter III, where I presented a few reasons why it is reasonable to believe that demand curves may take this shape. Yet, although it is reasonable to believe that customers do have this sort of constant elasticity demand curves, in fact their demand curves may be somewhat different. Thus, the descriptions of the market provided by my system cannot be expected to be entirely accurate.

If we do make these assumptions, then it will be possible for plant managers to define the demand curves the program will require in terms of two fairly simple concepts: customer size

and price sensitivity.<sup>1</sup> Customer size is determined by the quantity of the relevant item of linen a customer receives per month at the current laundering fee per unit. Price sensitivity is simply the customer's price elasticity or the percentage change in quantity which would result from a one percent change in the laundering fee.

The procedure the program will follow to obtain a description of the market in terms of customer types may be divided into four steps.

First, the program will require the plant manager to estimate the size of the total market. To do this, the manager specifies the current laundering fee charged and the total quantity of linen which is rented per month at that laundering fee. Also, the plant manager will be asked to estimate the total number of customers in the market. Finally, in order that the program will be able to calculate the revenue received under an Inventory Charge Plus Laundering Fee system, the manager will be asked to specify the inventory multiple to use in calculating the inventory the linen supply must hold to service a customer receiving a given amount of linen per month.

Second, the program will ask the plant manager to define three sizes of customers. To do this, the manager simply specifies the quantity of the relevant type of linen typically received by small medium and large customers. Ideally,

the quantity of linen that the linen supply manager specifies for the typical small customer will be the median quantity of linen received by all customers the manager considers small, and similarly for medium and large customers.

Third, the program will require the plant manager to define three degrees of price sensitivity among customers. That is, the manager will specify the median price elasticity for customers he considers very price sensitive, moderately price sensitive, and not-so price sensitive.

By following steps two and three, the plant manager will, in effect, have divided the total market for the item of linen he is attempting to price into the nine customer types given in the following table.

Types of Customers

Size of Customer	Very Price Sensitive $E = e_1$	Moderately Price-Sensitive $E = e_2$	Not Very Price Sensitive $E = e_3$
Small $S_1$			
Medium $S_2$			
Large $S_3$			

In the final step, the plant manager will be asked to decide what percent of the total market is best described by each customer type. Thus, the manager will be required to fill in the above table by asking himself, "What percent of my total potential customers are small and very price sensitive?" "What percent are small and moderately price sensitive?" and so on.

Thus, at this stage of the program, the plant manager will have completed the analysis of the market for a particular item of linen into nine customer types. The computer program will then be able to use this description of the market to generate the demand curve of each of the nine customer types, and to weight each of those demand curves by the estimated number of customers of that type. We might represent what the computer "knows" from performing these calculations by way of the following equation:

$$X = m_1 a_1 P^{b_1} + m_2 a_2 P^{b_2} + \dots + m_q a_q P^{b_q}$$

Where:

- X = Total quantity of linen soiled
- $m_i$  = Estimated number of customers of the  $i^{\text{th}}$  customer type.
- = Estimated total number of customers, times estimated percent of the market represented by the  $i^{\text{th}}$  customer type



$a_i P^{b_i}$  = Demand curve of the  $i^{\text{th}}$  customer type

### Calculations by the Program

Up to this point, the computer program has merely accepted input from the plant manager and performed calculations to derive approximations of various demand curves. The program must now use this data to decide which of the two most preferred pricing systems to use, and at what level to set prices. More specifically, the program must now calculate for both the Inventory Charge Plus Laundering Fee system and the Delivery Charge Plus Laundering Fee system the following:

- (1) whether it is worthwhile to deal with all customer types;
- (2) the optimal right-to-buy tariff to charge (inventory fee or delivery charge);
- (3) the optimal per-unit of output price (laundering fee) to charge;
- (4) the profit the linen supply would expect to earn from dealing with optimal mix of customer types and charging the optimal right-to-buy tariffs and laundering fees.

The plant manager will then compare the expected profits earned under both systems and select the most profitable.

To make the program perform these calculations, we need only construct the program so that it will follow the rule for maximizing profit under the Inventory Charge Plus Laundering

Fee system and the rule for maximizing profit under the Delivery Charge Plus Laundering Fee system. For each mixture of customer types which the linen supply might choose to serve, these rules generate the optimal right-to-buy tariffs and laundering fees. Also, by calculating the estimated profit for each mix of customer types, these rules will allow the program to determine the optimal mix of customer types to serve under each pricing system and the level of profits the linen supply can expect to earn from doing so.

Specifically, the calculations the program will perform are the following.

Calculation of Profits under a Delivery Charge Plus Laundering Fee Pricing System

Following the rules for maximizing profit under a uniform two-part tariff, the program will first assume that all customers are to remain in the market. It will then calculate the optimal laundering fee, delivery charge and the profit the linen supply could expect to earn from serving all customers. Next, the program will assume that the type of customer with the smallest consumer surplus, the smallest, most price sensitive type of customer will be excluded from the market. The program will then recalculate the optimal laundering fee and delivery charge and compute the profit the linen supply could expect to earn from serving all but the smallest, most price sensitive customers.

The program will continue to exclude customer types and calculate the linen supply's expected profits until the expected profits begin to decline. The program will then determine for what mix of customer types the linen supply's expected profits are the greatest. Finally, the program will report to the plant manager using the program the optimal customer mix, laundering fee, delivery charge and the profit the linen supply could expect to earn by serving that optimal mix of customers. These calculations will involve three sub-routines.

First, the optimal laundering fee  $P^*$  will be calculated from the formula:

$$p^* = \frac{MC}{1 + \frac{1 - NS_k}{E}}$$

where:

$N$  = Number of customers remaining in the market

$s_k$  = Market share demanded by customers of the customer type with the smallest consumer surplus of any customer type remaining in the market

$E$  = Elasticity of the market demand curve, taking account of the fact that some types of customers may have been driven from the market.<sup>2</sup>

$MC$  = Marginal cost.

Second, the optimal delivery charge  $D^*$  is calculated from the formula

$$D^* = \int_{P^*}^{\infty} a_k P^{b_k} = \frac{a_k P^{b_k+1}}{b_k+1}$$

where  $a_k P^{b_k}$  is the demand curve of the customer type with the smallest consumer surplus of any customer type in the market at a laundering fee price of  $P^*$ , and where  $b_k$  is less than negative one.

Finally, the profit the linen supply can expect to earn by serving the customers remaining in the market is calculated. To perform this calculation, the program substitutes the values for  $P$  and  $D$  into the profit equation of the linen supply. The general form of the profit equation for a linen supply using a Delivery Charge Plus Laundering Fee Pricing system is the following:

$$\pi = D \cdot N + P \cdot X - TC$$

where:

$N$  = Number of customers remaining in the market

$X$  = Total quantity of linen soiled by customers remaining in the market

$TC$  = Total cost.

#### Calculation of Profits Under an Inventory Charge Plus Laundering Fee Pricing System

Following the rules for maximizing profit under an Inventory Charge Plus Laundering Fee system, the program will again at first assume that all customers are to remain in the market. It will then calculate the optimal laundering fee and

inventory fee and compute the profit the linen supply could expect to earn from serving all customers. Next, the program will assume that the type of customer with the smallest consumer surplus per-unit of inventory will be excluded from the market. The program will then recalculate the optimal laundering fee and inventory fee, and compute the profit the linen supply could expect to earn from serving all customers but that type with the smallest consumer surplus per unit of inventory.

The program will continue to exclude customer types and calculate the linen supply's expected profits until the expected profits begin to decrease. The program will then determine for what mix of customer types the linen supply's expected profits are greatest. Finally, the program will report to the plant manager using the program the optimal customer mix, inventory fee, laundering fee and the profit the linen supply could expect to earn by serving that optimal mix of customers.

These calculations will again involve three subroutines.

First, the optimal laundering fee  $P^*$  will be calculated from the formula:

$$P^* = \frac{MC \cdot E}{1 + E + \frac{E+1}{b_j+1}}$$

Where:

$b_j$  = Price elasticity of the customer type with the smallest consumer surplus per unit of

inventory of any customer type remaining in the market ( $b_j < -1$ )

E = Elasticity of market demand curve, taking account of the fact that some types of customers may have been driven from the market.

MC = Marginal cost

The second subroutine calculates the optimal inventory fee  $T^*$  to charge per unit of inventory from the following formula

$$T^* = \frac{\int_{P^*}^{\infty} a_j P^{b_j} dP}{C(a_j P^{b_j})} = - \frac{P^*}{(b_j + 1)C}$$

where C is the inventory multiple and  $b_j$  is the price elasticity of the customer type with the smallest consumer surplus per unit of inventory of any customer type remaining in the market.

Finally, the last subroutine calculates the profit the linen supply can expect to earn by serving all customers remaining in the market. To perform this calculation, the program substitutes the optimal values for the laundering fee P and inventory fee T into the profit equation of the linen supply. For the Inventory Charge Plus Laundering Fee this equation is the following:

$$\pi = T \cdot C \cdot X + P \cdot X - TC$$

where

C = Inventory multiple

X = Total quantity of linen soiled by customers  
remaining in the market at a laundering fee price  
of P

TC = Total Cost

D. Possible Extensions of the Program

I would like now to discuss two ways in which the computer program might be changed to provide more subtle determinations of optimal customer mix, optimal prices and project profits.

The first change that might be made in the program is to require the plant manager to provide a more detailed description of the potential market for the item of linen he is attempting to price. One way in which a more detailed description of the market might be obtained would be to ask the plant manager to define twelve, sixteen or twenty customer types instead of only nine. For example, the plant manager might be asked to define the twenty customer types, and give estimates of the population of each type in the following table:

Price Sensitivity

Size of Customer	Extremely Price Sensitive	Very Price Sensitive	Moderately Price Sensitive	Not Very Price Sensitive
Very Large				
Large				
Medium				
Small				
Very Small				

Another way in which the description of the market might be made more detailed is for the program to ask the manager for a more detailed description of certain customer types. For example, a first run through the program might tell the manager not to deal with large, very price sensitive customers under an Inventory Charge Plus Laundering Fee system. However, since not all large very price sensitive customers are equally large or equally price sensitive, in reality it may be worthwhile for the linen supply to provide service to a few of those customers. Thus, the program could



be constructed so as to ask the manager to describe the large, very price sensitive customer type in more detail. This the manager would do by means of the same mechanism he used to describe the market as a whole: He would divide the large, very price sensitive customer type into sub-types, specify the customer size and price sensitivity of each sub-type and estimate the population of each sub-type. The program would then substitute these customer sub-types for the large, very price-sensitive customer type and recalculate an optimal customer mix, inventory fee, laundering fee and expected level of profit.

We might represent this procedure by way of the following two tables. If the program is designed to recognize only nine customer types, then on the first run through the program, the manager will be asked to define the customer types and estimate the population of each type in the following table.

All Potential CustomersPrice Sensitivity

Size of Customer	Very Price Sensitive	Moderately Price-Sensitive	Not Very Price-Sensitive
Small			
Medium			
Large			

Upon finding that the program recommends that he not deal with large, price sensitive customers, the manager then describes the population of large, very price sensitive customers in terms of nine sub-types of customers.

Large, Very Price Sensitive CustomersPrice Sensitivity

Size of Customer	(Relatively) Very Price Sensitive	(Relatively) Moderately Price Sensitive	(Relatively) Not Very Price Sensitive
(Relatively) Small			
(Relatively) Medium			
(Relatively) Large			

The program will then substitute this more detailed description for the description of the large, price sensitive customer type it used on its first run through the program. After this substitution is made, the program will in effect recognize seventeen customer types: nine sub-types of customers and the eight customer types remaining from the first run through the program. The program will then solve for the optimal mix of these seventeen customer types, the optimal inventory and laundering fees and the linen supply's expected profits.

The difficulty with implementing these two procedures

for improving the market descriptions provided by plant managers is that they may well ask the manager for information he cannot provide. Managers, in my experience, can distinguish nine types of customers. Beyond nine customer types, however, managers' eyes tend to glaze and their responses when forthcoming, are not uttered with much conviction. Thus, except for those few managers who know the markets for their products very well, these options for more detailed market descriptions might well go unused or, worse yet, be misused.

A second possible extension of the computer program would be to build the program so that it will recognize customer types having demand curves of a different form than  $x = a P^b$ , the constant elasticity demand curve I have required the program to use. For example, in addition to the constant elasticity demand curve,  $x = a P^b$ , the program might also recognize demand curves which are linear:  $x = mP + c$ . If this were the case, then the manager would be asked to estimate for each size and price sensitivity of customer type, what percent of that customer type has straight line demand curves and what percent has constant elasticity demand curves. The program would then calculate for each customer type the appropriate straight line demand curve and constant elasticity demand curve, assign the manager's population estimates to each type of demand curve for each customer type.<sup>3</sup> The

program would then continue with its calculations to determine the optimal customer mix, optimal right-to-buy tariffs and laundering fees, and project the linen supply's profit under both pricing systems.

In effect, the information the plant manager would be asked to provide is represented by the following table.

Types of Customers

Size of Customer	Very Price Sensitive	Moderately Price Sensitive	Not Very Price Sensitive
Small	% straight line % constant elasticity	% straight line % constant elasticity	% straight line % constant elasticity
Medium	% straight line % constant elasticity	% straight line % constant elasticity	% straight line % constant elasticity
Large	% straight line % constant elasticity	% straight line % constant elasticity	% straight line % constant elasticity

However theoretically more preferable this way of describing the market, it seems to me to be rather impractical. Many plant managers would find it difficult to understand the difference in customer behavior implied by a straight

line rather than a constant elasticity demand curve. Nearly all plant managers would find it difficult to say with any degree of confidence what percent of customers of a given customer type are likely to have straight line rather than constant elasticity demand curves. Thus again, except for those few managers who know the markets for their products very well, this option for a more detailed market description might well go unused or be misused.

## FOOTNOTES

## CHAPTER FIVE

<sup>1</sup>If we assume that a customer has a demand curve of the form  $X = aP^b$  and we are given estimates of his price elasticity of demand,  $b$ , and the quantity of linen  $X_1$  he receives at the current laundering fee  $P_1$ , we may calculate the parameter  $a$  as follows:

$$X_1 = aP_1^b$$

$$a = \frac{X_1}{P_1^b}$$

Thus, the customer's demand curve is given by:

$$q = \frac{X_1}{P_1^b} P^b$$

<sup>2</sup>If customers are driven from market, then the resulting market demand curve need not retain the same elasticity as the market demand curve when all customers are allowed to remain in the market. If for example, all customers of the smallest most price sensitive type were driven from the market, the resulting demand curve would be:

$$X = \sum_{i=1}^9 m_i a_i P^{b_i} - m_1 a_1 P^{b_1}$$

The price elasticity of this demand curve would be calculated as follows:

$$\begin{aligned} \text{Elasticity} &= \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} \\ &= \frac{\Delta X}{X} \cdot \frac{P}{\Delta P} \\ &= \frac{\Delta X}{\Delta P} \cdot \frac{P}{X} \end{aligned}$$

Or, when the change in price is very small:

$$\text{Elasticity} = \frac{dX}{dP} \cdot \frac{P}{X}$$

$$\begin{aligned}
 &= - \sum_{i=1}^9 m_i b_i a_i P^{b_i-1} - m_1 b_1 a_1 P^{b_1-1} \quad \frac{P}{X} \\
 &= \frac{\sum_{i=1}^9 m_i b_i a_i P^{b_i} - m_1 b_1 a_1 P^{b_1}}{\sum_{i=1}^9 m_i a_i P^{b_i} - m_1 a_1 P^{b_1}}
 \end{aligned}$$

This quantity is the price elasticity of the market demand curve when the smallest most price sensitive customers are driven from the market. Obviously, this quantity need not be the same as the price elasticity of the market demand curve when all customers are allowed to remain in the market.

In general, if  $s$  customer types are excluded from the market, the resulting market demand curves will be:

$$X = \sum_{i=1}^9 m_i a_i b_i P^{b_i} - \sum_{j=1}^s m_j a_j b_j P^{b_j}$$

This market demand curve will have an elasticity equal to

$$E = \frac{\sum_{i=1}^9 m_i a_i b_i P^{b_i} - \sum_{j=1}^s m_j a_j b_j P^{b_j}}{\sum_{i=1}^9 m_i a_i P^{b_i} - \sum_{j=1}^s m_j a_j P^{b_j}}$$

<sup>3</sup> How an estimate of the appropriate straight line demand curve could be made given only the size of customer and the level of price sensitivity which define a customer type may require some explanation.

Customer size, as you recall, is the quantity of linen soiled by a customer type at the current piece-rate or laundering fee. Thus, given a customer size, we have one point on the demand curve of that customer type.

The index of price sensitivity is the estimated price elasticity of the customer type. That is, the price sensitivity of the customer type is the percentage change in quantity of linen received per month which would result from a 1% change in the laundering fee. For a straight line demand curve the price elasticity will change as the quantity of linen received changes. Nonetheless, at any given quantity of linen received the price elasticity is fixed. Thus, if the manager were to estimate the price elasticity at current



prices, the program could calculate the quantity of linen that a customer with a straight line demand curve would receive if prices were increased 1%. (That calculation would simply be to multiply the customer's current quantity received by his price elasticity at the current level of prices of quantity received.) The program would then have two points on the demand curve of the customer type. Of course, since two points on a straight line define that line, the program would be able to define the straight line demand curve of that customer type.

## CHAPTER VI

### SUMMARY

#### A. One Part Pricing Systems

There are two types of one part pricing systems: discriminating systems and non-discriminating systems. Under a discriminating one part pricing system, each customer is charged the maximum amount he is willing to pay for each unit of output he purchases. Thus, each customer will end up paying a different price for each unit of output he purchases. A discriminating one part pricing system will provide a given firm with the maximum profit it can earn by operating in a given market. Yet, because discriminating one part pricing systems are not legal, no linen supply will wish to use one.

Under a non-discriminating one-part pricing system, all customers are charged the same price for all units of output they purchase. The Piece Rate pricing system is a sort of non-discriminating one-part pricing system, since all customers are charged the same piece rate for all pieces of linen they choose to receive. Non-discriminating one part pricing systems, such as the Piece Rate system, are in general, not the most profitable type of system. As long as a linen supply has some degree of monopoly power, it will earn larger profits under a two part pricing system

than it would under the piece rate system. However, in a purely competitive market, the piece rate system will provide profits as large as those earned under any two part pricing system.

#### B. Two Part Pricing Systems

A two part pricing system charges customers a per-unit price for each unit of output they purchase and a price for the right-to-buy output at that per-unit price. We may distinguish two types of two part pricing systems, according to the form in which the right-to-buy tariff is charged. A uniform two part pricing system charges all customers the same right-to-buy tariff. Examples of uniform two part pricing systems that could be used by linen supplies are the Piece-Rate With a Minimum system and the Delivery Charge Plus Laundering Fee system. In all but the most unusual circumstances, the Delivery Charge Plus Laundering Fee system will provide larger profits than the Piece-Rate With a Minimum system. Also, for monopolistic markets which include a sizeable segment of large, very price sensitive customers, the Delivery Charge Plus Laundering Fee system is the most profitable, legal pricing system a linen supply can use.

The second type of two part pricing system charges larger customers larger right-to-buy tariffs. Pricing systems of this type are called variable two part pricing systems. Two examples of variable two part pricing systems

which are used by linen supplies are the Flat Rate System and the Inventory Charge Plus Laundering Fee system. In all cases, the Inventory Charge Plus Laundering Fee system, will provide profits at least as large as those earned under a Flat Rate system. Furthermore, for monopolistic markets in which large customers are not any more price sensitive than smaller customers, the Inventory Charge Plus Laundering Fee system is the most profitable, legal pricing system a linen supply can use.

### C. Applications

Deciding which type of pricing system is best is only part of the problem facing a linen supply. The linen supply must also determine at what level to set prices under the optimal type of pricing system.

A computer program can be designed to help plant managers estimate the optimal level at which to set prices. Such a program would ask plant managers to define several types of customers and then to describe the market for an item of linen in terms of those customer types. From this data, and the rules for the optimal use of the various pricing systems, the program will calculate whether the linen supply should serve all customer types, the optimal level at which to set prices under each pricing system, and the profit the linen supply could expect to earn under each system.