Preliminary Forming Limit Analysis for Advanced Composites

by

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**ABSTRACT**

Advanced composite materials are attractive in situations where high performance materials are required, such as in aerospace, military, medical applications and sporting goods industries. However, because they are difficult to shape into complex geometries, currently, most composite parts are manufactured by the hand lay-up process, which is time-consuming and labor-intensive. The main motivation of this research is to explore new techniques which would lead to automation of the manufacturing process. Diaphragm forming proves to be a cost effective way of producing complex shaped parts with aligned-fiber composite materials. The work presented in this thesis is focused on the limits of the process.

The main problem in diaphragm forming is laminate wrinkling which occurs under inappropriate processing conditions. Through a large amount of experiments, it is found that the factors which determine the forming limits are: composite material properties, part geometry, diaphragm properties, temperature, forming rate, etc..

Current analyses show that the conformance of laminates to complex geometries is achieved by viscous shearing mechanisms, among which the two most important such modes are in-plane shear, where adjacent fibers slide past one another, and inter-ply shear where plies slide relative to each other. In order to calculate the shear strains required to form a given part, a kinematics analysis has been carried out for a range of different shapes. A three point bending test is employed to understand the constitutive laws governing the deformation. The nonlinear elastic behavior of the diaphragm materials is modeled by using the established bi-axial stress theory of rubbers. The balance between the mechanisms that cause and prevent wrinkling leads to the preliminary forming limit diagrams, which allow us to predict the occurrence of undesirable modes such as laminate wrinkling.

Thesis Supervisor: Dr. Timothy. G. Gutowski
Professor, Department of Mechanical Engineering
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Chapter 1

INTRODUCTION

1.1 Background

The main motivation of using composite materials in engineering practice is to integrate the properties of their component parts to obtain composite properties which may be impossible or unaffordable with conventional materials. By advanced composites, we mean materials composed of continuous or long discontinuous, aligned or woven fibers impregnated by polymer resin matrix. Because of their high strength and stiffness-to-weight ratios, corrosion resistance, wear resistance and many other advantages, they are especially attractive in situations where high performance materials are required, such as in aerospace, military, medical applications and sporting goods industries.

However, these prospective applications not only raise serious challenges to material scientists, but also, and even more so, to both design and manufacturing engineers, because advanced composites are quite expensive and difficult to shape into complex geometries. Currently, the majority of such parts are manufactured by hand lay-up process, which is time-consuming and labor-intensive, even though relatively flexible. In this
process, "prepregs", composite materials in the tape form of preimpregnated fibers in polymer matrix, are hand-moulded, layer by layer, onto the tools. Human error is also common in the production lines. All these will make the cost of the parts even higher and impede the competitiveness of composites.

Because of a demand for automation and cost reduction, some new manufacturing techniques have been developed. However, each of them has drawbacks. For example, filament winding and pultrusion have been very successful in reducing cost, but are quite limited in the geometries they can produce, and in the fiber paths that can be generated; resin transfer molding (RTM) requires expensive tooling, and local defects such as porosity may ruin the whole part; drape forming is quite efficient for single curvature parts but is very difficult to apply to double curvature ones. Although it is often mentioned that one advantage of composites is that they provide the possibility of integrating the design and the manufacturing processes, which means they allow tailoring of shape and microstructure to meet design requirements, we can see that this kind of integration is severely limited by the manufacturing techniques.

By comparison, diaphragm forming has shown strong potential of efficiently producing complex shaped parts with advanced composites. In the first step of this process, prepreg laminae are laid-up in various directions according to design requirements on a platform and trimmed into a preform shape. Then, the preform is placed between two elastic diaphragms, which are the supporting materials. Effective contact of the diaphragms with the preform is realized by drawing vacuum between them. Finally, by applying vacuum from beneath the bottom diaphragm and/or positive pressure on the
top, the preform is deformed over the tool. Figure 1.1 shows a schematic of the process.

(i) prepreg  (ii) lay up  (iii) compaction  (iv) preform

(v) assembly  (vi) forming process by pulling vacuum under the tool

Figure 1.1: Schematic of diaphragm forming process.
1.2 Research Objectives

Although diaphragm forming has shown promise, some failure modes have been found both in experimental work and practical application. Among them, laminate wrinkling is the dominant problem. Figure 1.2 shows a comparison between good and failed parts formed under different conditions.

![Comparison between good parts and wrinkled parts.](image.png)

**Figure 1.2:** Comparison between good parts and wrinkled parts.
In this project, we focus on continuous aligned fiber thermoset composite materials.

Through a large amount of experiments, it is found that the factors which determine the forming limits are: composite material properties, part geometry, diaphragm properties, temperature, forming rate, etc. Since the mechanisms of the forming process are very complicated and there is no ready general model for it, usually the prediction of whether a specific part could be made or not before forming, and the control during the process are both empirical. Obviously this will extend the design and redesign processes.

The objective of our forming limit analysis is to find the balance between the mechanisms that cause laminate wrinkling and those that prevent wrinkling. The means by which we approach this goal is based on the development of forming limit diagrams, which are convenient for engineering application. From the point of view of cost reduction, the forming limit diagrams will help us design and control the process more efficiently. From the standpoint of innovation, they will suggest appropriate modifications of the process.

The basic idea of the approach is schematically shown in Figure 1.3.
1.3 Thesis Overview

The research mainly consists of three components:

1. Kinematic analysis, which is directly related to the design of part geometry.


3. Analysis of diaphragm support.
Chapter 2 introduces the deformation modes of laminates in the forming process and the differentiation between necessary and undesirable modes. Quantification of the deformation modes leads to the concept of shear. Based on differential geometry, ideal mappings for various parts are obtained, and ideal shears are calculated.

In Chapter 3, the drape test is used as an experimental means to study constitutive behavior of composite prepregs. Test results show that the prepregs demonstrate viscoelastic behavior. However, since both the drape test and the forming experiment are dominated by viscous properties, we can simplify them using a viscous model, though sometimes with a time invariant correction term. A power law relationship exists between the stress and strain rate. This means that the resin system behaves like a non-Newtonian fluid. The viscosity is influenced by temperature and strain rate.

Chapter 4 uses a bi-axial stress theory of rubber elasticity to explain the performance of diaphragms in the forming process. Uniaxial test results are fitted to the theory.

In Chapter 5, forming limit diagrams for parts with different shapes, materials and lay-ups are presented. Experimental results show that laminate wrinkling is essentially induced by the inter-ply shear stress, and the main support comes from diaphragm tension. The forming limit diagrams are developed based on the balance between these mechanisms.

Finally, Chapter 6 presents some concluding remarks with suggestions for future work.
Chapter 2

KINEMATIC ANALYSIS

2.1 Introduction

Forming experiments show that the quality of a specific part is strongly influenced by the geometry of the part surface. Even for parts with single curvature, under inappropriate conditions, simple bending may induce failure modes such as in-plane buckling and tow-splitting. However, these problems are minor when compared with laminate wrinkling which occurs on double curvature parts.

In order to find the intrinsic relationship between part geometry and laminate wrinkling, we will use the method of kinematic analysis to study the mapping of fibers onto the tool.

According to [Pipkin and Rogers], an ideal fiber mapping is defined as one that maintains even fiber spacing. Since the whole laminate is assumed incompressible, part thickness will be constant. The reason for this definition is that fiber alignment and even thickness are very important for predictable mechanical performance.
2.2 Coordinate Systems

We define two local coordinate systems as shown in Figure 2.1.

![Diagram of coordinate systems](image)

**Figure 2.1**: Illustration of the material coordinate system, 1, 2, 3; and part coordinate system, $\mu, \nu, \zeta$.

In the material coordinate system, axis 1 lies along the fiber direction; axis 2 is transverse to the fibers and within the plane of a lamina; and axis 3 is perpendicular to the ply. (Hereinafter the terms "ply" and "lamina" are used interchangeably.)

In the part coordinate system, axis $\zeta$ is normal to the laminate; axis $\mu$ and $\nu$ could be in any two orthogonal directions within the laminate.
2.3 Kinematic Constraints

The unique kinematic performance of aligned fiber composites is directly related to their structure. Hence, although constitutive properties of the composites will be discussed in more detail in Chapter 3, in order to begin the categorization of deformation modes, we will make a qualitative description of them here.

Anisotropy and inhomogeneity are the fundamental differences of fiber reinforced composite laminates from conventional engineering materials, which are basically isotropic and homogeneous. An isotropic continuum has material properties that are the same in all directions at a point in the body. A homogeneous material has uniform properties throughout the body. In other words, properties of such a material are not a function of position or orientation.

By contrast, aligned fiber composites are anisotropic and inhomogeneous. Since the fibers are long and thin with high elastic moduli, while the resin matrix is basically a viscous fluid, the fibers can be regarded as inextensible and incompressible in their axial direction.
28 CHAPTER 2

2.4 Deformation Modes

Under the above constraints, it has been determined that the allowed deformation modes are quite limited. [Tam] divided them into three categories:

1. Between-plane modes;

2. Out-of-plane modes;

3. In-plane modes.

However, Tam's work was mainly focused on the deformation of a single lamina with unidirectional fibers, although one kind of ply-slip resulted from through-thickness bending was presented. Using his categorization, we extend the analysis to more deformation modes. We consider the deformation of a lamina and laminated plies respectively.

2.4.1 Deformation of a Lamina

For a lamina, all three categories of deformation may occur, as shown in Figures 2.2, 2.3 and 2.4.

Longitudinal shear and transverse shear may occur on an ideal mapping as long as undesirable thickness variation is not induced. These two kinds of shear can be caused by through-thickness bending, and hence are considered relatively easy to induce. In-plane shear is desirable for conformance to double curvature surfaces and is more difficult to achieve.
All the remaining deformation modes for a lamina are undesirable. For instance, fiber splitting and bunching-up lead to uneven fiber spacing, and out-of-plane buckling may cause, or be part of, the laminate wrinkling.

Figure 2.2: In-plane deformation modes of a lamina.
2.4.2 Deformation of Laminated Plies

It is obvious that for laminated plies, other than the deformation of each ply as described above, the only additional modes are between-plane and out-of-plane modes. They are shown in Figures 2.5 and 2.6.
Inter-ply shear caused by simple bending

Inter-ply shear caused by double curvature

Figure 2.5: Between-plane deformation modes of laminated plies.

Figure 2.6: Out-of-plane deformation mode of laminated plies
(Laminate wrinkling).
Among the between-plane modes, inter-ply shear caused by simple bending is relatively easy to induce compared to gross inter-ply shear occurring on double curvature parts.

When the between-plane modes cannot be realized, the out-of-plane mode — laminate wrinkling can result. This is the major problem that exists in the diaphragm forming process.

It should be emphasized that the deformation of a laminate is a combination, although not a simple summation, of the deformations of each of its plies. The deformation modes discussed above are different aspects of the same displacement, but not separate steps.

2.4.3 Sequence of Kinematic Approach

In summary of the above subsections, in-plane shear within a lamina and inter-ply shear between plies are the most desirable deformation modes. From experiments, it is observed that in-plane shear may occur in a similar fashion to trellising with little inter-ply shear. Trellising is a deformation mode of fabrics (see [Chey]). This means that the fundamental reason for deviation from the ideal fiber mapping is that inter-ply shear is very difficult to induce.

However, since ideal inter-ply shear is directly determined by the ideal in-plane shear mapping of the corresponding plies, we must begin with a detailed study of ideal in-plane shear mappings. Then we evaluate inter-ply shear and, in the latter part of this thesis, interpret how it affects the forming limits of composite laminates.
2.5 Quantitative Definition of In-plane Shear

We will adopt a definition for large strain shear provided by Pipkin. As shown in Figure 2.7, assume A and B are points on two parallel fibers and start on the same normal line. According to the ideal mapping assumption, the distance between the two fibers, $h$, will be preserved. After deformation, point B has moved a distance $\delta$ relative to point A along the fiber direction. Then the in-plane shear between the fibers is defined as:

$$\Gamma_{12} = \frac{\delta}{h}. \quad (2.1)$$

![Figure 2.7: Quantitative definition of in-plane shear.](image-url)
2.6 Analysis of In-plane Shear with Differential Geometry

In this section, we will employ the theory of differential geometry (refer to [Struik]) to find the relationship between part shape and in-plane shear of ideal mapping. We assume that the part surface is sufficiently smooth.

2.6.1 Surface Geometry

As shown in Figure 2.8, let P be a point on part surface S.

Figure 2.8: Surface geometry.
The vector $\vec{N}$ is the surface normal at point $P$, which means that it is perpendicular to the tangent plane of the surface at the point. The normal plane is defined as any plane that contains the surface normal. The intersection of a normal plane with the surface determines a curve on the surface called the normal curve, which has a curvature of $\kappa$ at $P$. Among all the normal curves on the surface through $P$, maximum and minimum curvatures $\kappa_1$ and $\kappa_2$ can be obtained in orthogonal directions, and are called the principle curvatures. The product of the principle curvatures:

$$K = \kappa_1 \cdot \kappa_2$$

(2.2)

is defined as the Gaussian curvature, which is an important property of a point on the surface.

Next, for a surface region $R$ with area $A$, we can introduce the definition of total curvature $K_T$:

$$K_T = \iint_R K \, dA.$$  

(2.3)

### 2.6.2 Geodesic Curvature and In-plane Shear

If we consider a fiber on the part surface to be a curve $C$, its curvature at a point $P$ can be decomposed as:

$$\kappa = \kappa_n \vec{N} + \kappa_g \vec{u}.$$  

(2.4)

where $\kappa_n$ is the normal curvature on the direction of surface normal $\vec{N}$, and $\kappa_g$, the geodesic curvature, is on the tangent plane of the surface at point $P$, with the direction of the unit vector $\vec{u}$. 
[Tam and Gutowski] showed that the incremental in-plane shear of a fiber relative to its neighboring fibers is related to its local geodesic curvature by:

\[ \mathrm{d} \Gamma_{12} = \kappa_g \, \mathrm{d}s, \]

(2.5)

where \( \mathrm{d}s \) is the length increment along the fiber. In the form of integration along the whole fiber of length \( L \), we get:

\[ \Gamma_{12} = \int_0^L \kappa_g \, \mathrm{d}s. \]

(2.6)

This means that the in-plane shear along a fiber path is solely determined by its geodesic curvature.

### 2.6.3 Gauss-Bonnet Theorem

The Gauss-Bonnet theorem is an application of Green's theorem on surfaces. It relates the line integral of \( \kappa_g \) along a closed path with the area integral of the Gaussian curvature in the enclosed region.

Referring to Figure 2.9, this theorem can be expressed as follows:

If the Gaussian curvature \( K \) of a surface is continuous in a simply connected region \( R \) bounded by a closed curve \( C \) of \( n \) smooth arcs making at the vertices exterior angles \( \theta_1, \theta_2, \ldots, \theta_n \), then:

\[ \oint_C \kappa_g \, \mathrm{d}s + \iint_R K \, \mathrm{d}A = 2\pi - \sum_{i=1}^{n} \theta_i, \]

(2.7)

where \( \kappa_g \) represents the geodesic curvature of the arcs.
Figure 2.9: Gauss-Bonnet theorem.

Figure 2.9 also illustrates how to choose the contour C to calculate the in-plane shear of a fiber. For instance, if we let $C_1$, $C_3$ be fiber paths, and $C_2$, $C_4$ be orthogonal geodesic paths, since

$$
\sum_{i=1}^{4} \theta_i = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} = 2\pi,
$$

from Equation (2.7), we get

$$
\oint_{C_1+\bar{C}_1} \kappa_g \, ds + \iint_{R} K \, dA = 0.
$$

Furthermore, if $C_1$ is taken as the initial fiber which is also a geodesic path, the in-plane shear of the fiber $C_3$ will be

$$
\Gamma_{12} = \int_{C_3} \kappa_g \, ds = -\iint_{R} K \, dA = -K_T(R).
$$

Thus, the in-plane shear of a fiber is related to the surface geometry.
The simplest application of the Gauss-Bonnet theorem is on planar curves. As shown in Figure 2.10, for any a smooth planar curve $C$, assume $\theta$ is the enclosed angle of its normals at the ends. Then, since the total curvature is always zero on any planar region, the in-plane shear of curve $C$ is:

$$\Gamma_{12} = \int_C \kappa_g \, ds = 2\pi - (\frac{\pi}{2} + \frac{\pi}{2} + \pi - \theta) = \theta.$$  \hspace{1cm} (2.11)

This result will be very useful later.

### 2.6.4 In-plane Shear of Hemisphere

For a hemisphere, it is easy to see that the ideal fiber paths are parallel semicircles. As shown in Figure 2.11, $C_1$, the semicircle passing through the top of the hemisphere, is half of a great circle, i.e. a geodesic path. Semicircle $C_3$ represents the fiber which we are interested in. $C_2$ and $C_4$, the paths...
connecting the ends of C1 and C3, are arcs of a great circle, and perpendicular to the fibers, hence are orthogonal geodesics.

![Diagram of geodesics on a sphere]

**Figure 2.11:** In-plane shear calculation for a hemisphere.

Now, since the geodesic curvature of arcs C1, C2 and C4 are all zero, the line integral in the Gauss-Bonnet theorem reduces to that of the fiber path C3. Considering the fact that the exterior angles sum to $2\pi$ and the Gaussian curvature for a sphere is constant:

$$K = \frac{1}{R^2},$$  \hspace{1cm} (2.12)

we find that the absolute value of the In-plane shear of fiber C3 is:

$$\Gamma_{12} = K \cdot A = \frac{1}{R^2}(\pi R b) = \pi \sin \theta,$$  \hspace{1cm} (2.13)

where A is the enclosed region of the path C1+C2+C3+C4; b is the distance between the projections of C1 and C3 onto the base of the hemisphere; and $\theta = \sin^{-1}\left(\frac{b}{R}\right)$. See Figure 2.11.
2.6.5 In-plane Shear of Curved C-Channel

A c-channel surface is composed of a flat top joined with two developable flanges, each along a contour. In order to use the Gauss-Bonnet theorem, the bending region along the contours should be smooth, in other words, should have a continuous Gaussian curvature. In an actual part, this requirement is satisfied by rounding of the bending edge. For convenience, we will regard the contour region to have continuous curvature with infinitesimal round radius. Figure 2.12 shows the kind of c-channel we are concerned with in this work, the two contours are arcs of concentric circles with radii $R_1$ and $R_2$.

Figure 2.12: C-channel with edge contours of concentric arcs.
When a fiber bends over an edge, to maintain even spacing with the neighboring fibers, it must have the same tangent angle to the edge on both sides of the contour. This idea is illustrated in Figure 2.13.

\[\text{Figure 2.13: Fiber bending over the edge.}\]
0 Degree Mapping

At first, let us study the case where the initial fiber is on the top and in the direction of the chord of the contour arcs. (Hereinafter this will be referred to as "0 degree mapping"). No in-plane shear is required on the top. In Figure 2.14, $O_1$ is the center of the contour created by intersection of the top and the inner flange. We assume the shear to be zero at the center of each fiber. When a fiber bends over onto the inner flange at point $P$, if the enclosed angle of arc $O_1P$ is $\alpha$, then the tangent angle at $P$ will also be $\alpha$. Since unrolling of the flange will not change the geodesic curvature of any path on it, and considering the result for planar curves in § 2.6.3, we can see that the shear at point $P$ is simply $\alpha$.

A more general expression for in-plane shear at any point on the flange can be obtained as shown in Figure 2.15.

Fiber $f_0$ is on the tangent line to the edge at point $O_1$; fiber $f_P$ passes through point $P$. The distance between these two fibers is:

$$\Delta = R_1 \cos \alpha,$$

(2.14)

where $R_1$ is the radius of the inner contour. For any fiber $f_A$ on the inner flange, its distance to the fiber $f_0$ is:

$$s_n = \overline{OA_0}.$$

(2.15)

Then, the normal distance between $f_P$ and $f_A$ is given by:

$$\overline{PA} = s_n - \Delta = s_n - R_1(1 - \cos \alpha).$$

(2.16)
Figure 2.14: C-channel 0 degree mapping:
Fiber path passing along both the top and the inner flange.
(a) Top view

(b) Unrolled inner flange

Figure 2.15: C-channel 0 degree mapping:
In-plane shear along a fiber on the inner flange.
If we set up an x-y coordinate system on the unrolled flange, the coordinates of the point A are:

\[ x_A = x_p + PA \sin \alpha = R_1 \alpha + [s_n - R_1(1 - \cos \alpha)] \sin \alpha, \]  
\[ y_A = PA \cos \alpha = [s_n - R_1(1 - \cos \alpha)] \cos \alpha. \]  

(2.17)  
(2.18)

The length of the fiber segment A_0A is given by,

\[ l_f = \int_0^\alpha \sqrt{\left( \frac{dx_A}{d\alpha} \right)^2 + \left( \frac{dy_A}{d\alpha} \right)^2} \, d\alpha \]

\[ = \int_0^\alpha [(s_n - R_1) + 2R_1 \cos \alpha] \, d\alpha \]

\[ = (s_n - R_1) \alpha + 2R_1 \sin \alpha. \]  

(2.19)

Notice that on the unrolled flange, planar arc A_0A encloses an angle of \( \alpha \), then \( \Gamma_{12}(P) = \alpha \). Therefore the in-plane shear at any location along the fiber on the inner flange can be related to the fiber length by the expression:

\[ l_f = 2R_1 \sin \Gamma_{12} + (s_n - R_1) \Gamma_{12}. \]  

(2.20)

A similar analysis yields the following expression for the outer flange:

\[ l_f = 2R_2 \sin \Gamma_{12} - (s_n + R_2) \Gamma_{12}. \]  

(2.21)

Based on either (2.20) or (2.21), when \( R_1 \) (or \( R_2 \)) and \( s_n \) are known, for any given fiber length \( l_f \) we can calculate the in-plane shear \( \Gamma_{12} \) using Newton’s iterative method. If we know the position of the point, i.e. \( x_A \) and \( y_A \), instead of \( s_n \), Equations (2.17) and (2.18) can be used to evaluate \( s_n \) and \( \alpha = \Gamma_{12} \). A useful result however, is that the maximum shear of 0 degree mapping onto a c-channel is simply \( \alpha \), half the angle of the enclosing arc.
90 Degree Mapping

If the initial fiber is at the center of the c-channel and perpendicular to the chord of the contour arcs, as shown in Figure 2.16, we refer to the mapping as being in the "90 degree" direction. Mathematically, we can prove that it is impossible to strictly meet the even spacing requirement without "singular" points on the fibers, where the paths are not smooth. However, when

\[ R_2 - R_1 \ll R_1, \]

and \[ \alpha \ll 1, \] (2.22)

this "unsmoothness" is trivial and in practice could be moderated by other deformation modes such as transverse shear.

The analytical description of the ideal 90 degree mapping is rather complicated. It will be demonstrated without recourse to unnecessary mathematical detail. Because of the symmetry about the initial fiber, we will study half of the part surface on one side of the center line.

On the inner flange, the fibers are parallel with each other and perpendicular to the edge. See Figure 2.17. Apparently, there is no in-plane shear.

On the top, as shown in Figure 2.18, \( O_1O_2 \) is the segment of the initial fiber; point \( O \) is the center of contour arcs; point \( P \) is on the edge of inner flange, and \( OP \) is at an angle of \( \alpha_1 \) to the \( x \) axis.
Figure 2.16: C-channel 90 degree mapping: Schematic of initial fiber.

Figure 2.17: C-channel 90 degree mapping: Inner flange.
[Kim] proved that if the enclosed angle of the half c-channel is less than:

$$\alpha_{\text{crit}} = \cos^{-1}\left(\frac{R_1}{R_2}\right),$$  \hspace{1cm} (2.23)

which is true in our experimental research, there are two regions on the top. In region I, fibers are parallel to the initial fiber, while in region II the curved segment AB can be expressed by the following parametric equations:

$$x = R_1 \cos \theta_1 - R_1 (\alpha_1 - \theta_1) \sin \theta_1$$ \hspace{1cm} (2.24)

$$y = R_1 \sin \theta_1 + R_1 (\alpha_1 - \theta_1) \cos \theta_1$$

where \(0 \leq \theta_1 \leq \alpha_1\).
The length of segment PB can be obtained by:

\[ \int_{0}^{\alpha} \sqrt{\left(\frac{dx}{d\theta_1}\right)^2 + \left(\frac{dy}{d\theta_1}\right)^2} \, d\theta_1 = \frac{R}{2} \alpha^2. \]  

(2.25)

This result will be used when calculating inter-ply shear.

Assume a fiber bends onto the outer flange at point C, while OC is at an angle of \( \alpha_2 \) to the x axis. (See Figure 2.19) The fiber path on the outer flange is parallel to the initial fiber except along a curved segment CD, which can be described by parametric equations:

\[
egin{align*}
u &= R_2 (\sin \alpha_2 - \sin \theta_2) \cos \theta_2, \\
v &= R_2 (\sin \alpha_2 - \sin \theta_2) \sin \theta_2,
\end{align*}
\]

(2.26)

where \( \alpha_2^* \leq \theta_2 \leq \alpha_2 \),

in which \( \alpha_2^* \) can be determined by:

\[ u(\theta_2 = \alpha_2^*) = R_2 \sin \alpha. \]

(2.27)

Figure 2.20 is a schematic of the fiber on the outer flange.

Again, considering the result for planar curves in §2.6.3, we can see that the maximum shear of 90 degree mapping is also equal to the enclosing angle of the half c-channel.
Figure 2.19: C-Channel 90 degree mapping: Top view of the fiber bends over onto the outer flange at point C.

Figure 2.20: C-channel 90 degree mapping: Outer flange.
2.7 Inter-ply Shear

2.7.1 Quantification

Consider two adjacent plies with different fiber directions. As shown in Figure 2.21, points A and B, one on each ply, are coincident in x and y before forming. During forming, there is a relative displacement \( \delta_{\text{int}} \) between the two points. (See Figure 2.22.) We can assume the direction of this displacement to be \( v \). If the normal distance between the plies is \( h_{\text{int}} \), we can define the inter-ply shear to be:

\[
\Gamma_{3v} = \frac{\delta_{\text{int}}}{h_{\text{int}}}
\]

(2.28)

Figure 2.21: Preform for hemisphere with radius R.
Before forming:
points A and B coincident on adjacent cross plies.
One important characteristic of inter-ply shear is that it is a function of the fiber directions of the plies involved. Before a general evaluation method of the function is obtained, we will study a few cases to illustrate the basic idea of the calculation.
2.7.2 Inter-ply Shear of Hemisphere

Figure 2.21 shows a preform for a hemisphere with plies of orthogonal fiber directions x and y. Assume the radius of the hemisphere is R. For the plies with fibers in y direction, the preform shape can be described by:

\[ y = \pm \frac{\pi R}{2} \cos \left( \frac{x}{R} \right) \quad \text{where} \quad -\frac{\pi R}{2} \leq x \leq \frac{\pi R}{2}, \quad (2.29) \]

while for x direction plies, the corresponding expression is:

\[ x = \pm \frac{\pi R}{2} \cos \left( \frac{y}{R} \right) \quad \text{where} \quad -\frac{\pi R}{2} \leq y \leq \frac{\pi R}{2}. \quad (2.30) \]

We choose an arbitrary point \((x_0, y_0)\) on the flat preform. As shown in Figure 2.23, after forming, the point on a y direction ply is located at \(A(X_1, Y_1, Z_1)\) on the hemisphere surface; and the corresponding point on an x direction ply is transformed to \(B(X_2, Y_2, Z_2)\). For clarity, point \(B(X_2, Y_2, Z_2)\) is not shown in the figure.

Let

\[ \theta_1 = \frac{x_0}{R}, \]
\[ \theta_2 = \frac{y_0}{R}, \quad (2.31) \]

then

\[ \phi_1 = \frac{y_0}{R \cos \theta_1} = \frac{\theta_2}{\cos \theta_1}, \]
\[ \phi_2 = \frac{x_0}{R \cos \theta_2} = \frac{\theta_1}{\cos \theta_2}. \quad (2.32) \]
Figure 2.23: Illustration of how to locate a point in certain ply onto hemisphere surface.

The coordinates of points A and B can be expressed as:

\[
X_1 = R \sin \theta_1
\]
\[
Y_1 = R \cos \theta_1 \sin \phi_1 = R \cos \theta_1 \sin \left( \frac{\theta_2}{\cos \theta_1} \right)
\]
\[
Z_1 = R \cos \theta_1 \cos \phi_1 = R \cos \theta_1 \cos \left( \frac{\theta_2}{\cos \theta_1} \right)
\]  
(2.33)

and

\[
X_2 = R \cos \theta_2 \sin \phi_2 = R \cos \theta_2 \sin \left( \frac{\theta_1}{\cos \theta_2} \right)
\]
\[
Y_2 = R \sin \theta_2
\]
\[
Z_2 = R \cos \theta_2 \cos \phi_2 = R \cos \theta_2 \cos \left( \frac{\theta_1}{\cos \theta_2} \right)
\]  
(2.34)
The relative displacement between the two points is:

\[ \delta = \alpha R, \]  

(2.35)

where \( \alpha \) is the azimuthal angle between points A and B:

\[
\alpha = \cos^{-1} \left[ \frac{1}{R^2} (X_1X_2 + Y_1Y_2 + Z_1Z_2) \right] 
\]

\[
= \cos^{-1} \left[ \sin \theta_1 \cos \theta_2 \sin \left( \frac{\theta_1}{\cos \theta_2} \right) + \sin \theta_2 \cos \theta_1 \sin \left( \frac{\theta_2}{\cos \theta_1} \right) + \right] 
\]

\[
\cos \theta_1 \cos \theta_2 \cos \left( \frac{\theta_2}{\cos \theta_1} \right) \cos \left( \frac{\theta_1}{\cos \theta_2} \right) \right] 
\]

(2.36)

The distribution of \( \alpha \) in a quadrant is shown in Figure 2.24.

The maximum value of \( \delta \) happens at \( x=y=0.934R \), where \( \alpha=0.297 \). Hence

\[ (\delta_{int})_{\text{max}} = 0.297R \]  

(2.37)

and the ideal maximum inter-ply shear is

\[ (\Gamma_{3v})_{\text{max}} = \frac{(\delta_{int})_{\text{max}}}{h_{int}} = \frac{0.297R}{h_{int}} \]  

(2.38)

where \( h_{int} \) is the thickness of the resin rich layer between the plies. Assume that \( h_{int} = 10^{-3} \) inch, for a hemisphere with \( R=3.5 \) inch:

\[ (\delta_{int})_{\text{max}} = 1.04 \text{ inch}, \]  

(2.39)

and

\[ (\Gamma_{3v})_{\text{max}} = \frac{(\delta_{int})_{\text{max}}}{h_{int}} = 1.04 \times 10^3. \]  

(2.40)

The relative magnitude of inter-ply shear and in-plane shear for this size of hemisphere can be estimated by:

\[ \frac{(\Gamma_{3v})_{\text{max}}}{(\Gamma_{12})_{\text{max}}} = \frac{1.04 \times 10^3}{\pi / 2} = 6.6 \times 10^2, \]  

(2.41)

However, the most significant point worthy notice is that the inter-ply shear scales with the part size, whereas the in-plane shear does not.
Figure 2.24: Relative displacement between orthogonal plies on a quarter hemisphere with radius $R$. 
2.7.3 Inter-ply Shear of Curved C-Channel

We will determine the inter-ply shear between 0 and 90 degree plies. At first, for the inner flange, as shown in Figure 2.25, we use the same convention as in Figure 2.15.

![Diagram](image)

Figure 2.25: Illustration of how to calculate inter-ply displacement at point A on the inner flange of c-channel.

At point A, in-plane shear of the 0 degree fiber is $\alpha$; fiber length is $l_f$. The normal distance between $A_0A$ and $O_1$ is $s_n$. We assume:

$$\alpha << 1 \quad \text{and} \quad s_n << R_1, \quad (2.42)$$

which is true in our experiments. The relative displacement in the direction of 0 degree fiber can be evaluated through:

$$\delta_0 = l_f - (R_1\alpha + PA \sin \alpha)$$

$$= \frac{1}{6}(R_1 + s_n)\alpha^3 \quad (2.43)$$

$$= \frac{1}{6}R_1\alpha^3,$$
while in the 90 degree direction, the corresponding value is:

\[
\delta_{90} = \frac{R_1}{2} \left( \alpha + \frac{PA \sin \alpha}{R_1} \right)^2 + \frac{PA \cos \alpha - s_n}{R_1} \\
= \frac{1}{2} s_n \frac{R_1 + s_n \alpha^2}{R_1} \\
= \frac{1}{2} s_n \alpha^2.
\]  

(2.44)

Hence the relative displacement between 0 and 90 plies at point A is:

\[
(\delta_{\text{int}})_A = \sqrt{\delta_0^2 + \delta_{90}^2} \\
= \sqrt{\left( \frac{1}{6} R_1 \alpha^3 \right)^2 + \left( \frac{1}{2} s_n \alpha^2 \right)^2} \\
= \frac{1}{6} \alpha^2 \sqrt{R_1^2 \alpha^2 + 9s_n^2}.
\]  

(2.45)

For outer flange, for a point where in-plane shear of 0 degree fiber is \( \alpha \), a similar analysis leads to the following result:

\[
\delta_0 = \frac{1}{6} (R_2 - s_n) \alpha^3 \\
= \frac{1}{6} R_2 \alpha^3, \\
\delta_{90} = \frac{1}{2} s_n \frac{R_2 - s_n \alpha^2}{R_2} \\
= \frac{1}{2} s_n \alpha^2, \\
\delta_{\text{int}} = \frac{1}{6} \alpha^2 \sqrt{R_2^2 \alpha^2 + 9s_n^2}.
\]  

(2.46) (2.47) (2.48)

When \( R_2 - R_1 << R_1 \), (2.49)

the following expression can be applied to both flanges:

\[
\delta_{\text{int}} = \frac{1}{6} \alpha^2 \sqrt{R^2 \alpha^2 + 9s_n^2}.
\]  

(2.50)
Again, the inter-ply shear for c-channel scales with the part size, while in-plane shear does not.

In our experiments, one size of c-channel is:

\[
R = 96 \text{ inch,} \\
-0.125 \leq \alpha \leq 0.125, \\
0 \leq s_n \leq 4 \text{ inch.}
\] 

(2.51)

The magnitude of the interply displacement on either flange is illustrated in Figure 2.26.

![Inter-PLY Displacement](image)

**Figure 2.26:** Inter-PLY displacement on a flange of c-channel.
CHAPTER 2

The maximum value \( (\delta_{\text{int}})_{\text{max}} = 0.044 \text{ inch}, \) \hfill (2.52)

and the ideal maximum inter-ply shear is:

\[
(\Gamma_{3v})_{\text{max}} = \frac{(\delta_{\text{int}})_{\text{max}}}{h_{\text{int}}} = \frac{0.044 \text{ in}}{10^{-3} \text{ in}} = 44
\] \hfill (2.53)

The relative magnitude of inter-ply shear and in-plane shear for this size of c-channel can be estimated by:

\[
\frac{(\Gamma_{3v})_{\text{max}}}{(\Gamma_{12})_{\text{max}}} = \frac{44}{0.125} = 3.5 \times 10^2.
\] \hfill (2.54)
Chapter 3

DRAPE TEST

3.1 Introduction

In the previous chapter, it has been shown that, because of the unique structure of aligned fiber composites, their conformance to complex geometries is predominantly achieved by shearing mechanisms, either within a lamina or between plies. If the shearing mechanisms are prohibited, failure modes such as in-plane buckling and laminate wrinkling may occur. From forming experiments, we can see that the ability of fibers and plies to shear relative to each other is not only determined by part geometry and laminate lay-up, which are related to the magnitude of the required shears, but also by many other factors. Among them the most important ones are temperature and deformation rate, both of which are a reflection of viscous properties. In order to understand the deformation behavior of a composite material, we must study its rheological characteristics.

Again, due to the structure of aligned fiber composites, the experimental means that we can use to study the material properties are quite limited by comparison to those for isotropic materials. For example, for an isotropic continuum, the tensile creep test is widely used to measure viscoelastic
tensile modulus; while for aligned fiber composites, in the fiber direction, the
tensile response will be dominated by the elasticity of the fibers, and if the
loading has a component in the transverse direction, the fibers tend to spread
or may even be torn apart.

[Neoh] used a three point bending test to measure the drape properties of
prepregs, where "drape" is defined as the ability of a material to conform to
complex curvatures. The test results show the viscoelastic behavior of the
thermoset prepregs which we are interested in, and the viscous phenomena
dominate the dynamic response, especially when the deformation rate is not
too low. Because of its simplicity and repeatability, we use this drape test to
study the rheological properties of a broader range of composite materials.

3.2 Experimental Set-up

Figure 3.1 shows a schematic of the experimental set-up. The three point
bending apparatus is attached to an Instron 1125 universal testing machine,
which is used primarily for rate controlled displacement.

For more details of the set-up, please refer to [Neoh]'s thesis.
Figure 3.1: Drape test set-up: (a) 3 point bending apparatus, (b) data acquisition.
Unless specified otherwise, all the experiments presented are done on a single ply of prepreg and the drape force is measured as a function of time and deflection. The dimensions of the prepreg samples are 2 in. wide and 2.25 in. long along the fiber direction. The distance between the supports is 2 in. and the punch is centered between the supports. To avoid friction, a layer of non-porous teflon is placed on the surface of the supports. The maximum deflection is 0.3 in. and the range of constant crosshead velocities is from 0.1 in/min to 10 in/min.

3.3 Viscoelastic Response and Non-Newtonian Viscosity

Figure 3.2 shows typical drape test results for different crosshead speeds.

![Figure 3.2: Typical external load evolution in drape test.](image)
It is easily seen that the response is not only determined by the magnitude of deflection, but also by the speed of deformation. This indicates that the deformation has a significant viscous component. If we keep the prepreg at a certain deflection and let the internal stress relax, there is some elastic residual (Figure 3.3).

![Graph showing stress relaxation](image)

**Figure 3.3:** Stress relaxation within the prepreg while keep certain deflection.

Therefore, the overall response is viscoelastic. However, we will use the drape test results to determine the viscous properties of the resin matrix, based on the following:

1. From the experimental data, it can be seen that there is a power law relationship between the maximum load and the crosshead speed (see Figure 3.4), which suggests that the dynamic response of the material to three point bending is dominated by viscous deformation, and the viscosity is non-Newtonian;
2. The elastic component is basically related to the fiber bending, while what we are concerned with is the shearing deformation between fibers;

3. In the forming process, the conformance of the laminate to complex shapes is achieved by viscous shearing mechanisms between fibers or plies.

\[ r^{-n} = m \cdot \gamma^{0.35700} \quad R^2 = 0.999 \]

**Figure 3.4:** Power law relationship between the maximum load and the crosshead speed.

Using a simple model, the relationship between shear stress and shear strain in a non-Newtonian fluid can be expressed by:

\[ \tau = m \dot{\gamma}^n, \quad (3.1) \]
where \( m \) is called the non-Newtonian viscosity function, and \( n \) is a dimensionless exponent.

However, the experimental observation only gives us a macro view and suggests a model for the materials behavior. In order to find the relationship between the external response and the internal constitutive behavior, a deformation model and stress analysis are introduced in §3.4 and §3.5 respectively.
3.4 Deformation Modeling

Due to the viscous shearing mechanism between the fibers, when a prepreg sample is under bending moment, all the individual fibers deform into the same shape to attain dynamic steady state. This is quite different from the bending of a isotropic beam. The comparison is illustrated in Figure 3.5.

![Isotropic beam and Fiber reinforced beam](image)

**Figure 3.5:** Comparison between isotropic beam and fiber reinforced beam under bending moment.

Experimental results show that the shape of the prepreg sample (i.e. the shape of individual fibers) during three point bending can be approximated by that of a pure elastic beam (see [Neoh]). As shown in Figure 3.6, since the deflection is symmetric about the center line along which the loading force $F$ acts, we may consider only half of the beam on one side, say segment AB.
According to elasticity theory, the deflection along AB is given by:

\[ y = \Delta \left( \frac{x^3}{2a^3} - \frac{3x}{2a} \right), \quad 0 \leq x \leq a, \tag{3.2} \]

where \( \Delta \) is the deflection at the center point B.

\[ \frac{dy}{dx} = \frac{3\Delta}{2a} \left( \frac{x^2}{a^2} - 1 \right), \quad 0 \leq x \leq a, \tag{3.3} \]

and the net shear at the point is:

\[ |\Gamma| = \theta = \tan^{-1} \left( \frac{dy}{dx} \right) = \tan^{-1} \left[ \frac{3\Delta}{2a} \left( 1 - \frac{x^2}{a^2} \right) \right], \quad 0 \leq x \leq a. \tag{3.4} \]
The shear distribution along the prepreg for $\Delta=0.3$ inch and $a=1$ inch is shown in Figure 3.7. Notice that the shear is antisymmetric about the center point B.

![Figure 3.7: Shear distribution along a prepreg under three point bending with length $2a=2$ inch and deflection $\Delta=0.3$ inch.](image)

The derivative of the shear with respect to time leads to the shear rate:

$$|\dot{\Gamma}| = \left| \frac{d\Gamma}{dt} \right| = \frac{3\dot{\Delta}}{2a} \left( \frac{1-x^2}{a^2} \right) \left( 1 + \frac{3\dot{\Delta}}{2a} \left( \frac{1-x^2}{a^2} \right) \right)^{-2},$$

where $\dot{\Delta}$ is the speed of the crosshead.

[Neoh] has shown that about 40% of the prepreg experiences a shear rate lower than the average value over the whole length, and the remaining 60% undergoes a slightly higher shear rate. This is shown in Figure 3.8.
Figure 3.8: Shear rate distribution along a prepreg under three point bending with length $2a=2$ inch, deflection $\Delta=0.171$ inch and deflection rate $\dot{\Delta}=1$ in/min.

Notice that even though Figure 3.8 is based on a specific deflection $\Delta$ and deflection rate $\dot{\Delta}$, the pattern of the shear rate distribution is the same for other values of $\Delta$ and $\dot{\Delta}$. This is very important for the analysis of force and moment balances in §3.5.
3.5 Force and Moment Balances

[Tam] developed a linear viscoelastic model to simulate the response of composite material to three point bending. The material is assumed to consist of elastic layers with viscous slip layers between them. Elastic layers may be discrete fibers, tows or both, depending on the problem involved; and viscous layers are assumed to behave as a Newtonian fluid (Figure 3.9).

![Diagram showing material model with elastic layers and viscous slip layers](image)

**Figure 3.9:** Material model: elastic layers with viscous slip layers between them.
Numerical simulation shows that the stress profile through the thickness is quite different from that in an elastic material. The majority of axial stress is distributed in the outer elastic layers. In steady state, the axial stress is singularly distributed in the outmost layers. This is schematically shown in Figure 3.10. [Neoh] extended this model to the case where the resin is regarded as a non-Newtonian fluid. The corresponding simulation gives a more accurate stress evolution during the three point bending test.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure3.10.png}
\caption{(a) Linear through thickness stress distribution for an elastic beam. (b) Singular stress distribution for a pure shear beam.}
\end{figure}

Based on the assumption of a singular stress distribution for a pure shear beam, we can investigate the relationship between the drape force $F$ and the shear rate $\dot{\gamma}$. Suppose the shear stress between the resin and the fibers is $\tau$. In
addition, we define the sample width as constant \( b \) and the thickness as constant \( \varepsilon \). Making use of the shear rate distribution described in §3.4, [Neoh] inferred the following approximate expression:

\[
F = 2b\varepsilon m\dot{\Gamma}_{\text{max}}. \tag{3.6}
\]

Since

\[
\dot{\Gamma} = \frac{\frac{3\dot{\Delta}}{2a}\left(1 - \frac{x^2}{a^2}\right)}{1 + \left[\frac{3\Delta}{2a}\left(1 - \frac{x^2}{a^2}\right)^2\right]^2}, \tag{3.7}
\]

the maximum shear happens at \( x = \Delta = 0 \):

\[
\dot{\Gamma}_{\text{max}} = \frac{3\dot{\Delta}}{2a}. \tag{3.8}
\]

Notice that (3.8) holds only when the deflection \( \Delta \) is small relative to the sample length. In our experiments, \( a = 1 \) inch and \( 0 \leq \Delta \leq 0.3 \) inch.

For given \( \dot{\Delta} \) and \( a \), i.e. certain \( \dot{\Gamma}_{\text{max}} \), we can measure the maximum load \( F_{\text{max}} \) when the viscous flow is fully developed, which is indicated by a leveling off of the load-deflection curve. The experimental data give us a power law curve fit on the plot of \( F_{\text{max}} \) versus \( \dot{\Gamma}_{\text{max}} \):

\[
F_{\text{max}} = p\dot{\Gamma}^q_{\text{max}}. \tag{3.9}
\]

Comparing with Equation (3.6), we get:

\[
n = q \quad \text{[dimensionless]}, \tag{3.10}
\]

\[
m = \frac{p}{2b\varepsilon} \quad \text{[psi \cdot sec}^n]. \tag{3.11}
\]
3.6 Experimental Results

The two kinds of materials we studied are the ones being used in diaphragm forming experiments: Hercules AS4/3501-6 and Toray T800H/3900-2. For each material, a series of experiments has been carried out with different temperatures and shear rates. The results give us strong evidence that the deformation is dominated by the viscous properties of the resin matrix, and this viscosity is non-Newtonian.

For a non-Newtonian fluid, the shear stress $\tau$ can be related to shear rate $\dot{\gamma}$ in the following way:

$$\tau = m\dot{\gamma}^n = \eta \dot{\gamma}.$$  \hfill (3.12)

where

$$\eta = m\dot{\gamma}^{n-1} \text{ [psi} \cdot \text{sec}].$$ \hfill (3.13)

is called the apparent viscosity. When $n<1$, the material exhibits shear thinning. Figure 3.11 clearly shows this kind of relationship.

![Graph showing shear thinning phenomenon in drape tests.](image)

Figure 3.11: Shear thinning phenomenon in drape tests.
(At room temperature 72±1 °F.)
Furthermore, the variation of apparent viscosity with temperature can be approximated by the Arrhenius equation:

\[ \eta = \eta_0 \exp\left(\frac{\Delta E}{RT}\right) \]

where \( \Delta E \) is the activation energy, \( R \) is the gas constant and \( T \) is the absolute temperature. For a Newtonian fluid, \( \eta_0 \) and \( \Delta E \) are material constants, while for a non-Newtonian material, they vary with shear rate. This can be seen from Figures 3.12 and 3.13.

For more detailed drape test results, please refer to Appendix A.

---

**Figure 3.12:** Variation of apparent viscosity with temperature for Hercules AS4/3501-6.
Figure 3.13: Variation of apparent viscosity with temperature for Toray T800H/3900-2.
Chapter 4

DIAPHRAGM RUBBER ELASTICITY

4.1 Introduction

There are two mechanisms that tend to oppose laminate wrinkling: the inherent elastic resistance of the material itself and the restraining force supplied by diaphragm tension. To compare their relative effects, we must quantitatively analyze the support they supply to the laminate. The elastic resistance of the laminate was investigated by means of a buckling test. The work shown in this chapter is focused on the diaphragm rubber elasticity. The comparison of the two will be discussed in Chapter 5.

Unidirectional tensile tests have been carried out and the nonlinear elastic properties observed, as shown in Figure 4.1.
The tension state is much more complex in the forming process, since when a diaphragm is clamped around the periphery and forced to conform to a part surface, a multidirectional strain distribution is induced. However, at any point on the diaphragm, we can locally approximate the stress state by bi-axial tension. On the other hand, since laminate wrinkling always happens at specific locations and in corresponding specific directions, we can define the axis along the direction perpendicular to the wrinkle (also called the critical direction) as axis 1, and the one parallel to the wrinkle direction as axis 2. Axis 3 is through the thickness. This is schematically shown in Figure 4.2. In the subsequent sections, we will introduce a theory of bi-axial rubber elasticity and apply it to our evaluation of the diaphragm support.
4.2 Deformation Modeling

Consider a finite element in the diaphragm material (See Figure 4.3). Assume that axes 1,2,3 are in the directions of principal strain. The principal extension ratios are defined as:

\[
\begin{align*}
\lambda_1 &= \frac{L_1}{L_{10}}, \\
\lambda_2 &= \frac{L_2}{L_{20}}, \\
\lambda_3 &= \frac{L_3}{L_{30}}, \\
\end{align*}
\] (4.1)

while \( L_{i0} \) is the length in the \( i \)th direction in the unstrained state, and \( L_i \) is the corresponding length after deformation \((i=1,2,3)\).
A rubber's bulk modulus is high compared to its other moduli. It may therefore be considered incompressible. By conservation of volume, then:

$$\lambda_1 \lambda_2 \lambda_3 = 1. \quad (4.2)$$

[Ward] demonstrated that, for an isotropic incompressible solid undergoing a pure homogeneous deformation, the strain energy $U$ is given by:

$$U = U(I_1, I_2), \quad (4.3)$$

where

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad (4.4)$$

and

$$I_2 = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} \quad (4.5)$$
are invariants. Accordingly, when a diaphragm is under bi-axial tension, as shown in Figure 4.3, the stresses can be expressed as:

\[
\sigma_{11} = 2 \left( \lambda_1^2 - \frac{1}{\lambda_1^4 \lambda_2^2} \right) \left( \frac{\partial U}{\partial I_1} + \lambda_2^2 \frac{\partial U}{\partial I_2} \right),
\]

\[
\sigma_{22} = 2 \left( \lambda_2^2 - \frac{1}{\lambda_1^4 \lambda_2^2} \right) \left( \frac{\partial U}{\partial I_1} + \lambda_1^2 \frac{\partial U}{\partial I_2} \right).
\]

(4.6)  
(4.7)

We define the nominal stress as:

\[
\sigma^* = \frac{\sigma A}{A_0},
\]

(4.8)

where \(A\) is the area of the cross-section on which the stress acts, and \(A_0\) is its initial value in the unstrained state. Then for direction 1, since

\[
A = A_0 \lambda_2 \lambda_3 = \frac{A_0}{\lambda_1},
\]

(4.9)

\[
\sigma_{11}^* = \frac{\sigma_{11}}{\lambda_1} = 2 \left( \lambda_1 - \frac{1}{\lambda_1^2 \lambda_2^2} \right) \left( \frac{\partial U}{\partial I_1} + \lambda_2^2 \frac{\partial U}{\partial I_2} \right).
\]

(4.10)

From (4.6) and (4.7), it can be shown that

\[
\frac{\partial U}{\partial I_1} = \frac{\left\{ \frac{\lambda_2^2 \sigma_{11}}{\lambda_1^2 - 1 / \lambda_1^2 \lambda_2^2} - \frac{\lambda_1^2 \sigma_{22}}{\lambda_2^2 - 1 / \lambda_2^2 \lambda_1^2} \right\}}{2(\lambda_1^2 - \lambda_2^2)},
\]

\[
\frac{\partial U}{\partial I_2} = \frac{\left\{ \frac{\sigma_{11}}{\lambda_1^2 - 1 / \lambda_1^2 \lambda_2^2} - \frac{\sigma_{22}}{\lambda_2^2 - 1 / \lambda_2^2 \lambda_1^2} \right\}}{2(\lambda_2^2 - \lambda_1^2)}.
\]

(4.11)  
(4.12)

Thus if we can measure the stresses and the extension ratios in both directions, we can experimentally determine the functions \(\frac{\partial U}{\partial I_1}\) and \(\frac{\partial U}{\partial I_2}\).
[Rivlin and Saunders] studied vulcanized rubber and got the following conclusions:

1. $\frac{\partial U}{\partial I_1}$ is approximately a material constant and independent of $I_1$ & $I_2$;

2. $\frac{\partial U}{\partial I_2}$ is independent of $I_1$, but is a weak linear function of $I_2$.

Hence if let

$$\sigma_0 = 2 \frac{\partial U}{\partial I_1},$$

$$\varepsilon = \frac{\partial U / \partial I_2}{\partial U / \partial I_1},$$

(4.10) can be written as:

$$\dot{\sigma}_{11} = \left( \lambda_1 - \frac{1}{\lambda_1^3 \lambda_2^2} \right) \left( 1 + \varepsilon \lambda_2^2 \right) \sigma_0,$$

where

$$\varepsilon = 0.152 - 0.00368 \times I_2.$$ (4.16)

We can also write (4.15) as:

$$\dot{\sigma}_{11} = g(\lambda_1, \lambda_2, \varepsilon) \cdot \sigma_0.$$ (4.17)

When unidirectional tension is applied,

$$\lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}},$$ (4.18)

Equation (4.15) becomes:

$$\dot{\sigma}_{11} = \left( \lambda_1 - \frac{1}{\lambda_1^2} \right) \left( 1 + \frac{\varepsilon}{\lambda_1} \right) \sigma_0.$$ (4.19)
Using this formula, we can simulate the uniaxial tensile tests of the rubber used in the forming process. As shown in Figure 4.4, the results fit with the experimental data very well.

![Graph showing stress vs. extension ratio for different tests and calculations.](image)

**Figure 4.4:** Simulation of uniaxial tensile response of diaphragm rubber.
4.3 Evaluation of $\sigma_0^*$ and $g(\lambda_1, \lambda_2, \varepsilon)$ from Measurement

From the simulation, we find that for the same material, $\sigma_0^*$ varies with thickness. Four different diaphragm thicknesses are used in our forming experiments. Their $\sigma_0^*$ values are evaluated from the uniaxial tensile test results and listed as follows:

<table>
<thead>
<tr>
<th>Diaphragm Thickness (in.)</th>
<th>1/64</th>
<th>1/32</th>
<th>1/16</th>
<th>1/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0^*$ (psi)</td>
<td>83.3</td>
<td>108</td>
<td>130</td>
<td>138</td>
</tr>
</tbody>
</table>

As mentioned in §4.1, for a given part, laminate wrinkling tends to happen in the same region and in the same pattern. For example, on a hemisphere, wrinkles occur perpendicular to the edge at locations where the preform stiffness is lowest; for a c-channel, wrinkles tend to occur on the flanges and perpendicular to the edges. Therefore, for individual parts, we can define the axes 1,2,3 in the manner outlined above, and then measure the average $\lambda_1$ and $\lambda_2$ in the critical regions. By using (4.2), we get

$$\lambda_3 = \frac{1}{\lambda_1 \lambda_2},$$

(4.20)

and consequently $I_2$. 
Finally, we evaluate

\[
g(\lambda_1, \lambda_2, \varepsilon) = \left( \lambda_1 - \frac{1}{\lambda_1^2 \lambda_2^2} \right) \left( 1 + \varepsilon \lambda_2^2 \right),
\]

(4.21)

where \( \varepsilon \) is related to \( I_2 \) by (4.16).

Some examples of \( g(\lambda_1, \lambda_2, \varepsilon) \) are listed as follows:

**Table 4.2:** Examples of \( g(\lambda_1, \lambda_2, \varepsilon) \) evaluation for diaphragm forming.

<table>
<thead>
<tr>
<th></th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \varepsilon )</th>
<th>( g(\lambda_1, \lambda_2, \varepsilon) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemisphere R=2.0 in.</td>
<td>1.026</td>
<td>2.25</td>
<td>0.128</td>
<td>1.390</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hemisphere R=2.5 in.</td>
<td>1.042</td>
<td>2.5</td>
<td>0.123</td>
<td>1.593</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hemisphere R=3.5 in.</td>
<td>1.167</td>
<td>2.8</td>
<td>0.110</td>
<td>2.020</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hemisphere R=4.5 in.</td>
<td>1.286</td>
<td>3.1</td>
<td>0.091</td>
<td>2.318</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-Channel Length=12 in.</td>
<td>1.22</td>
<td>2.25</td>
<td>0.121</td>
<td>1.792</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-Channel Length=24 in.</td>
<td>1.017</td>
<td>1.42</td>
<td>0.139</td>
<td>0.698</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-Channel Length=48 in.</td>
<td>1.091</td>
<td>1.68</td>
<td>0.135</td>
<td>1.130</td>
</tr>
</tbody>
</table>
Chapter 5

FORMING LIMIT DIAGRAM

5.1 Introduction

The diaphragm forming process has shown great potential to produce advanced composite parts with complex geometries. However, failure modes may occur under improper forming conditions. Among them the most serious one is laminate wrinkling. To prevent redundant design and unsuccessful process control, a way to determine the forming limits is imperatively required.

In the last three chapters, we have discussed the important factors that influence the forming limits: kinematic analysis introduces the desirable deformation modes and quantifies them; the drape test is used to reveal the viscous properties that determine the shearing stresses developed during forming (this actually includes the effects of temperature and forming rate, since they are related to the viscous properties); rubber elasticity is investigated to evaluate the support supplied by the diaphragms.
In this chapter, we will explore the balance between the mechanisms that induce and suppress wrinkling, and graphically present it as a forming limit diagram.

5.2 Compressive Force

5.2.1 Comparison between In-plane and Inter-ply Viscous Shear Forces

In the kinematic analysis, it was mentioned that experimental observations suggest that inter-ply shear is more difficult to induce than in-plane shear. In this subsection, we will employ a relative-order-of-magnitude analysis to confirm this point. It should be noticed that although ideal shears are not obtained in the forming process, it's quite reasonable to use them to evaluate which mode is more difficult to achieve and hence more dominant in the laminate wrinkling problem.

Consider a small element on the laminate. As shown in Figure 5.1, L is the specific length of a wrinkled region (in the critical direction which is perpendicular to the wrinkle); w is the width of the region (parallel to the wrinkle). The number of plies is \( N_p \), and the thickness of each ply is \( H \).

Then at the same point on the laminate, the ratio of inter-ply and in-plane viscous shear forces can be estimated by:

\[
\frac{F_{3v}}{F_{12}} \sim \frac{N_p L w m \Gamma_{3v}^n}{N_p H w m \Gamma_{12}^n} \sim \frac{L \Gamma_{3v}^n}{H \Gamma_{12}^n} \sim \frac{L}{H} \left( \frac{\Gamma_{3v}}{\Gamma_{12}} \right)^n.
\]  

(5.1)
We know that:

\[ H \approx 10^{-3} \text{ in.,} \]
\[ \frac{\Gamma_{3v}}{\Gamma_{12}} \approx 10^2 \text{ to } 10^3 \text{ (see Chapter 2),} \]
\[ n \approx 0 \text{ to } 0.45 \]

From experimental observations, \( L \approx 10^0 \text{ in.} \).

Hence

\[ \frac{F_{3v}}{F_{12}} \approx \frac{L}{H} \left( \frac{\Gamma_{3v}}{\Gamma_{12}} \right)^n \approx \frac{10^0}{10^{-3}} \cdot (10^2 \text{ to } 10^3)^{0.45} = 10^3 \text{ to } 10^4. \] (5.2)

According to this analysis of relative magnitude, we will regard the interply viscous shear stress as the main source of compressive force within a laminate.
5.2.2 Evaluation of Equivalent Compressive Force

Now we know that inter-ply shear stress will induce laminate wrinkling. However, inter-ply displacement is determined by part geometry and laminate lay-up. Generally, this relationship is a complicated function. The magnitude and direction of the viscous shear stress may change from point to point in the laminate body. Hence the shear stress itself is not a good measure to use as a forming limit criterion.

On the other hand, from the macro view, when a laminate is formed onto the double curvature tool, there is always some region where the material is compressed in some direction within the laminate. Perpendicular to this direction the laminate tends to wrinkle when the resultant effect of the shear stress is too strong to be relaxed at the given deformation rate. Therefore, there should be an equivalent compressive force acting on the region, and it is an appropriate function in the forming limit criterion, since it is directly related to the laminate wrinkling.

To evaluate this compressive force, we will use the analysis of compressive work. Consider a region as described above, in the compressed direction.

As shown in Figure 5.2, suppose the length contracts by a value $\Delta S$, then the equivalent compressive force $F_{eq}$ can be related to the compressive work in the following way:

$$F_{eq}\Delta S = \int \int \tau(\Gamma_{3v})d\Gamma_{3v}dV,$$

where $V$ is the volume of the material enclosed in the concerned region.
Again, since the inter-ply shear stress is the dominant source of compressive force, in the above energy integral, only \( \int_0^{r_{3v}} \tau(\dot{r}_{3v}) d\Gamma_{3v} \) is included in the calculation.

If we use a viscous model for the inter-ply shear stress:

\[
\tau_{3v} = m \dot{r}_{3v}^n, \tag{5.4}
\]

then

\[
\int_0^{r_{3v}} \tau(\dot{r}_{3v}) d\Gamma_{3v} = \frac{m}{n+1} \frac{\Gamma_{3v}^{n+1}}{t^n}, \tag{5.5}
\]

and \( F_{eq} \) can be estimated by:

\[
F_{eq} = \frac{1}{\Delta S} \int_V \frac{m}{n+1} \frac{\Gamma_{3v}^{n+1}}{t^n} dV = \frac{V}{\Delta S} \frac{m}{n+1} \frac{\Gamma_{3v}^{n+1}}{t^n}. \tag{5.6}
\]
Furthermore, since

\[ V \sim L w N p h_{int}, \]
\[ \Delta S \sim L, \]  

we get

\[ F_{eq} \sim w N p h_{int} \frac{m}{n+1} \frac{\bar{p}^{n-1}_{3v}}{t^{n}}. \]  

This result proves very useful later where we obtain the relative compressive force in the forming limit diagrams.

5.3 Mechanisms that Resist Laminate Wrinkling

5.3.1 Diaphragm Tension

Diaphragm rubber elasticity has been discussed in detail in Chapter 4. Consider the element in §5.2.1. For the corresponding element on the diaphragm, the tension in the critical direction is given by:

\[ T_d = D w \sigma_{11}^{*}, \]  

where \( D \) is the diaphragm thickness, and \( \sigma_{11}^{*} \) is given by (4.15).

5.3.2 Elastic Resistance of Composite

For the elastic resistance of the laminate itself, we will refer to the buckling test as described below. The test geometry was based on an Euler buckling test column. A schematic of the experimental set-up is shown in Figure 5.3.
Samples were either 8 or 16 plies thick and the test direction was chosen to be that which was weakest, i.e. in the $45^\circ$ direction on a ($0^\circ/90^\circ$) laminate and at $22.5^\circ$ to the $0^\circ$ fibers on a ($0^\circ/90^\circ/+45^\circ/-45^\circ$) lay-up. A series of typical test results are shown in Figure 5.4.

![Diagram of experimental set-up](image)

**Figure 5.3:** Experimental set-up used to determine the wrinkling resistance of the composite.
(For $0^\circ/90^\circ$ samples $\alpha = 45^\circ$, and for $0^\circ/90^\circ/+45^\circ/-45^\circ$ samples $\alpha = 22.5^\circ$).

We can estimate the critical elastic resistant force for the element in §5.2.1 as:

$$F_{\text{elastic}} = \frac{4\pi^2 E(t)I}{L^2} = \frac{4\pi^2 E(t)wN_p^3H^3}{12L^2}, \quad (5.10)$$

where $E(t)$ is the time dependent stiffness of the composite material; $1 \leq a \leq 3$ for the composite beams in the tests, whereas $a=3$ for an isotropic and homogeneous beam.
Figure 5.4: Typical buckling test results.
5.3.3 Comparison between Diaphragm Support and Elastic Resistance of Composite

The relative magnitude of the two resistance forces is approximated by:

\[
\frac{T_d}{F_{\text{elastic}}} = \frac{12Dw\sigma_{11}^*L^2}{4\pi^2E(t)wN_p^aH^3} = \frac{3D\sigma_{11}^*L^2}{\pi^2E(t)N_p^aH^3}.
\]  (5.11)

Since

\[
D \sim 10^{-2} \text{ in.},
\]
\[
\sigma_{11}^* \sim 10^2 \text{ psi},
\]
\[
L^2 \sim 10^0 \text{ in.}^2,
\]
\[
E(t) \sim 10^3 \text{ psi},
\]
\[
N_p^a \sim 10^0 \text{ to } 10^3,
\]
\[
H^3 \sim 10^{-7} \text{ in.}^3,
\]

we get

\[
\frac{T_d}{F_{\text{elastic}}} \sim 10^1 \text{ to } 10^4.
\]  (5.12)

Apparently, diaphragm tension provides the main support against laminate wrinkling.

5.4 Preliminary Forming Limit Diagrams

The preliminary forming limit diagrams are based on the balance between the equivalent compressive force $F_{\text{eq}}$ and the diaphragm tension $T_d$:

\[
F_{\text{eq}} \sim wN_p h_{\text{int}} \frac{m}{n+1} \frac{f^{n+1}}{t^n},
\]  (5.8)

\[
T_d = Dw\sigma_{11}^*.
\]  (5.9)
The common factor $w$ can be eliminated from both expressions and thus the two axes of the forming limit diagrams are:

$$F_{eq} \sim N_p h_{int} \frac{m}{n+1} r_{3v}^{n+1} t^n,$$

(5.13)

$$T_d \sim D_{11}^*.$$  

(5.14)

Furthermore, the value of $F_{eq}$ will be normalized with respect to its range in each plot.

For the $(0^\circ/90^\circ)$ hemispheres, all the experiments were carried out using Hercules AS4/3501-6 material; on the other hand, the part sizes are relatively small (maximum radius=4.5 in.). A fairly clear boundary can be seen between good parts and wrinkled parts. (Figure 5.5)

For the $(0^\circ/90^\circ/+45^\circ/-45^\circ)$ lay-up c-channels, both Hercules AS4/3501-6 and Toray T800H/3900-2 materials are used and the range of the part sizes are relatively wide (with length from 12 to 48 in.). The data cannot be divided neatly into regions where there are good parts only and bad parts only. (Figure 5.6)

Actually, it is not surprising that the above diagrams don't work consistently, because the following trends are clearly observed in the experiments:

Firstly, the actual shears that occur during forming are different from the ideal shears, and the discrepancy is related to the part size and geometry.

Secondly, the results for high temperature drape tests do not show as clear a trend as those carried out at room temperature (see Figures in Appendix A).
Figure 5.5: Forming limit diagram based on a simple viscous model for $(0^\circ/90^\circ)$ hemispheres.

Figure 5.6: Forming limit diagram based on a simple viscous model for $(0^\circ/90^\circ/+45^\circ/-45^\circ)$ c-channels.
In other words, the temperature effect has a limit not completely accounted for in a simple viscous model which is based around the reliable data we obtain at room temperature.

There is also a limit to the rate effect for the Toray material. This means that some time invariant factors are involved in the forming limits.

In the next section, some reasonable corrections will be made, and the revised diagrams will show fairly clear forming limits.

### 5.5 Corrections to the Forming Limit Diagrams

#### 5.5.1 Length Scale

Inter-ply shear is very difficult to measure, while the in-plane shear measurements are presented in Figure 5.7. We can see that the ideal shear is never achieved, and the ratio of actual shear to ideal shear increases with the part size. In other words, some relationship of the form below should be used to scale the compressive force calculation:

$$\Gamma_{\text{actual}} = f_L(L) \cdot \Gamma_{\text{ideal}}, \quad (5.15)$$

where $f_L(L)$ is a monotonically increasing function.

Please notice that if we assume that

$$f_L(L) \sim L^0 \quad (5.16)$$
then from (5.13) and (5.14) the length scales of the two axes (after \( w \) is crossed out) in the forming limit diagram are:

\[
F_{eq} \sim (L^{1+p})^{1+n},
\]

\[
T_d \sim L^0.
\]

![Graph showing forming limit diagram with ideal shear comparison](image)

**Figure 5.7:** Comparison between actual and ideal shears for (a) hemisphere; (b) c-channel.
We assume that $f_L(L)$ will have the same trend for inter-ply shear as for in-plane shear. According to the actual in-plane shear measurement, the $f_L(L)$ values we use for different parts are listed below.

<table>
<thead>
<tr>
<th>Hercules Hemisphere ($0^\circ/90^\circ$)</th>
<th>C-Channel ($0^\circ/90^\circ/+45^\circ/-45^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius [in.]</td>
<td>Length [in.]</td>
</tr>
<tr>
<td>0.38</td>
<td>Hercules</td>
</tr>
<tr>
<td>0.5</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>Toray</td>
</tr>
<tr>
<td>3.5</td>
<td>12</td>
</tr>
<tr>
<td>4.5</td>
<td>24</td>
</tr>
</tbody>
</table>

### 5.5.2 Limit of Temperature Effect

From the drape tests, we can see that the viscous stress tends to level off from the Arrhenius equation when temperature is high, as shown in Figure 5.8.

Forming experiments show that the effect of higher temperature is even weaker than what we see from the drape test. If we assume a nominal temperature $T_{\text{nominal}}$, there should be a monotonically decreasing function:

$$f_T(T) = \frac{T_{\text{nominal}}}{T}.$$  \hspace{1cm} (5.19)
Figure 5.8: Drape tests show the limit of temperature effect.

However, for simplicity we will assume:

\[ T_{\text{nominal}} = \min(T, T^*) \]

(5.20)

where

\[ T^* = 100 \, ^\circ F \quad \text{for Hercules AS4 / 3501 - 6,} \]
\[ T^* = 120 \, ^\circ F \quad \text{for Toray T800H / 3900 - 2.} \]

(5.21)
5.5.3 Limit of Rate Effect

The Hercules and the Toray materials are specially designed for aerospace applications. Their fibers are very stiff, and their resin behavior is more complicated than that of a simple viscous fluid. When the forming time is very long, i.e. when shear rate is very low, the drape test cannot give us a clear picture of the material constitutive properties. This can be seen from Figure 5.9, which shows typical results of the drape tests with both low and high shear rates.

![Figure 5.9: Scattering of drape test data at low shear rates.](image)

However, from the forming process, we find that the rate effect is weakened when the forming time is very long. This is especially true for the Toray material. On the other hand, severe spring back could happen if a Toray part is formed but not cured on the tool (see Figure 5.10). This suggests that
non-viscous properties of the Toray material cannot be omitted for the forming limit analysis.

![Figure 5.10: A Toray part showing severe spring back.](image)

A time-invariant correctional item is added to $F_{eq}$ for the Toray material. Considering there is a $\frac{1}{t^n}$ factor in the expression of $F_{eq}$, we can write the corrected $F_{eq}$ as:

$$F'_{eq} = F_{eq}(1 + \omega t^n).$$

For the Toray material, $\omega=0.1$ is a good choice for the constant.

The sensitivity of the forming limit diagrams to the parameters such as $f_L(L), T^*$ and $\omega$ will be discussed in §5.7.
5.6 Improved Forming Limit Diagrams

Based on the above corrections, the improved forming limit diagrams for \((0^\circ/90^\circ)\) hemispheres and \((0^\circ/90^\circ/+45^\circ/-45^\circ)\) c-channels are plotted as Figures 5.11 and 5.12 respectively. In both diagrams, a clear boundary of good parts and wrinkled parts can be determined. The experimental data and the terms evaluated for the forming limit diagrams are listed in Appendix B.

![Figure 5.11: Improved forming limit diagram for \((0^\circ/90^\circ)\) hemispheres.](image)
5.7 Sensitivity of the Forming Limit Diagrams to Certain Parameters

In the improved forming limit diagrams shown in the last section, there are some parameters whose values are not determined by using material test but estimated from the forming observation. Now, we discuss the sensitivity of the forming limit diagrams to these parameters by studying the case of \((0^\circ / 90^\circ / +45^\circ / -45^\circ )\) c-channels.
5.7.1 Sensitivity to $f_L(L)$

The $f_L(L)$ values used for different parts were listed in §5.5.1. For c-channels, $f_L(L)$ is different for the Hercules and the Toray materials. If we let them be the same as Hercules values, i.e. as listed below.

**Table 5.2:** $f_L(L)$ values used for Hercules c-channels.

<table>
<thead>
<tr>
<th>Length (in.)</th>
<th>12</th>
<th>24</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_L(L)$</td>
<td>0.52</td>
<td>0.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

the forming limit diagram will become as in Figure 5.13.

![Figure 5.13: Sensitivity to $f_L(L)$: Forming limit diagram for $(0^\circ/90^\circ/±45^\circ/±45^\circ)$ c-channels.](image)

We can see that the boundary becomes vague, and there is a wide transition region between the good parts and the wrinkled parts.
5.7.2 Sensitivity to $T^*$

In the improved forming limit diagrams, we selected

$$T^* = 100 \, ^\circ F \quad \text{for Hercules AS4 / 3501-6,}$$
$$T^* = 120 \, ^\circ F \quad \text{for Toray T800H / 3900-2.} \quad (5.21)$$

Since the temperature effect seems stronger in the drape test, we will try to raise $T^*$ for both materials. Let

$$T^* = 120 \, ^\circ F \quad \text{for Hercules AS4 / 3501-6,}$$
$$T^* = 150 \, ^\circ F \quad \text{for Toray T800H / 3900-2,} \quad (5.23)$$

then the forming limit diagrams are as in Figure 5.14. Actually, it can be seen that the forming limit is not clear for either material respectively.

![Diagram](image)

Figure 5.14: Sensitivity to $T^*$: Forming limit diagram for (0°/90°/+45°/-45°) c-channels.
5.7.3 Sensitivity to $\omega$

In current stage of the forming limit analysis, $\omega$ is completely an empirical constant. To see if the time-invarint term could be less important, we try

$$\omega = 0.05 \quad (2.24)$$

instead of $\omega = 0.1$

which was used in the improved diagram in §5.6. The result is shown in Figure 5.15. The boundary is not as clear because the data for Toray material are not well arranged.

Figure 5.15: Sensitivity to $\omega$:
Forming limit diagram for $(0^\circ/90^\circ/+45^\circ/-45^\circ)$ c-channels.
Chapter 6

DISCUSSION

6.1 Conclusion

The main conclusion of this thesis is that the complicated problem of determining the forming limits of advanced composites can be solved using a combination of simplified models.

Advanced composite materials are anisotropic and inhomogeneous, with a resin matrix which is non-linear viscous, hence the detailed mathematical description of the material behavior may be computationally intense. In addition, the deformation process is also anisotropic and inhomogeneous, therefore the exhaustive stress analysis of the system cannot be achieved by analytical means. This research started with simple models for different aspects of the forming process: the kinematic analysis illustrated the concept of shear deformations and allow us to decompose the intricate three dimensional displacements; an easily manipulated three point bending test was used to model the principal constitutive relationship of the materials; the nonlinear elastic behavior of the diaphragm materials was modeled by using the established bi-axial stress theory of rubbers.
CHAPTER 6

After some reasonable modifications, the resulting forming limit diagrams clearly demonstrated the boundary between good parts and wrinkled parts of different sizes and made from different materials. We found that the length scale is not only determined by the ideal mapping of the fibers, but also by the relative amount of shears realized, which is related to the part size. It was also revealed that the temperature and rate effects are not as strong as in a simple viscous model. Therefore, the size and complexity of the parts that can be made are not expected to be considerably increased if temperature and rate are the only parameters varied. Innovations must be explored to expand the range of parts that can be made by the diaphragm forming technique.

The forming limit diagrams can also be used to analyze how much improvement an innovation can achieve, and thus determine if the innovation is a worthwhile investment. In the next section, we will show an example of using the forming limit diagram to assess innovation.

6.2 Illustration of the Effect of Innovation

One of the innovations developed in the M.I.T. composites manufacturing program is "reinforced diaphragm forming". In this process, a stiff, directional reinforcement is placed adjacent to one or both diaphragms to suppress the out-of-plane wrinkling (see [Gutowski, et al]). The basic idea of the innovation is illustrated in Figure 6.1. An unwrinkled part formed with this technique is shown in Figure 6.2.
Figure 6.1: Illustration of reinforced diaphragm forming.
Experiments show that the "reinforced diaphragm forming" process has led to a great improvement in the ability to produce thicker parts with small and medium sizes. For instance, we cannot make good 24 inch long Toray parts with \((0^\circ/90^\circ/+45^\circ/-45^\circ)\) lay-up without the reinforcement, whereas with the reinforcement we can make \((0^\circ/90^\circ/+45^\circ/-45^\circ)_{4s}\) parts, i.e. with 32 plies. To show these successful parts, the forming limit diagram presented in §5.6 is modified in Figure 6.3.
Figure 6.3: Forming limit diagram for \((0^\circ/90^\circ/+45^\circ/-45^\circ)\) c-channels modified to show the effect of reinforced diaphragm forming.

We can see that the critical \(F_{eq}\) is at least about a factor of 10 larger than for the unreinforced process. According to (5.17), in the forming limit diagram:

\[
F_{eq} \sim (L^{1+p})^{1+n},
\]

(5.17)

hence the new process has the potential to make much larger parts, perhaps by a factor of \((10)^{1/(1+p)(1+n)} \) . Assume \(p=0.5\) and \(n=0.3\), the factor will be:

\[
(10)^{1/(1+0.5)(1+0.3)} = 3.3.
\]

(6.1)

This means that using the reinforced forming we may make a part whose length is about:

\[
24\text{ in. } \times 3.3 = 79\text{ in. } = 6.6\text{ ft.}
\]

(6.2)
Though this part size is beyond our current machine capability, the forming limit diagram illustrates the possible impact of using a reinforced diaphragm forming process.

### 6.3 Suggestions for Future Work

The forming limit analysis presented in this thesis has supplied an explicit and logical way of evaluating the formability of a given part. On the other hand, it is preliminary because different shapes cannot be fitted into a universal forming limit diagram, and there are some empirical parameters about which we do not have a clear enough understanding. In this section, we will make some suggestions for the future work.

The current kinematic analysis has utilized the general theory of differential geometry, but the approach for different shapes are isolated and inefficient. This may be the main reason that the general forming limit of different parts is not obtained. A CAD model was developed by [Gonzalez-Zugasti] for mapping continuous aligned fiber composite materials over arbitrary geometries. It should be improved to map the different plies of a whole laminate so that both the in-plane and inter-ply displacements can be directly determined by inputing the laminate lay-up and geometric specifications of a part. In addition, the difference between the actual and ideal shears, along with its relationship to the part geometry and size must be studied more thoroughly. For this purpose, a method of measuring the inter-ply displacement is required.
Although it is suggested that the viscous phenomenon dominates the forming process, the simple viscous model was not successful in collapsing all the experimental data. Some modifications have been made but are quite empirical. More sophisticated experimental and theoretical approaches must be carried out to give us a more clear constitutive model of the materials. However, the detailed analytical solution of the problem is not suggested because it will lead to a calculation too complicated to be convenient for engineering practice.

The work method we used to evaluate the compressive force is still at a conceptual level. A more detailed analysis should be accompanied by a clear kinematic description so that we can make a distinction between energies associated with different deformation modes. This will also help us to further identify the dominant mechanisms that cause laminate wrinkling.

The tensile state of the diaphragm material is described by a bi-axial theory of rubber elasticity. The accuracy of this simplification needs some experimental verification, because the diaphragms interact with both the laminate and the tool. A theoretical analysis of these interactions are also necessary to find out how the compressive and supporting mechanisms are balanced.

Finally, it should be emphasized that the object of the forming limit analysis is to supply a practical tool for design and manufacturing engineers. So the balance between sophistication and convenience must be well managed. For such a problem with considerable complexity, this is the real challenge.
Bibliography


[Rivlin and Saunders]  

[Struick]  

[Tam]  

[Tam and Gutowski]  

[Ward]  
DRAPE TEST RESULTS

The relationship between shear stress and shear strain in a non-Newtonian fluid can be expressed by:

\[ \tau = m \dot{\gamma}^n = \eta \dot{\gamma} \]  

(3.12)

where \( m \ [\text{psi} \cdot \text{sec}^n] \) is the non-Newtonian viscosity function; \( n \) is a dimensionless exponent; and

\[ \eta = m \dot{\gamma}^{n-1} \ [\text{psi} \cdot \text{sec}] \]  

(3.13)

is the apparent viscosity.

Drape test data give us a power law curve fit on the plot of maximum load \( F_{\text{max}} \) versus maximum shear rate \( \dot{\gamma}_{\text{max}} \):

\[ F_{\text{max}} = p \dot{\gamma}_{\text{max}}^q. \]  

(3.9)

According to the discussion in Chapter 3, the maximum shear rate is determined by:

\[ \dot{\gamma}_{\text{max}} = \frac{3 \dot{\Delta}}{2a}. \]  

(3.8)

Notice that (3.8) holds only when the deflection \( \Delta \) is small relative to the sample length \( 2a \). In our experiments, \( a = 1.0 \ \text{inch}, \ 0 \leq \Delta \leq 0.3 \ \text{inch}, \) and \( 0.1 \ \text{in/min} \leq \dot{\Delta} \leq 10 \ \text{in/min}. \)

The maximum load \( F_{\text{max}} \) is measured when the viscous flow is fully developed, which is indicated by a leveling off of the load-deflection curve.
The force and moment balances yields:

\[ n = q \quad [\text{dimensionless}], \quad (3.10) \]

\[ m = \frac{P}{2b\varepsilon} \quad [\text{psi} \cdot \text{sec}^n]. \quad (3.11) \]

For all the experiments, \( b = 2.0 \) inch, while the thickness of prepreg \( \varepsilon = 0.0064 \) inch for the Hercules AS4/3501-6 material, and \( \varepsilon = 0.0085 \) inch for the Toray T800H/3900-2 material.

In the following sections, the plots of \( F_{\text{max}} \) versus \( \dot{\gamma}_{\text{max}} \) are presented with a power law curve fit. The power law coefficients \( m \) and \( n \) are calculated by using (3.10) and (3.11). The shear thinning phenomenon can be observed through the plots of \( \eta \) versus \( \dot{\gamma}_{\text{max}} \). The variation of \( \eta \) with temperature is also shown.

### A.1 Drape Test Results for Hercules AS4/3501-6 Prepreg

![Graph showing maximum load versus maximum shear rate](image)

**Figure A.1:** Maximum load versus maximum shear rate for AS4/3501-6 prepreg tested at 72°F.
DRAPE TEST RESULTS

Figure A.2: Maximum load versus maximum shear rate for AS4/3501-6 prepreg tested at 85°F.

\[ y = 7.0790e-2 \times x^{0.27423} \quad R^2 = 0.938 \]

- Max. Shear Rate [1/sec]
- Max. Load [lb]

Figure A.3: Maximum load versus maximum shear rate for AS4/3501-6 prepreg tested at 100°F.

\[ y = 3.5554e-2 \times x^{0.13998} \quad R^2 = 0.924 \]

- Max. Shear Rate [1/sec]
- Max. Load [lb]
Figure A.4: Maximum load versus maximum shear rate for AS4/3501-6 prepreg tested at 115°F.

Table A.1: Power law coefficients for Hercules AS4/3501-6 prepreg tested at different temperatures.

<table>
<thead>
<tr>
<th>Temperature [°F]</th>
<th>m [psi·sec^n]</th>
<th>n [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>11.5</td>
<td>0.442</td>
</tr>
<tr>
<td>85</td>
<td>2.77</td>
<td>0.274</td>
</tr>
<tr>
<td>100</td>
<td>1.39</td>
<td>0.140</td>
</tr>
<tr>
<td>115</td>
<td>1.00</td>
<td>0.123</td>
</tr>
</tbody>
</table>
DRAPE TEST RESULTS

Figure A.5: Shear thinning phenomenon for Hercules AS4/3501-6.

Figure A.6: Variation of apparent viscosity with temperature for Hercules AS4/3501-6.
A.2 Drape Test Results for Toray T800H/3900-2 Prepreg

Figure A.7: Maximum load versus maximum shear rate for Toray T800H/3900-2 prepreg tested at 73°F.

\[ y = 1.3963 \times x^{0.32323} \quad R^2 = 0.997 \]

Figure A.8: Maximum load versus maximum shear rate for Toray T800H/3900-2 prepreg tested at 85°F.

\[ y = 0.45022 \times x^{0.26363} \quad R^2 = 0.981 \]
Figure A.9: Maximum load versus maximum shear rate for Toray T800H/3900-2 prepreg tested at 100°F.

Figure A.10: Maximum load versus maximum shear rate for Toray T800H/3900-2 prepreg tested at 120°F.
Figure A.11: Maximum load versus maximum shear rate for Toray T800H/3900-2 prepreg tested at 140°F.

Table A.2: Power law coefficients for Toray T800H/3900-2 prepreg tested at different temperatures.

<table>
<thead>
<tr>
<th>Temperature [°F]</th>
<th>m [psi · sec(^n)]</th>
<th>n [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>41.1</td>
<td>0.323</td>
</tr>
<tr>
<td>85</td>
<td>13.2</td>
<td>0.264</td>
</tr>
<tr>
<td>100</td>
<td>7.31</td>
<td>0.191</td>
</tr>
<tr>
<td>120</td>
<td>4.06</td>
<td>0.184</td>
</tr>
<tr>
<td>140</td>
<td>1.48</td>
<td>0.071</td>
</tr>
</tbody>
</table>
Figure A.12: Shear thinning phenomenon for Toray T800H/3900-2.

Figure A.13: Variation of apparent viscosity with temperature for Toray T800H/3900-2.
As discussed in Chapter 5, basically the two axes of the forming limit diagrams are:

\[
F_{eq} \sim N_p h_{int} \frac{m}{n+1} \frac{f_3}{n} t^n, \quad (5.13)
\]

\[
T_d \sim D\sigma_{11}, \quad (5.14)
\]

and the value of \( F_{eq} \) will be normalized with respect to its range in each plot.

The following corrections are made based on experimental observations:

(1) Length Scale

\[
\Gamma_{\text{actual}} = f_L(L) \cdot \Gamma_{\text{ideal}}, \quad (5.15)
\]

where \( f_L(L) \) is a monotonically increasing function. The \( f_L(L) \) values used for different parts are listed in Table 5.1.
(2) **Temperature Correction**

\[
T_{\text{nominal}} = \min(T, T^*),
\]

where

\[
T^* = \begin{cases} 
100^\circ F & \text{for Hercules AS4 / 3501-6}, \\
120^\circ F & \text{for Toray T800H / 3900-2}.
\end{cases}
\] (5.20)

(3) **Rate Correction**

For the Toray T800H/3900-2 material,

\[
F'_{\text{eq}} = F_{\text{eq}} \left( 1 + \omega t^n \right),
\]

where \( \omega = 0.1 \).

The experimental data and the terms evaluated for the forming limit diagrams are listed in the following tables.
<table>
<thead>
<tr>
<th>Expt #</th>
<th>N_p</th>
<th>Geometry</th>
<th>Radius (in)</th>
<th>t (min)</th>
<th>T (F)</th>
<th>T_{nominal} (F)</th>
<th>D (in)</th>
<th>m</th>
<th>n</th>
<th>F_{eq}</th>
<th>Normalized</th>
<th>T_d</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>16</td>
<td>(0/90)4S</td>
<td>3.5</td>
<td>240</td>
<td>70</td>
<td>70</td>
<td>0.0625</td>
<td>10.47</td>
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<td>0.1641</td>
<td>16.25</td>
<td></td>
</tr>
<tr>
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<td>16</td>
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<td>180</td>
<td>70</td>
<td>70</td>
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</tr>
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<td>70</td>
<td>0.0625</td>
<td>10.47</td>
<td>0.449</td>
<td>0.1166</td>
<td>16.25</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>0/90</td>
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<td>5</td>
<td>70</td>
<td>70</td>
<td>0.0625</td>
<td>10.47</td>
<td>0.449</td>
<td>0.1166</td>
<td>16.25</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>0/90</td>
<td>3.5</td>
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<td>70</td>
<td>70</td>
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<td>0/90/0</td>
<td>3.5</td>
<td>5</td>
<td>70</td>
<td>70</td>
<td>0.0625</td>
<td>10.47</td>
<td>0.449</td>
<td>0.1749</td>
<td>16.25</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>0/90/0</td>
<td>3.5</td>
<td>5</td>
<td>100</td>
<td>100</td>
<td>0.0625</td>
<td>1.737</td>
<td>0.212</td>
<td>0.0463</td>
<td>16.25</td>
<td></td>
</tr>
<tr>
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<td>3</td>
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<td>5</td>
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<td>16</td>
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<td>0/90</td>
<td>4.5</td>
<td>5</td>
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<td>70</td>
<td>0.0625</td>
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<td>0.1678</td>
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<td>100</td>
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<td>1.737</td>
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</tr>
<tr>
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<td>1.737</td>
<td>0.212</td>
<td>0.0308</td>
<td>16.25</td>
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</tr>
<tr>
<td>19</td>
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<td>0/90</td>
<td>3.5</td>
<td>5</td>
<td>120</td>
<td>100</td>
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<td>1.737</td>
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<td>5</td>
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<td>70</td>
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</tr>
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<td>70</td>
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<td>0.0871</td>
<td>13.9286</td>
<td></td>
</tr>
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<td>24</td>
<td>16</td>
<td>(0/90)4S</td>
<td>3.5</td>
<td>30</td>
<td>70</td>
<td>70</td>
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Table B.2: Data for Hercules (0°/90°) hemisphere wrinkled parts.

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Table B.3: Data for Hercules (0°/90°/+45°/-45°) c-channel good parts.

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Table B.4: Data for Hercules (0°/90°/+45°/-45°) c-channel wrinkled parts.

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Table B.5: Data for Toray (0°/90°/+45°/-45°) c-channel good parts.

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Table B.6: Data for Toray (0°/90°/+45°/-45°) c-channel wrinkled parts.

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### Table B.7: Data for Toray (0°/90°/45°−45°) c-channel good parts (reinforced diaphragm forming)

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<th>t(min)</th>
<th>T(F) T normal (F) D(in)</th>
<th>n</th>
<th>F₀₀ Normalized</th>
<th>Tₐ (s)</th>
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<tr>
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