Calibration of a Physically-based Distributed Rainfall-Runoff Model with Radar Data

by

Yanlong Zhang

Submitted to the Department of Civil and Environmental Engineering
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Abstract

A physically-based distributed rainfall-runoff model is presented and tested in two river basins. The model discretizes the terrain of a river basin into rectangular elements, thus exploiting topographic information available from digital elevation maps (DEM). Soil properties and rainfall input are also represented as rectangular grids. Initial water table depth is used to describe the prestorm basin soil moisture conditions, and both infiltration-excess and saturation-excess mechanisms of runoff generation are taken into account.

A method to derive the prestorm water table depth across the basin is also implemented. This method relates the water table depth with the prestorm streamflow and the basin topographical and soil characteristics. The model calibration and verification in the Arno river basin in Italy is against three streamflow gauges simultaneously. Both raingauge and meteorological radar data are used as rainfall input. In the case of the Souhegan river basin in New England, only radar data is used as rainfall input to the model. It was found that the model does fairly well in these two vastly different basins, with two different rainfall measuring methods. This demonstrates that with spatially distributed DEM and radar rainfall data, a physically-based distributed rainfall-runoff model can be a very useful tool in flood forecasting.

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Contents

1 Introduction 10

2 The Rainfall-Runoff Model 17

2.1 The One-dimensional Infiltration Model ......................... 17

2.1.1 Assumptions ........................................ 17
2.1.2 Unsaturated Flow .................................. 20
2.1.3 Saturated Flow ..................................... 22
2.1.4 Wetting and Top Front Evolution ........................ 27

2.2 Basin Scale Model ....................................... 30

2.2.1 Equivalent rainfall rate ................................ 31
2.2.2 Moisture Balance .................................... 32
2.2.3 Runoff Generation .................................... 36
2.2.4 Surface flow routing .................................. 39

3 Initial Basin Conditions and Model Calibration Procedure 42

3.1 Initial Basin Moisture Conditions ............................... 42

3.1.1 Relating Spatially Distributed Water Table Depth to Basin Averaged Water Table Depth ............... 43

3.1.2 Relating Basin Averaged Water Table Depth with Prestorm Streamflow ................................. 47

3.2 Application to Souhegan River Basin ............................ 52

3.3 Initial Water Table for Arno River Basin ....................... 57

3.4 Model Calibration Procedure .................................. 58
List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Soil column representation</td>
<td>18</td>
</tr>
<tr>
<td>2-2</td>
<td>Infiltration in the unsaturated soil</td>
<td>21</td>
</tr>
<tr>
<td>2-3</td>
<td>Pore pressure profile in the perched saturated zone</td>
<td>25</td>
</tr>
<tr>
<td>2-4</td>
<td>Integration domain $\Omega$ for the continuity equation</td>
<td>27</td>
</tr>
<tr>
<td>2-5</td>
<td>Different pixel states</td>
<td>37</td>
</tr>
<tr>
<td>3-1</td>
<td>Typical sectional view of a hillslope</td>
<td>44</td>
</tr>
<tr>
<td>3-2</td>
<td>Schematic representation of an unconfined aquifer</td>
<td>48</td>
</tr>
<tr>
<td>3-3</td>
<td>Recession flow at Souhegan river. Linear regression fitted line and 15% vertical threshold</td>
<td>53</td>
</tr>
<tr>
<td>3-4</td>
<td>Recession flow at Souhegan river. The 4 straight lines correspond to 5% and 10% envelopes with slopes 3/2 and 3 respectively.</td>
<td>54</td>
</tr>
<tr>
<td>3-5</td>
<td>Sensitivity of water table depth to whether 5% or 10% envelope is used to estimate $a_1$</td>
<td>55</td>
</tr>
<tr>
<td>3-6</td>
<td>Sensitivity of water table depth to critical flow rate $Q_c$</td>
<td>56</td>
</tr>
<tr>
<td>3-7</td>
<td>Sensitivity of water table depth to a priori estimated drainable porosity $n_e$</td>
<td>56</td>
</tr>
<tr>
<td>4-1</td>
<td>DEM of Rosano</td>
<td>63</td>
</tr>
<tr>
<td>4-2</td>
<td>Channel network of Rosano</td>
<td>64</td>
</tr>
<tr>
<td>4-3</td>
<td>Soil map of Rosano</td>
<td>66</td>
</tr>
<tr>
<td>4-4</td>
<td>Rain gauges in Rosano</td>
<td>69</td>
</tr>
<tr>
<td>4-5</td>
<td>Hydrographs at Rosano for storm of Feb.20-22, 1977 using hourly rain gauge data</td>
<td>71</td>
</tr>
</tbody>
</table>
4-6 Hydrographs at Subbiano for storm of Feb.20-22, 1977 using hourly rain gauge data ................................................................. 72
4-7 Hydrographs at Fornacina for storm of Feb.20-22, 1977 using hourly rain gauge data ................................................................. 72
4-8 Hydrographs at Rosano for storm of Feb.20-22, 1977 using daily rain gauge data ................................................................. 75
4-9 Hydrographs at Subbiano for storm of Feb.20-22, 1977 using daily rain gauge data ................................................................. 75
4-10 Hydrographs at Fornacina for storm of Feb.20-22, 1977 using daily rain gauge data ................................................................. 76
4-11 Hydrographs at Rosano for storm of Jan. 9-10, 1979 ...................... 77
4-12 Hydrographs at Subbiano for storm of Jan. 9-10, 1979 ...................... 78
4-13 Hydrographs at Fornacina for storm of Jan. 9-10, 1979 ...................... 78
4-14 Hydrographs at Subbiano for storm of Nov. 13-14, 1982 ................... 80
4-15 Hydrographs at Fornacina for storm of Nov. 13-14, 1982 ................... 80
4-16 Hydrographs at Rosano for storm of Nov. 24-26, 1987 .................... 82
4-17 Hydrographs at Subbiano for storm of Nov. 24-26, 1987 .................... 82
4-18 Hydrographs at Fornacina for storm of Nov. 24-26, 1987 .................... 83
4-19 Hydrographs at Rosano for storm of Oct. 30-31, 1992 using rain gauge data ................................................................. 84
4-20 Hydrographs at Subbiano for storm of Oct. 30-31, 1992 using rain gauge data ................................................................. 85
4-21 Hydrographs at Fornacina for storm of Oct. 30-31, 1992 using rain gauge data ................................................................. 85
4-22 Rainfall volume at spatial points measured by radar and rain gauges 87
4-23 Hydrographs at Rosano for storm of Oct. 30-31, 1992 using radar and rain gauge data ................................................................. 89
4-24 Hydrographs at Subbiano for storm of Oct. 30-31, 1992 using radar and rain gauge data ................................................................. 90
4-25 Hydrographs at Fornacina for storm of Oct. 30-31, 1992 using radar and rain gauge data

5-1 DEM of Souhegan
5-2 Channel network of Souhegan
5-3 Soil map of Souhegan
5-4 Rain gauges within radar coverage
5-5 Radar and rain gauge comparison for the storm Sept. 19-20, 1987
5-6 Measured and simulated hydrographs for the storm Sept. 19-20, 1987
5-7 Radar and rain gauge comparison for the storm June 27, 1987
5-8 Measured and simulated hydrographs for the storm June 27, 1987
5-9 Radar and rain gauge comparison for the storm June 22-23, 1987
5-10 Measured and simulated hydrographs for the storm June 22-23, 1987
5-11 Radar and rain gauge comparison for the storm June 22-23, 1988
5-12 Measured and simulated hydrographs for the storm June 22-23, 1988
5-13 Radar and rain gauge comparison for the storm October 21-22, 1988
5-14 Measured and simulated hydrographs for the storm October 21-22, 1988
5-15 Radar and rain gauge comparison for the storm August 29-30, 1988
5-16 Measured and simulated hydrographs for the storm August 29-30, 1988
List of Tables

3.1 Results with different drainable porosity values .................. 57
3.2 Basin averaged water table depth for the 6 storms .................. 57

4.1 Soil properties in Rosano ........................................ 67
4.2 Soil properties in Rosano(continued) .......................... 68
4.3 Simulations for the storm Feb.20-22, 1977 using hourly rain gauge data 71
4.4 Simulations for the storm Feb.20-22, 1977 using daily rain gauge data 74
4.5 Simulations for the storm Jan. 9-10, 1979 .......................... 77
4.6 Simulations for the storm Nov. 13-14, 1982 ........................ 79
4.7 Simulations for the storm Nov. 24-26, 1987 ........................ 81
4.8 Simulations for the storm Oct. 30-31, 1992 using rain gauge data .......................... 84
4.9 Simulations for the storm Oct. 30-31, 1992 using radar and rain gauge data .................. 89

5.1 Soil properties in Souhegan ....................................... 97
5.2 Simulation for the storm Sept. 19-20, 1987 ...................... 103
5.3 Simulation for the storm June 27, 1987 ........................... 104
5.4 Simulation for the storm June 22-23, 1987 ........................ 106
5.5 Simulation for the storm June 22-23, 1988 ........................ 108
5.6 Simulation for the storm October 21-22, 1988 ..................... 112
5.7 Simulation for the storm August 29-30, 1988 ...................... 113
Chapter 1

Introduction

A watershed consists of a complex three-dimensional mosaic of soils, vegetation and bedrock, all of which are highly heterogeneous over space. The hydrological processes occurring in a watershed are very complicated. A drop of water may follow an infinite number of pathways between its precipitation on the land surface and its subsequent discharge through stream channels or evapotranspiration (Woolhiser, 1981). However, with the increasing understanding of the hydrological processes and the increasing availability of computer resources, significant advances have been made in developing physically-based, distributed hydrological models during the last two decades.

Unlike the traditional lumped conceptual models which take lumped input parameters and simulate total runoff at the basin outlet, physically-based distributed models simulate the hydrological processes at every point within a basin. In these models, hydrological processes are modeled with the governing principles, either by partial differential equations of mass, momentum and energy conservation, or by empirical equations derived from independent experimental researches, such as Darcy’s equation for subsurface flow and Penman-Monteith equation for evapotranspiration.

Runoff generation mechanisms

The main components into which precipitation may be partitioned are evapotranspiration, overland flow, unsaturated subsurface flow and saturated subsurface flow. Runoff is influenced by the rainfall intensity and duration, antecedent basin condi-
tions and the basin characteristics, especially the basin topography. In most cases, over a basin scale, the major component of the hydrograph is overland flow.

The generation of overland flow occurs under at least three sets of conditions (Kirkby, 1985). When rain falls faster than it can be infiltrated into the soil, then the excess rainfall will form Hortonian or infiltration-excess overland flow. When the soil is saturated, any further rainfall will generate saturation-excess overland flow. Return overland flow occurs where subsurface flow is forced up to the surface by the soil or slope configuration.

Since the seventies, the concept of variable source area overland flow generation has been gaining more and more attention. This concept implies that during a storm, most rainfall infiltrates into the soil and migrates as subsurface flow downslope to produce saturated areas, mostly near the channel. From these areas overland flow is generated either as saturated overland flow directly from rainfall or as return overland flow. Saturated contributing areas can expand or shrink in response to the changing rainfall intensity. In contrast to the “Hortonian” overland flow concept, the variable source area concept incorporates the entire range of the hillslope processes, rather than only the infiltration process in a vertical soil column.

In urban or other developed watersheds, the Hortonian overland flow may dominate the hydrograph, but in forested or wildland watersheds, it is the saturated overland flow and return flow that dominate. Numerous observations have been reported to support the variable source area theory. For example, Pilgrim et al. (1978) reported on a field evaluation of surface/subsurface runoff processes under natural and artificial rainfall. They found that the surface flow was mostly a combination of return flow and saturated overland flow, and the Hortonian overland flow was discontinuous downslope and only local in nature.

**Development of physically based distributed models**

Given the complex three-dimensional nature of the hydrological processes, it is extremely difficult to model them mathematically. Not all of the conceptual understanding of the way hydrological systems work is expressible in formal mathematical
terms (Beven, 1985). Sometimes hydrological processes are expressed mathematically in the form of non-linear partial differential equations that can not be solved analytically for the cases of practical interest. Numerical solutions must be found, which will involve the discretization of the space-time coordinates. And when the mathematical models are applied to a basin, some simplifications are usually necessary (such as reducing the dimensionality of the processes) to make the problem manageable.

The Institute of Hydrological Distributed Model (IHDM) classifies the watershed into a number of spatial zones of like attributes with respect to vegetation type and microclimate (Calver, 1988). The basic structure of the model calculations is a two dimensional representation of each hillslope section in the vertical slice sense; changes of hillslope width within each section can be incorporated, allowing the effects of converging and diverging hillslope flows to be taken into account. The IHDM assumes that overland flow results from Hortonian runoff and return flows. Channel flow and any overland flow are modeled by the one dimensional Kinematic Wave equation, which is solved using a 4-point implicit finite difference scheme. The unsaturated and saturated flow are modeled together using two-dimensional Richards equation, incorporating Darcy’s law and consideration of mass conservation. This equation is solved using a finite element scheme, the Galerkin method of weighted residuals for the two space dimensions and an implicit finite difference method for the time dimension. The interception and evapotranspiration may be derived from modeling or observation, and must be subtracted from rainfall to get net rainfall input.

The Systeme Hydrologique European (SHE) model is one of the most comprehensive physically-based distributed hydrological models. It simulates the entire land phase of the hydrological cycle (Abbot et al., 1986). In the SHE model, spatial distribution of watershed parameters, rainfall input and hydrological response is achieved in the horizontal by an orthogonal grid network and in the vertical by a column of horizontal layers at each grid square. Grid spacing in the horizontal can vary across the network, but must remain the same for a given row or column within the network array. Node spacing in the vertical is a function of the vegetation type which characterizes a given grid square and can vary between the root zone and the soil
layer below it. Each primary process of the land phase of the hydrological cycle is modeled as a separate component: interception is modeled by the Rutter accounting procedure; evapotranspiration is modeled by the Penman-Monteith equation; overland and channel flow are modeled by simplified St. Venant equations; unsaturated zone flow is modeled by the one dimensional Richards equation; saturated zone flow is represented by the two dimensional Boussinesq equation; and snowmelt is obtained using an energy budget method.

The assumptions made in the SHE model are: flow in the unsaturated subsurface zone is essentially vertical and flow in the saturated subsurface zone is essentially horizontal. Flows in the soil macropores are secondary details and thus are not explicitly but implicitly modeled. Flow in the confined aquifer is of secondary importance and is not modeled. The one dimensional unsaturated flow columns of variable depths act as a link between the two dimensional overland flow component and the two dimensional saturated flow component. These equations are solved numerically for each node in space and at each time step. The time step can be different for different components, but must remain constant in a component.

Most distributed models partition the watershed into small grids like the SHE model. The grid structure typically restrains the flow from one node to one of the eight possible directions and flow paths take on a zigzag shape (Moore and Hutchinson, 1991). Vieux et al. (1988) used a TIN (Triangular Irregular Network) to provide a framework for solving the kinematic forms of the two dimensional overland flow equations using the finite element method. In this application, the quadrilateral and triangular elements were aligned so that the principal slope direction was parallel to one of the sides of the element. This made the flow path representation more realistic.

Recognizing the importance of the topography on the basin hydrological responses, the THALES model (Moore and Hutchinson, 1991; Grayson et al., 1992a) assumes that the contour lines are equipotential lines and pairs of orthogonals (streamlines) to the equipotential lines form the "stream tubes". Adjacent contour lines and streamlines define irregularly shaped elements. Surface runoff enters an element orthogonal to the upslope contour line and exits orthogonal to the downslope contour line, with
the adjacent streamlines being no-flow boundaries. Flow from one element can then be successively routed to downstream elements within the same stream tube.

The model also assumes that the infiltrated water flows downslope in a saturated layer overlaying an impermeable base, and this flow is modeled as one dimensional kinematic flow. If the subsurface flow rate exceeds the capacity of the soil profile to transmit the water, surface saturation occurs and the rain falling on the saturated areas becomes direct runoff. Runoff can also be generated by Hortonian overland flow, which is also modeled as one dimensional kinematic flow. Infiltrated water is assumed to flow vertically through an unsaturated zone to become part of the saturated layer.

More recently, Paniconi and Wood (1993) developed a numerical model based on the 3D transient Richards equation describing flow in variably saturated porous media. The equation is solved using the finite element method to simulate the hydrological processes at subcatchment and catchment scale. The model can be applied to catchments of arbitrary geometry and topography. The model automatically handles both soil driven and atmosphere driven surface fluxes, and both saturation excess and infiltration excess runoff production.

Runoff generated within a basin varies considerably over space, and this variation occurs at different spatial scales. However, if small area variability is integrated over a large enough area, the effects of the small variations are often attenuated or completely submerged (Grayson et al., 1992b). The representative Elementary Area (REA), defined as the spatial scale at which the runoff spatial variability disappears, was first proposed by Wood et al (1988). They argue that the internal physical processes within an REA should be studied in detail, and distributed models can use REAs as building blocks to simulate the basin runoff generation. Although there is still an ongoing debate on whether REA really exists, this concept does bring some insight into the issue of spatial variability of runoff generation.

Our understanding and ability to model the hydrological processes is very limited compared to the spatially heterogeneous physical reality. In most of the current distributed models, physical processes are modeled based on the small scale physics of homogeneous systems. In application, we are forced to lump up the small scale
physics to the model element scale (Beven, 1989). The implicit assumption is that the same small scale homogeneous physical equations can be applied at the model element scale with the same parameters. This is a very strong assumption because at a larger scale, some parameters could totally lose their physical meanings. Beven (1989), after examining the fundamental problems with distributed models, concluded that the current generation of physically-based distributed models are, in fact, LUMPED conceptual models. He argued that we need the theory for lumping subgrid processes to spatially heterogeneous grid squares.

In principle physically-based distributed models do not need calibration, because all the parameter values have their physical meaning and are measurable in the field. However, two primary reasons make calibration indispensable for almost all the models before they are applied to a real basin. First, every model has approximations in its representation of the physical processes; second, oftentimes not all the parameter values required by the model are available for a whole basin, some parameter values have to be guessed from experience. Even if they are all available, considerable uncertainty is always associated with these data. Soil properties seem to be most difficult to obtain, not only because that their measurement is financially prohibitive, but also because that they are highly heterogeneous, and the measured values depend strongly on the measurement scale (Beven, 1985).

Physically-based distributed models should be able to predict the internal flow behavior of the basin. However, although there are many models which have reported satisfactory results in representing real basins, most of these “satisfactory results” mean only a good fit of the predicted and observed hydrographs at the basin outlet, or, at most, at only a few other points along the basin channel. But lumped models can also easily achieve a “good fit” with the observed hydrograph at the basin outlet. Different combinations of parameter values and process presentations could produce similar results at the basin outlet. Thus, it is quite possible to obtain a “good fit” at the basin outlet when representing the wrong physical processes.

With all the problems, distributed models still have some advantages over lumped conceptual models. One of the advantages is that we can use a distributed model as a
tool for hypothesis testing (Grayson et al., 1992b), i.e., to assist in the understanding of physical systems by providing a framework within which to analyze data. For instance, the effects of land use change of a basin can be forecasted with a distributed model. Because of the lack of physical significance, lumped models usually need a lot of historically measured data for calibration. Thus in forecasting the hydrological responses of ungauged basins, distributed models will usually perform better than lumped models.

Distributed models represent the future. Spatially distributed topographic and rainfall data (Digital Elevation Maps, or DEM, and radar rainfall data) are becoming more and more available. Remote sensing as a tool has the potential to measure spatially distributed soil parameters (for example, antecedent soil moisture content distribution). Computer resources are expanding every day. Distributed models, which make the best use of the distributed topographic, soil and rainfall certainly be the choice of the future, and new ways to solve the existing problems will be found.

In the following chapters, we demonstrate the applications of a physically based distributed rainfall-runoff model to two river basins of vastly different characteristics. The first river basin, the Arno, is located in Central Italy, we have three streamflow gauges at different parts of the river channel, and rainfall data from raingauges and radar are available. The second river basin, the Souhegan, is located in New England. We use only radar data as rainfall input. It can be shown that with real-time radar data, a physical based distributed model can do a very good job in flood forecasting.
Chapter 2

The Rainfall-Runoff Model

The physically based distributed runoff model used in this work is DBS (Distributed Basin Simulator). It was first proposed by Cabral et al. (1990) and further developed by Garrote (1992). Garrote also wrote all the computer code to make the model actually a flood forecasting system with a very friendly user interface. The system can be run either on-line for real time flood forecasting or off-line for model calibration and basin hydrological behavior studies. This chapter is based on Cabral et al. (1990) and Garrote (1992).

2.1 The One-dimensional Infiltration Model

2.1.1 Assumptions

Consider a vertical soil column of horizontal size $dX \times dY$ and infinite depth with surface slope angle $\alpha$ (figure 2.1). The reference system is formed by the axes $n$ and $p$, where $n$ is perpendicular to the terrain surface and positive downward, and $p$ is parallel to the terrain surface and positive downslope. The other axis is $Y$ perpendicular to both $n$ and $p$.

The flow in the soil is assumed Darcian. The full equations are considered in the saturated area, but the kinematic approximation (Beven, 1984) is adopted in the unsaturated zone, where the contribution of the capillary pressure to the hydraulic
gradient is neglected (Garrote, 1992). Flow through the macropores in the soil is also neglected. Under the kinematic approximation, the infiltration capacity of the soil column is equal to the saturated hydraulic conductivity at the surface.

Further assumptions include:

(1) The soil is anisotropic with one primary anisotropic direction parallel to the terrain surface \( p \) and the other primary anisotropic direction normal to the terrain surface \( n \). The anisotropy ratio is defined as

\[
a_r = \frac{K_p(\theta_s, 0)}{K_n(\theta_s, 0)}
\]

where \( K_n(\theta_s, 0) \) and \( K_p(\theta_s, 0) \) are the soil surface saturated hydraulic conductivities in the \( n \) and \( p \) directions respectively. \( a_r \) is assumed to be a constant larger than 1.

(2) In the directions \( p \) and \( Y \), the soil is homogeneous within the column \( dX \times dY \), but in the direction \( n \) normal to the soil surface, the soil is heterogeneous with the
soil saturated hydraulic conductivity decreasing exponentially

\[ K_n(\theta_s, n) = K_n(\theta_s, 0)e^{-fn} \]  \hspace{1cm} (2.2a)

\[ K_p(\theta_s, n) = K_p(\theta_s, 0)e^{-fn} \]  \hspace{1cm} (2.2b)

where \( K_n(\theta_s, n) \) and \( K_p(\theta_s, n) \) are the \( n \) and \( p \) direction soil saturated hydraulic conductivities at normal depth \( n \) respectively; \( f \) is a constant expressing the conductivity exponential decay rate.

The decrease of saturated hydraulic conductivity with normal depth is a key assumption that leads to the formation of perched saturated zone to be described later. Although the decrease may take different functional forms, the exponential form is adopted here. Beven (1982) finds that a number of soil data sets from a variety of basins are well represented by the exponential decay form of saturated hydraulic conductivity.

The Brooks-Corey model (Brooks and Corey, 1964) is used to relate the unsaturated hydraulic conductivity to the saturated hydraulic conductivity and moisture content. Using equations (2.2a) and (2.2b), the Brooks-Corey model gives

\[ K_n(\theta, n) = K_n(\theta_s, 0)e^{-fn\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^e} \]  \hspace{1cm} (2.3a)

\[ K_p(\theta, n) = K_p(\theta_s, 0)e^{-fn\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^e} \]  \hspace{1cm} (2.3b)

where \( K_n(\theta, n) \) and \( K_p(\theta, n) \) are the \( n \) and \( p \) direction soil hydraulic conductivities at normal depth \( n \) and moisture content \( \theta \) respectively; \( \theta_r \) is the soil residual moisture content defined as the value below which moisture cannot be extracted by capillary forces; \( \theta_s \) is the soil saturated moisture content (porosity); and \( e \) is the soil pore size distribution index.

Parameters \( \theta_r, \theta_s \) and \( e \) are assumed to be constant with depth although hydraulic conductivity is a function of depth. The same approach has been adopted by other authors (e.g., Yeh et al., 1985).
2.1.2 Unsaturated Flow

Using Darcy's equation, the flow vector \( q \) can be expressed as (Cabral et al., 1990)

\[
q = -K_n J_n i_n - K_p J_p i_p
\]  

(2.4)

where \( J_n \) and \( J_p \) are the \( n \) and \( p \) direction soil hydraulic gradients respectively, and \( i_n \) and \( i_p \) are the unit vectors in the \( n \) and \( p \) directions respectively. Under the kinematic approximations, gravity is the only driving force for moisture flow in the unsaturated zone. Hydraulic gradient is only gravitational,

\[
J = J_n i_n + J_p i_p = -\cos(\alpha) i_n - \sin(\alpha) i_p
\]  

(2.5)

Substituting equation (2.5) into equation (2.4) and from equations (2.1) and (2.3), we have

\[
q = K_n \cos(\alpha) i_n + a_r K_n \sin(\alpha) i_p
\]  

(2.6)

Because of the soil anisotropy, the unsaturated flow is not in the vertical direction \( z \), but rather it deflects at an angle \( \beta_u \) with the vertical direction \( z \) (figure 2.1).

It is clear from figure 2.1 that

\[
\tan(\alpha + \beta_u) = \frac{q_p}{q_n} = a_r \tan(\alpha)
\]  

(2.7)

Or we can write

\[
\beta_u = \tan^{-1}(a_r \tan(\alpha)) - \alpha
\]  

(2.8)

Angle \( \beta_u \) is constant with depth in the unsaturated zone, and it increases with \( a_r \).

Consider rainfall at rate \( R \) smaller than the surface normal saturated hydraulic conductivity (figure 2.2), from continuity we can write for steady state unsaturated flow

\[
q = \frac{\cos(\alpha)}{\cos(\alpha + \beta_u)} R
\]  

(2.9)
Figure 2-2: Infiltration in the unsaturated soil

The steady normal flow is

\[ q_n = \cos(\alpha + \beta_u)q = R \cos(\alpha) \]  

(2.10)

From equations (2.6) and (2.10) we have

\[ K_n(\theta, n) = R \]  

(2.11)

Combining equations (2.11) and (2.3a) yields

\[ \theta(R, n) = \left( \frac{R}{K_n(\theta_s, 0)} \right)^{\frac{1}{\gamma}} (\theta_s - \theta_r) \exp\left( \frac{f_n}{\epsilon} \right) + \theta_r \]  

(2.12)

The above equation shows that under steady state, the moisture content in the unsaturated zone increases exponentially with normal depth in order to maintain the normal hydraulic conductivity equal to the rainfall rate \( R \) (Cabral et al., 1990). At
a critical depth $N^*$, the soil becomes saturated and a perched saturated zone starts to develop. This depth $N^*$ also corresponds to where the saturated normal hydraulic conductivity equals to the rainfall rate.

$$K_n(\theta_s, N^*) = R \quad (2.13)$$

Substituting into equation (2.2a), and solving for $N^*$ yields

$$N^*(R) = \frac{1}{f} \ln\left(\frac{K_n(\theta_s, 0)}{R}\right) \quad (2.14)$$

the above equation applies only for the case $R \leq K_n(\theta_s, 0)$. For $R > K_n(\theta_s, 0)$, the saturation is at the surface, no unsaturated zone exists above the wetting front.

Substituting equation (2.11) into equation (2.6) yields

$$q = R \cos(\alpha)i_n + a_r R \sin(\alpha)i_p \quad (2.15)$$

Vertical and horizontal components of the flow in the unsaturated zone are, respectively (Cabral et al., 1990),

$$q_z = q_n \cos(\alpha) + q_p \sin(\alpha) = R[\cos^2(\alpha) + a_r \sin^2(\alpha)] \quad (2.16)$$

and

$$q_x = -q_n \sin(\alpha) + q_p \cos(\alpha) = R \cos(\alpha) \sin(\alpha)(a_r - 1) \quad (2.17)$$

Both vertical and horizontal components of the flow are constant with depth in the unsaturated zone. For anisotropy ratio greater than one, horizontal flow goes in the downslope direction (Cabral et al., 1990).

### 2.1.3 Saturated Flow

For a given rainfall rate, as the wetting front penetrates beyond the critical depth $N^*$, the infiltration capacity (the saturated normal conductivity at that depth) of the soil
becomes less than the recharge rate from above, and moisture starts to accumulate above the wetting front. A zone of perched saturation develops and grows both downward and upward. The top of the perched saturation zone is defined as the top front. It is assumed that both the wetting front and the top front proceed perpendicular to the soil surface.

Within the zone of saturation, the soil moisture content is constant. Since we also assumed that all derivatives in the $p$ direction are zero, the continuity equation becomes (Cabral et al., 1990)

$$\frac{\partial q_n}{\partial n} = 0 \quad (2.18)$$

Equation (2.18) means that the $q_n$ is constant in the $n$ direction within the saturated zone. Since the gravitational gradient is constant and hydraulic conductivity decreases with depth, constant normal flow implies a positive pressure buildup within the saturated zone to compensate for the different hydraulic conductivities of different soil layers.

$$J = J_n i_n + J_p i_p = (-\cos(\alpha) + \frac{\partial \Psi}{\partial n}) i_n - \sin(\alpha) i_p \quad (2.19)$$

where $\Psi$ is a positive pore pressure varying only with normal depth.

The flow equation in the saturated zone is

$$q = q_n i_n + q_p i_p = -K_n(\theta_s, n) J_n i_n - K_p(\theta_s, n) J_p i_p = -K_n(\theta_s, 0)e^{-fn} J_n i_n - K_p(\theta_s, 0)e^{-fn} J_p i_p$$

or

$$q_n(n) = (\cos(\alpha) - \frac{\partial \Psi}{\partial n}) K_n(\theta_s, 0)e^{-fn} \quad (2.21)$$

Substituting equation (2.21) into equation (2.18) yields

$$\frac{\partial}{\partial n}[(\cos(\alpha) - \frac{\partial \Psi}{\partial n}) K_n(\theta_s, 0)e^{-fn}] = 0$$

or

$$\frac{\partial^2 \Psi}{\partial n^2} - f \frac{\partial \Psi}{\partial n} + f \cos(\alpha) = 0 \quad (2.22)$$
This equation governs the pressure distribution within the saturated zone.

Pressure at both top and wetting fronts can be assumed atmospheric given that they are in contact with the unsaturated zones. Integration of equation (2.22) with this boundary conditions $\Psi(N_f) = \Psi(N_t) = 0$ leads to

$$\Psi(n) = \cos(\alpha)[(n + \frac{1}{f}) - \frac{e^{fN_f} - e^{fN_t}}{e^{fN_f} - e^{fN_t}}(N_t + \frac{1}{f}) - \frac{e^{fN_n} - e^{fN_t}}{e^{fN_f} - e^{fN_t}}(N_f + \frac{1}{f})]$$  \hspace{1cm} (2.23)$$

where $N_f$ and $N_t$ are the normal wetting and top front depths respectively.

Substituting equation (2.23) into equation (2.19) yields

$$J = -\left[\frac{f(N_f - N_t)}{e^{fN_f} - e^{fN_t}}\right] \cos(\alpha)i_n - \sin(\alpha)i_p$$  \hspace{1cm} (2.24)$$

And substituting equation (2.23) into equation (2.21) yields

$$q_n = K_n(\theta_s, 0) \left[\frac{f(N_f - N_t)}{e^{fN_f} - e^{fN_t}}\right] \cos(\alpha)$$  \hspace{1cm} (2.25)$$

We can define an "equivalent hydraulic conductivity" for the saturated zone, $K_{eq}$, as the normal hydraulic conductivity of a homogeneous soil column with the same normal flow $q_n$ given by equation (2.25) (Garrote, 1992),

$$K_{eq}(N_f, N_t) = K_n(\theta_s, 0) \left[\frac{f(N_f - N_t)}{e^{fN_f} - e^{fN_t}}\right] \cos(\alpha)$$  \hspace{1cm} (2.26)$$

This $K_{eq}$ also corresponds to the harmonic mean of the soil normal hydraulic conductivities over the saturated depth

$$K_{eq}(N_f, N_t) = \frac{\int_{N_f}^{N_t} dn}{\int_{N_f}^{N_t} \frac{dn}{K_n(\theta_s, n)}}$$

We may also define "equivalent depth", $N_{eq}$, the normal depth where the normal saturated hydraulic conductivity equals to $K_{eq}(N_f, N_t)$. From equations (2.2a) and (2.26),

$$N_{eq}(N_f, N_t) = -\frac{1}{f} \ln\left[\frac{f(N_f - N_t)}{e^{fN_f} - e^{fN_t}}\right]$$  \hspace{1cm} (2.27)$$
Figure 2-3: Pore pressure profile in the perched saturated zone

$N_{eq}$ is also the depth at which the pore pressure is maximum (equation (2.23)) (figure 2.3). For $N_t < n < N_{eq}$, the saturated hydraulic conductivity is greater than $K_{eq}$, and the pressure gradient is positive downward to compensate for the excessive hydraulic conductivity and maintain constant normal flow. For $N_{eq} < n < N_f$, the saturated hydraulic conductivity is smaller than $K_{eq}$, and constant normal flow implies a negative pressure gradient downward.

The parallel component of the flow is

$$q_p = K_p(\theta_s, 0)e^{-fn}\sin(\alpha)$$  \hspace{1cm} (2.28)

Between the wetting and the top fronts, normal flow is affected by the pressure gradient in the normal direction while parallel flow is not. And also due to the soil anisotropy, the saturated flow is deflected laterally. The angle of flow with the vertical
direction $z$ is designated $\beta$, which is a function of normal depth:

$$\tan(\alpha + \beta(n)) = \frac{q_p}{q_n} = a_r \tan(\alpha) \frac{e^{f N_f} - e^{f N_t}}{f(N_f - N_t)} e^{-f n}$$

(2.29)

Solving for $\beta(n)$ gives

$$\beta(n) = \tan^{-1}[a_r \tan(\alpha) \frac{e^{f N_f} - e^{f N_t}}{f(N_f - N_t)} e^{-f n}] - \alpha$$

(2.30)

The vertical and horizontal components of the flow in the zone of saturation are, respectively.

$$q_z(n) = q_n \cos(\alpha) + q_p \sin(\alpha) = [K_n(\theta, 0) \frac{f(N_f - N_t)}{e^{f N_f} - e^{f N_t}} \cos(\alpha)] \cos(\alpha) + [K_p(\theta, 0) e^{-f n} \sin(\alpha)] \sin(\alpha)$$

or

$$q_z(z) = K_n(\theta, 0) [a_r e^{-f z \cos(\alpha)} \sin^2(\alpha) + \frac{f(N_f - N_t)}{e^{f N_f} - e^{f N_t}} \cos^2(\alpha)]$$

(2.31a)

and

$$q_x(n) = -q_n \sin(\alpha) + q_p \cos(\alpha) = -[K_n(\theta, 0) \frac{f(N_f - N_t)}{e^{f N_f} - e^{f N_t}} \cos(\alpha)] \sin(\alpha) + [K_p(\theta, 0) e^{-f n} \sin(\alpha)] \cos(\alpha)$$

or

$$q_x(z) = K_n(\theta, 0) \sin(\alpha) \cos(\alpha) [a_r e^{-f z \cos(\alpha)} - \frac{f(N_f - N_t)}{e^{f N_f} - e^{f N_t}}]$$

(2.31b)

Because of the flow deflection in the saturated zone, the vertical infiltration is not constant with depth. It is the sum of a constant term and a term which decays exponentially with depth. The resulting horizontal flow may be negative (upslope) as well as positive (downslope), depending on the relative values of $a_r, f, z, \alpha, N_f$ and $N_t$. 

26
2.1.4 Wetting and Top Front Evolution

Wetting front before perched saturation

Under steady infiltration, the soil moisture profile in the unsaturated zone is given by equation (2.12). Prior to a storm the initial moisture profile is assumed to be given also by equation (2.12) with $R$ equal to a very small initial recharge rate $R_i$. Consider a constant rainfall rate $R, R > R_i$ over a given interval, a sharp discontinuity in the moisture content separates the area affected by the propagation of the infiltration wave and the undisturbed area below the front.

Assume: (1) equation (2.15) is valid for the area just above and below the wetting front; (2) the wetting front is parallel to the surface and advances perpendicular to it.
The continuity equation can be written as

$$\frac{\partial \theta}{\partial t} + \frac{\partial q_n}{\partial n} + \frac{\partial q_p}{\partial p} = 0$$  \hspace{1cm} (2.32)$$

Integration of equation (2.32) in the domain $\Omega$ (enclosed by $p = p_1, p = p_2, n = n_1$ and $n = n_2$) (figure 2.4) gives

$$\int_{\Omega} \left( \frac{\partial \theta}{\partial t} + \frac{\partial q_n}{\partial n} + \frac{\partial q_p}{\partial p} \right) d\Omega = \int_{\Omega} \left( \frac{\partial \theta}{\partial t} \right) d\Omega + \int_{\Omega} \left( \frac{\partial q_n}{\partial n} + \frac{\partial q_p}{\partial p} \right) d\Omega = 0$$  \hspace{1cm} (2.33)$$

Interchanging the integral and the derivative in the first term and applying Green’s theorem to the second term gives

$$\frac{d}{dt} \left( \int_{\Omega} \theta d\Omega \right) + \int_{s\Omega} (\vec{q} \cdot \vec{n}) d(s\Omega) = 0$$  \hspace{1cm} (2.34)$$

where $\vec{n}$ is the unit vector normal to the boundary of $\Omega$. The time rate of change in moisture content within $\Omega$ is balanced by the flux $\vec{q}$ across its boundary $s\Omega$.

Above and below the wetting front, the soil moisture profiles are given by equation (2.12) with infiltration rate equal to $R$ and $R_i$ respectively. Thus

$$\int_{\Omega} \theta d\Omega = \int_{p_1}^{p_2} \int_{n_1}^{n_2} \theta(n) dn dp = (p_2 - p_1)[\int_{n_1}^{n_n} \theta(n) dn + \int_{N_f}^{n_2} \theta(n) dn]$$

$$= (p_2 - p_1)[\left( \frac{R}{K_n(\theta_s, 0)} \right) \left( \theta_s - \theta_r \right) \left( \frac{e^{\frac{T}{n_2}}} {\int} \right)(e^{\frac{T}{n_2}} - e^{\frac{T}{n_1}}) + \theta_r(N_f - n_1) +$$

$$\left( \frac{R_i}{K_n(\theta_s, 0)} \right) \left( \theta_s - \theta_r \right) \left( \frac{e^{\frac{T}{n_2}}} {\int} \right)(e^{\frac{T}{n_2}} - e^{\frac{T}{n_f}}) + \theta_r(n_2 - N_f)]$$  \hspace{1cm} (2.35)$$

and

$$\frac{d}{dt} \left( \int_{\Omega} \theta d\Omega \right) = (p_2 - p_1)[\left( \frac{R}{K_n(\theta_s, 0)} \right) \left( \theta_s - \theta_r \right) e^{\frac{T}{n_f}} + \theta_r]$$

$$- \left( \frac{R_i}{K_n(\theta_s, 0)} \right) \left( \theta_s - \theta_r \right) e^{\frac{T}{n_f}} + \theta_r] \left( \frac{dN_f}{dt} \right)$$

$$= (p_2 - p_1)[\theta(R, N_f) - \theta(R_i, N_f)] \left( \frac{dN_f}{dt} \right)$$  \hspace{1cm} (2.36)$$
The second term of equation (2.34) can be expanded as
\[
\int_{s\Omega} (\vec{q} \cdot \vec{N})d(s\Omega) = -\int_{n_1}^{n_2} q_p|_{p=p_1} dn - \int_{p_1}^{p_2} q_n|_{n=n_1} dp + \int_{n_1}^{n_2} q_p|_{p=p_2} dn + \int_{p_1}^{p_2} q_n|_{n=n_2} dp
\]

Since the flow parallel to the hillslope is constant with depth
\[
q_p|_{p=p_1} = q_p|_{p=p_2} \forall n
\]
we have
\[
\int_{s\Omega} (\vec{q} \cdot \vec{N})d(s\Omega) = -\int_{p_1}^{p_2} q_n|_{n=n_1} dp + \int_{p_1}^{p_2} q_n|_{n=n_2} dp \quad (2.37)
\]
For \( n < N_f \), \( q_n \) is given by equation (2.10), and for \( n > N_f \), \( q_n \) is also given by equation (2.10) with \( R = R_i \). It then follows that
\[
\int_{s\Omega} (\vec{q} \cdot \vec{N})d(s\Omega) = -R \cos(\alpha)(p_2 - p_1) + R_i \cos(\alpha)(p_2 - p_1) \quad (2.38)
\]
Substituting equations (2.36) and (2.38) into equation (2.34) and after manipulations yields
\[
\frac{dN_f}{dt} = \frac{(R - R_i) \cos(\alpha)}{\theta(R, N_f) - \theta(R_i, N_f)}, \quad N_f < N^*(R) \quad (2.39)
\]
Equation (2.39) governs the advancement of the wetting front before perched saturation develops, that is, before \( N_f \) reaches the critical depth \( N^*(R) \). Since when \( N_f < N^*(R) \), the top front is the same as the wetting front, equation (2.39) is also valid for the top front evolution.

It can be seen that the wetting front advancement velocity depends on the difference between the rainfall rate and the initial recharge rate and the difference between the moisture distributions corresponding to \( R \) and \( R_i \) at the wetting front.

**Wetting and top fronts in perched saturation**

After the wetting front reaches the critical depth \( N^*(R) \), the evolution of \( N_f \) can be similarly obtained through integration of the continuity equation. The domain of integration is now defined by \( N_i < n_1 < N_f < n_2 \). Normal flow is given by equation
(2.25) at \( n = n_1 \) (saturated zone) and by equation (2.10) at \( n = n_2 \) (initial recharge zone). The result is

\[
\frac{dN_f}{dt} = \frac{K_n(\theta_s, 0) e^{f(N_f - N_t)} - R}{\theta_s - \theta(R_t, N_f)} \cos(\alpha), N_f > N^*(R) \tag{2.40}
\]

Derivation of the evolution of the top front is similar to that of the wetting front. The domain of integration is now defined by \( n_1 < N_t < n_2 < N_f \). Normal flow is given by equation (2.10) at \( n = n_1 \) and by equation (2.25) at \( n = n_2 \). The result is

\[
\frac{dN_t}{dt} = \frac{K_n(\theta_s, 0) e^{f(N_t - N_f)} - R}{\theta_s - \theta(R, N_t)} \cos(\alpha), N_t > 0 \tag{2.41}
\]

Since \( R \cos(\alpha) > q_n = K_n(\theta_s, 0) e^{f(N_f - N_t)} \cos(\alpha) \), the time derivative of \( N_t \) is negative. \( N_t \) normally decreases until it reaches the terrain surface. Eventually when \( N_t \) reaches the surface, we have

\[
\frac{dN_t}{dt} = 0, N_t = 0 \tag{2.42}
\]

## 2.2 Basin Scale Model

The one-dimensional infiltration model is formulated for a constant rainfall intensity in a uniform slope of infinite length. Water moves in the plane defined by the vertical direction and the direction of maximum slope (\( p \) direction). Lateral inflow is balanced by lateral outflow for each vertical section. When this one-dimensional model is applied to a basin scale, many difficulties arise. First, water does not move just in a plane, but rather, the flow is truly 3-dimensional. Second, even if the flow is confined in a plane, water will accumulate or deplete at certain points because of different slopes and different soil properties, and this accumulation and/or depletion will affect the wetting and top front evolutions. Third, rainfall rarely, if ever, occurs with constant intensity in time or space. To deal with these difficulties, further assumptions and modifications have to be made.
2.2.1 Equivalent rainfall rate

The basin is horizontally discretized into small square elements called pixels. Each pixel is treated as a soil column within which the one-dimensional model is applicable. The temporal variation of the rainfall intensity in one pixel can be treated by assuming that water gets redistributed in the normal direction so quickly that only a single moisture wave propagates downwards despite the variability of the rainfall intensity during a storm. This assumption, although strong, is supported to some extent by the way the unsaturated infiltration mechanism redistributes moisture (Garrote, 1992). With a variable rainfall intensity, if moisture content at some depth is higher, the hydraulic conductivity will be higher, and moisture will tend to migrate from that point. Conversely, if moisture content is lower at some depth, moisture will tend to accumulate.

For each computation step and for each pixel, a moisture balance is performed to derive the moisture content (see next section). We define an equivalent uniform rainfall rate $R_e$ as that which would lead to the same moisture content in the unsaturated part of the soil column. The soil moisture profile is given by equation (2.12). Integrating equation (2.12) above the top front and equating it to the unsaturated moisture content $M_u$, we get

$$\int_0^{N_t} \left( \frac{R_e}{K_n(\theta_s, 0)} \right) \left( \theta_s - \theta_r \right) e^{\frac{t}{\theta_s}} + \theta_r \right) dn = M_u$$

and solving for $R_e$ yields

$$R_e = K_n(\theta_s, 0) \left[ \frac{M_u - \theta_r N_t}{(\theta_s - \theta_r) \left( e^{\frac{t_{NT}}{\theta_s}} - 1 \right)} \right]$$

where $M_u$ is the total storm moisture content inside the soil column above the top front.

Equation (2.43) is valid only when there is an unsaturated part in the pixel. When the top front is at the surface ($N_t = 0$), the equivalent rainfall rate is the actual rainfall rate at that time step.
2.2.2 Moisture Balance

The lateral moisture fluxes between neighboring pixels pose another difficulty. The lateral moisture flux, the vertical flux, the moisture content and the hydraulic potential of the soil within a pixel are functions of one another. The three-dimensional equation of moisture flow is needed to fully account for the moisture exchange between pixels. However, to reduce the computation burden to the extent bearable for real-time flood forecasting, simplifications must be introduced.

The simplifications adopted are based on the idea of decoupling the vertical and horizontal moisture flow equations. Two mechanisms of lateral moisture transfer are considered. First, when the one-dimensional model is applied to a bounded domain, the horizontal component of flow produces a net moisture flow at the downslope boundaries, which is transmitted to the contiguous element. Additionally, the application of the one-dimensional model independently to each element gives different pressure and moisture distribution to every element, which, in turn, leads to horizontal hydraulic gradients that drive lateral flow between elements (Garrote, 1992).

Lateral flow in homogeneous terrain

In the one-dimensional kinematic model of infiltration, local slope, anisotropy and vertical heterogeneity produce a diversion of the infiltration from the vertical direction. Thus lateral moisture movement exists. For an infinite homogeneous slope, the moisture flow is not affected by the boundary conditions. However, for a finite domain, the boundary conditions will in general have great influence on the flow. The influence is treated rather simply here.

The one-dimensional infiltration equations are assumed to be valid at the subgrid scale. The horizontal component of the flow can be integrated vertically from the surface to the wetting front to obtain the net flow across the boundary (Garrote, 1990).

\[ Q_h = W \int_0^{N \cos(\alpha)} q_z(z) \, dz \]

where \( Q_h \) is the total discharge from the upslope element, and \( W \) is the width of the
cross-section. This equation can be evaluated as

\[ Q_h = \int_0^{N_f \frac{\cos \alpha}{\cos \alpha}} qz(z)\,dz + \int_{N_f \frac{\cos \alpha}{\cos \alpha}}^{N_t \frac{\cos \alpha}{\cos \alpha}} qz(z)\,dz \]  

(2.44)

Substituting equations (2.17) and (2.31) into equation (2.44) yields

\[ Q_h = W \sin(\alpha) \left\{ [N_t R(a_t - 1)] + [K_n(\theta_s, 0) \frac{\alpha_r}{f} (e^{-f N_t} - e^{-f N_f})] - [K_n(\theta_s, 0) \frac{f(N_f - N_t)^2}{e^{f N_f} - e^{f N_t}}] \right\} \]

(2.45)

For a rainfall rate lower than the initial infiltration capacity \((R < K_n(\theta_s, 0))\), lateral discharge is given only by the first term of equation (2.45) while \(N_t = N_f < N^*(R)\). After perched saturation has developed \((N_t < N_f)\), lateral discharge is given by all the three terms of equation (2.45). And eventually when saturation reaches the surface \((N_t = 0)\), the lateral discharge is given only by the second and the third term of equation (2.45).

The lateral flow across the cross-section is evaluated for every element in the basin. Subsurface inflow into a given element is given by the sum of the outflow from all its upstream elements draining directly into it. The computations are carried out recursively according to the relation “drains to” (Garrote, 1992). This means that the outflow from all the upstream elements should be computed before computing the outflow from a given element.

The simplifications made are: (1) Boundary effects are neglected. The one-dimensional model is applied to the central point of every element, and each element is considered effectively an infinite extension of soil. (2) The influence of lateral moisture flow on the momentum equations is neglected, although the mass conservation is taken into account. The equations governing the evolution of wetting and top fronts are affected only indirectly by the lateral moisture balance (through the equivalent rainfall rate). (3) The flow entering every element is assumed parallel to its line of maximum slope, irrespective of the orientation of the slopes of the upstream elements.
Lateral flow due to spatial variability

The horizontal hydraulic gradients between elements are usually very small because the horizontal distances between elements are typically very large. Therefore, the lateral flow due to spatial variability of pressure is small compared to the lateral flow due to topography and anisotropy (Garrote, 1992).

Again, the one-dimensional model is applied to the central point of every element independently. The pressure distribution along the normal direction is estimated for every element. The lateral hydraulic gradient between two contiguous elements is given by

\[ J_x(z) = \frac{\partial \Psi}{\partial x} \approx \frac{\Delta \Psi(z)}{\Delta x} = \frac{\Psi_2(z) - \Psi_1(z)}{x_2 - x_1} \]  \hspace{1cm} (2.46)

where \( \Psi \) is the pore pressure, \( z \) is the vertical depth, and \( x \) is the horizontal distance.

The assumptions made are: (1) The effects of different depths of the perched saturation at contiguous elements are neglected. (2) The effects of different slopes at contiguous elements are neglected. Thus the normal directions to the terrain surface at two contiguous elements are assumed parallel to each other, and \( \Delta x \) is independent of \( z \). Hence the vertical depth \( z \) can be substituted by \( \frac{\Delta z}{\cos(\alpha)} \), where \( \alpha \) is the average slope angle of the two elements.

Using Darcy's equation, we get the lateral flow

\[ q_x(z) = -K_m(z)J_x(z) \]  \hspace{1cm} (2.47)

Because of the spatial heterogeneity, two contiguous elements may have different soil hydraulic conductivities inside a basin. The equivalent hydraulic conductivity in the horizontal direction, \( K_m(z) \), for the inter-element distance \( \Delta x \) is approximated by the equivalent hydraulic conductivity in the parallel direction, which, in turn, corresponds to two elements of length \( \Delta x/2 \) connected in series

\[ \frac{1}{K_m(z)} = \frac{1}{2K_p^1(\theta_s, z)} + \frac{1}{2K_p^2(\theta_s, z)} \]  \hspace{1cm} (2.48)

The total lateral flow resulting from the spatial variability of pressure can be
obtained by integrating equation (2.47) along the saturated depth

\[ Q_x = - \int_{z_{inf}}^{z_{sup}} K_m(z) J_x(z) dz \]  

(2.49)

where

\[ z_{inf} = \min\left( \frac{N_{t_1}}{\cos(\alpha)}, \frac{N_{t_2}}{\cos(\alpha)} \right) \]

and

\[ z_{sup} = \min\left( \frac{N_{t_1}}{\cos(\alpha)}, \frac{N_{t_2}}{\cos(\alpha)} \right) \]

Substituting equation (2.46) into equation (2.49) yields,

\[ Q_x = \int_{z_{inf}}^{z_{sup}} K_m \frac{\Psi_1(z) - \Psi_2(z)}{\Delta x} dz \]

(2.50)

The lateral flow of moisture due to the spatial variability of pressure can be computed as the difference between the flows that would result considering pressure distribution in both pixels independently. Substituting equations (2.23) and (2.48) into the first term of equation (2.50) and considering a cross-section of width \( W \) yields the moisture outflow from element 1

\[ Q_{out} = K_p(\theta_s, 0) \frac{W}{x_2 - x_1} (N_f - N_t) \left[ \frac{N_{t_1} e^{f N_f} - N_{t_2} e^{f N_t}}{e^{f N_f} - e^{f N_t}} + \frac{1}{f} - \frac{N_f + N_t}{4} \right] \]  

(2.51)

All the variables in the above equation refer to element 1 only. The second term of equation (2.50) which represents the moisture inflow into element 1 can be derived similarly.

The total moisture outflow from a given element is the sum of equations (2.45) and (2.51), and the total moisture inflow to a given element is the sum of the moisture outflow from all the upstream elements that drain directly to it.
2.2.3 Runoff Generation

Two modes of runoff generation are represented in DBS: infiltration excess runoff and return flow (Garrote, 1992). The infiltration excess runoff is a direct consequence of pixel states, and return flow is the result of the global moisture balance in the soil column.

Pixel states

Depending on the positions of the top and wetting fronts, a soil element can be in any of the four different moisture states, which have different runoff generation potentials (figure 2.5)

- Unsaturated state: The wetting front is at some depth above the water table, and perched saturation has not developed yet, the wetting and top fronts are the same. The soil column generate infiltration excess runoff only.
- Perched saturated state: The wetting front has penetrated beyond the critical depth $N^*$, and the top front is separated from the wetting front but has not reached the surface. The soil column can generate infiltration excess runoff.
- Surface saturated state: The wetting front has not reached the water table, but the top front has reached the soil surface. The soil column generates infiltration excess runoff. It may also generate return flow if infiltration plus subsurface inflow exceed subsurface outflow plus the storage increment due to the progression of the wetting front.
- Fully saturated state: The wetting front has reached the water table and the top front has reached the soil surface. Both infiltration excess runoff and return flow are generated.

Infiltration excess runoff

The infiltration capacity of a soil column is a function of the wetting and top front positions. When the rainfall intensity is higher than the infiltration capacity, infiltration excess runoff is generated. For unsaturated ($N_f = N_t$) and perched saturated
Figure 2-5: Different pixel states
(0 < \( N_t < N_f \)) elements, the infiltration capacity is only controlled by the surface saturated hydraulic conductivity,

\[
I_{\text{max}} = K_n(\theta_s, 0) \cos(\alpha) \quad (2.52)
\]

where \( I_{\text{max}} \) is the infiltration capacity of the soil column.

For surface saturated elements \((N_t = 0, N_f < N_{w_t})\), the infiltration capacity is the harmonic mean of the saturated normal hydraulic conductivities of the saturated depth. From equation (2.26) we have

\[
I_{\text{max}} = K_n(\theta_s, 0) \frac{fN_f}{e^{fN_f} - 1} \cos(\alpha) \quad (2.53)
\]

\( I_{\text{max}} \) is controlled by the whole perched saturated zone from the surface down to the wetting front, rather than just by the surface layer.

For fully saturated elements \((N_t = 0, N_f = W_{w_t})\), only the inter-storm recharge rate \( R_i \) can be maintained, no storm rainfall can infiltrate to the soil column.

\[
I_{\text{max}} = 0 \quad (2.54)
\]

The actual infiltration \( I \) is given by

\[
I = R, R \leq I_{\text{max}}
\]

\[
I = I_{\text{max}}, R > I_{\text{max}} \quad (2.55)
\]

And the infiltration excess runoff rate \( R_{\text{inj}} \) is given by

\[
R_{\text{inj}} = R - I \quad (2.56)
\]
Return flow

The moisture balance equation for one element during one time step can be written as

\[
\frac{dM_t}{dt} = \frac{dN_f}{dt} \theta(R_t, N_f) + I + \frac{Q_{lin} - Q_{lout}}{A}
\]  

(2.57)

where \( M_t \) is the total moisture content above the wetting front, \( Q_{lin} \) is the lateral moisture inflow, \( Q_{lout} \) is the lateral moisture outflow, and \( A \) is the horizontal area of one element. The change of \( M_t \) in an element is the result of infiltration, lateral moisture exchange, and the incorporation of the initial moisture due to the vertical displacement of the wetting front. \( \theta(R_t, N_f) \) can be evaluated with equation (2.12).

It is assumed that all the moisture inflow accumulates above the wetting front. Therefore, \( M_t \) has an upper limit set by \( N_f \theta_s \) corresponding to surface saturation (Garrote, 1992). Whenever the sum of the previous moisture content plus the net moisture inflow exceeds this limit, return flow is generated, and the return flow rate during \( t_1 - t_0 \) is

\[
R_r = \{N_f(t_1)\theta_s - M_t(t_0) - \int_{t_0}^{t_1} \left[ \frac{dN_f}{dt} \theta(R_t, N_f) + I + \frac{Q_{lin} - Q_{lout}}{A} \right] dt \}/(t_1 - t_0) 
\]  

(2.58)

The total runoff generation rate \( R_f \) by one element is

\[
R_f = R_{inf} + R_r
\]  

(2.59)

2.2.4 Surface flow routing

The runoff generated by each grid inside the basin has to be routed to the basin outlet to get the actual hydrograph of a storm event. The path that the runoff generated at every grid point follows can be derived from the DEM according to the rule that water drains to the lowest of its 8 neighboring grids. For a hillslope pixel, the path consists of two parts: the hillslope part and the channel part; and for a channel pixel, the runoff only follows a channel path.

Although theoretical equations exist for both the overland flow and open channel
flow, these equations are very difficult to apply because detailed knowledge about the
gometry and hydraulic characteristics of the overland and channel flow paths is hard
t to obtain. For a distributed rainfall-runoff model, the “two velocities” assumption is
still the most convenient and widely used runoff routing method. For example, Wyss
oruted the overland flow to the stream with a uniform velocity.

For the DBS model, both the overland flow and streamflow velocities are assumed
constant throughout the basin for a given time step, and these two velocities are
allowed to vary with the streamflow rate at the basin outlet as the storm progresses,
because the streamflow rate at the basin outlet is, to some extent, an indication of the
changing flow conditions in the basin. It is also assumed that once it is generated, the
runoff does not get infiltrated again although it may travel through an unsaturated
element on its way to the stream.

Specifically, we assume the streamflow velocity \( v_s \) at time \( \tau \) is a power function of
the streamflow rate at the basin outlet \( Q \) at time \( \tau \),

\[
    v_s(\tau) = c_v [Q(\tau)]^r
\]

where \( c_v \) and \( r \) are both constant coefficients. And the hillslope velocity \( v_h \) at time \( \tau \)
is assumed proportional to the streamflow velocity at time \( \tau \),

\[
    v_h(\tau) = \frac{v_s(\tau)}{K_v}
\]

where \( K_v \) is another constant coefficient.

The uniform travel velocities of the runoff allow for a simple computation of the
hydrograph at the basin outlet. The instantaneous response function of an element
\((x, y)\) at time \( \tau \) is assumed to be a Dirac delta function, with a delay equal to the
travel time from the element to the basin outlet,

\[
    h_\tau(x, y, t) = \delta \left( \frac{l_h(x, y)}{v_h(\tau)} + \frac{l_s(x, y)}{v_s(\tau)} \right)
\]
where $l_h(x, y)$ is the travel distance from element $(x, y)$ to the nearest stream, and $l_s(x, y)$ is the travel distance from the nearest stream point to the basin outlet.

An incremental basin response is estimated independently for every time step $\tau$ routing the runoff produced at every element of the basin,

$$ q_\tau(t) = \sum_{(x,y)\in \text{basin}} R_f(x, y, \tau) h_\tau(x, y, t) \Delta x \Delta y $$

(2.63)

where $(R_f(x, y, \tau)$ is the runoff produced at element $(x, y)$ at time $\tau$ given by equation (2.59), $\Delta x \Delta y$ is the area of an element. The hydrograph at the basin outlet up to time $T$ is obtained by adding up these incremental responses since the beginning of the storm

$$ Q(t) = \sum_{\tau=0}^{\tau=T} q_\tau(t) $$

(2.64)
Chapter 3

Initial Basin Conditions and Model Calibration Procedure

3.1 Initial Basin Moisture Conditions

Runoff produced from one storm depends on the storm precipitation and the prestorm basin moisture conditions — initial water table depth and the moisture profile in the unsaturated zone above the water table. In the DBS model, the prestorm basin moisture condition is parameterized only by the water table depth. The unsaturated zone moisture profile is assumed to be given by equation (2.12) with inter-storm recharge rate $R_i$ such that the soil is saturated at the water table depth. Garrote (1992) showed how sensitive the DBS model is to the initial water table depth. However, only in rare cases are the spatially distributed water table depth data available, as they have to be observed by wells or piezometers. Oftentimes, people will just assume that the water table is parallel to the terrain surface, as Bathurst (1986) did when he applied SHE to a upland watershed in mid-Walse. But this is only a very coarse approximation. If we ignore the spatial soil changes for the time been, common sense tells us that water table is the deepest at the watershed boundaries, and the closer to the perennial channels, the shallower the water table depth will be.

The water table evolves as the combined result of precipitation, infiltration, deep water percolation, subsurface water flow and evaporation. During the inter-storm
period, the water in the soil has three destinations: evaporation, runoff and deep percolation. Sometimes deep percolation can be neglected for a short period water balance calculation, but inter-storm evaporation is very important except in very cold and humid regions. The detailed history of the meteorological conditions (temperature, wind speed, etc.) between storms are usually not available, thus making it hard to estimate the amount of water that has been evaporated. To estimate water table prior to a storm, a water balance model is seldom possible.

It would seem very useful to relate the water table depth with the streamflow at the basin outlet prior to a storm. First because conceptually the amount of flow at the basin outlet indeed depends on the water table position in the basin, second because the streamflow data is usually easy to measure.

Previously, the initial water table generation model used for DBS was by Cabral et al. (1990). The model assumes constant recharge rate across the basin, which is taken to be in long term equilibrium with the outflow at the basin outlet. Evaporation and deep percolation are not considered, all the recharged water comes out as outflow. Only saturated flow in the soil is considered, and Darcian flow equations are solved numerically. Water table depth is initialized at zero everywhere (whole basin saturated), and the saturated zone is allowed to drain under the constant recharge rate. Flow equations are also solved recursively, each time with an updated water table depth file. As time goes by, the discharge from the saturated zone reduces until a steady state is reached when the computed discharge is equal to the inter-storm streamflow at the basin outlet. This model does a fairly good job, but it is too sensitive to the soil parameters and the initial recharge rate. Sometimes it can generate unrealistic results at some pixels.

3.1.1 Relating Spatially Distributed Water Table Depth to Basin Averaged Water Table Depth

We use a new water table generation model based on Sivapalan et al. work (1987) and Troch et al. work (1993a). This model also assumes a constant recharge rate to
the water table across the basin, but the flow equations are solved analytically rather than numerically. Figure (3.1) is a typical sectional view of a hillslope. With the Dupuit approximation (assuming horizontal flow only and neglecting vertical flow), the downslope flow beneath a water table at depth $z_i$ is approximated by

$$q_i = T(z_i) \tan \beta$$ (3.1)

where $\beta$ is the slope of the terrain surface, and $T(z_i)$ is the transmissivity of the water table aquifer: the integration of hydraulic conductivity from the profile bottom to the water table. With saturated hydraulic conductivity an exponential function of depth, we have

$$T(z_i) = \int_{z_i}^{Z} K_p(\theta, z) \, dz = \frac{K_p(\theta_s, 0)}{f} \left[ \exp(-f z_i) - \exp(-f Z) \right]$$ (3.2)
where $K_p(\theta_s, z)$ is the saturated hydraulic conductivity at depth $z$ and $K_p(\theta_s, 0)$ is the saturated hydraulic conductivity at the soil surface.

For large $f$ or $Z$, $\exp(-fZ)$ can be assumed small (Sivapalan et al., 1987). Substituting (3.2) into (3.1), we get

$$q_i = \frac{K_p(\theta_s, 0)}{f} \tan \beta \exp(-fz_i)$$

$$= T_0 \tan \beta \exp(-fz_i) \tag{3.3}$$

where $T_0$ is the transmissivity coefficient.

Under quasi steady state conditions with a spatially uniform recharge rate $R$ to the water table (Sivapalan et al. 1987),

$$aR = T_0 \tan \beta \exp(-fz_i) \tag{3.4}$$

where $a$ is the area draining through location $i$ per unit contour length. Equation (3.4) results in the following expression for water table depth

$$z_i = -\frac{1}{f} \ln \left( \frac{aR}{T_0 \tan \beta} \right) \tag{3.5}$$

Integrating over the total area gives the watershed mean water table depth (Sivapalan et al., 1987)

$$\bar{z} = \frac{1}{A} \int_A z_i \, dA$$

$$= \frac{1}{fA} \int_A \left[ -\ln \left( \frac{a}{T_0 \tan \beta} \right) - \ln R \right] \, dA \tag{3.6}$$

where $A$ is the total area of the watershed.

Substituting (3.4) into (3.6) yields (Sivapalan et al., 1987)

$$\bar{z} = \frac{1}{f} \left[ -\frac{1}{A} \int_A \ln \left( \frac{a}{T_0 \tan \beta} \right) \, dA + f z_i + \ln \left( \frac{a}{T_0 \tan \beta} \right) \right] \tag{3.7}$$
or

\[ f(\bar{z} - z_i) = [\ln(\frac{a}{\tan\beta}) - \lambda] \] - \[\ln T_0 - \ln T_e\] \quad (3.8)

where

\[ \lambda = \frac{1}{A} \int_A \ln(\frac{a}{\tan\beta}) dA \]

and

\[ \ln T_e = \frac{1}{A} \int_A \ln T_0 dA \]

We can also write equation (3.8) as

\[ z_i = \bar{z} - \frac{1}{f}[\ln(\frac{a T_e}{T_0 \tan\beta}) - \lambda] \quad (3.9) \]

where \(\ln(\frac{a T_e}{T_0 \tan\beta})\) is the combined topography-soil index. Equation (3.9) predicts the local water table depth relative to the basin averaged water table depth according to soil and topographic properties of the basin. Of particular interest are the predicted local water table depths less than the capillary fringe. These locations represent the initially saturated areas of the watershed.

If we also assume that the constant recharge rate is in equilibrium with the base recession flow prior to the storm, integrating equation (3.3) along the perennial channels and substituting equation (3.9) for \(z_i\), yields the base flow of the watershed (Sivapalan et al., 1987)

\[ Q = 2 \int_L q_i dL \]

\[ = 2 \int_L a T_e \exp(-\lambda - f\bar{z}) dL \]

where \(L\) is the total length of perennial channels of the watershed. Here the channel routing time of the flow is not considered. Equation (3.10) can be rewritten as

\[ Q = Q_0 \exp(-f\bar{z}) \quad (3.11) \]

where

\[ Q_0 = A T_e \exp(-\lambda) \quad (3.12) \]
Equation (3.11) can be inverted to get the basin averaged water table depth prior to a storm event given the prestorm base flow rate (Sivapalan et al., 1987)

\[ z = -\ln(Q/Q_0)/f \] (3.13)

Equation (3.11) means that \( Q_0 \) and \( f \) are both parameters of the basin recession flow. We can use equation (3.13) and equation (3.9) together to get the spatially varied water table depth across the watershed prior to a storm event. All we need are the prestorm base flow at the outlet, \( Q_0 \) and \( f \) values. If the previous storm occurred a sufficiently long time ago, we can assume that the streamflow at the outlet is only due to base flow, as people often do.

However, it is not easy to get reliable soil parameter values across the whole basin. Thus it is not easy to get an estimate of \( Q_0 \), since \( Q_0 \) depends on DEM and soil parameters of the watershed. Oftentimes the soil data we get are very coarse, based only on a few laboratory sample measurements. Furthermore, \( Q_0 \) is a basin scale parameter, it should depend on the basin scale soil hydraulic conductivities rather than the hydraulic conductivities measured at the laboratory, as these two different scale conductivities are often very different due to the macropores and so on. Due to the high sensitivity of the basin averaged water table depth to the value of \( Q_0 \), this model may results in very unrealistic initial water table depths.

Troch et al. (1993a), recognizing this problem, developed a method to find the values of \( z \) from the watershed base flow recession curves. This makes \( z \) depend on basin scale parameters rather than on laboratory measured small local scale parameters.

### 3.1.2 Relating Basin Averaged Water Table Depth with Prestorm Streamflow

The water table height \( h(x, t) \) is a function of space and time. Under the Dupuit approximation, for a homogeneous rectangular unconfined aquifer above an impermeable layer and draining to a stream (Figure 3.2), we can use Boussinesq's equation
to describe the dynamics of the water table

\[ \frac{\partial h}{\partial t} = \frac{k}{n_e} \frac{\partial}{\partial x} (h \frac{\partial h}{\partial x}) \]  

(3.14)

where \( t \) is the time, \( n_e \) is the drainable porosity of the aquifer, and \( k \) is the hydraulic conductivity of the aquifer. Here the slope effect of the impermeable layer is neglected.

For the initial condition of complete saturation of the aquifer and for the case of fully penetrating stream with initial \( D_c = 0 \) (figure 3.2), two exact solutions of equation (3.14) can be achieved, one valid for small time \( t \), the other valid for large time \( t \) (Troch et al, 1993a). For small \( t \), the effect of the impermeable wall at \( x = B \) is negligible as if \( B = \infty \). Polubarinova-Kochina (1962) has an exact solution for this case

\[ h(x, t) = 2.365D(Y - 2Y^4 + 3Y^7 - 4/11Y^{10} - ...) \]  

(3.15)
where \( Y = 0.487 \sqrt{\eta} \), and \( \eta = (x \sqrt{\eta_e})/(2 \sqrt{kD}t) \). The resulting outflow rate at \( x = 0 \) is (Troch et al, 1993a)

\[
q(t) = 0.332(k \eta_e)^{1/2} D^{3/2}t^{-1/2} \tag{3.16}
\]

The response of the aquifer to a sudden drainage at \( x = 0 \) is like the propagation of a wave. As soon as the wave reaches the impermeable wall, the small time solution is no longer valid (Troch et al. 1993a). Boussinesq obtained an exact solution of equation (3.14) by assuming that the initial water table has the shape of an inverse incomplete beta function (See Polubarinova-Kochina, 1962, P.515-517)

\[
h(x, t) = D(317(x/B)^{1.115(x/B)})t \tag{3.17}
\]

where \( \phi(x/B) \) is the initial shape of the water table surface. The outflow rate to the channel will be (Troch et al, 1993a)

\[
q(t) = \frac{0.862kD^2}{B[1 + 1.115(kD_{n_e}B^2)t]^2} \tag{3.18}
\]

Equation (3.18) becomes valid only after certain time, when the water table shape starts to resemble the assumed inverse incomplete beta function \( \phi(x/B) \).

When the previous storm event has occurred a sufficiently long time ago, the large time solution is applicable in a watershed. We can get the averaged water table height for the aquifer by integrating equation (3.17)

\[
\bar{h}(t) = \frac{1}{B} \int_0^B h(x, t) \, dx \\
= \frac{0.773D}{[1 + 1.115(kD_{n_e}B^2)t]} \tag{3.19}
\]

Substituting (3.19) into (3.18) yields

\[
q = 1.443 \frac{k}{B} \bar{h}^2 \tag{3.20}
\]

In a true watershed, the effect of the slope of the terrain surface and the slope of
the underlying impermeable layer on the water table dynamics has to be considered. However, except in extremely rugged terrain areas, this effect is ephemeral and limited to a few days at most after a storm because after precipitation the water table always tends to become flatter with time (Troch et al, 1993a). Thus, the homogeneous rectangular unconfined aquifer seems to be a good approximation of the hillslope in a watershed as far as the water table shape is concerned. Integrating equation (3.20) along the perennial channels of the watershed yields the watershed base flow \( Q \) (Troch et al, 1993a)

\[
Q = 2.886 \frac{k}{L} \bar{h}^2 L \\
= 5.772k\bar{h}^2 D_dL \\
= 5.772k(D - \bar{z})^2 D_dL
\]

(3.21)

where \( D_d \equiv 2/B \) is the watershed drainage density, and \( L \) is the total length of the perennial channels of the watershed. Here again the channel routing time of flow is neglected.

Equation (3.21) can be used to determine the basin averaged water table depth prior to a storm event. While the prestorm base flow \( Q \), drainage density \( D_d \) and perennial channel length \( L \) are all easy to measure, the basin scale effective values of hydraulic conductivity \( k \) and soil depth \( D \) are not. Troch et al. (1993a) defined a critical base flow value \( Q_c \) corresponding to a situation where the aquifer starts to behave in accordance with the solution for large time. At the critical time \( t_c \), the water table shape becomes an inverse incomplete beta function \( h(x, t_c) = D\phi(x/B) \). Substituting into equation (3.17) yields

\[
1 + 1.115\left(\frac{kD}{n_e B^2}\right)t_c = 1 
\]

(3.22)

And substituting equation (3.22) into equation (3.18) gives the critical outflow rate at \( x = 0 \)

\[
q_c = \frac{0.862kD^2}{B} 
\]

(3.23)
Integrating $q_c$ along the prennial channels of the watershed gives

$$Q_c = 3.450kD^2D_dL$$

(3.24)

From (3.24), the basin scale soil depth $D$ can be estimated once $k$ is known.

To estimate $D$ and $k$, Brutsaert and Nieber (1977) proposed to analyze the basin recession flow hydrograph in differential form

$$\frac{dQ}{dt} = \phi(Q)$$

(3.25)

where function $\phi(.)$ is a characteristic of the watershed. They showed that for several solutions based on the Dupuit-Boussinesq hydraulic theory, $\phi(.)$ can be written as a power function

$$\frac{dQ}{dt} = -aQ^b$$

(3.26)

where $a$ and $b$ can be related to the topographic and soil properties of the watershed.

It follows from equation (3.18) that for large time

$$a_1 = \frac{4.804k^{1/2}L}{n_eA^{3/2}}$$

(3.27)

$$b_1 = 3/2$$

(3.28)

and from equation (3.16) we have for small time

$$a_2 = \frac{1.133}{kn_eD^3L^2}$$

(3.29)

$$b_2 = 3$$

(3.30)

Theoretically for a watershed, a log-log plot of $dQ/dt$ vs. $Q$ of the recession base flow should follow two straight lines, one with a slope of $3/2$ representing the large time behavior of the watershed, the other with a slope of 3 representing small time behavior. The intersection of these two lines corresponds to the critical flow rate $Q_c$.

The drainage area $A$, drainage density $D_d$ and perennial channel length $L$ can be
measured from the map or DEM. Thus, based on this plot, with a priori estimation of the basin scale drainable porosity, we can estimate $k$ from equation (3.27), $D$ from equation (3.24), and eventually, $\bar{z}$ from equation (3.21). With the estimated $\bar{z}$ value, equation (3.9) can be used to get the spatially varied water table depths across the whole watershed.

It should be noted that hydraulic conductivity $k$ and soil depth $D$ are properties of a given watershed. They do not change from storm event to storm event. Thus, we need to estimate them only once for a given watershed. Then for each storm, the prestorm base flow value is all we need to get the initial water table depth across the watershed.

### 3.2 Application to Souhegan River Basin

Since $k$ and $D$ are basin characteristics, we should expect to get the same estimated values for them using recession flow data measured at different times. However, the streamflow measurements, as (or more than) any other hydrological measurements, are full of small or large errors. In order to minimize the effect of these errors, we should use as many recession flow data as possible to estimate $k$ and $D$. In the Souhegan case, we use the recession parts of the continuous hourly streamflow data (sometimes 4 or 5 measurements per day) from 1985 through 1993.

Figure (3.3) shows the relationship between $dQ/dt$ and $Q$ of the recession flow in the Souhegan river. A log-linear regression to minimize the sum of the squared differences between the observed and fitted values of $\ln(-dQ/dt)$, results in $dQ/dt = -5.6775 \times 10^{-7}Q^{1.6068}$. That is to say, the slope of the fitted line is 1.6068, very close to 3/2 as predicted by equation (3.28) for large time behavior of the aquifer. The correlation coefficient of $dQ/dt$ and $Q$ is 0.76.

As mentioned earlier, to get a good estimation of the basin-averaged water table depth prior to a storm, we need a good estimation of $a_1$ and $Q_c$ from the $dQ/dt - Q$ plot and a reasonable a priori estimation of $n_e$ based on our knowledge about the basin. As it turns out, when applied to an actual basin, some of the theoretical
Figure 3-3: Recession flow at Souhegan river. Linear regression fitted line and 15% vertical threshold estimation scheme has to be modified a little.

Although we are using only the recession flow data, we can not guarantee that we are dealing only with base flow. Inevitably, there will be some overland flow component, interflow component, etc.. The base flow component should correspond to the smallest $|dQ/dt|$ for a given $Q$. Troch et al. (1993a) hypothesized to use a lower envelope with a slope of 3/2 to exclude 5% or 10% of the data points. this envelope would correspond to the theoretical large time watershed behavior (Equation (3.27) and (3.28)). Above this envelope, the data points may have been “polluted” by interflow, overland flow, etc.. In the Souhegan river basin, the 10% envelope corresponds to $a_1 = 10^{-6.53}$, and the 5% envelope corresponds to $a_1 = 10^{-6.85}$ (figure 3.4).

The critical base flow value $Q_c$ represents the upper limit for the applicability of the large time solution. Troch et al. (1993a) suggested that another envelope of slope 3 excluding 5% or 10% of the data points be used, the intersection of the envelope of slope 3 and the envelope of slope 3/2 represents the critical flow value.
Figure 3-4: Recession flow at Souhegan river. The 4 straight lines correspond to 5% and 10% envelopes with slopes 3/2 and 3 respectively.

They argued that in the large $Q$ value region, it is highly possible that some data points represent the small time behavior of the watershed. In their case of the Zwalm watershed in Belgium, the intersection position was quite insensitive to whether the 5% or 10% envelopes are used. Later in another study of two watersheds in eastern Pennsylvania, Troch et al. (1993b) found that the slope of 3 is not apparent in the $dQ/dt - Q$ plots. Therefore they estimated the critical base flow $Q_c$ to be the maximal observed base flow value in the plots. In our case of the Souhegan river basin, the intersection point is very sensitive to whether the 5% or 10% envelopes are used (Figure 3.4). With the 10% envelopes, $Q_c = 3.8019$ and with the 5% envelopes, $Q_c = 3.1623$.

The drainable porosity value $n_e$ should be estimated based on pumping tests, ideally at many locations in the watershed, because $n_e$ is a basin scale parameter, laboratory soil sample tests would not be reliable. Unfortunately, we don’t have any pumping test data in the Souhegan river basin. The range 0.02 to 0.07 are typical drainable porosity values for the watershed under study (Freeze and Cherry, 1979,
Figure 3-5: Sensitivity of water table depth to whether 5% or 10% envelope is used to estimate $a_1$

P.61). So we choose a middle value in this range $n_e = 0.04$ as the drainable porosity for the Souhegan river basin.

The actual water table depth for a given prestorm base flow is very sensitive to whether 5% or 10% envelope is used to determine $a_1$ value (figure 3.5), and it is also very sensitive to the a priori estimated drainable porosity value $n_e$ but not so sensitive to the critical flow rate $Q_c$ (figures 3.6 and 3.7). Whether the 5% or the 10% or any other envelope is chosen should really depend on the data available — its quality, its range and how it is spreaded in the parameter space, etc. Likewise, the way to derive $Q_c$ value should also depend on the data available.

Here the 5% envelope is used to determine $a_1$ value. We also suggest the use of a vertical threshold excluding 15% of the large $Q$ data points. This vertical line would represent the critical flow value $Q_c$, because for large $Q$, it is more likely that the streamflow includes other components (besides base flow) than that the streamflow represents the small time behavior of the base flow in watershed. The critical flow value corresponding to this 15% threshold is $Q_c = 4.48$. 

55
Figure 3-6: Sensitivity of water table depth to critical flow rate $Q_c$

Figure 3-7: Sensitivity of water table depth to a priori estimated drainable porosity $n_e$
Table 3.1: Results with different drainable porosity values

<table>
<thead>
<tr>
<th>Drainable Porosity $n_e$</th>
<th>Hydraulic Conductivity $k(cm/s)$</th>
<th>Soil Depth $D(m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.01431</td>
<td>4.373</td>
</tr>
<tr>
<td>0.03</td>
<td>0.03221</td>
<td>2.915</td>
</tr>
<tr>
<td>0.04</td>
<td>0.05726</td>
<td>2.186</td>
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<tr>
<td>0.05</td>
<td>0.08947</td>
<td>1.749</td>
</tr>
<tr>
<td>0.06</td>
<td>0.12883</td>
<td>1.458</td>
</tr>
<tr>
<td>0.07</td>
<td>0.17535</td>
<td>1.249</td>
</tr>
</tbody>
</table>

Table 3.2: Basin averaged water table depth for the 6 storms

<table>
<thead>
<tr>
<th></th>
<th>Storm 1</th>
<th>Storm 2</th>
<th>Storm 3</th>
<th>Storm 4</th>
<th>Storm 5</th>
<th>Storm 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q(m^3/s)$</td>
<td>1.87</td>
<td>2.49</td>
<td>1.70</td>
<td>2.18</td>
<td>1.5576</td>
<td>1.954</td>
</tr>
<tr>
<td>$\bar{z}(mm)$</td>
<td>1084.4</td>
<td>926.3</td>
<td>1145.2</td>
<td>1007.4</td>
<td>1189.8</td>
<td>1070.2</td>
</tr>
</tbody>
</table>

For the Souhegan river basin, the total drainage area $A = 443.1km^2$, total perennial channel length $L = 458.45km$, and drainage density $D_d = 1.0346423km^{-1}$. With the values $a_1 = 10^{-6.85}$ and $Q_e = 4.48$ estimated from the plot, we can solve equations (3.24) and (3.27) simultaneously to get the basin scale effective hydraulic conductivity $k$ and soil depth $D$ for different drainable porosity values $n_e$ (table 3.1). Corresponding to $n_e = 0.04$, we have $k = 0.05726cm/s$ and $D = 2.186m$ for the Souhegan river basin.

Table 3.2 gives the basin averaged water table depths for the six summer storms we will use in our model calibration and verification in the Souhegan basin. Distributed water table depth values can be generated from the resulting average depths.

### 3.3 Initial Water Table for Arno River Basin

Due to the lack of long term streamflow data in the Arno river, the basin scale effective hydraulic conductivity and soil depth values cannot be derived. Furthermore, as we will see later, for most of the storms we simulated, the prestorm streamflow data at the basin outlet are missing, the stage gauge starts recording data in the middle of
the storm. Thus this initial water table derivation scheme can not be used, we have to resort to the original initial water table generation model developed by Cabral et al. (1990).

In the Cabral et al. model, the initial water table depth is determined by the uniform inter-storm recharge rate, which equals to the inter-storm streamflow rate divided by the total drainage area. The recharge rate is assumed to be in long-term equilibrium with the inter-storm streamflow. Although the long term streamflow data are not available for the Arno river basin, they are available for the Sieve basin, which is a subbasin of the Arno. From these data, a value of inter-storm recharge rate can be assigned to a given probability of exceedance for each month. A recharge rate with low probability of exceedance represents wet initial basin states, and a recharge rate with high probability of exceedance represents dry initial basin states. For each month, three probability levels are selected: 0.1, 0.5 and 0.9, and the corresponding inter-storm recharge rates are obtained (see Cabral et al., 1990).

It is assumed that the inter-storm uniform recharge rate value in the Sieve subbasin is representative of the whole Rosano basin (see chapter 4 for the definition of Rosano basin). Thus for each month, corresponding to the 3 states (wet, medium and dry), water table depths are derived for the Rosano basin using the recharge values from the Sieve subbasin.

3.4 Model Calibration Procedure

A good match between simulated and observed hydrographs at the basin outlet and a few other locations along the channel is a necessary but not sufficient condition for a good model performance. As a physically-based distributed model, a good model performance also means a good internal model behavior — a good match between simulated and observed hydrological behavior at every time and every point within the basin. The hydrological behavior includes subsurface flow rates, overland flow rates, soil moisture profiles, etc.. However, the lack of enough field measured hydrological data to check the internal "matches" makes it impossible to be 100% sure that the
parameter combination we find is indeed the best combination. There may be some other parameter combinations that can give better basin internal behavior and the same match of hydrographs. This is uncertainty type 1 — wrong processes but right output.

Even if a good match of hydrographs at one or a few locations is our only goal, there is still a second type of uncertainty introduced because usually it is not realistic to search through every point in the parameter space. It is quite possible that another parameter combination in a complete different part of the parameter space can give the same or better "matches". We call this uncertainty type 2 — overlooked parameter combination.

Both uncertainty type 1 and type 2 are due to the fact that when hydrological processes are integrated over a large area, small area hydrological variations tend to be submerged. If only the hydrographs at a few locations are checked, these small area hydrological variations are not detected. However, both uncertainty type 1 and type 2 can be reduced when more streamflow gauges are available and when more storm events of various kinds are used to do the calibration, especially when hydrographs at the outlets of very small subbasins are checked. Also since all the parameters have physical meaning, we should expect them to fall into certain ranges. If our calibrated parameter values do fall into these ranges, we would think our parameter combination is, maybe not the best, but at least a good, combination.

The idea to quantify the calibration quality has been explored by many people, and many calibration quality indicators have been proposed. For example, Calver (1988) used the sum of squared differences between observed and predicted catchment outflows at 0.5-hour intervals as a measure of the calibration quality. More recently, Beven and Binley (1992) proposed a Generalized Likelihood Uncertainty Estimation strategy (GLUE procedure) to quantify the calibration and uncertainty estimation of distributed rainfall-runoff models. It is based on the premise that prior to the introduction of any quantitative or qualitative information to a modeling exercise, any model/parameter set combination that predicts the variable or variables of interest must be considered equally likely as a simulator of the system. However,
this procedure requires a large number of model simulation runs, thus the demand on computer resources is huge.

Our goal is to test our DBS model, and to give a qualitative, not a quantitative assessment of the model. Thus we just compare the simulated and the measured values of total runoff volume, peak time and peak flow for each storm and at each stage gauge, and plot the observed and simulated hydrographs to give a qualitative estimate of the overall goodness of the hydrograph “match”.

In the DBS model, spatially and temporally distributed rainfall data are assumed to be correct although there are some inevitable measurement errors. We do not modify rainfall data unless another measurement is convincingly more accurate than the one we are using. Measured spatially distributed soil parameters (surface saturated hydrological conductivity in the normal direction, saturated moisture content, residual moisture content and pore size distribution index) are also assumed correct but we are prepared to consider a hydrologically realistic range around the single numbers in case that our simulations are totally off, for we know that our data sources are not so reliable. The parameters left for calibration purpose are: anisotropy ratio $A_r$, soil hydraulic conductivity decay rate $f$, routing parameters $C_v$, $K_v$ and $r$.

For small river basins, it may be reasonable to assume uniform values for these calibration parameters across the basin. As the basin size increases, this assumption may not be valid. One way to address this problem is to consider different values for the calibration variables in different subbasins, especially when these subbasins show different soil and/or terrain characteristics. However, to keep things simple, we stay with uniform variable values in our calibration effort.

The sensitivity analysis done by Garrote (1992) for the DBS model reveals that total runoff volume is controlled only by $A_r$ and $f$. Hydrograph shape, especially peak time is controlled by the routing parameters. Peak volume is controlled by all the parameters. Since linearity is still one of the most handy assumptions in basin hydrological response, we will set $r = 0$ unless absolutely necessary to do otherwise.

Thus, our task is to seek the parameter combination of $A_r$, $f$, $C_v$ and $K_v$ that gives the best match of simulated hydrographs with measured hydrographs. The
procedure we follow is a trial and error procedure. For each storm, we first fix $A_r$ and $f$, and only vary $C_v$ and $K_v$ to match the peak time. Then we fix $C_v$ and $K_v$, and let $A_r$ and $f$ change values to match the total runoff volume. The changes of $A_r$ and $f$ will affect the peak time slightly, so $C_v$ and $K_v$ need to be adjusted again. This procedure is repeated until an overall best match of the hydrographs is achieved. For basins that have more than one streamflow gauges, higher priority is given to gauges with larger drainage areas. Due to our assumption of uniform calibration parameter values, it is possible that the hydrograph match at one gauge contradicts the hydrograph match at another. If this happens, the gauge with smaller drainage area has to be compromised.
Chapter 4

Calibration for the Arno River Basin

The first watershed selected for this model calibration and application is the part of the Arno river basin upstream of the Nave-di-Rosano streamflow gauge station. Nave-di-Rosano is very close to the city of Florence. We will refer to this part of the basin as the Rosano basin from now on. The Arno river flows through the city of Florence and Pisa, and drains an area of about 8000 km$^2$ on the northwest of the Italian peninsula. The rainy season in the Arno basin lasts from October to April, with the largest amount of rainfall occurring in October and February. Winter storms are usually the result of frontal systems. During fall, the mountain may also produce orographic precipitation when moist air from the nearby Mediterranean sea is advected by westerly winds.

There are three streamflow gauges in the basin for which we have hourly streamflow measurements: Fornacina, Subbiano and Nave-di-Rosano. Upstream of the Fornacina gauge is the Mountainous basin of 820 km$^2$ called Sieve; upstream of the Subbiano gauge is also a mountainous basin of 730 km$^2$ called Subbiano. Nave-di-Rosano gauge collects outflow from both basins, with a drainage area of about 3660 km$^2$. While the northern part of the basin is mountainous, the southern part is very flat. In fact the boundaries in the south is not so clearly defined, you may even see different boundaries in different maps (For instance, the lithology map, the channel network...
map and the DEM all have different boundaries.). The highest peak in the basin is 1657\textit{m} above sea level (figure 4.1).

### 4.1 The Data

The DEM data with a resolution of $400\text{m} \times 400\text{m}$ is the basic building block of the modeling effort. The channel network image is derived by setting a threshold contributing area $1.28\text{ km}^2$. Any pixel with a contributing area larger than this threshold value is believed to be a channel pixel (figure 4.2). From each pixel water drains to one of its 8 neighbor pixels that has the lowest elevation, thus only one flow direction is assigned to each pixel. The distance to the closest stream can be derived for each pixel by tracing a drop of water according to its flow direction until it gets to the channel. For a channel pixel, the distance to a stream is zero. The slope of a pixel is computed as the elevation difference between this pixel and its down-stream pixel divided by the distance between them.
Figure 4-2: Channel network of Rosano
Soil data was provided by the University of Florence in Italy. There are 63 classes of soil in the Rosano basin (figure 4.3). For each class, hydraulic conductivity, effective porosity, and lithology information were given. However, the hydraulic conductivity and effective porosity values were not very reliable, as they were only estimated from the geolithological map. In particular, the 63 classes of soils had only 7 different hydraulic conductivity values: 0.00036mm/hr, 0.0036mm/hr, 0.036mm/hr, 0.36mm/hr, 3.6mm/hr, 360mm/hr and 360000mm/hr. After a few real storm data simulations, it was evident that the range was too wide. When the hydraulic conductivity values within the Sieve subbasin were compared with the values also for Sieve subbasin but from another source used by Garrote(1992) and Cabral et al. (1990), it was found that these two sets of data roughly corresponded to each other in the sense that although the absolute values of conductivity were different, the relative magnitudes of the conductivity for different soil classes were maintained. Since the hydraulic conductivity data from the other source range from 0.25 to 45 mm/hr, and had been successfully used for runoff simulations (Garrote, 1992), we rescaled all hydraulic conductivity values to make their range comparable with Garrote's data (1992).

The residual moisture content and pore size distribution index values for the 63 soil classes were estimated based on the lithology map and Mualem’s work (1978). The resultant parameter values of the 63 soil classes are shown in table 4.1 and table 4.2.

The rainfall data come in two forms: rain gauge measurements and radar measurements. Although there are many rain gauges existing inside or around the Rosano basin, they are not all in operation during every storm. Accumulated rainfall volume data were read mostly every 20 minutes, but sometimes every 10 or 15 minutes or every hour. The rain gauge locations are shown in figure 4.4. For rain gauge data, we use a program developed by Garrote (1992) to interpolate the point rain rate data into spatially distributed rain rate data. The interpolation is a weighted average,
Figure 4-3: Soil map of Rosano
Table 4.1: Soil properties in Rosano

<table>
<thead>
<tr>
<th>Soil class</th>
<th>$K_n(\theta_s, 0)$ (mm/hr)</th>
<th>$\theta_s$</th>
<th>$\theta_r$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>12.0</td>
<td>0.600</td>
<td>0.10</td>
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<td>0.04</td>
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</tr>
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Table 4.2: Soil properties in Rosano (continued)

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<td>0.050</td>
<td>0.03</td>
<td>20.0</td>
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</table>
with the weight being the inverse of the squared distance, that is

$$\text{Rain}(i,j) = \frac{\frac{1}{d_1^2} R_{g_1} + \frac{1}{d_2^2} R_{g_2} + \ldots + \frac{1}{d_n^2} R_{g_n}}{\frac{1}{d_1^2} + \frac{1}{d_2^2} + \ldots + \frac{1}{d_n^2}}$$  \hspace{1cm} (4.1)$$

where \( \text{Rain}(i,j) \) is the interpolated rain rate at location \((i, j)\) \(R_{g_k}(k = 1, 2, \ldots, n)\) is the measured rain rate at gauge \(k\), and \(d_k(k = 1, 2, \ldots, n)\) is the distance from \((i, j)\) to gauge \(k\). This program actually can also do interpolation with weight being the inverse of the distance to any power, but the weight being the inverse of the squared distance is the most commonly used.

There are a total of five storm events for which both streamflow and rainfall data are available, we chose three for model calibration and the other two for model
4.2 Model Calibration

Three is probably the minimum number of storms for a runoff model calibration. For each storm with the rainfall input, we run the model using three different initial water table depth files corresponding to the three initial basin states: dry (90% probability of exceedance), medium (50%) and wet (10%). After the calibration process described in chapter 3, we found that the following parameter combination gives the best overall performance for the three storm simulations:

\[ f = 0.0007 \text{mm}^{-1} \]
\[ a_r = 100 \]
\[ C_v = 4800 \text{m/hr} \]
\[ K_v = 16.0 \]
\[ r = 0 \]

During the calibration process, it often occurs that a change in the calibration parameter values improves the simulation for one storm but makes the simulations worse for the other storms. For the Arno basin, because streamflow data are available at three gauges, the conflict of simulation quality also occurs among different gauges for the same storm. The important thing is to find a compromise, taking into account the simulations at all the streamflow gauges for all the storms.

4.2.1 Storm February 20-22, 1977

With rainfall data from 11 hourly rain gauges, the simulation underestimates both the total runoff and the peak flow rate at all the three streamflow gauges (figures 4.5-4.7 and table 4.3). However changing the calibration parameter values would make the other two storm simulations worse.

The 11 hourly rain gauges prove to be inadequate to capture the spatial characteristics of the rainfall field in the Rosano basin, especially within the Subbiano subbasin, where only 2 hourly rain gauges provide hourly rainfall data, and both of
Table 4.3: Simulations for the storm Feb.20-22, 1977 using hourly rain gauge data

<table>
<thead>
<tr>
<th>Location</th>
<th>Total runoff (m³)</th>
<th>Peak flow rate (m³/s)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosano</td>
<td>Measured</td>
<td>(not available)</td>
<td>1110.0</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>4.01 x 10⁷</td>
<td>882.9</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>3.59 x 10⁷</td>
<td>798.3</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>2.91 x 10⁷</td>
<td>653.5</td>
</tr>
<tr>
<td>Subbiano</td>
<td>Measured</td>
<td>2.45 x 10⁷</td>
<td>695.1</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>1.56 x 10⁷</td>
<td>416.6</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>1.42 x 10⁷</td>
<td>381.7</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>1.10 x 10⁷</td>
<td>300.8</td>
</tr>
<tr>
<td>Fornacina</td>
<td>Measured</td>
<td>1.86 x 10⁷</td>
<td>550.0</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>1.79 x 10⁷</td>
<td>528.3</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>1.62 x 10⁷</td>
<td>482.6</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>1.30 x 10⁷</td>
<td>379.0</td>
</tr>
</tbody>
</table>

Figure 4-5: Hydrographs at Rosano for storm of Feb.20-22, 1977 using hourly rain gauge data
Figure 4-6: Hydrographs at Subbiano for storm of Feb.20-22, 1977 using hourly rain gauge data

Figure 4-7: Hydrographs at Fornacina for storm of Feb.20-22, 1977 using hourly rain gauge data
them are close to the boundaries. The total rainfall volume in the Subbiano subbasin computed from the 11 hourly rain gauges data interpolation is 27516705.6 m$^3$, while the total measured runoff at Subbiano gauge is 28414800 m$^3$, larger than the total rainfall!

Measured daily rainfall data from another source are available at 9 gauges for this storm within the Subbiano subbasin. Of the 9 daily gauges, 7 are at different locations from the hourly gauges. When the measured daily rainfall data on February 21 at these 7 daily gauges are compared with the daily rainfall at the corresponding locations derived from our interpolation using the 11 hourly gauges, it is found that the interpolation greatly underestimates the rainfall. For example, the daily rainfall at *Salutio* (noted as ‘S’ in figure 4.4) and *Montemignaio* (noted as ‘M’ in figure 4.4) are interpolated as 8 mm and 22 mm respectively but are actually measured as 24 mm and 45 mm respectively.

The problem of misrepresenting the spatial characteristics of the rainfall field using gauge data often occurs when the number of gauges are not enough compared to the spatial variation of the rainfall field. This is clearly the case for this storm. The heavy precipitation zone between hourly gauges *Arezz* and *Camaldoli*, which both have low precipitation, are completely missed by the interpolation. Radar has the potential to solve this problem, but unfortunately no radar data are available for this storm.

Since we have the daily rainfall data, and since the total rainfall volume is the single most important input to ensure successful runoff simulations, we decided to use this extra information. Both hourly and daily rainfall data are available at gauge *La-Verna*. The daily rainfall at the 7 daily gauges was distributed over time to get the hourly rainfall; the spreading is according to the hourly rainfall distribution at gauge *La-Verna*. In other words, we assume that the temporal rainfall distributions at the 7 daily gauges are the same as the temporal rainfall distribution at gauge *La-Verna*.

With hourly rainfall data at the 7 extra gauges, we redo the rainfall interpolation and runoff simulation, and the results are shown in figures 4.8-4.10 and table 4.4. We can see that the simulated hydrographs at both the Subbiano and Rosano gauges improve a lot.
Table 4.4: Simulations for the storm Feb.20-22, 1977 using daily rain gauge data

<table>
<thead>
<tr>
<th>Site</th>
<th>Type</th>
<th>Total runoff ( (m^3) )</th>
<th>Peak flow rate ( (m^3/s) )</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosano</td>
<td>Measured &amp; not available</td>
<td>1110.0</td>
<td>1245:00</td>
<td></td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>5.01 ( \times 10^7 )</td>
<td>1086.7</td>
<td>1243:00</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>4.51 ( \times 10^7 )</td>
<td>986.2</td>
<td>1243:00</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>3.70 ( \times 10^7 )</td>
<td>816.4</td>
<td>1243:00</td>
</tr>
<tr>
<td>Subbiano</td>
<td>Measured</td>
<td>2.45 ( \times 10^7 )</td>
<td>695.1</td>
<td>1241:00</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>1.83 ( \times 10^7 )</td>
<td>516.5</td>
<td>1243:00</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>1.66 ( \times 10^7 )</td>
<td>478.7</td>
<td>1243:00</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>1.28 ( \times 10^7 )</td>
<td>376.6</td>
<td>1242:00</td>
</tr>
<tr>
<td>Fornacina</td>
<td>Measured</td>
<td>1.86 ( \times 10^7 )</td>
<td>550.0</td>
<td>1241:00</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>2.01 ( \times 10^7 )</td>
<td>578.4</td>
<td>1242:00</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>1.84 ( \times 10^7 )</td>
<td>534.2</td>
<td>1242:00</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>1.48 ( \times 10^7 )</td>
<td>423.6</td>
<td>1242:00</td>
</tr>
</tbody>
</table>

The final calibration is based on the daily rain gauge data.

4.2.2 Storm January 9-10, 1979

Hourly rainfall data are provided at 10 rain gauges. Gauge *Siena* was not in operation. The model does well in the Sieve subbasin, where the characteristics of the basin response to the storm are well captured (figures 4.13 and table 4.5). The simulated and the measured hydrograph shape, total runoff volume, peak flow time and peak flow value all match very well. For the Subbiano subbasin, however, the simulated total runoff volume and peak flow value are much smaller than the measured ones (figures 4.12 and table 4.5). This may also be due to the fact that not enough gauges were in operation, but no daily rainfall data was available to verify this hypothesis. Another possible reason is that the actual initial state in the Subbiano subbasin was much wetter than that defined by the uniform inter-storm recharge rate. Considering that the Rosano basin is quite large (3660km\(^2\)), it is highly possible that a particular part of the basin is wetter than the rest of the basin due to the spatial variations of the storm rainfall. But again lack of data renders this explanation unverifiable.

The streamflow data at Nave-di-Rosano are available only at a few times. It is
Figure 4-8: Hydrographs at Rosano for storm of Feb.20-22, 1977 using daily rain gauge data

Figure 4-9: Hydrographs at Subbiano for storm of Feb.20-22, 1977 using daily rain gauge data
Figure 4-10: Hydrographs at Fornacina for storm of Feb.20-22, 1977 using daily rain gauge data

hard to evaluate the simulations for the whole Rosano basin (figures 4.11 and table 4.5).

4.2.3 Storm November 13-14, 1982

Hourly rainfall data are available at all the 11 hourly gauges. No streamflow data are available at Nave-di-Rosano gauge station. The simulations at both the Sieve and Subbiano subbasins are reasonable (figures 4.14-4.15 and table 4.6). Peak flow time at these two gauges are well produced, and the total storm runoff volumes are very close to the measured. For the Subbiano subbasin, the rising limb and the first half of the recession limb of the simulated hydrograph corresponding to the medium initial basin state match very well with those of the measured hydrograph. Only the tail of the measured hydrograph is higher than the simulated hydrographs. For the Sieve subbasin, however, the simulation that has the best match with the measured hydrograph corresponds to the dry initial basin state. Again since the Rosano basin is a relatively large basin, it is possible that different subbasins have different initial
Table 4.5: Simulations for the storm Jan. 9-10, 1979

<table>
<thead>
<tr>
<th>Location</th>
<th>Type</th>
<th>Total Runoff ($m^3$)</th>
<th>Peak Flow Rate ($m^3/s$)</th>
<th>Peak Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosano</td>
<td>Measured</td>
<td>(not available)</td>
<td>(not available)</td>
<td>(not available)</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>$8.45 \times 10^7$</td>
<td>1157.0</td>
<td>223:00</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>$7.57 \times 10^7$</td>
<td>1056.7</td>
<td>223:00</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>$6.36 \times 10^7$</td>
<td>905.0</td>
<td>223:00</td>
</tr>
<tr>
<td>Subbiano</td>
<td>Measured</td>
<td>$3.63 \times 10^7$</td>
<td>600.6</td>
<td>221:00</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>$2.23 \times 10^7$</td>
<td>395.3</td>
<td>223:00</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>$1.99 \times 10^7$</td>
<td>368.9</td>
<td>223:00</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>$1.63 \times 10^7$</td>
<td>312.2</td>
<td>223:00</td>
</tr>
<tr>
<td>Fornacina</td>
<td>Measured</td>
<td>$2.80 \times 10^7$</td>
<td>490.0</td>
<td>222:00</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>$3.09 \times 10^7$</td>
<td>533.7</td>
<td>222:00</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>$2.84 \times 10^7$</td>
<td>508.7</td>
<td>222:00</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>$2.43 \times 10^7$</td>
<td>448.9</td>
<td>222:00</td>
</tr>
</tbody>
</table>

Figure 4-11: Hydrographs at Rosano for storm of Jan. 9-10, 1979
Figure 4-12: Hydrographs at Subbiano for storm of Jan. 9-10, 1979

Figure 4-13: Hydrographs at Fornacina for storm of Jan. 9-10, 1979
4.3 Model Verification

With the calibrated parameter values unchanged, we run the model with another two storms for model verification. Again for each storm, three different initial water table depth files corresponding to the wet, medium and dry initial basin states are used. We realized that two storm simulations may not be enough to conclusively demonstrate the capabilities of the model, but we do get a qualitative sense of the model performance from two storm simulations.

4.3.1 Storm November 24-26, 1987

Hourly rainfall data are available at 17 gauges. The simulated hydrographs agree quite well with the measured hydrographs except at gauge station Fornacina, where the measured hydrograph has a much sharper peak than the simulated ones and the simulated total storm runoff is much larger than the measured (figures 4.16-4.18 and table 4.7). Nonetheless, the measured flow peak at gauge Fornacina falls well between the ranges set by the simulations corresponding to the dry and wet initial conditions, and the measured peak flow time at gauge Fornacina is also very close to the simulated.

<table>
<thead>
<tr>
<th></th>
<th>Total runoff ($m^3$)</th>
<th>Peak flow rate ($m^3/s$)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subbiano</td>
<td>2.61 x 10^7</td>
<td>597.5</td>
<td>7614:00</td>
</tr>
<tr>
<td>Measured</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% exceedance</td>
<td>2.60 x 10^7</td>
<td>714.0</td>
<td>7614:00</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50% exceedance</td>
<td>2.21 x 10^7</td>
<td>603.9</td>
<td>7614:00</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90% exceedance</td>
<td>1.66 x 10^7</td>
<td>438.2</td>
<td>7614:00</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fornacina</td>
<td>2.75 x 10^7</td>
<td>630.0</td>
<td>7868:00</td>
</tr>
<tr>
<td>Measured</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% exceedance</td>
<td>3.67 x 10^7</td>
<td>1016.4</td>
<td>7869:00</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50% exceedance</td>
<td>3.23 x 10^7</td>
<td>899.0</td>
<td>7869:00</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90% exceedance</td>
<td>2.49 x 10^7</td>
<td>702.1</td>
<td>7869:00</td>
</tr>
<tr>
<td>Simulated</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4-14: Hydrographs at Subbiano for storm of Nov. 13-14, 1982

Figure 4-15: Hydrographs at Fornacina for storm of Nov. 13-14, 1982
Table 4.7: Simulations for the storm Nov. 24-26, 1987

<table>
<thead>
<tr>
<th>Location</th>
<th><em>Runoff</em></th>
<th>Peak flow rate ($m^3/s$)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosano</td>
<td>Measured (not available)</td>
<td>1585.7</td>
<td>7869:00</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>$2.35 \times 10^8$</td>
<td>1918.2</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>$2.06 \times 10^8$</td>
<td>1737.7</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>$1.62 \times 10^8$</td>
<td>1380.3</td>
</tr>
<tr>
<td>Subbiano</td>
<td>Measured</td>
<td>$5.77 \times 10^7$</td>
<td>680.3</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>$5.72 \times 10^7$</td>
<td>756.6</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>$4.98 \times 10^7$</td>
<td>685.6</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>$3.80 \times 10^7$</td>
<td>554.3</td>
</tr>
<tr>
<td>Fornacina</td>
<td>Measured</td>
<td>$2.21 \times 10^7$</td>
<td>498.0</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>$5.36 \times 10^7$</td>
<td>691.2</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>$4.85 \times 10^7$</td>
<td>633.2</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>$3.77 \times 10^7$</td>
<td>477.9</td>
</tr>
</tbody>
</table>

At the Nave-di-Rosano gauge station, streamflow data are missing for many time steps. We could not calculate the total runoff. From the existing data, though, we can see that the shape of the hydrograph looks like an inverted "U". The peak is rather flat. The simulation does capture this overall shape of the measured hydrograph.

### 4.3.2 Storm October 30-31, 1992

**Using Rain gauges**

This was one of the big storms that caused a lot of floods in the city of Florence. Rainfall data for this storm are available at 44 gauges that well spread over the basin and at places outside but close to the basin. For the most of the time and for the most of the gauges, rainfall data are read every 15 minutes. If for some rain gauges there are no rainfall records at one time step, a linear interpolation over time is used to get the missing rainfall records.

Because this storm occurred right at the end of October, we simulate it with two sets of initial conditions, one corresponds to the October recharge rates, and the other corresponds to the November recharge rates. Shown in figures 4.19-4.21 and table 4.8 are the simulation results with the November initial conditions. The simulation with
Figure 4-16: Hydrographs at Rosano for storm of Nov. 24-26, 1987

Figure 4-17: Hydrographs at Subbiano for storm of Nov. 24-26, 1987
the October initial conditions makes the hydrographs slightly lower than that with the November initial conditions, but the overall shapes are similar. We can see from the hydrographs that the simulated results are reasonable except that the peak flow times are a few hours behind the measured peak flow times at all the three gauges.

For this storm there was a period of light rain between two heavy rain periods, resulting in two well defined hydrograph peaks at all the three gauges, especially at the upstream gauges Subbiano and Fornacina. The simulations capture the overall shapes of the hydrographs including these two peaks fairly well. At all the three gauges, the simulated total storm runoff and the peak flow rates compare favorably with the measured.

From the measured hydrograph at Nave-di-Rosano, it is obvious that there was a storm shortly before this storm because the prestorm streamflow rate is more than 400 m³/hr. Thus base flow is added in our simulation. This large prestorm streamflow might be one of the reasons why our simulated peaks lag behind the measured peak. The large amount of prestorm streamflow made the flow faster than otherwise.
Table 4.8: Simulations for the storm Oct. 30-31, 1992 using rain gauge data

<table>
<thead>
<tr>
<th>Location</th>
<th>Measured/Simulated</th>
<th>Total runoff (m³)</th>
<th>Peak flow rate (m³/s)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosano</td>
<td>Measured</td>
<td>2.78 x 10⁸</td>
<td>2896.8</td>
<td>7300:00</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>2.51 x 10⁸</td>
<td>2628.6</td>
<td>7304:15</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>2.30 x 10⁸</td>
<td>2374.1</td>
<td>7304:15</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>1.98 x 10⁸</td>
<td>2007.8</td>
<td>7304:15</td>
</tr>
<tr>
<td>Subbiano</td>
<td>Measured</td>
<td>6.81 x 10⁷</td>
<td>980.7</td>
<td>7301:00</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>7.40 x 10⁷</td>
<td>1321.4</td>
<td>7303:45</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>6.64 x 10⁷</td>
<td>1205.0</td>
<td>7303:30</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>5.51 x 10⁷</td>
<td>1006.6</td>
<td>7303:15</td>
</tr>
<tr>
<td>Fornacina</td>
<td>Measured</td>
<td>6.12 x 10⁷</td>
<td>717.0</td>
<td>7299:00</td>
</tr>
<tr>
<td></td>
<td>10% exceedance</td>
<td>6.92 x 10⁷</td>
<td>848.1</td>
<td>7302:15</td>
</tr>
<tr>
<td></td>
<td>50% exceedance</td>
<td>6.41 x 10⁷</td>
<td>806.0</td>
<td>7302:15</td>
</tr>
<tr>
<td></td>
<td>90% exceedance</td>
<td>5.38 x 10⁷</td>
<td>671.9</td>
<td>7302:15</td>
</tr>
</tbody>
</table>

Figure 4-19: Hydrographs at Rosano for storm of Oct. 30-31, 1992 using rain gauge data.
Figure 4-20: Hydrographs at Subbiano for storm of Oct. 30-31, 1992 using rain gauge data

Figure 4-21: Hydrographs at Fornacina for storm of Oct. 30-31, 1992 using rain gauge data
Using Radar

For the storm of 1992, radar rainfall data are available for six hours, with a total of 18 radar images, but they are not in regular time intervals, some of them are 10 minutes apart, and some of them are 15, 30, or 45 minutes apart. The radar images are already converted into Cartesian coordinate, with the pixel size $617m \times 617m$, and the data are given in reflectivities (dBZ).

There are some ground clutter problems with these radar data, which come from the reflection of the radar beam by ground targets. While rain drops usually move horizontally due to the horizontal winds during a storm, ground clutters are stationary. The standard way to filter out ground clutters with a Doppler radar is to set up a threshold radial velocity value. Pixels with radial velocities below this threshold are assumed to represent ground clutters.

Doppler velocity data are not available for this storm, however. Thus a simple method is used to remove the ground clutters. First we calculate the mean and standard deviation of reflectivity values over time (18 images) for each pixel. A new image is generated when the mean is divided by the standard deviation for each pixel. In this new image, some pixels have extremely high values, contrasting sharply with their neighbors. These pixels are considered to represent ground clutters and are removed using a threshold method.

After the clutter removal, a spatial interpolation is performed to assign new reflectivity values to the clutter pixels. From each clutter pixel, we seek its 8 nearest non-clutter neighboring pixels in the 8 directions. The weighted mean of the reflectivity values of these 8 pixels is assigned to the clutter pixel, with the inverted distances to the clutter pixel as the weights.

A $Z-R$ relationship ($Z$—reflectivity, $R$—rainfall rate) is necessary to convert the radar reflectivity data into rainfall rate data. However, this $Z-R$ relationship usually depends on the radar features, the storm type and the climatic characteristics of the area. After the installation of a weather radar, many storms of various types are often needed to calibrate a specific $Z-R$ relationship for that area and for that radar.
However, the radar in the Arno area has not been calibrated, and with only 18 radar images, it is not realistic to calibrate our own $Z-R$ relationship. In fact, we did a linear regression in the logarithmic space for each radar image using the rain gauge data and the radar reflectivity data at locations corresponding to the rain gauge positions. The results were quite discouraging: for each of the radar images, we got quite different coefficients $a$ and $b$ values ($Z = aR^b$).

The Palmer-Marshall $Z-R$ relationship $Z = 200R^{1.6}$ is the most widely used $Z-R$ relationship, and it has been proven to be valid in many different geographical areas and under a wide variety of meteorological conditions. However, when the Palmer-Marshall $Z-R$ relationship was used for the Rosano radar data, over the six hours that radar reflectivity data are available, the radar-derived accumulated rainfall values at the 44 gauge locations were consistently lower than those measured from the gauges (figure 4.22). On average, the radar-derived accumulated rainfall values were about $1/4.4$ of those measured by the gauges at the 44 gauge locations. This demonstrated that there is a system bias in the radar rainfall measurement using the Palmer-Marshall $Z-R$ relationship, and this bias need to be corrected.
Again since it is the total rainfall volume that is the most important for a successful runoff simulation, we decide to simply choose $Z = 200 \times (R/4.4)^{1.6}$. that is, $Z = 18.7R^{1.6}$, to get the correct accumulated rainfall volume. Due to the quality of the original reflectivity data, this $Z-R$ relationship will make rainfall rates at some positions at some times unrealistically high (like more than 100 mm/hr).

After the conversion of reflectivities to rainfall rates, the rainfall images are re-sampled to make the pixel size 400m × 400m, consistent with the DEM data, soil data, etc.. The resampling strategy is again a weighted average of the rainfall values from the old image, with the weights being the area of the old image pixels falling into the new image pixel.

A storm simulation is performed using the combination of gauge and radar recorded rainfall data. During the six hours that radar rainfall data are available, radar rainfall data are used, and during the other period of the storm, gauge rainfall data are used. The results are shown in figures 4.23-4.25 and table 4.9. Compared with the simulations using gauge rainfall data only, which have quite smooth hydrographs, the simulations using radar rainfall data make the hydrographs fluctuate a lot. The reason is that radar records instantaneous rainfall while gauge records temporally averaged rainfall. Since rainfall fields often display large temporal variations, radar recorded rainfall consequently also has large temporal variations.

Due to the large temporal fluctuations of the radar recorded rainfall, the simulated streamflow peaks are sharper than those using rain gauge data only. At gauge Fornacina, each simulated hydrograph actually has three large peaks compared with only two peaks of the measured and of the simulated using rain gauge data only. However, the overall shapes of the simulated hydrographs are quite similar to those using rain gauge data only.

4.3.3 Summary

Three storms of similar magnitude were used to calibrate the model. The identified parameters performed reasonably well for the two verification storm simulations. In particular, the second verification storm (October 30-31, 1992) is of much larger
Table 4.9: Simulations for the storm Oct. 30-31, 1992 using radar and rain gauge data

<table>
<thead>
<tr>
<th>Location</th>
<th>Runoff</th>
<th>Peak rate</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosano</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>$2.78 \times 10^8$</td>
<td>$2896.8$</td>
<td>7300:00</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>$2.79 \times 10^8$</td>
<td>$2991.5$</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>$2.57 \times 10^8$</td>
<td>$2736.6$</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>$2.26 \times 10^8$</td>
<td>$2412.2$</td>
</tr>
<tr>
<td>Subbiano</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>$6.81 \times 10^7$</td>
<td>$980.7$</td>
<td>7301:00</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>$7.81 \times 10^7$</td>
<td>$1566.9$</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>$7.06 \times 10^7$</td>
<td>$1465.4$</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>$6.00 \times 10^7$</td>
<td>$1324.1$</td>
</tr>
<tr>
<td>Fornacina</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>$6.12 \times 10^7$</td>
<td>$717.0$</td>
<td>7299:00</td>
</tr>
<tr>
<td>10% exceedance</td>
<td>Simulated</td>
<td>$7.04 \times 10^7$</td>
<td>$903.1$</td>
</tr>
<tr>
<td>50% exceedance</td>
<td>Simulated</td>
<td>$6.52 \times 10^7$</td>
<td>$859.1$</td>
</tr>
<tr>
<td>90% exceedance</td>
<td>Simulated</td>
<td>$5.56 \times 10^7$</td>
<td>$736.3$</td>
</tr>
</tbody>
</table>

Figure 4-23: Hydrographs at Rosano for storm of Oct. 30-31, 1992 using radar and rain gauge data
Figure 4-24: Hydrographs at Subbiano for storm of Oct. 30-31, 1992 using radar and rain gauge data

Figure 4-25: Hydrographs at Fornacina for storm of Oct. 30-31, 1992 using radar and rain gauge data
magnitude than any of the three calibration storms and the measured hydrographs show large variations (several peaks), which usually makes the simulation difficult. Yet the hydrograph comparisons at all the three streamflow gauges are reasonable. The good simulation quality may be partly due to the fact that a much denser rain gauge network, which can capture well the structure of the spatial rainfall field, was in operation. More importantly, however, the good simulation quality shows that the physical processes are well represented in the model, and these physical processes are the same irrespective of the storm magnitudes.

Due to its instantaneous nature, the radar rainfall measurement often shows large temporal variations. Consequently, the runoff simulation with radar rainfall input will also show large variations in the hydrographs.
Chapter 5

Calibration for the Souhegan River Basin

The second watershed selected for this model calibration is the Souhegan river basin, located across the border between Massachusetts and New Hampshire in Northeast USA. The Souhegan River is a tributary of the Merrimack River. The centroid of the basin is about 80km from the MIT radar station. Mean annual rainfall in this area is 1150mm to 1200mm, and rainfall is distributed almost evenly over the year.

The terrain is primarily the result of glacial action. There are several mountains above 500m high. One streamflow gauge located not exactly at, but very close to, the basin outlet provides hourly streamflow measurements throughout most time of the year. Only that part of the basin upstream of this stage gauge is considered, which has an area of about 450km$^2$ (figure 5.1).

5.1 The Data

The DEM we are using was made by the Defense Map Agency, with grid size 3'' x 3''. This size corresponds to roughly 67.5m x 92.5m at this latitude. There are a total of 72046 pixels in the watershed. This grid size serves as the basic grid size on which the model operates. All other spatially distributed data are converted into this grid size. As in the Arno case, the channel network is also derived from the DEM by setting a
threshold contributing area 0.9366km² (figure 5.2). Flow is taken only in the direction of the steepest gradient. Slope value and distance to stream value are calculated for each pixel.

Soil maps made by the United States Department of Agriculture, Soil Conservation Service are available. These maps are at the scale of 1:20,000, thus detailed information about soil physical and chemical properties, landuse type, etc. is provided. For instance, each soil class has a permeability value (from which we can infer a conductivity value) for each of the three layers. Each soil class has a bulk moisture density value (oven dried density of the soil), which contains information about the soil porosity. Also the soil classes are grouped into the 4 standard hydrological groups A, B, C and D.

The soil maps are digitized into the computer, however, not all the details of the soil maps are digitized, but rather, the soil classes are grouped into the four standard hydrological groups A, B, C and D. There are two reasons to do so. First, for a hydrological model simulation, the most important things about soil are the
Figure 5-2: Channel network of Souhegan
hydraulic properties of the soil. While in the soil maps many different soil classes actually have similar hydraulic properties. For instance, given everything else the same, two soils with different surface slopes will belong to two different soil classes. Second, the soil maps are in such detail that many soil patches are even smaller than our pixel size $67.5m \times 92.5m$.

The boundaries between different soil groups are digitized as a series of point coordinates using a digitizing table. However, the digitized boundaries are not exactly the boundaries in the map. Manual work always tends to err, and some seemingly very small errors can cause tremendous trouble when the vector data are converted into raster data. So if this happens, manual corrections are necessary to remove those errors.

There are many different algorithms to convert vector data into raster data, each corresponding to different digitizing schemes. The algorithm we use is very simple. We start from a point $(i, j)$ which has a quantity $(A, B, C$ or $D)$.

1. $Q(i, j)=A$ (or $B, C, D)$;
2. if $(i+1, j)$ not on the border then $i = i+1$; goto (1)
3. if $(i-1, j)$ not on the border then $i = i-1$; goto (1)
4. if $(i, j+1)$ not on the border then $j = j+1$; goto (1)
5. if $(i, j-1)$ not on the border then $j = j-1$; goto (1)

This algorithm basically starts from a point inside a closed boundary, and spreads the property of that point to the whole area inside the boundary. Thus, for each soil patch, its closed boundary and an initial point inside it are digitized into the computer, and the associated soil group name is recorded. The raster form of the digitized soil data is shown in figure 5.3.

Surface saturated hydraulic conductivity $K_n(\theta_s, 0)$, saturated moisture content $\theta_s$, residual moisture content $\theta_r$ and pore size distribution index $\epsilon$ are the soil parameters we use in our model. They have to be estimated for each soil group. $K_n(\theta_s, 0)$ is estimated based on the soil permeability information from the soil survey manuals. However, difficulty arises in the estimation of $\theta_s$, $\theta_r$ and $\epsilon$. These three soil parameters should be estimated based on the soil texture and density information, but it turns
Figure 5-3: Soil map of Souhegan
Table 5.1: Soil properties in Souhegan

<table>
<thead>
<tr>
<th>Soil class</th>
<th>$K_n(\theta_s, 0)(mm/hr)$</th>
<th>$\theta_s$</th>
<th>$\theta_r$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
<td>60.0</td>
<td>0.50</td>
<td>0.05</td>
<td>3.6</td>
</tr>
<tr>
<td>Class B</td>
<td>45.0</td>
<td>0.50</td>
<td>0.05</td>
<td>3.6</td>
</tr>
<tr>
<td>Class C</td>
<td>35.0</td>
<td>0.50</td>
<td>0.05</td>
<td>3.6</td>
</tr>
<tr>
<td>Class D</td>
<td>25.0</td>
<td>0.50</td>
<td>0.05</td>
<td>3.6</td>
</tr>
</tbody>
</table>

out that soil classes belonging to the same hydrological group ($A$, $B$, $C$ or $D$) may have very different textures. On the other hand, soil classes belonging to different hydrological groups may have very similar texture.

Fortunately, $\theta_s$, $\theta_r$ and $\epsilon$ are not as important as the soil hydraulic conductivity in the model, as shown by Garrote (1992). Therefore, we just assign basin averaged $\theta_s$, $\theta_r$, and $\epsilon$ values to all the four soil groups (table 5.1). These basin averaged parameter values are estimated based on the soil texture and density information and Mualem’s work (1978).

All the rainfall data used for the model simulation are from the WR-66 radar at MIT. The radar is an S-band radar, with wavelength 11cm, beam width 1.45 degree. pulse length 1 $\mu$s, and recording precision 0.5 dBZ. The radar can measure the rainfall field at several elevation angles (0.7, 1.4, 5.0, 10.0, etc.). However, usually only two elevation angles are available during a storm. We always use the measurements with the lowest elevation angle to avoid the possible bright band which occurs when the radar beam is so high that it hits the snow/ice melting layer in the atmosphere.

At large ranges, ground clutters exist when the radar beam hits the mountain tops. Close to the radar site, echoes coming from the high buildings and detected by the radar side lobes is a major source of ground clutter contamination. Rain drops usually have a horizontal component of movement because of the winds while ground clutters are stationary, so ground clutters can be removed by setting up a threshold Doppler velocity value, which measures the mean radial velocity of the raindrops inside a radar resolution volume. Any radar echo with Doppler velocity smaller than the threshold velocity is considered a ground clutter echo. This was done by a program in the MIT
radar system. The program allows the user to choose the threshold Doppler velocity value.

Radar reflectivity images are converted from polar coordinate into Cartesian coordinate through a program called POLKA in the MIT radar system. The converted Cartesian radar data have a spatial resolution of $1km \times 1km$.

While radar has the advantage of measuring the rainfall field at high spatial resolutions, it suffers from several sources of error. The radar beam overshooting the rainfall height, the ground clutter, the bright band, the radome attenuation, and the evaporation of raindrops before reaching the ground are but a few of these sources. Needless to say, a careful calibration of the radar after its installation against the rain gauge measured "ground truth" is very important to ensure the accuracy of quantitative rainfall measurement.

Within the radar coverage, we have four rain gauges that have hourly rainfall data available: Boston Airport, Concord, Durham and Macdowell (figure 4.4). But we did not go through a $Z-R$ relationship calibration process because: (1) Four rain gauges within an area of $31400km^2$ are not enough to capture the spatial structure of the rainfall field, and (2) Austin (1987) has done comprehensive work using MIT radar to study New England storms. So the $Z-R$ relationships we use are based on the work of Austin. $Z = 230R^{1.4}$ is used for storms without strong convection, and $Z = 400R^{1.3}$ is used for strong convective storms.

Nevertheless, before we run the runoff model, we first compared the radar measured rainfall rates at Boston Airport, Concord, Durham and Macdowell with the gauge measured rainfall rates. We realized that these two are not quite comparable because radar measures the instantaneous rainfall rate averaged within its resolution volume while gauge measures temporally averaged rainfall at a single point. Rainfall has been shown to exhibit tremendous variability within a very short spatial distance and time. These two measurements could be quite different sometimes. But again since the rainfall volume is very important in runoff simulations, we just want to get a rough idea about our radar performance and make sure that our radar rainfall data are not too different from the assumed ground truth measured by gauges.
Figure 5-4: Rain gauges within radar coverage
For the most part, the radar records rainfall every 5 minutes, although it sometimes records rainfall every 10 minutes. If the model runs at a shorter time step than the radar interval, a linear interpolation of radar rainfall data over time is performed to get the rainfall at the missing time step. If the model runs at a longer time step than the radar interval, the model can automatically average the rainfall rates within one time step to get the mean rainfall rate.

5.2 Model Calibration

Although the precipitation in the Souhegan Basin is distributed evenly within a year, the streamflow is not. During winter the river is basically frozen. In spring, a lot of floods occur when the snow accumulated during the winter starts to melt, and the floods can become even larger when storms come with the snow melting. In summer, due to the large amount of evaporation, the streamflow is usually very small, but floods also occur when storms occur.

The spring floods are usually larger than the summer floods, and the peak flow rates during spring floods are often 2 to 5 times larger than the peak flow rates during summer floods. However, since spring floods are often complicated by the snow melting, we choose six storms all occurring in summer for the model simulation. Three of the six storms are used for model calibration, and the rest are for model verification.

After a trial-and-error calibration process, it is found that the following combination of parameter values gives the best overall performance for the three storm simulations:

\[ f = 0.003 \text{mm}^{-1} \]
\[ a_r = 5 \]
\[ C_v = 120.0 \text{m/hr} \]
\[ K_v = 80.0 \]
\[ r = 0 \]

As can be seen from the routing parameters, the basin response to storm rainfall
is quite slow. This is because that the soils in the basin are porous, the slopes are not steep, and the basin is well covered by vegetation. It takes long time for rainfall to become runoff and flow down to the basin outlet.

5.2.1 Storm 1: September 19-20, 1987

No strong convection occurred in this storm. The $Z-R$ relationship used is $Z = 230R^{1.4}$. The radar measured rainfall at Concord, Durham and Macdowell matches reasonably well with the gauge measured rainfall (figure 5.5). At Boston airport, however, the radar greatly underestimates the rainfall. The major reason might be because that the Boston airport is in an urban area and very close to the radar site, thus the high buildings around could reflect the beam to the radar side lobes and thus contaminate the radar echoes.

The overall shape of the simulated hydrograph matches quite well with that of the measured hydrograph (figure 5.6). The simulated total runoff volume and peak flow rate are close enough to the measured ones (table 5.2). However, the peak time is a few hours earlier, and if the routing parameters are changed, the peak times for the other two calibration storms will mismatch the measured peak times. In fact, the entire simulated hydrograph seems to be occurring a few hours earlier. The rainfall started at time 6287:00, and it became quite heavy right around the basin outlet at time 6289:00, yet the streamflow continued to decline until about 6300:30. An error in the streamflow data file is possible but could not be confirmed.

Actually there was a small storm right before this storm, which raised the streamflow rate from around $2m^3/hr$ to a peak around $4.6m^3/hr$. Shortly after the streamflow started to recess, the second storm occurred. But the rainfall data for the first storm are not available. Had we simulated starting from the first storm, we might have reached different results.
Figure 5-5: Radar and rain gauge comparison for the storm Sept. 19-20, 1987
Table 5.2: Simulation for the storm Sept. 19-20, 1987

<table>
<thead>
<tr>
<th></th>
<th>Total runoff (m$^3$)</th>
<th>Peak flow rate (m$^3$/s)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>2576076</td>
<td>7.90128</td>
<td>6322:59-6332:24</td>
</tr>
<tr>
<td>Simulated</td>
<td>2507014</td>
<td>7.702718</td>
<td>6310:00</td>
</tr>
</tbody>
</table>

Figure 5-6: Measured and simulated hydrographs for the storm Sept. 19-20, 1987
Table 5.3: Simulation for the storm June 27, 1987

<table>
<thead>
<tr>
<th>Total runoff (m³)</th>
<th>Peak flow rate (m³/s)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>2149371</td>
<td>10.08192</td>
</tr>
<tr>
<td>Simulated</td>
<td>1956782</td>
<td>11.294686</td>
</tr>
</tbody>
</table>

5.2.2 Storm2: June 27, 1987

For this storm $Z = 230R^{1.4}$ is used to convert reflectivity into rainfall rate. There was no rainfall occurring at the Boston airport, which has been confirmed from both the radar and the rain gauge. For the other three rain gauges, the radar and gauge rainfall measurements are shown in figure 5.7. We can see that the radar measured rainfall rate is higher than the gauge measured rainfall rate at some time steps and lower than the gauge measured rainfall rate at other time steps at Concord, but overall, the comparison is reasonable. At Durham and Macdowell, where rainfall occurred only during a short period, the radar and gauge comparison is not so satisfactory.

The simulated peak flow rate and peak flow time both match quite well with the measured ones (table 5.3). Only the simulated total runoff volume is slightly lower than the measured. While the overall shape of the simulated hydrograph is reasonable, the simulated hydrograph is a little spiky compared with the measured hydrograph (figure 5.8). This again could be due to the fact that radar measures instantaneous rainfall and thus could have very large temporal variations.

5.2.3 Storm3: June 22-23, 1987

Again $Z = 230R^{1.4}$ is used for this storm. The comparison between radar and rain gauges is shown in figure 5.9. The radar measured rainfall rate is lower than the gauge measured rainfall rate almost all the time and at all the four rain gauges. This consistent underestimate of rainfall could be one reason why the simulated total runoff is slightly lower than the measured (table 5.4). Otherwise, the overall shape of the simulated hydrograph and the peak flow rate and peak flow time all match well with those of the measured (figure 5.10). And again, the simulated hydrograph is a
Figure 5-7: Radar and rain gauge comparison for the storm June 27, 1987
Figure 5-8: Measured and simulated hydrographs for the storm June 27, 1987

<table>
<thead>
<tr>
<th></th>
<th>Total runoff ($m^3$)</th>
<th>Peak flow rate ($m^3/s$)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>1356234</td>
<td>6.40032</td>
<td>4184:02-4189:53</td>
</tr>
<tr>
<td>Simulated</td>
<td>1280534</td>
<td>6.28704</td>
<td>4184:00</td>
</tr>
</tbody>
</table>

little spiky.

5.3 Model Verification

5.3.1 Storm4: June 22-23, 1988

Very strong convection occurred during this storm, thus $Z = 400R^{1.3}$ is chosen to convert radar reflectivity to rainfall rate. Still the radar greatly overestimates the rainfall intensity at all the four rain gauge locations, as can be seen from figure 5.11. Actually, rainfall intensity higher than 100$mm/hr$ is recorded at quite a few pixels and at quite a few time steps in the radar images. Following Austin's suggestion
Figure 5-9: Radar and rain gauge comparison for the storm June 22-23, 1987
(1987), a cap is put which limits the maximum rainfall intensity to 100mm/hr.

The strong convection may produce hail in the air which, in turn, can greatly enhance the reflectivity. Thus the total rainfall volume can be greatly overestimated by radar even with the maximum rainfall intensity limited to 100mm/hr.

With the overestimated rainfall as input, the simulated total runoff is almost twice the measured total runoff and the simulated peak flow rate is twice the measured peak flow rate (table 5.5). Only the simulated peak time matches the measured (figure 5.12).

This storm simulation shows once again that the correct rainfall input is of vital importance to a successful model simulation.

Table 5.5: Simulation for the storm June 22-23, 1988

<table>
<thead>
<tr>
<th></th>
<th>Total runoff (m³)</th>
<th>Peak flow rate (m³/s)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>1772859</td>
<td>7.5048</td>
<td>4182:01-4183:03</td>
</tr>
<tr>
<td>Simulated</td>
<td>3304378</td>
<td>14.9215</td>
<td>4184:00</td>
</tr>
</tbody>
</table>
Figure 5-11: Radar and rain gauge comparison for the storm June 22-23, 1988.
5.3.2 Storm 5: October 21-22, 1988

$Z = 230R^{1.4}$ is used to convert the radar reflectivity to rainfall rate for this storm. Again the radar measured rainfall intensity matches reasonably well with the gauge measured rainfall intensity except at Boston airport where the radar overestimates rainfall (figure 5.13). But again the radar data at close to the radar site should not be trusted.

Both the simulated total runoff volume and peak time match the measured ones, but the simulated peak flow rate is much higher than the measured (table 5.6). However, the streamflow were measured only once in many hours (2-19 hours) for this storm. In particular, there was no measurement 2 hours before and 13 hours after the peak time (6368:28) (figure 5.14). So there may well be another “peak” during this interval that are just missed by our measurements, and there is a great chance that this missed “peak” flow rate could be higher than the actually measured peak flow rate.
Figure 5-13: Radar and rain gauge comparison for the storm October 21-22, 1988
Table 5.6: Simulation for the storm October 21-22, 1988

<table>
<thead>
<tr>
<th></th>
<th>Total runoff (m³)</th>
<th>Peak flow rate (m³/s)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>3978939</td>
<td>12.149</td>
<td>6368:28</td>
</tr>
<tr>
<td>Simulated</td>
<td>3785737</td>
<td>17.76</td>
<td>6369:00</td>
</tr>
</tbody>
</table>

Figure 5-14: Measured and simulated hydrographs for the storm October 21-22, 1988
Table 5.7: Simulation for the storm August 29-30, 1988

<table>
<thead>
<tr>
<th></th>
<th>Total runoff ($m^3$)</th>
<th>Peak flow rate ($m^3/s$)</th>
<th>Peak time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>3779495</td>
<td>15.1512</td>
<td>5803:21</td>
</tr>
<tr>
<td>Simulated</td>
<td>3563751</td>
<td>15.3838</td>
<td>5807:00</td>
</tr>
</tbody>
</table>

5.3.3 Storm 6: August 29-30, 1988

$Z = 230R^{1.4}$ is used for this storm. The comparison between radar measured rainfall intensity and gauge measured rainfall intensity at the four gauge locations is shown in figure 5.15. The match is reasonable.

The simulated hydrograph matches almost perfectly with the measured hydrograph except the peak time, for which the simulated is less than four hours later than the measured (table 5.7 and figure 5.16). The overall shape, the peak flow rate and the total runoff of the simulated hydrograph are all very close to those of the measured.

5.3.4 Summary

Both the calibration and verification processes are much easier when streamflow data are available at only one gauge. For a distributed model, on the other hand, it also means that more calibration storms are needed in order not to miss the better parameter combinations. In the Souhegan river basin, with only one streamflow gauge and only three calibration storms, the verification results are reasonable except for the storm 4, where the overestimation of rainfall by radar results in the overestimation of runoff by the model.

Radar as a tool for spatial rainfall measurement may not match well with the gauge rainfall measurements, because it is an instantaneous measurement of spatially averaged rainfall over a resolution volume while a gauge measures the temporally averaged rainfall at a single spatial point. However, as long as there is no system bias in the radar measurement, in other words, as long as the total rainfall volume is measured correctly, the model simulation will be all right. In five out of six storms,
Figure 5-15: Radar and rain gauge comparison for the storm August 29-30, 1988
Figure 5-16: Measured and simulated hydrographs for the storm August 29-30, 1988 we have shown that the radar rainfall measurements are satisfactory.
Chapter 6

Conclusion

A physically-based distributed rainfall-runoff model DBS has been described. A new method to derive prestorm water table depth across a river basin has been presented. This initial water table generation method relates the water table depth with the prestorm streamflow at the basin outlet, as a function of the topography and soil properties of the basin in question. When applied to the Souhegan river basin in New England, it was demonstrated that the recession flow closely follows the theoretical curve suggested by this method.

The DBS model was applied to two river basins: the Arno river basin in Italy and the Souhegan river basin in New England. In the Arno river basin, which has a drainage area of 3660 km$^2$, streamflow data were available at three gauges. Three storms were chosen for model calibration and two storms were chosen for model verification. Except for the six hour radar measurement, most of the rainfall data came from gauge measurements. In the Souhegan river basin, which has a total drainage area of 450 km$^2$, only one gauge provided streamflow measurements. Three storms were chosen for model calibration and three storms were chosen for model verification. Radar data were used as model rainfall input. Overall, the model did reasonably well as far as the simulated hydrographs are concerned.

In each of these two river basins, the number of storms used for model verification may not be enough to judge conclusively the quality of the model performance. But considering that these two river basins are vastly different in terms of soil, topography,
climate, basin size and so on, the simulation results are encouraging.

From the simulation results, the following conclusions can be drawn:

(1) With the increasing availability of computer resources, a physically-based distributed rainfall-runoff model can do very well in simulating the hydrological responses of a watershed.

(2) Correct rainfall input is very important for a successful rainfall-runoff model simulation. However, problems exist in both gauge and radar rainfall measurements. When rain gauge data are used, the gauge density has to be high enough to capture the spatial variation of the rainfall field. When radar data are used, the radar has to be calibrated with various kinds of storms before actually put into use.

(3) Although problems exist as when strong convection produces hail during a storm, generally a well-calibrated radar can measure reasonably well the spatial rainfall field.

(4) A physically-based distributed rainfall-runoff model, combined with DEM data and real-time radar rainfall measurements, can be a very useful tool for real-time flood forecasting.

Suggestions for future research include:

(1) There is still a lot of arbitrariness in determining $a_1$ and $Q_c$ from the $dQ/dt - Q$ plot of the recession flow in a basin. More robust schemes should be developed to estimate these parameter values, which may include developing an objective way to filter out the unreliable recession flow data.

(2) As has been shown in chapter 3, the basin averaged initial water table depth is very sensitive to the drainable porosity of the basin $n_e$. However, pumping data are usually not available to estimate $n_e$ for most of the basins, so $n_e$ was estimated a priori in the Souhegan basin. A method should be developed to estimate $n_e$ from the basin hydrological data, maybe from a simple basin scale water balance calculation.

(3) The uniform inter-storm recharge rate assumption makes the initial water table derivation method applicable only to small river basins.

(4) The assumption of uniform values for the calibration parameters clearly will meet difficulties in large basins, especially for $f$ and $a_r$ values. On the other hand, it
may not be realistic to assign distributed $f$ and/or $a_r$ values since field measurements are rarely available. Parameter $f$ is in some sense a description of soil depth, and it has been observed that soil tends to be deeper in the valley and shallower at the mountain top. It may be worthwhile to try to find a loose relationship between $f$ and some topographical characteristics (like distance to stream or terrain slope).

(5) It would be interesting to actually measure the basin internal behavior at certain locations during a storm (the soil moisture content, wetting front depth, etc.) and compare them with the model simulation. Thus we can get a feeling how well the physically-based model captures the physical processes.
Bibliography


