A Propeller Blade Design Method Using Generalized Geometry and Viscous Flow Computations

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James Gregory Diggs

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Signature of Author

James Gregory Diggs
May 1994

Certified By

Justin E. Kerwin
Professor of Naval Architecture

Henry S. Marcus
Professor of Ocean Systems Management

Accepted By

A. Douglas Carmichael
Chairman, Departmental Committee on Graduate Students
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ABSTRACT

Traditional propeller design techniques have poorly handled propellers with high conicity and/or strong viscous interactions with the body. A vortex-lattice propeller design code is coupled to axisymmetric RANS codes to model the effective wake. The MIT code, Propeller Blade Design 10.2 couples the propeller design with Navier-Stokes solutions of the effective wake problem.

The propeller geometry is generalized from cylindrical streamtubes to arbitrary streamtubes. Modern designs and full stems have pushed the limits of cylindrical propeller definition methods. In addition, the geometry definition methods were modularized to increase the design flexibility. Finally, the overall design method from preliminary ship design to construction design is examined.

The improved design program is called PBD10.2. The geometry modules are called D2XYZ, XYZ2B, B2XYZ, and XYZ2D, where “D” stands for designer parameters and “B” stands for a B-spline Surface. This document describes the development of these codes.

Thesis Supervisor: Justin E. Kerwin
Title: Professor of Naval Architecture
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1. GOALS

The goal of this thesis was to improve the existing design code, Propeller Blade Design version 10.1, (PBD10.1). The improved code is called PBD10.2. Geometry definitions were generalized to handle highly conical bodies. A RANS code is coupled with the design method to solve the effective wake problem. Improvements in search strategy are suggested to speed the design process. The result is a new propeller design code, PBD10.2, two new geometry codes, D2XYZ and XYZ2D, and a strategy for future improvements to the MIT family of propeller codes.

The code has been expanded to include highly conical geometries, with hub and duct imaging. Geometrical tools were developed to handle non-cylindrical geometry conventions in a modular fashion. The new tools allow designers to define the propeller geometry along arbitrary streamlines.

Previously, the designer needed to supply the effective wake to the code. Now, with axisymmetric RANS modeling, the code can calculate the effective wake from the propeller and hull geometry. Because of this improvement, an experimental step in the design process can now be modeled by a computer.

The overall philosophy governing these improvements was to simplify and generalize the approach. It is anticipated that portions of this thesis will be building blocks in an overall design method that may eventually be fully automated. By modularizing the subroutines and using graphical data files, the code is more transportable and robust. The graphical interfaces and geometrical processes developed here will be useful if the MIT design and analysis codes are grouped into a single integrated, automated design tool. The code improvements were designed to allow for such large-scale automation, while maintaining stand-alone capabilities.
2. INTRODUCTION

2.1. HISTORY OF PROPELLERS

Propellers are the primary movers on most sea vessels, and as such, propeller design has been a crucial aspect in the performance of vessels. Originally, propellers were simple constant pitch designs based upon design experience and tradition. Over the last 100 years, propellers have become more complicated and more critical to the ship. The performance of propellers has been improved through the use of ducts, pre-swirl vanes, and contra-rotating designs. Rising fuel costs have motivated the current propeller designers to squeeze every percentage of efficiency out of the designs. The demands of noise reduction have also driven design practices. Now, a propeller must meet thrust, vibration, cavitation and noise criteria. The costs of propellers and propeller experiments are high, and designers are trying to shave iterations off the design spiral, while meeting strict performance criteria. This thesis aids in the process by automating some of the more time-consuming and uncertain aspects of propeller designs, such as the effective wake problem. This thesis also improves the accuracy of designs with high conicity by generalizing the definition of propeller geometry to include arbitrary streamsurfaces.

2.2. LATTICE CODES

Propellers were originally designed using empirical charts and elementary theory derived from aerodynamics. As early as the 1920's, designers were starting to apply theoretical potential-flow methods to the design and analysis process\(^1\). The design process differs from the analysis process in the direction of calculations, in that it uses performance criteria as input, and comes up with a suitable geometry. The analysis process uses the geometry of the propeller as input, and determines the performance. The design process was improved by early lifting line calculations. The loading and the

\(^1\)Glauert, Elements of Aerofoil and Airscrew Theory, Cambridge University Press, 1926.
chord of the propeller can be roughly determined by modeling the three-dimensional propeller blade as a single lifting line with trailing vortices. The first improvements to lifting line calculations were empirical three-dimensional corrections. Lifting surface methods further improve the propeller model, by using a three-dimensional mesh to represent the blade.

Three-dimensional corrections are needed as the propeller shape has increasingly incorporated more skew, thickness, camber, chord, and rake: basically, anything that makes the propeller not resemble a lifting surface. For a long thin airplane propeller, the lifting line closely represents the actual propeller, but most marine propellers have lower aspect ratios and therefore, a single vortex does not represent the details of the flow accurately. The flow is not as simple as a two-dimensional analysis model and therefore, marine propellers are not precisely handled by lifting line theory.

PBD10.2 is based upon vortex lattice methods that expand upon a lifting line calculation by distributing the circulation streamwise along a blade section. The calculation is a potential based method, using vortex segments distributed along the blade to represent the blade shape. The vortex segments form a gridwork that discretely represent the circulation distribution spanwise and chordwise across the blade by building blocks of constant strength vortices. The wake of the propeller is modeled as a gridwork of free vorticity convected downstream with the circumferential mean flow. Image vorticity lattices are used to model the boundary of a hub or a duct, and corresponding wake lattices are aligned for the images.

The effective wake problem has been a difficult point in the propeller design process. With all of the PBD10.2 grids and lattices, and with all of the induced velocities calculated, PBD10.2 would still be inaccurate if the nominal wake was used as the inflow. The effective wake accounts for the vortical interactions between the hull and the propeller. In the presence of the propeller, the boundary layer of the hull becomes thinner, increasing the vorticity in the wake. The effect is rotational and is missed in a
potential flow calculation like PBD10.2. Currently, designers test the model of the hull in a towing tank with a generic stock propeller. By measuring the flow behind the self-propelled hull, the total wake is determined. However, this measurement is still not the effective wake, since the flow includes the induced velocities of the propeller. By using an analysis code to subtract out the induced velocities of this stock propeller, the effective wake is indirectly measured. If the stock propeller and the final design are similar "enough," then the effective wake will be the same for the final propeller. However, the use of a RANS code to calculate the effective wake will help to minimize the time spent testing. This is the motivation for coupling PBD10.2 with an axisymmetric RANS code. The RANS code calculates the effective wake, and helps PBD10.2 to determine the design of the propeller.

PBD10.2, the version developed for this thesis, has the improvements of generalized geometry and coupling with a viscous flow-solver to solve the effective wake problem. Full stems magnify the coupling between the propeller and the hull, which is reflected in the effective wake. To handle this, the flow around the hull is calculated by an axisymmetric RANS flow solver. A circumferential mean representation of the propeller is included in this body problem. A vortex-lattice method is used to model the fluctuating components of the propeller problem which can not be handled axisymmetrically. The blade-to-blade variations in the flow are fluctuations from a fixed frame of reference. Because of these blade-to-blade variations, this treaty is called the blade problem.

The geometry definitions were generalized to better solve the design problem for sterns with high conicity. The non-cylindrical definitions is general and flexible enough to handle a wide variety of axisymmetric streamsurfaces. Trimming problems at the hub and tip of the propeller can be eliminated. The more flexible convention gives the designer more accurate control of the loading and the pressure distribution, particularly near the root and tip of the propeller. B-spline surfaces are used to represent the
propeller in a more continuous fashion than conventional geometry parameters. The B-
spline algorithm helps the alignment procedure in PBD10.2. Finally, the geometry codes
were modularized to make the design method more flexible.
3. GEOMETRY

3.1. INTRODUCTION

Blade designers have traditionally defined the geometry of propellers with parameters along the radial axis of the propeller. The chord, pitch, rake and skew of the propeller are tabulated at a discrete number of points along the radial axis. The camber and thickness are defined across the face of the blade in a meshwork of axial and radial lines. The meshwork results from the cylindrical coordinate system that designers have assumed.

In two ways, this thesis improves upon the current geometry system. The changes are motivated by the need for a continuous, universal definition of the blade, and a need for a non-cylindrical coordinate system.

Design parameters describe the blade at a series of radial stations. At each of these stations, the foil section is also specified. Fairing between each of the sections defines the entire blade, except at the root, where special definitions of fillet geometry are used. Parameters such as pitch and camber are used as indicators of the hydrodynamics of the blade, and as such they are very useful. Unlike xyz coordinates however, a plot of these dimensions does not give the shape of the blade, until xyz points are calculated. For this reason, the blade manufacturers are using xyz coordinates and a mesh of points that is magnitudes finer than the designers' information. The successful design of a modern propeller requires the knowledge of experts in each of these fields, but translating the geometry at each step is redundant and can be inaccurate.

The first step in the PBD series of codes is to represent the blade in Cartesian coordinates based upon the definitions of chord, pitch, camber, rake, skew, and thickness. Now PBD10.2 receives the input as a B-spline surface, which defines the xyz points everywhere. There are several advantages to this scheme. First, the alignment procedure in PBD10.2 manipulates the B-spline itself, so the need for a translation from design
parameters to geometry is eliminated. Second, the output geometry is a B-spline, which can be used to start PBD10.2 again, which removes another translation from the traditional design cycle. Thickness and fillet geometry can also be added to the B-spline to produce the manufacturer's geometry. If a designer wants to further examine the blade, parameters of chord, pitch, and skew can be extracted from the B-spline, so the designer can interpret the new shape. Finally the B-spline surface can be easily plotted and viewed.

The second improvement was the addition of the generalized geometry convention. Current design methods and definitions of propeller geometry use cylindrical coordinates to define blade sections. Modern hull shapes often have high conicity. The conicity can be problematic in developing and understanding the hydrodynamics of the blade. Blade sections defined along cylindrical streamlines obscure the importance of chord, pitch, and camber in conical flows. The blade shape that the flow sees is not the same as the sections that the designer defined. Situations such as these indicate that a more general coordinate system is preferable.

Even without these hydrodynamic reasons, there is a practical reason for using non-cylindrical blade geometries. The hub and duct of a propeller are often not cylindrical or even conical, but they are streamlines. The designer has to trim the excess propeller at the hub and tip streamlines in order to generate the proper geometry for construction. The problem can be simplified by a streamwise definition system.

PBD10.2 improves upon existing methods by using generalized geometry definitions and by representing the blade as a B-spline surface. PBD10.2 now takes a B-spline surface as the representation of the blade. The blade geometry that PBD10.2 converges upon is also represented as a B-spline surface. As far as the code knows, there are no parameters like chord, pitch and skew. Preprocessing and postprocessing geometry codes generate B-spline surfaces from designer parameters, and translate the B-spline surface back into chord, rake, camber, and skew. Figure 3-1 represents the process.
This new method makes the design method more general and modular. The definition of the geometry can be changed easily, without modifying the other steps in the design process. Currently, only PBD10.2 uses B-splines as the representation of the geometry. When the other MIT codes can handle B-splines, the codes will communicate the geometry through B-spline surfaces. This will reduce the translations between the codes, and provide a uniform, continuous definition of the blade. At any time, the designer can translate the blade into the parameters of pitch and camber, in order to closely examine the blade.

In the future, it is anticipated that the MIT design and analysis codes will work closely together in an automated, unified, graphical based design program. The individual codes will be coupled together by a driver program, which will use B-splines as the standard definition of blade geometry. To check on how the design is proceeding, the designer can query the program for the designer parameters. In this scenario, the geometry codes provide a window into the design environment.
3.2. GENERALIZED GEOMETRY DEFINITION

The first improvement made to the geometry methods was to generalize the cylindrical coordinate system. Neely\textsuperscript{2} at the Naval Surface Warfare Center developed the method that is used here. The non-cylindrical coordinate system uses axisymmetric tubes instead of cylinders to define the surfaces on which foil sections are superimposed. The axisymmetric tubes can be cylinders, but more often the tubes are streamsurfaces. The motivation for this generalization was high conicity sterns in current propeller designs. The limits of the cylindrical coordinate system were exceeded by the conical angle and large curvature of the new sterns.

Cylindrical coordinate systems can be forced to handle conical flow fields, but there are two areas in which the non-cylindrical methods are superior. First, the sections can be defined upon streamlines, so the effects of camber, pitch, and thickness are comparable in a qualitative sense to the results from two-dimensional theory. For a lightly loaded propeller with a cylindrical hub, the streamsurfaces approach cylinders. However, with heavily loaded propellers in conical flow, the streamsurfaces can not be represented as cylinders. In such cases, non-cylindrical sections help to eliminate flow across sections, and simplify the design and analysis of the propeller.

The second area of improvement concerns the tip and root intersection with hub and duct geometries. Non-cylindrical sections increase the accuracy of thrust predictions and the accuracy of loading and pressure distributions when designing the blade for highly tapered bodies with ducts. Because cylindrical propeller definitions can not match the conicity of the hub and tip, the propeller must be trimmed to match the surfaces, as shown in Figure 3.2.

\textsuperscript{2}Neely, S., "Approach for Non-Cylindrical Blade Sections," Memo, Nov. 17, 92, and an upcoming NSWC report.
In order to produce the specified thrust of the conical propeller, the designer matches the thrust of the cylindrical propeller and hopes that the end extrapolations are negligible. Without non-cylindrical design and analysis tools, the designer can not calculate the thrust of the conical propeller, so the success of the new design is indeterminable. The accuracy of the most critical design goal, thrust, is now reduced.

The streamwise definition of geometry does involve a reorientation of thinking. There is no longer a single hub or tip radius; radius varies with axial length. The cylindrical surfaces of traditional geometry definitions have been generalized to include any arbitrary axisymmetric surface, called generation tubes. The tubes are most effective if they resemble streamlines, but the tubes need not follow the flow. The advantages occur when the generation tubes are a family of surfaces defined by the hub streamsurface and the tip streamsurface.

The geometry system is a curvilinear coordinate system, as shown in Figure 3-3. In Figure 3-3, the meridional plane of the coordinate system is presented. The three curvilinear coordinates are the parameters \( v, w, \) and \( \theta \), instead of the traditional coordinates \( r/R, x/R, \) and \( \theta \). A point \( P(v,w,\theta) \) is represented by these three coordinates. A generating tube is defined by the family of points \( P(c,w,\theta) \), where \( c \) is a constant. A reference curve is defined by the family of points \( P(v,c,\theta) \). The third coordinate in the curvilinear coordinated system is \( \theta \), which represents the angle away from the meridional plane, as in the traditional coordinate system. In the case of cylindrical generating tubes
and radial reference lines, the curvilinear coordinate system collapses to the traditional cylindrical definitions.

![Curvilinear Coordinate System](image)

**Figure 3-3: Curvilinear Coordinate System**

Generating tubes are a family of axisymmetric surfaces that generally align with the streamlines of the propeller. The hub of the propeller describes the smallest generating tube, and the duct or tip of a propeller describes the largest generating tube. The distance between those two tubes is measured by \( v \), the normalized arclength along reference curves. The propeller designer chooses a main reference curve, \( P(v,0,0) \), and the rest of the reference curves follow from that specification. Typically, a main reference curve will be a radial line, but this is not always the best choice, so the main reference curve is left in a more general form.

The parameter \( v \) is used to measure the normalized distance across streamlines in the curvilinear coordinate system. At the hub of the propeller, the parameter \( v=0 \), and the family of points on the hub are described by \( P(0,w,\theta) \). Similarly, at the tip of the propeller, \( v=1.0 \), and the family of points along the tip are described by \( P(1.0,w,\theta) \). In the case of radial reference curves, \( v \) is comparable to \( r/R \) in the traditional coordinate system.
The parameter $w$ is used to measure the distance along streamlines in the curvilinear coordinate system. The main reference curve $P(v,0,0)$ is the origin for streamwise distance. The streamwise parameter $w$ corresponds to the measurement $x/R$ in the more traditional cylindrical coordinate system.

To establish the curvilinear coordinate system, two generating tubes and a main reference curve must be specified in cylindrical coordinates. The radii of the hub and tip generating tubes are defined at axial stations, and a main reference curve is specified from the hub streamline to the tip streamline by the designer. The value of $v$ at a point $P$ is determined by measuring the normalized arclength along a reference curve from the hub to the point $P$. The distance is non-dimensionalized by the arclength along the same reference curve from the hub to the tip of the propeller. Figure 3.4 shows the method for determining $v$ at a point $P$.

![Figure 3-4: Definition of $v$ at point $P$](image)

In Figure 3-4, the symbol (L) represents the arclength along a constant parameter curve. The value of $v$ is determined using the following equations.

$$v = \frac{\text{arclength}}{\text{arclength}} = \frac{L_{v=v_0}^{v=v_c}}{L_{v=v_0}^{v=v_1}}$$

arinlength = \int ds

$$ds = \sqrt{(dx/dt)^2 + (dr/dt)^2} \, dt$$

where $dt$ is tangent to the reference curve $P(v,c,0)$
In a similar fashion, the value of $w$ is defined as the arclength away from the reference curve along each generating tube. The parameter $w$ is non-dimensionalized by the total arclength along the main reference tube, $P(v,0,0)$, and not along the reference tube at point $P$. Figure 3-5 shows the method.

![Diagram](image)

**Figure 3-5: Definition of $w$ at point $P$**

The following equations describe how the parameter $w$ is measured.

$$w = \frac{\text{arclength}_{0}^{w=c}}{\text{arclength}_{0}^{v=1}} = \frac{L_{w=0}^{w=c}}{L_{v=1}^{v=0}}$$ (3.2)

$$\text{arclength}_{0}^{w=c} = \int ds$$

$$ds = \sqrt{(dx/dt)^2 + (dr/dt)^2} \, dt$$

where $dt$ is tangent to the generating tube $P(c, w, 0)$

$$\text{arclength}_{0}^{v=1} = \int ds$$

$$ds = \sqrt{(dx/dt)^2 + (dr/dt)^2} \, dt$$

where $dt$ is tangent to the main reference curve $P(v, 0, 0)$

The angle $\theta$ is the same for both cylindrical coordinates and curvilinear coordinates.

The traditional definition of a nose-tail line for propellers is a helix, which has the characteristics of constant pitch angle and constant pitch, and is a geodesic. When the cylindrical sections are generalized, all three properties can not be maintained. The nose-
The nose-tail curve can generally preserve only one of the three traits: constant pitch angle, constant pitch, or a geodesic. Figure 3-6 shows the resulting nose-tail line for a cone using the three methods.

**Nose-Tail Lines**

![Nose-Tail Lines Diagram]

**Figure 3-6: Three Definitions of Nose-Tail Curve for a Conical Hub**

The first definition of the nose-tail curve is the constant pitch angle method, described by Neely. Because the radius may not be constant along the chord of the section, the pitch of the nose-tail line may vary. In order for the pitch angle to be constant, the following relationship must hold.

$$\frac{\partial \theta}{\partial \omega} = \text{constant}$$

This is the simplest definition and the one that was chosen.

The second definition is a constant pitch method. The nose-tail is along a line that will advance a constant axial distance for each increment of angular displacement along the curve.³ This effect couples the radius of the nose-tail line, and the angle, $\theta$, as shown in the following relationship.

$$\frac{1}{r} \frac{d\theta}{d\omega} = \text{constant}$$
The pitch of the propeller section is kept constant even though the radius may be varying. As shown in Figure 3-6, for a conical hub the leading edge of the propeller is farther away from the axis of rotation than the trailing edge. Therefore, the pitch angle is shallower at the leading edge, even though the pitch is constant.

The final definition of a nose-tail line is a geodesic between the nose and tail of the section along the surface of the generating tube. A geodesic is the curve of minimum curvature and distance between two points on a surface. A geodesic will satisfy the following conditions.

\[
\frac{dt}{ds} = \kappa n
\]
\[
\frac{d\chi}{ds} = t
\]

where \( t \) is the tangent vector to the curve, \( s \) is the arclength along the curve, and \( n \) is normal to the generating tube. The locus of the curve is \( \chi \) and \( \kappa \) is the local curvature of the surface in the direction of the curve. The "Great Circle" routes of airplanes around the globe are geodesics because the locus of the curve is the center of the earth, and therefore, Great Circle routes are the shortest distance between two location on the globe. For the same reason, longitudinal lines around the earth are geodesics, while lines of constant latitude are not. For a conical hub, a geodesic would have the following characteristic.

\[
r \frac{d\theta}{dw} = \text{constant}
\]

As shown in Figure 3-6, a geodesic will have a larger pitch angle at the larger radii, because the arclength around the smaller radii is shorter.

---

After the nose-tail line has been defined, pitch, camber, and chord can be calculated. Camber is measured normal to the nose-tail curve, and the chord is measured along the nose-tail curve. Thickness is added to the camber line normal to the nose tail line and along a generating tube. Skew is defined as the angular coordinate $\theta_m$ of the mid-chord line. Rake is defined as the value of the parameter $w_m$ at the mid-chord line.

The proposed geometry system systematically generalizes the more traditional cylindrical convention. By doing so, a large family of radially symmetric bodies can be described without the difficulties associated with high conicity. The propeller sections can be defined along streamtubes, eliminating trimming problems and giving the designer more accurate control of the propeller loading and pressure distributions.

The current design method was modularized to incorporate differing geometry conventions. Though the current definition of geometry is simple and general, the definition of the geometry is largely a matter of convention and convenience for the individual design and designer. The simplest and most convenient system should be used in the design process. To facilitate this, the design method has separated the geometry definitions and the design problem. The B-spline surfaces are generated from a set of Cartesian coordinates, which describe the blade. Essentially, any code that produces Cartesian coordinates from the current definition of chord, pitch, rake, skew, camber, and thickness will suffice as the geometry module of the designer software. Therefore, any of the three definitions of the nose-tail curve can be used, so long as a module can generate xyz points from design parameters.
3. 3. B-SPLINES

B-splines are a mathematical family of curves that accurately and easily model an arbitrary surface, such as a propeller blade. More specifically, B-splines are a subset of the class of curves and surfaces termed Non-Uniform Rational B-splines (NURBS), as described by Kerwin. They have become popular in the computer-aided design community in recent years. For the purposes of this thesis, the characteristics of B-splines are given here.

A B-Spline curve has the following properties.

- The spline is defined by a control polygon.
- The beginning and end of the polygon net are the beginning and end of the spline.
- The slope of the curve at the endpoints is tangent to the polygon.
- Unlike the endpoints, the other points on the control polygon are generally not on the curve.

The B-spline representation of a curve in space is:

\[ P(w) = P(x(w), y(w), z(w)) = \sum_{j=1}^{N_n} V_j N_j^k(w) \]

where \( P \) is a point with the Cartesian coordinates \((x,y,z)\). \( V_j \) are the coordinates of the control polygon that set the shape of the curve and \( N_j^k \) are the B-spline basis functions of order \( k \). The parameter \( j \) steps through the individual points on the control polygon, and \( w \) is a monotonically increasing parameter along the arclength of the curve. The value of \( k \) has been chosen to be \( k=4 \) for this application. This means that the basis functions will consist of piecewise 4th order cubic polynomials in the parameter \( w \). These small polynomials are pieced together to form the curve. The discontinuous point at which two

\[^4\text{Kerwin, Geometry Class Notes, Chap 5. MIT 1994.} \]
4th order polynomials meet is called a knot. The polynomials are constructed in such a way as to match in value, slope, and curvature of adjacent polynomials at the knot points. Curves of this type are referred to as "uniform, integral B-splines." They possess the requirement of curvature continuity that is essential for a fair surface, but the continuity of higher derivatives is unnecessary. The final requirement is that at the beginning and end of the curve, the slope and value of the B-spline surface agrees with the control polygon. Figure 3-7 shows the fourth order basis functions for a four vertex control polygon and Figure 3-8 shows the B-spline curve and the B-spline control polygon. The family of curves shown in Figure 3-7 are outlined in the following set of equations.

\[
\begin{align*}
N_1^4(w) &= (1 - w)^3 \\
N_2^4(w) &= 3w(1 - w)^2 \\
N_3^4(w) &= 3w^2(1 - w) \\
N_4^4(w) &= w^3
\end{align*}
\]

A B-spline surface is described by a network of control polygons, with parameters \( u \) and \( w \). Points on the surface are obtained from the sum of the products of two basis functions, one for the parameter \( u \) and one for the parameter \( w \).

Unlike a polynomial, the coefficients in a B-spline curve (or surface) define a polygon curve (or net) that looks like the geometry, and therefore has intuitive meaning. In addition, by moving one control point, the curve can be locally modified. A similar perturbation of a cubic polynomial coefficient would have a global and unintuitive effect on the curve.
A B-spline surface is analogous to a spring system, like a trampoline bed. The starting and end points are fixed in slope and location by the polygon. The other points exert a spring-like pull upon the curve, without guaranteeing that the control point is on the curve itself, as shown in Figure 3-8.

B-splines are also convenient to use. The number of points required to define the B-spline surface is small, with a 7x7 grid of vertex points defining the propeller geometry satisfactorily. The B-spline surface is easy to manipulate during the design process, because each B-spline vertex has a local effect on the blade. Figure 3-9 shows how a perturbation in the B-spline net has an effect on the blade shape. The geometry is
intuitively linked to the position of the B-spline vertex. If a similar perturbation was applied to one coefficient in a cubic polynomial, the effect on the blade would be global, not local, and unintuitive.

Figure 3-9: Local Perturbation of a B-Spline Surface
3. 4. EXAMPLE

To demonstrate the non-cylindrical geometric definition method and the automated B-spline procedure, a sample propeller blade is generated. The blade will be fitted with a B-spline surface, which will then be queried to determine the Cartesian points of the blade. Kerwin’ developed the codes XYZ2B and B2XYZ to handle this step. Finally, the initial and final blade parameters will be compared to show consistency. For purposes of demonstration, the designer environment will never be entered, so that the blade shape will be preserved throughout the process. That is, we merely redescribe the blade shape using the various methods.

The first step is to process the designer parameters of chord, pitch, rake, camber and skew to create a Cartesian file of points, using the non-cylindrical parameter definitions. The output from this process is a Cartesian coordinate geometry file, as shown in Figure 3-10. The Cartesian points are fitted with a B-spline surface. The output from this process is a B-spline network, shown with the fitted surface in Figure 3-11.

![Figure 3-10: Cartesian Points of a Sample Propeller Blade](image-url)
At this point in the calculation, the B-spline network would then be modified by the propeller blade design process. For the purposes of this demonstration, we will skip the blade design process, so as to validate the geometry codes.

The B-spline geometry is then queried to determine the Cartesian coordinates at a network of streamtubes and chordwise positions. The B-spline surface is evaluated along streamlines to determine Cartesian points that represent the blade. The output from this code is a Cartesian coordinate file, as shown in Figure 3-12.
The final step is to use the Cartesian points to develop the section shapes. From the section shapes, the chord, pitch, camber, skew, and rake are measured. The output from this step is the design parameters, shown compared to the input design parameters in Figure 3-13 through Figure 3-15. The geometry is the same after all of the translations, except for the fairing that the B-splines did. A greater number of B-spline vertices would more closely represent the blade, if fairing of the blade not desired.

If this was an actual PBD10.2 design iteration, and after examining the output, further tuning was needed, then a restart file from PBD10.2 can be used as the starting point for the next iteration. The restart file contains the B-spline surface representation of the blade. The geometry can be further tuned, without a translation step between

Figure 3-12: Cartesian Point of a Sample Blade from a B-Spline Surface
designer parameters and B-spline surface. If the design is satisfactory, the B-spline surface would be passed to the manufacturing codes or structural computations. Because the blade shape is now universally and continuously defined, communication of geometries has been improved.

Figure 3-13: Initial and Final Distributions of Chord

Figure 3-14: Initial and Final Distributions of Maximum Camber
Figure 3-15: Initial and Final Distributions of Pitch
4. PROPELLER DESIGN

Propeller design methods have continually evolved and become more sophisticated as the requirements on propellers have grown. Originally, empirical methods were used to design propellers; but these methods were more art than science. Lifting line methods represented the blade with a vortex-line in potential flow theory.1 Lifting line analyses are useful in determining global quantities like optimum circulation, but local quantities are not as well defined. More recently, a vortex-lattice model of the propeller blade described by Kerwin and Greeley,6 expanded the lifting line into a mesh of vortices, forming a lattice. The method proved to be very capable of describing the local flow about the blade. The current method extends this vortex lattice method to include rotational effects.

Propeller blade design methods develop propeller geometry from requirements of thrust in two steps. First, the spanwise circulation required to produce thrust is determined from a lifting line model of the propeller. Secondly, one finds the geometry that will produce this circulation most effectively. In this thesis, the second step is improved.

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4. 1. DESIGN METHODS

Vortex-lattice methods, originally developed by Faulkner\(^7\) and Kerwin,\(^8\) are potential flow methods that discretize the vorticity across a propeller blade and in the wake. These methods are considered intuitive because the geometry is represented by the blade latticework, and because the theory is similar to lifting line theory. The methods are robust and flexible for a variety of propeller shapes and aspect ratios. Because of these advantages, vortex lattice methods have become a valuable tool in the design of propellers.

PBD10.2 is an extension of a vortex lattice method developed by Greeley and Kerwin.\(^6\) and expanded upon by Leibman.\(^9\) The vortex-lattice method is used to solve the blade to blade flows for designing the propeller. An axisymmetric flow solver has been coupled to the design process to solve the axisymmetric, viscous propeller flows. The design method is described here.

Figure 4-1 is a simplified flow diagram of PBD10.2, demonstrating the design process for a propeller blade. The design method is divided into two problems, a potential flow blade problem and a vortical viscous axisymmetric flow problem. A vortex lattice method is used to solve the blade problem, and a vortex flow solver is used to solve for the axisymmetric problem. The vortex lattice method consists of four steps, a lattice generation step, an induced velocity calculation, an iterative blade shape manipulation, and finally, a blade force calculation. The design method is meant to be modular, to facilitate improvements in the viscous flow solver and changes in the definition of the geometry.


4.1.1. Coupling Propulsor and Hull Flows

Dividing the problem into an axisymmetric vortical hull problem and a blade-to-blade flow calculation is an extension to current propeller design methods that has been made by several researchers.\textsuperscript{10,11,12} The motivation for this work has been the troublesome issue of effective wake. Kerwin,\textsuperscript{5} laid the problem out in detail, but the

\textsuperscript{11}General Electric, at Groton, presentation on April 22, 1994
\textsuperscript{12}Uhlman, J., NUWC presentation at MIT on April 29, 1994
problem will also be outlined here. Scott Black$^{13}$ and William Milewski$^{14}$ were helpful
teammates in coupling these flows. They studied the body problem, while I studied the
propulsor flows.

The inflow to highly conical, heavily loaded propulsors is greatly influenced by
the interactions between the hull and the propeller. An extreme example would be a
submarine with a separated nominal wake, that relies upon the propulsor to reattach the
flow. In such a case, the effects of the propeller upon the vorticity in the boundary layer
are exacerbated by the high conicity of the hull. The effective wake is needed to model
this coupling between the vorticity in the wake and the effects of the propeller.

The total wake of the hull includes the viscous and potential effects of the
propeller. The nominal wake is the wake of the hull without the presence of a propeller.
The effective wake differs from the nominal wake because it includes the influence of the
propeller upon the vorticity in the wake. Propeller acceleration of the flow compresses
streamlines to maintain mass continuity. As a result, vorticity in the wake, such as the
vorticity caused by the boundary layer, is pressed in closer to the flow. Since overall
circulation is conserved, the vortical component to the flow increases. Figure 4-2 shows
a typical boundary layer for a conceptual hull. Though a boundary layer is small in
relation to the length of the hull, the boundary layer is significant when compared to the
diameter of the propeller.

\footnotesize
$^{13}$Black, S.D. "An Integrated Lifting Surface/Navier Stokes Propeller Design Method", S.M., MIT Dept of
Virginia Beach, Va.
In coupling the flow between the body and the blades of the propeller, it is helpful to define two problems, a body problem, and a blade problem. The body problem is axisymmetric, and consists of the flow around the body and the axisymmetric contribution of the propeller. The body may be an axisymmetric underwater vehicle, or may be the shaft and hub of a surface ship propeller. To signify quantities from this flow, the superscript \((o)\) is used. Typical tools for solving this problem are axisymmetric Euler and viscous flow codes.

The second flow is the blade problem, which is the flow produced by a set of blades operating behind the axisymmetric body. This problem is traditionally solved by lifting line or vortex lattice methods. The superscript referring to these flows is \((\oplus)\).

Using this notation, the total velocity field can be represented as

\[
V^\oplus = V_e^\oplus + \bar{V}_i^\oplus + \tilde{V}_i^\oplus
\]

where \(V_e^\oplus\) is the effective inflow, and \(V_i^\oplus\) is the total induced velocity, which can be broken into a mean induced velocity, \(\bar{V}_i^\oplus\), and a fluctuating velocity, \(\tilde{V}_i^\oplus\). Figure 4.3 demonstrates the radial distribution of velocity for a one-bladed propeller, admittedly a hypothetical case.
The induced velocities can be calculated using traditional potential flow methods, such as vortex-lattices. Unfortunately, $V_\infty$ is not measurable, and cannot be calculated from potential flow approaches. The effective wake is a model of the propeller’s influence on vorticity in the wake, and as such, must be treated with a flow solver.

Because of the complexity of three-dimensional flow solvers, it is useful to assume an axisymmetric representation of the effective wake, $V_\infty^e = \bar{V}_\infty^e = V_\infty^o$, as was shown in Figure 4-3. Since the inflow (nominal wake) is restricted to axisymmetric, this assumption is correct from potential flow theory. However, vorticity in the wake is modified by the induced velocities of the propeller. These induced velocities are not axisymmetric and therefore, the effective wake loses its radial symmetry in the presence of blade fluctuations. If the propeller had an infinite number of blades, the induced velocities and the effective wake would be axisymmetric, but for finite blades, the effective wake has a fluctuating term, $V_\infty^e = \bar{V}_\infty^e + \tilde{V}_\infty^e$.

The strength of the blade variations, $\tilde{V}_\infty^e$, changes spatially through the wake. The fluctuations are largest at the blade, and disappear rapidly as the distance away from the plane of the propeller increases. To account fully for these fluctuations, a three-
dimensional viscous flow solver is needed. For this thesis, the effective wake was assumed to consist only of an axisymmetric component, \( V_e^\circ \approx \bar{V}_e^\circ = V_e^\circ \).

4.1.2. Equivalent Force Calculations

To couple the RANS solver and the vortex-lattice code, the mean flow must be the same between the blade problem and the body problem. Kerwin\(^7\) shows that for the mean flow to be identical, the force on the blade distributed axisymmetrically (i.e. multiplied by the number of blades and divided by \(2\pi r\)) will, in general, be different from the force necessary in the axisymmetric flow solver. The equivalent force in the body problem differs from the force on the blade because of the fluctuations in induced velocities. Figure 4-3 indicates the radial distribution of velocities at the plane of the propeller for a single blade. The varying induced velocities are generally greatest at the blade and create asymmetries which can not be accounted for by the RANS code. Only in the case of an infinite number of blades will the two forces be the same, because the induced velocities no longer contain a fluctuating component.

The proper equivalent force for the body problem is derived by keeping the circulation in the flow constant for both problems. Figure 4-4 shows the velocities for a single-bladed propeller, at two radial locations of Figure 4-3. The first set of velocities is across the blade, or the 0° case. The second set of velocities are 180° away from the propeller blade, and finally, the circumferential mean velocities are shown. The effective wake is identical for all three cases because of axisymmetry. However, the induced velocities are not axisymmetric; the effects of the blade are strongest at the blade and weakest 180° away from the blade.
Figure 4-4 : Velocities for a One-Bladed Propeller

Figure 4-4 demonstrates the blade to blade variations in a propeller induced velocity field. Because of the extra energy wasted in these variations, the one bladed propeller will be less efficient than an infinite bladed propeller. For a constant circulation, an infinite bladed propeller will have slightly higher thrust and significantly lower torque than a finite bladed propeller, a result which is anticipated by the Kramer diagram, in Figure 4-5.\textsuperscript{15}

Because the blade generally induces the largest velocities upon itself, the blade operates in the peak of the induced velocity fluctuations. The differences are greatest for a highly loaded single blade. The force upon the blade is determined through Kutta-Joukowski's theorem.

\[ F = \rho VT \]  \hspace{1cm} (4.1)

Figure 4-5: Kramer Diagram

Because the velocity at the blade contains the fluctuations in induced velocities, as shown in Figure 4-3, the force on the blade is different from the force necessary to model the axisymmetric flow with the RANS solver. The blade forces are slightly different to account for the fluctuations. However, if the velocity $V$ in equation 4.1 is the mean velocity and does not include the blade-to-blade fluctuations, then the equivalent force is computed. This equivalent force is the correct force to use in the body problem because then the circumferential mean induced velocities are modeled in both problems. The idea
of an equivalent force is further developed in the upcoming sections on induced velocity and forces.

Now that the two mean flows are identical between the body problem and the blade problem, we must decide on the division of labor between body problem solver and the blade problem solver. The circumferential mean velocities can be calculated in either the RANS code or the vortex-lattice solver, as outlined below.

1. The circumferential mean induced velocities can be calculated from the induction of the vortex lattices. Using the total velocity from the RANS code and the circumferential mean velocity from the vortex-lattices, an effective wake is found.

\[
V_e = V^0 - \overline{V}_i
\]

Using the effective wake as an input, the blade solver finds the total induced velocities and aligns the blade.

2. The circumferential mean induced velocities are calculated by the RANS solver, using forces developed in the vortex-lattice calculations. Therefore, the vortex lattice code is calculating the blade-to-blade velocities only, and the RANS solver is modeling the rest of the velocities.

\[
V^\circ = V^0 + \overline{V}_i
\]

Using the axisymmetric solution from the RANS code, the blade solver adds in only the fluctuating induced velocities for aligning the blade. The mean induced velocities have been modeled already by the body solver.

Of the two methods, the second was chosen, though others have typically chosen the first route. This choice was motivated by the fact that the RANS flow solver is a better model of hub and duct effects than simple vortex-lattice images.
4. 2. BLADE PROBLEM

4. 2. 1. Input

Figure 4-1 shows the design method, which consists of a blade problem and a body problem. The blade problem solves for the fluctuating induced velocity components, and aligns the blade to the total flow. The body problem solves for the effective wake and the mean induced velocities around the hull. The effective wake and the propeller geometry are needed as input to the blade problem. From the propeller geometry, vortex lattices models of the blade, hub and duct images, and transition wakes are created. The fluctuating induced velocities are calculated and added to the axisymmetric solution of the body flow. Using the total velocities, the propeller blade is aligned. Axisymmetric forces are calculated using the axisymmetric velocities, and total blade forces are calculated to get the thrust and torque of the blade.

An initial estimate of the propeller geometry is needed to start the design process. The mean camber surface of the propeller blade is represented by a B-spline surface, which is created from initial estimates of chord, pitch, rake, skew, and camber. Initial guesses for geometry generally come from a lifting line analysis of the propeller. From a lifting line code, the chord, camber, and design spanwise loading distribution are determined. The initial chordwise distribution of loading can be based upon a NACA 66 a=0.8 mean-line camber distribution. The experience of the designer is needed to determine the skew and rake of the propeller, because the design method does not make predictions about unsteady forces.

The propeller effective wake field is also needed as input. A generally acceptable starting point for effective wake is the nominal wake of the propeller. If an experimentally determined effective wake is not available, the viscous flow solver can be used. The method then converges upon the correct flow field. For highly conical bodies, the propeller is often used to reattach the separated flow, and in this case, the nominal
wake is not a good starting point. The negative velocities at the root of the blade are too severe for a successful completion of the vortex-lattice code. In such a case, a rough estimate of the propeller forces is used to accelerate the flow in the RANS model, and reattach the separated flow near the hub. The technique is analogous to experimental techniques which use a stock propeller to simulate the effective wake of the final design.\textsuperscript{13}

4.2.2. Vortex-Lattice Discretization

The first step in the design process is to discretize the blade and wake. The distribution of bound and free vorticity in the blade is represented by a vortex-latticework, as shown in Figure 4-6. This gridwork is aligned approximately along and orthogonal to the streamlines in the flow. Therefore, the bound vorticity is represented mostly by spanwise vortices, and the free vorticity is represented by streamwise vortex segments.

![Figure 4-6: Propeller Vortex-Lattice](image)

Figure 4-6: Propeller Vortex-Lattice
The wake of the propeller is divided and modeled in two parts, the transition wake and the ultimate wake. The transition wake is a more detailed model than the ultimate wake, but since the ultimate wake is “far” away from the propeller, the simplifications speed the design process without impacting the accuracy. In the transition wake, the free vorticity is represented by a vortex latticework. Bound vorticity does not exist in the wake. The transition wake extends two propeller diameters downstream from the trailing edge of the blade, shown to be sufficient by Leibman. At the end of the transition wake, the ultimate wake begins. The effect of free vorticity in the ultimate wake is modeled a family of generalized actuator disks, developed by Hough and Ordway. Figure 4-7 demonstrates the structure of the wake.

Figure 4-7: Wake Structure

---

Duct and hub image vortex lattices model the effects of the hub and duct. For propellers with loading at the root or tip, the effects of the hub and duct are significant. Figure 4-8 shows a circulation distribution with and without loading at the hub. When a hub is not modeled, the circulation at the tip of the propeller must be zero. Analysis has shown that with hub loading, the slope of the circulation at the wall should be zero. Kerwin and Leopold\textsuperscript{17} showed that the method of images could be used to represent the effects of the hub in the potential flow analysis of propellers. The method of images is based on the axisymmetric result that two point vortices of equal but opposite strength induce zero normal velocity on a circle of radius $r_h$ when the following relationship holds true.

\[
r_i = \frac{r_h^2}{r_v}
\]

The subscript (h) refers to the hub, but is equally applicable to the duct image. From this equation, an image grid models the wall effects of the hub and duct. For a three-dimensional helix, the method of images is acceptable at modeling the hub, so long as the pitch of the helix is large.\textsuperscript{9} Figure 4-9 shows the case of a three-dimensional vortex and its image about a cylinder.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4-8.png}
\caption{Circulation Distributions With and Without Hub Loading}
\end{figure}

The simple image lattice method models the effects of the hub and duct very accurately, as long as the pitch angle is high. Though the pitch angle at the root is generally high, the rotational velocities near the duct can drive the pitch angle down. In addition, conicity in the flow can lessen the accuracy of the hub image. Because of these inaccuracies, the RANS code was used to model the circumferential mean effects of the hub and the duct.

Panel methods can model hubs and ducts very accurately. Also known as boundary integral methods, panel methods use singularities distributed over the surface of the panel and duct to represent a wide range of geometries. Hess and Valazero\textsuperscript{18} applied sources to model the effects of a hub in 1985. Lee\textsuperscript{19} and Hsin\textsuperscript{20} developed PSF10, which uses a potential based panel method to model the hub. Caja\textsuperscript{21} illustrated that panel methods modeling the hub and duct of the propeller closely agree with the image method of Kerwin and Leopold.

Leibman classified the interactions for a propeller with no duct as *hub on hub*, *hub on propeller*, and *propeller on hub*. The effect of the hub on the propeller is the most significant, and can be accurately modeled by a vortex image lattice. The effect of the hub on the hub is negligible for cylinders and becomes more important for full-sterns. Finally, the effect of the propeller on the hub can be ignored unless the hub flow is changed enough to affect the propeller flow. In essence, this is a second order effect that is ignored in simplifying the problem. Leibman then concludes that a simple image lattice is the best compromise between efficiency and accuracy.

The use of a RANS code to solve the body problem improves the image model in several ways. First, the hub on hub problem can be important if the stern is full. Boundary layers and separation can not be modeled by the potential flow image, but are handled by the axisymmetric body solution. Secondly, the body problem has been formulated to include the mean induced velocities of the propeller. Using the categorizations of Leibman, the hub on propeller problem is now divided into a axisymmetric hub on propeller problem and a blade-to-blade hub on propeller problem. The axisymmetric problem is modeled by the RANS code, which accurately represents the hub and duct. The blade-to-blade fluctuations are solved using vortex-lattice images, which are very good at modeling the effects of the wall locally. Because the fluctuations are small and local, image lattices are sufficient. This was one of the major motivations for solving for the mean induced velocities in the body problem and not the blade problem.

Figure 4-10 shows an image hub grid, along with the hub image wake. Experience has shown that only a third of the blade gridwork needs to be modeled. The rest of the vorticity is too far away to contribute to the induced blade-to-blade variations at the hub. Taking advantage of this feature can help to speed the blade solution.
4.2.3. Induced Velocities

Induced velocities are calculated by the vortex-lattice method and by the RANS code. The induced velocities are broken into circumferential mean velocities, $\bar{V}_i^\circ$ and fluctuating induced velocities, $\tilde{V}_i^\circ$. The mean induced velocities are totaled from forces passed to the RANS code. The fluctuating component is calculated from the radial distribution of induced velocities is found using the vortex-lattice methods of PBD10.2.
The RANS code calculates the axisymmetric flow about the body, including the mean induced velocities of the propeller. The propeller blades are modeled by passing the effective forces on the propeller lattice to the RANS code. These effective forces are converted to axisymmetric body forces and interpolated onto the RANS gridwork of the flow. This process is described by Black.\textsuperscript{13}

For a straight vortex segment of strength $\Gamma$, the induced velocity is shown by Kerwin\textsuperscript{15} to be:

$$w(y) = \frac{\Gamma}{4\pi} \int_{s_1}^{s_2} \frac{y d\zeta}{(\zeta^2 + y^2)^{3/2}}$$

$$w(y) = \frac{\Gamma}{4\pi y} \left[ \frac{\zeta}{(\zeta^2 + y^2)^{1/2}} \right]_{s_1}^{s_2}$$

$$w(y) = \frac{\Gamma}{4\pi y} \left[ \frac{x_2}{\sqrt{x_2^2 + y^2}} - \frac{x_1}{\sqrt{x_1^2 + y^2}} \right]$$ \hspace{1cm} 4.3

The variables in Equation 4.3 are defined in Figure 4-11.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4_11.png}
\caption{Calculation of Induced Velocities for a Straight Line Vortex Segment}
\end{figure}

Using this equation, the contributions of each vortex segment in the lattice are summed to find the total induced velocities. However, the final results are the total velocities, and
not simply the blade-to-blade fluctuations that are necessary for coupling the blade and body problems.

To carry out the coupling between the body and blade problem, the radial distribution of induced velocities must be determined, and then averaged. In the blade solution, the induced velocities contributions of the blades, wakes, hub images, and duct images are calculated at several angular positions. Using a Romberg integration scheme, the mean induced velocities are calculated. Many thanks are owed to Professor Kerwin for developing this scheme. A ring of calculation points is shown in Figure 4-12. The mean induced velocities are then subtracted from the total induced velocities to compute the blade-to-blade fluctuations in induced velocities. It is these varying velocities which are added to the RANS code axisymmetric solution, before the blade is aligned.

Figure 4-12 : Computation Domain for Calculation of Average Induced Velocities
4. 2. 4. Blade Alignment

The blade shape is determined by aligning the blade with the flow. In this scheme, the velocity is calculated at the center of a vortex ring, i.e. at the control point in the grid. The inflow, the key blade lattice, the other blade lattices, the wake lattices, and the image hub and duct lattices contribute to the velocity at the control point. If the blade is properly aligned, then the velocity will be purely tangential to the blade; there will be no velocity through the blade surface. If the blade is improperly aligned, the velocity normal to the blade will indicate the direction and magnitude that the blade needs to be displaced to maintain the boundary. In this way, the pitch and the camber of the blade shape can be determined. For the equations to be uniquely solved, an arbitrary reference line must be established. In PBD10.2, the reference line is generally the leading edge of the blade. With the leading edge fixed, the code acts like a flag simulator, pushing and pulling the propeller into alignment like a stiff flag behind a flag pole. The leading edge is useful because of its importance in unsteady performance. However, any radial lattice can be used as the fixed axis, including the trailing edge of the propeller.

The alignment is accomplished by modifying the B-spline vertices of the blade surface as described in Leibman\textsuperscript{3}, and as shown in Figure 4-13. The amount to move the B-spline vertices is determined by solving an over-constrained system of linear equations in a method of least-squares. First the normal velocity is computed at each control point, shown in vector form below.

\[
[(\tilde{V} \circ \hat{n})_{\text{base}}] = (\tilde{V} \circ \hat{n})_{\text{base, i}} \quad \text{for } i = 1, \text{ number of control points} \quad (4.4)
\]

Next, each vertex in the B-spline net is perturbed a small amount, \(\Delta s_j\), for \(j=1\), to the number of control points, to find the change in the normal velocity vector, \(\Delta (\tilde{V} \circ \hat{n})\). From this, the influence coefficient matrix, \([A]\), is found.
\[ A_{ij} = \frac{\Delta(\bar{V} \circ \hat{n})_{i}}{\Delta s_j} \]

\[ [A] \equiv \begin{bmatrix} \frac{\partial (\bar{V} \circ \hat{n})}{\partial s} \end{bmatrix} \] (4.5)

The resulting linear system of equations is in the form
\[ Ax + b = 0 \]
\[ As + (\bar{V} \circ \hat{n})_{base} = 0 \] (4.6)

The system of equations is over-constrained if the number of control points is greater than the number of B-spline network vertices, which is almost always the case. Experience has proven that the method is robust and reliable for a large range of geometries.

The direction of the perturbance \( \Delta s_j \) has a large effect on the alignment procedure. Because the alignment procedure is used to determine pitch and camber, it would be useful for the perturbance to lie along a streamtube, normal to the camber of the blade. If the blade surface normal vector does not lie on the streamtube, the perturbance direction will be the projection of this normal onto the streamtube, as shown in Figure 4-13. If the true normal is used, instead of the projected normal, the tip and root propeller sections could move off their respective streamtubes. Therefore, to preserve the tip and root, the projection is used.
Figure 4-13: Normal and Perturbation Vectors

Figure 4-14: B-spline Polygon Mesh and Blade Surface. The Second Lattice has been Locally Perturbed.
Figure 4-14 shows how powerful this blade alignment scheme is. The B-spline network represents the overall geometry intuitively. By moving one vertex, the local shape of the blade has changed, in an intuitive manner. If one of the coefficients were similarly perturbed for a high-order polynomial, the effect would be global and unintuitive.

4.2.5. Force Calculation

There are two sets of forces that need to be calculated. The first type of force is the blade forces, which are summed into thrust and torque coefficients. The second type of forces are the effective forces. The effective forces are passed to the Navier-Stokes code. Both forces are calculated using the equation

\[ F = \rho \vec{V} \times \vec{\Gamma} \]

If the effective forces are being calculated for the RANS code, the velocity includes only the effective wake and the mean induced velocities from the axisymmetric body solution. However, if the total forces are being calculated, then the blade-to-bade induced velocities from PBD10.2 are also included.

Figure 4-15 shows the blade forces and the circumferential mean force contours across the blade. The first comparison shows the axial forces, and the second shows the tangential forces. The mean axial forces and blade tangential forces are larger than their counterparts. From Figure 4-10, it is clear that the mean induced velocities are less than the induced velocities at the blade. Figure 4-10 also shows that the mean axial velocity is less than the axial velocity at the blade, while the mean tangential velocity is greater than the tangential velocity at the blade. Since the force is related to the cross-product of velocity and a fixed circulation, one would expect that the mean axial force would be greater than the blade axial force, because of the differences in tangential velocity. Likewise, the mean tangential force would be less than the blade tangential force. These results agree with Figure 4-15. Figure 4-16 shows the circulation on the blade extracted
from a body solution, as measured from the tangential velocities in the flow. The circulation is higher than the prescribed circulation if blade forces are used, but agrees nicely when the effective forces are used in the RANS code.

Figure 4-15: Comparison of Blade and Circumferential Mean Forces

Figure 4-16: Circulation Distributions Based on Blade and Effective Forces
5. STUDY OF THE DESIGN PROCESS

The propeller design process is studied here to analyze the critical tools in the method. Propeller designers search through a variety of designs for acceptable solutions to the design requirements and for an optimal design. Future propeller designs will take advantage of integrated and automated design tools to help reduce design time and to find more efficient solutions. A global optimization tool will first define the acceptable design space, and then efficiently search the design space to determine the optimal acceptable solution. As a first step in this, the design process is analyzed to determine the tools that most define the geometry. These critical design codes are targeted as the first tools to automate and optimization techniques.

Propeller design consists of defining a propeller geometry that maximizes efficiency while meeting performance criteria in the categories of thrust, cavitation, weight, and unsteady forces. It is an iterative process without a unique solution. Stated formally, the problem is this:

\[
\text{Maximize } \eta = \frac{TV_a}{\omega Q},
\]

where \( \eta \) = efficiency, \( T \) = thrust, \( V_a \) = velocity, \( \omega \) = rotative speed, and \( Q \) is the torque. The independent variables are chord, pitch, rake, skew, camber and thickness, and the constraints are on thrust, diameter, structure, cost, cavitation, unsteady forces and weight. The length of the list of constraints indicates that the acceptable solution space is small. In addition to meeting all of the requirements and optimizing efficiency, a successful design will minimize the time and cost of the design process.
5. 1. PLAYERS

The design of a Naval propulsor makes a particularly good example of the design process. A Naval propeller design involves four parties; NAVSEA, NSWC, the Manufacturer, and the Fleet. The design requirements are specified by the Naval Sea Systems Command, (NAVSEA), the engineering agent of the Navy. The Naval Surface Warfare Center (NSWC), is chosen as the design agent in this example of the design process. NSWC is a design and research department of the Navy, and is a typical design agent for Naval propellers. Four different departments in NSWC design and test the propeller before the final geometry is decided upon. NAVSEA receives the final propulsor geometry from NSWC and issues a contract to a manufacturer. NSWC works closely with the propeller manufacturers to insure the design is successfully delivered to the fleet and is successful in sea trials. At any point, a reiteration could occur in the design of the propeller. The flow of information is shown in Figure 5-1,

Figure 5-1 : Flow of Information Through the Design Process
5.2. DESIGN SPIRAL

The design spiral demonstrates the iterative nature of propeller design. It is a method of searching a design space for a solution that meets the design specifications and optimizes efficiency. At each station of the design, the information about the design is improved, based upon the previous calculations. The method is a mixture of computational models and physical testing, used to determine an optimal propeller geometry. Final testing is used to confirm the performance of the design.

![Design Spiral for Naval Propellers Using MIT Codes](image)

Elimination of extra design spiral iterations saves time and money in the design process. Testing is particularly time-consuming and expensive. Therefore, modern propeller design methods try to improve the accuracy of the computer models. As these codes become more reliable, testing and design spiral reiterations are reduced.
5. 2. 1. NAVSEA

The design spiral starts with a list of performance requirements specified by the owner, in this case NAVSEA. From the design of the hull, NAVSEA will have determined the resistance curve. The available power and shaft rpm will be fixed by NAVSEA's choice of a power plant. In addition, the limiting criteria on propeller diameter, cost, weight, cavitation performance, and unsteady forces will be specified from the design and mission requirements of the ship. If an acceptable design can not be found, trade-off studies will be carried out between NSWC and NAVSEA to determine feasible performance goals.

5. 2. 2. Naval Surface Warfare Center

5. 2. 2. 1. Experimentalists

While most of the data necessary for starting the design comes from NAVSEA, a few parameters must be determined through testing and RANS calculations. The next step in the design spiral is to determine the thrust reduction factor \((1-t)\), the nominal wake harmonic analysis, and the effective wake. As RANS codes improve in accuracy and reliability, expensive and time-consuming efforts in the towing tank are reduced. The nominal wake is measured in the towing tank or is calculated by three-dimensional RANS simulations. The nominal wake is analyzed spatially to find the dominant wake harmonics. From this harmonic analysis, the number of blades on the propeller is chosen to minimize vibrations. The thrust deduction factor is a measure of the increase in resistance of a ship hull that is subject to the pressure field of a propeller. This can be either measured in towing tank testing or determined through RANS simulations. Finally, the total wake of the hull is measured when self-propelled by a stock propeller. The induced velocities of the stock propeller are subtracted off of the total wake to
calculate an effective wake. The coupling of a RANS code with PBD10.2 potentially removes the effective wake testing.

5.2.2.2. Propeller Designers

The Propeller Lifting Line code PLL is used in parametric design studies to determine optimum circulation and chord distributions. It is a good preliminary design tool for searching for acceptable solutions because the code quickly predicts the hydrodynamic and cavitation performance of a propeller. PLL gives a good indication of the propeller loading needed to produce the required thrust while meeting the other constraints. Typically, the cavitation performance a propeller with optimum loading is unacceptable. Therefore, an constrained optimal propeller design must be found, taking into account all of the design requirements. From the propeller loading, preliminary estimates of blade chord, thickness, and camber are determined. The preliminary blade shape and the wake are used with cavitation bucket diagrams to determine the preliminary cavitation performance of the blade. At angular increments, the characteristics of the foil are plotted on the cavitation bucket diagram. By doing this over a full revolution, estimates of cavitation performance are determined. Thrust breakdown estimates are made at this point in the design spiral. Tip vortex cavitation prediction is done by a simple algorithm added in PLL. After the parametric studies in PLL have been completed, the designer has a handful of candidate propeller geometries, which should be further developed using PBD10.2, unsteady force analyses, and structural analyses.

Before the next step is taken in the design spiral, preliminary estimates of rake and skew are needed. The rake is used to fit the propeller into the ship aperture and therefore to reduce ship vibrations. The skew distribution is determined from the harmonic wake analysis. At various radii, the wake deficit has an associated phase angle.
The skew distribution is designed to insure that the leading edge of each propeller blade section does not reach the deficit simultaneously.

Now that a preliminary geometry estimate has been made, a B-spline representation of the blade is created, using the code D2XYZ and XYZ2B. The B-spline distribution and the nominal wake file are used with PBD10.2 and the RANS code to determine final pitch and camber distributions. The effective wake is calculated in this process.

The next step is to predict the unsteady performance of the blade. Propeller Unsteady Force prediction codes time-step through a revolution of the propeller to determine the unsteady forces. If these forces are too large, a new skew distribution is determined, and PBD10.2 is run again. If the cavitation performance is unacceptable, a new loading and chord distribution may be needed. This involves more PLL iterations.

5. 2. 2. 3. Structural Analysis

The structure is analyzed to determine the strength of the blade. The propeller designers work with the structural analysts to determine the proper fillets for the connection of the blade to the hub. The thickness distribution is optimized to meet structural requirements and weight restrictions. If the two requirements can not be simultaneously fulfilled, then the blade geometry needs to be redesigned. Often the weak point in the blade is strengthened through slightly modified skew and rake distributions. Because of the changes, new lifting-surface predictions are needed.

5. 2. 2. 4. Experimentalists

A model of the proposed propeller is tested at NSWC for cavitation and open water performance. The open-water performance and self-propelled testing are done in the towing tank, and the cavitation performance is studied in the water tunnel. If the design is unsuccessful here, a new design iteration is needed.
5. 2. 3. Manufacturer and Fleet

Once the design has been approved internally at NSWC, the design is sent to NAVSEA for approval and contracting. NAVSEA sends the geometry to a propeller manufacturer, and then the finished propeller is tested in sea trials.

5. 3. DESIGN PROCESS ANALYSIS

The design process can be improved in two related ways. Overall, a reduction in the number of iterations is very cost efficient. This involves accurately defining the limits of the feasible design space. The second way to improve the design process is to improve the strategy by which the design space is searched for feasible and optimal solutions.

The number of iterations can be reduced by increasing the accuracy of the design predictions, so that the final testing step is a routine confirmation of the predicted performance. Design problems that are discovered at the testing stage are the most expensive to fix. The improvements in PBD10.2 increase the accuracy of the lifting surface step in the design spiral.

The second approach to improving the design method is to improve the strategy of the search for an optimal design. The choice of the proper search method to find the optimal solution has received a large amount of attention recently. Because of the large number of interrelated constraints on propeller designs, an efficient search algorithm can greatly enhance the design process. There are four popular search strategies that could be useful to propeller designs.

The first search strategy is the exhaustive search. The design space is searched through systematic and extensive parametric studies. Promising design neighborhoods are searched in a finer mesh to determine the best design. While this is a very thorough search of the design space, it can also be the most time-consuming. A knowledgeable
designer can direct the search, adding intelligence and speeding up the search for an efficient design. This is the method that is generally used.

Non-linear gradient methods calculate the change in the objective function for perturbations in each of the dependent variables. In the case of a propeller design, the change in efficiency would be calculated for incremental changes in chord, pitch, rake, skew, camber and thickness. Then the code would make the changes that prove most beneficial to the solution, as long as the constraints are satisfied. Gradient methods work best when the objective function is continuous with respect to the dependent variables. This approach is the most promising method for automated propeller design.

The third search strategy is called simulated annealing. In the annealing of steel, the metal is heated to a high temperature, and then allowed to cool slowly. The individual molecules crystallize into an organized structure. Another example is a box of blocks that is gently shaken until the blocks align into an organized pattern. In each case through random perturbations, the components organize into a compact structure because it is the "least-energy state." In a simulated annealing propeller design, the blade parameters would be randomly perturbed, and successful design improvements would be kept. Designs with high efficiency and that satisfy the designer requirements would have a lower state of energy than poor designs. This type of search is useful for design spaces with many local optima, but would not be suitable for propeller designs.

The fourth type of design search is based on genetics. The information describing the propeller is stored in a string of genes. Each individual gene would describe a particular characteristic (trait) of the blade, such as the chord distribution, or the number of blades. Successful propeller designs would "mate," crossing gene strings so that the next generation combines traits of previous successful designs in a random fashion. Successful designs are more likely to mate and produce offspring for the next generation than unsuccessful designs, which would be weeded out. This method also shows promise
as an efficient search algorithm, but only for the preliminary parametric searches of the design space.

5. 4. DESIGN AUTOMATION

Though all of the steps in the design spiral are important, certain steps define most of the blade geometry, while others only confirm that a single constraint is satisfied. At the beginning of the design spiral, the propeller geometry is not well known. As the design progresses, the geometry and the satisfaction of the performance criteria become more certain. Each step in the design contributes to the certainty of the final design, but in unequal amounts. Since design automation and optimization are a difficult process to apply to the whole design, the automation should focus on the most important steps in the design spiral.

To decide upon which steps were most important, Table 5-1 was created. Each column represents a step in the design spiral. The performance criteria and propeller geometry variables are listed in the first column. The numbers express on a scale of zero to five the certainty of the design at each step. For example, satisfaction of cavitation criteria can be initially determined in the lifting line analysis. The unsteady force analysis adds more certainty to the cavitation predictions. Finally, cavitation model testing determines the performance of the propeller. In this way, the certainty of knowledge was rated for each design variable and performance criteria.
Table 5-1: Certainty of Knowledge in Each Step of the Design Spiral

<table>
<thead>
<tr>
<th>Criteria and Variables</th>
<th>Testing</th>
<th>Line</th>
<th>Surface</th>
<th>Force</th>
<th>Perf.</th>
<th>Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Effective Wake</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>&lt; Weight</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>&lt; Cavitation</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>&lt; Unsteady</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>&lt; Diameter</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Section Type</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Circulation</td>
<td>0</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Chord</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Pitch</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Camber</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Thickness</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Skew</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Rake</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Total Gained Knowledge</td>
<td>7</td>
<td>33</td>
<td>17</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The total at the bottom of the table represents the amount that the design is developed in each step of the design spiral. From the total certainty of knowledge in one step, the previous step's total is subtracted to indicate how much the design has improved. Out of 70 possible points, 33 pieces of the puzzle fall into place during the lifting line analysis. Seventeen more pieces of knowledge are accumulated during the lifting surface analysis. This indicates that the lifting line and lifting surface analyses are the critical tools in the propeller design. Efficiency and accuracy gains in these steps would be most beneficial to the overall design process.

In addition, the similarities in theory between lifting surface methods and lifting line method make it easier to couple and automate these steps first. The procedures and inputs into the two codes are similar. The parametric studies are done at the lifting line and lifting surface steps of the design spiral. Parametric studies of the design space can
be improved with automated optimization techniques such as non-linear gradient searches.
6. VALIDATION and DEMONSTRATION

6.1. GEOMETRY MODELING

D2XYZ and a conventional cylindrical propeller geometry code were run on simultaneous cases to show the agreement between the two methods. The parameters of the propeller are shown below in Table 6-1. To make the comparison fair, the generating tubes are cylindrical and the main reference curve is a radial line.

<table>
<thead>
<tr>
<th>$r/R$</th>
<th>Chord</th>
<th>Pitch</th>
<th>Camber</th>
<th>Skew</th>
<th>Rake</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>0.080</td>
<td>66</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.35</td>
<td>0.090</td>
<td>62</td>
<td>0.02</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>0.40</td>
<td>0.098</td>
<td>58</td>
<td>0.03</td>
<td>0.40</td>
<td>0.02</td>
</tr>
<tr>
<td>0.50</td>
<td>0.104</td>
<td>50</td>
<td>0.035</td>
<td>1.63</td>
<td>0.04</td>
</tr>
<tr>
<td>0.60</td>
<td>0.106</td>
<td>42</td>
<td>0.036</td>
<td>3.67</td>
<td>0.06</td>
</tr>
<tr>
<td>0.70</td>
<td>0.104</td>
<td>34</td>
<td>0.036</td>
<td>6.53</td>
<td>0.08</td>
</tr>
<tr>
<td>0.80</td>
<td>0.094</td>
<td>26</td>
<td>0.035</td>
<td>10.20</td>
<td>0.10</td>
</tr>
<tr>
<td>0.90</td>
<td>0.086</td>
<td>18</td>
<td>0.03</td>
<td>14.69</td>
<td>0.12</td>
</tr>
<tr>
<td>0.95</td>
<td>0.074</td>
<td>14</td>
<td>0.02</td>
<td>17.24</td>
<td>0.13</td>
</tr>
<tr>
<td>1.00</td>
<td>0.061</td>
<td>10</td>
<td>0.0</td>
<td>20.00</td>
<td>0.14</td>
</tr>
</tbody>
</table>

A comparison of the geometry from the two blades is shown in Figure 6.1. The two geometries are identical in all views, which demonstrates that the curvilinear geometry method collapses to cylindrical coordinates when using cylinders and radial lines.
6. 2. PROPELLER DESIGN

To validate the propeller design, a blade was designed with PBD10 and PBD10.2. The initial geometry was identical. The final geometry is shown in Figure 6-2. The pitch and camber distributions are shown in Figure 6-3 and Figure 6-4. The differences at the end of the blade stem from the B-spline surface model of the blade. It is believed that the B-spline surface is more stable at the edges of the propeller than the polynomials used in PBD10. The PBD10.2 results should therefore be more accurate at the root and the tip than the PBD10 geometry.
Figure 6-2: Final Geometry Comparison between PBD10 and PBD10.2

Figure 6-3: Comparison of Pitch Distributions between PBD10 and PBD10.2
Figure 6-4: Comparison of Maximum Camber between PBD10 and PBD10.2

Figure 6.5 is a fanciful propeller designed using PBD10.2. The design is highly conical to demonstrate this feature of the design. Conceptual multiple blade row ducted propulsors have also been designed using this procedure.
Figure 6-5: Highly Conical Propeller Design
7. CONCLUSIONS

The propeller blade design process has been improved by the addition of generalized geometry definitions and the coupling with a Navier Stokes code. The geometry and the RANS code are necessary to explore fuller sterns. In the process, the design spiral is made more modular, to facilitate the creation of an integrated design tool, which couples the MIT array of codes in a graphical environment. Suggestions towards optimization routines are also made.

The geometry definitions are generalized from cylindrical coordinate systems to a curvilinear coordinate system. The modular programming of the geometry algorithms makes the propeller design method more flexible. If a different geometry definition is more appropriate, the D2XYZ and XYZ2D modules can be replaced easily. The non-cylindrical coordinate system augments the design process by making it possible to define the blade along streamlines, allowing better control of the hub and tip design.

Coupling PBD10.2 with a RANS code enables the design process to handle vorticity in the propeller wake. Typically, there is strong vorticity in the wake because of the hull’s boundary layer. This vorticity is influenced by the propeller’s induced velocities, but potential flow design tools can not handle the interactions. Therefore, PBD10.2 was coupled with an axisymmetric RANS code to improve the design of the propeller. The vortical interactions were handled in a modular fashion, so that any RANS or Euler code can be coupled to the blade design process. The hub and duct images are improved in the process.

Automation of the design process should concentrate on the lifting line and lifting surface tools. Improvements in the PLL parametric search strategies would be very beneficial. Because PLL and PBD10.2 are very similar design codes, and are a large part of the design, the two codes are ideal candidates for integration into an automated design tool.
Improvements can be made in the design process. Second order thickness corrections need to be incorporated into the design of the propeller blade. The blade boundary layer effects can be added into the design process. The prediction of the flow at the tip of the propeller can be improved for duct and band geometries.

Overall, experimental validation of the design method for a variety of propeller blades would be useful. To complete this effort, a propeller analysis routine needs to be developed, based mostly on the tools already in PBD10.2.

Finally, a propeller design environment would increase the productivity of the designers. The tools need to be integrated and automated into a graphical design tool with a friendly user-interface. Design optimization tools could be incorporated into the integrated design environment, which will speed the design process.
8. REFERENCES


General Electric, at Groton, presentation given to the MIT Dept. of Ocean Engineering on April 22, 1994.


Kerwin, J.E. Geometry Class Notes, Chap 5. MIT 1994.


9. APPENDIX

9.1. PROBLEM FORMULATION

Propeller blade design is based on lifting surface theory, so a cursory explanation is needed here. There are many places where the problem has been thoroughly treated, such as Kerwin\textsuperscript{15} and Newman.\textsuperscript{22} Here, the development will focus on the suitability of a vortex-lattice code for solving the problem.

A lifting surface is a thin arbitrary body designed to create a force perpendicular to the incoming flow. Wings, hydrofoils, control surfaces, and propeller blades are representative of these surfaces. All lifting surfaces are characteristically thin in the dimension of lift, so they can be approximated by assuming zero-thickness. For the purposes of this formulation, the linearized problem is examined. Further, let us consider the case of steady, uniform, irrotational inflow, where the flow is incompressible and inviscid.

9.2. ASSUMPTIONS

Before examining the problem, a word should be said about the assumptions being made. The assumptions are helpful in examining the lifting surface problem, but often these assumptions are too simple for the treatise of a complex, heavily loaded propulsor on a full afterbody. Therefore, the validity and appropriateness of these assumptions is examined. In cases where these assumptions are no longer valid, this thesis accounts for the inconsistencies. For example, the surface was assumed to be planar in the linearized theory. However, a change of coordinates applies the formulation to the case of non-planar propulsors.

The assumptions of steady, uniform, irrotational inflow are convenient for the formulation of the problem, but do not apply for marine propellers operating in the

highly vortical inflow of a ship's boundary layer. The wake is neither uniform, nor irrotational and inviscid. Because of this, RANS coupling in the design procedure is needed to correct for these simplifications. For the purposes of the formulation, the assumptions are helpful and allow potential flow to describe the problem. In the body of this document, the treatment of cases which are not inviscid, steady, or irrotational is described.

The assumption of inviscid flow is accurate for the large Reynolds numbers of most marine propellers, so long as the Kutta condition is preserved at the trailing edge of the surface. If the Reynolds number is large, then the thickness of the boundary layer is very small in comparison to the length of the lifting surface. In such a case, the viscous effects are very small, and the potential flow solution is accurate. However, viscosity is still necessary in solving the problem, because it gives rise to the familiar Kutta Condition at the trailing edge. Velocities around a sharp trailing edge would become infinite, if it were not for viscosity. Physical observations confirm that viscosity forces the flow to leave the trailing edge smoothly.

The leading edge is also sharp like the trailing edge in our formulation, and there will in general be flow around the leading edge. But once thickness is introduced, the leading edge will not be sharp, and the problem of infinite velocities will be resolved. Therefore, the Kutta condition is applied only at the trailing edge, and is not necessary at the leading edge.

9.3. PROBLEM DEFINITION

Consider the case of a three-dimensional lifting body operating in an unbounded, steady, uniform, inviscid, incompressible, inviscid inflow, as shown in Figure 9.1. The inflow is described by $V_{in}=(U_{in},0,0)$, in vector notation. The potential of the flow is
described as $\Phi$, where $\Phi = \nabla V$, which includes the potential of the free stream. The boundary equations to the problem then become:

- **Laplace Equation:** $\nabla^2 \Phi = 0$ throughout the fluid (Eq 1)
- **Kinematic Boundary Cond.** $\mathbf{V} \cdot \hat{n} = 0$ on the body. (Eq 2)
- **Infinitely away from the blade,** $V = V_{\text{in}}$ as $r \to \infty$ (Eq 3)
- **Kutta Condition** $V \neq \infty$ at the trailing edge. (Eq 4)

---

**Figure 9-1: Definition of the Lifting Surface Problem**

The material surface of the wing separates the flow into an upper and lower flow, with corresponding $V_u$ and $V_l$ velocities at the surface. The velocities at a point $P$ are shown in Figure 4-2. From these velocities, a mean velocity, $V_m = (V_u + V_l)/2$, and a differential velocity, $V_d = (V_u - V_l)/2$ are defined.
Figure 9-2: Bound and Free Vorticity Vectors

The discontinuity at the lifting surface generates vorticity, as described by $\gamma = 2V_d$.

From Bernoulli's equation, the change in pressure, $\Delta p$, is simply

$$\Delta p = p_1 - p_u = 1/2 \rho(V_m^2 - V_i^2) .$$

The jump in pressure can also be related to the vorticity, as shown in Figure 4-2. The vorticity vector, $\gamma$, is different from the direction of the mean flow, $V_m$, by the angle, $\delta$. From this, the jump in pressure can be rewritten as $\Delta p = \rho V_m \gamma \sin \delta$, a useful result. As the vorticity aligns with the flow, the jump in pressure goes to zero. This gives rise to the thought of free and bound vorticity. Free vorticity is aligned with the flow and does not generate any force. Bound vorticity does contribute to the change in pressure, which now can be defined as

$$\Delta p = \rho V_m \gamma_b$$

9.4. FREE AND BOUND VORTICITY

At this point, the lifting surface problem is almost defined, but first the free vorticity and the bound vorticity must be related. Kelvin's Theorem states that the circulation around any simply-connected contour must be constant. Vorticity can only be created or destroyed by shear forces; because of the inviscid flow assumption, there can
be no shear forces, and no changes in vorticity. In addition, the assumption of uniform inflow means that the inflow has no circulation. Therefore, the only circulation in the flow must be introduced by the presence of the lifting surface.

The vorticity on the surface can be resolved into free vorticity and bound vorticity, as shown previously in Figure 9-2. Heimholtz' Theorem states that a vortex filament must continue throughout the fluid. This implies that the bound vorticity on the foil must be shed from the foil as free vorticity, which extends downstream to infinity. Using both Kelvin and Heimholtz's Theorems, the free vorticity on the foil can be related to the bound vorticity on the foil.

\[
\gamma_f = \gamma_{b\text{le}} - \int_x^y \frac{\partial \gamma_b}{\partial y} d\xi \quad \text{on the foil} \\
\gamma_f = -\frac{\partial \Gamma}{\partial y} \quad \text{in the wake}
\]

(Eq 5)  

(Eq 6)

where \(\Gamma(y)\) is the circulation, \((\xi, \eta, \zeta)\) are dummy variables for \((x, y, z)\), and the subscript \((\text{le})\) refers to the leading edge of the lifting surface.

Using these equations for a simple wing, the bound and free vorticity is shown in Figure 9-3. The bound vorticity was specified by along the span and across the chord of the blade. From this distribution of loading, the shed vorticity contours are plotted. At the leading edge, there is no free vorticity, but the vorticity grows across the chord of the blade. At the center of the span, the free vorticity is zero. The distribution of free vorticity is symmetrical about the blade, except for the sign.
The idea of bound and free vorticity on the lifting surface makes a vortex-latticework a natural model of the potential flow solution. Because bound and free vorticity are orthogonal, a gridwork discretizing the free and bound vorticity is a natural and intuitive method of modeling the circulation on the blade. The circulation across the surface is discretized into segments of vorticity, to represent the loading on the blade. In addition, the wake of the surface is also represented by a grid of vortex elements. The most general gridwork of vortex segments would not be limited to an orthogonal gridwork of bound and free vortex segments, but this is one scheme. In such a scheme, the spanwise lattices represent bound vorticity across the blade, and have no circulation in the wake. The chordwise lattices are representing the free vorticity, and are aligned along streamlines of the flow.

By applying the previous boundary conditions to the flow, the problem can be solved. In particular, the kinematic boundary condition, (Eq 2), is needed to ensure that
the lifting surface is a material surface, i.e. there is no flow normal to the surface of the foil. The kinematic boundary condition is applied at the control point of vortex cells, and not everywhere across the flow. In approaching a vortex segment, the crossflow increases because of the local infinity at the core of the vortex. Figure 9.4 shows the velocity normal to a panel which is bounded by two equal parallel vortex segments. For a constant spacing grid, the control points are at the center of the panels, while for a cosine spacing, the control points are centered by the mapping. James\textsuperscript{23} and Stark\textsuperscript{24} have proven the robustness of vortex lattice methods with these respective griddings.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure9-4.png}
\caption{Crossflow Velocity Across the Span of a Panel}
\end{figure}

The fluid velocity at any point can be determined by the contributions of the bound and free vorticity to the free-stream velocity. In particular across the lifting surface, the perturbation velocity is specified by the induction of the blade vortex sheets and the wake vortex sheet through an application of Biot-Savart's law;

\[ w_{\text{bound}} = -\frac{1}{4\pi} \int \int_{\text{span,chord}} \gamma_b \frac{(x - \xi)}{r^3} d\xi d\eta \quad (4.7) \]

\[ w_{\text{free}} = \frac{1}{4\pi} \int \int_{\text{span,chord \\ \\ \& wake}} \gamma_f \frac{(y - \eta)}{r^3} d\xi d\eta \quad (4.8) \]

\[ W = w = w_{\text{bound}} + w_{\text{free}} \]

The velocity can now be determined anywhere on the surface of the foil or in the wake, and therefore, through momentum analysis, the lift and drag on the foil can be determined.

9.5. CONCLUSIONS

If the geometry and the incoming angle of attack are known, the load distribution can be found from the above integrals. This is known as the analysis problem. On the other hand, if the load distribution is given, then the equations can be integrated to find the slope of the surface, and therefore the geometry and angle of attack. In either case, the integrals are singular with no closed form solution, even for this simplified, linearized problem. Numerical methods such as the vortex-lattice method are necessary for finding the solution to either the design or the analytical problem.

A lifting line model simplifies the propeller blade design problem into a locally two dimensional strip-wise analysis. Because of this, a lifting line representation of a propeller blade is useful in the initial design iterations, though the low aspect ratios and skewed profiles of modern propellers require a more complicated design method, like vortex lattice methods for the final design.