A Comparison of the Airline Benefits of European-style and American-style Aircraft Purchase Options in Periods of High Demand.

by

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ABSTRACT

Commercial Aircraft are commonly acquired by an airline in a contract that specifies a fixed quantity of firm orders as well as options that allow the airline to obtain additional aircraft at a future time for an agreed price. This option purchasing process allows the customer to avoid taking delivery of aircraft if economic circumstances are not favorable. However, this model can have drawbacks in circumstances when the product in question is in very high demand. Exercising a rolling-style aircraft purchase option in such circumstances may result in a delivery lag of several years, during which profitable opportunities may be lost. Shortening the time between exercising an option and taking delivery of the aircraft would allow for better timing of the delivery and reduce the effect of a significant negative event such as an economic recession or a terrorist attack occurring between the exercise date and the delivery date. In this thesis, an alternative purchasing system using fixed-term aircraft purchase options is presented. In this system, customers are able to make the decision to exercise an option at a later point in time, with more information, yet receive the aircraft at the same time as the current system. Additionally, in situations where the decision to accept an aircraft has been made, it will allow the customer to acquire the aircraft sooner. The proposed alternative system is presented and the potential benefits and difficulties are discussed including potential incentives for manufacturers to offer such a system. A simulation is presented to compare the relative value of a fixed-term option to a rolling option under hypothetical circumstances. The simulation shows that under the circumstances modeled, the fixed-term option system is superior to the existing rolling option system in yield environments where profitability is close to zero. This advantage decreases as profitability increases and at very high profitability levels, such as in a very strong passenger yield environment, the existing option structure is superior. The results are consistent over a wide range of average consumer fares and discount rates.

Thesis Supervisor: John-Paul Clarke
Title: Principal Research Scientist, Department of Aeronautics and Astronautics
This thesis is dedicated to

Marilyn Malpica

George Miller

And

The Blizzard of 2005

For taking me in, inspiring me, and showing me that I really am half-Texan at heart
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Chapter 1

Introduction

Commercial airlines use their fleets of aircraft to transport passengers and cargo around the world. The aircraft are a significant and essential component of their business and great care is taken in matters related to these aircraft. Periodically it is necessary for an airline to acquire new aircraft in order to expand their business or to replace older units which may no longer be safe or economically viable to operate.

The decision making process for determining what type and what quantity of aircraft to obtain and which manufacturer to purchase them from is often complex. Each different aircraft type has its own capabilities and characteristics which may or may not make it suitable for a given airline or route. A new aircraft will likely be in service for 20 or more years and the purchase price for aircraft and the operating expenses associated with them are very high. Forecasts spanning the expected operating life of the aircraft can be created to aid in the decision. These forecasts are simplifications and educated guesses regarding the future of the
overall economic climate and the specific region in question. While limited by the assumptions included in them, they often provide the justification for purchasing new aircraft. An aircraft may cost as much as $200 million. Financing may be necessary to produce enough capital to purchase the aircraft and negotiations with creditors can add to the complex economic evaluation process.

Integral to the decision making process is the ability of the manufacturer to meet the time requirements of the customer airline. Aircraft production is limited to a specific rate, such as 100 per year for a given model, and each aircraft has a delivery position or delivery date when it will be delivered to the customer. These delivery positions are referred to as “slots”. The production rate for the manufacturer assembly line may potentially be increased to meet higher demand but this increase may be difficult to implement on short notice. Many different suppliers contribute components to the aircraft manufacturers and the final assembly line may only increase its rate if all the suppliers are able to meet the new production quota. Since this limited production volume must be shared by all customers purchasing that model of aircraft from a given manufacturer, it is not always possible for the customer to specify exactly when they would like to receive their aircraft. Because of the manufacturing time as well as crew training and other factors, most large orders (10 or more units) are delivered over a period of several years.

Once a specific aircraft type and desired quantity has been decided upon, and delivery positions have been examined, a portion of the order may be designated as aircraft purchase options. Contracts may include more than 1 type of aircraft but for simplicity we will assume only a single type. Purchase options are used to limit the liability of the customer. These provisions give the customer the right,
but not the obligation, to accept delivery of aircraft beyond the quantity specifically purchased in the original contract. These option aircraft would be available for purchase at the price specified in the terms of the option contract. The customer may exercise any of these options at any point after the contract is signed by selecting an available delivery position. These options are referred to as rolling options because they do not expire for many years and are not fixed to a specific delivery position. They figuratively “roll” along with the customer in time. Rolling options are purchased for a price which is small compared to the total purchase price of an aircraft and paid for at the time the contract is signed.

The option structure described above is commonplace in the commercial airline industry today. It provides airlines an opportunity to acquire additional aircraft to expand their fleet beyond the original order without negotiating a new contract. Most importantly, by designating a portion of the contract to be options rather than entirely firm orders, the airline may avoid taking delivery of those option aircraft if the economic circumstances were to deteriorate after signing the contract but before delivery.

While valuable, there are limitations to the type of aircraft purchase option described above which will be discussed later in this chapter. In this thesis, we will suggest an alternative option structure and will attempt to compare the two option types under a certain set of circumstances using a purchasing decision model which we have created. The proposed option system is intended to provide increased flexibility to the customer airline while creating or maintaining the incentive for the manufacturer to provide such options. We will begin with a description of the need for a new model, an introduction to our concept,
background material concerning real options and their applications, and the airline fleet-planning process.

If you want more background information about the airline industry or more on a particular aspect, *The Airline Handbook*, produced by the Air Transport Association is a comprehensive and accurate place to start. It is available for free on their website at www.airlines.org.

### 1.1 The Need for an Alternative Purchasing Model

Throughout the world, airlines are affected by the economic fortunes of the regions they operate in. In the US, the pattern of boom and bust in airline profits has been repeated and has escalated in amplitude since the US airline industry was deregulated in 1978. [Jiang, 2005] The problem created by the cyclical nature of the economy is exacerbated by the high fixed costs of an airline. Aircraft are capital intensive as is the special equipment needed to operate, service, and supply them. Equipment must be paid for whether or not it is used and often has an operational service-life spanning several economic cycles. Labor groups, which constitute a large percentage of industry costs, are often unionized. Organized labor groups commonly have contracts that make it difficult to adjust the size of the labor force to fit the economic climate. Adding to the high fixed costs has been the unprecedented increase in fuel costs in recent years. As a result the airlines are reluctant to take on aircraft and the resulting increases in equipment and personnel needed to operate them without the confidence that those aircraft will be profitable. This has led to the prevalent use of purchase options in the industry.
The time between a customer placing an order and that aircraft being delivered is referred to as the “lead time”. Manufacturers would like to keep this period sufficiently long so that they have a steady backlog of orders, meaning the assembly line is continuously active, and so that they can plan future production rates accordingly. Lead time therefore becomes an important component in the negotiation process between the manufacturer and the customer. In many situations a customer placing an order must wait several years before receiving their aircraft. Due to the tendency for some airlines to build their fleet gradually, the time between delivery of the initial and final units within an order may be several years. The combination of these two time elements can result in a very long gap between the decision to acquire an aircraft and delivery of that plane. Given the uncertainty and cyclical behavior of the airline industry, the likelihood that an aircraft will be delivered in an unfavorable economic climate increases as the order-delivery lag increases. Essentially, by the time the last plane of an order arrives, the airline may not want that many anymore.

Purchase options allow an airline to avoid acquiring a portion of those later deliveries by not exercising their options. For example, a customer decides it needs 30 aircraft and an order of 30 aircraft is made. Of these 30 units, 20 are guaranteed orders, referred to as “firm” orders, and 10 units are held as options. If the carrier decides not to exercise any of these 10 options, likely due to poor current or projected economic circumstances, only the 20 firm aircraft will be delivered. Without the capability to buy options, a customer wanting 30 aircraft would have needed to make all 30 units firm orders. If economic circumstances had deteriorated after their order, they would be forced to accept and pay for all 30 aircraft rather than only 20 in the option scenario. The 10 unwanted aircraft impose a financial burden on the airline which may be significant. If the aircraft
are operated, they may produce a loss for the airline. If the aircraft go unused, the airline has either paid for them entirely and sunk a large quantity of capital, or it continues to pay the creditor which lent it the purchase capital. While it is possible for an airline to lease or sell an unwanted aircraft, the individualized construction of that aircraft may make it less desirable for another airline. This reduces its value on the second-hand market.

The practice of using purchase options can be very beneficial to an airline as illustrated above. By offering the options, manufacturers keep their customers happy and loyal while collecting revenue from the sale price of the options which are separate from the aircraft purchase price. Stability in the commercial aircraft market is increased because fewer unwanted aircraft are delivered. This system is generally good for all parties but it has limitations.

The options in the above example are typically American-style call options known as rolling options. These options are not tied to any delivery date; they merely allow the customer to accept additional aircraft units beyond their firm order commitment without negotiating a new contract. If at some point after the original contract is signed, the airline wishes to exercise an option they may do so, at the price specified in the original contract. At the time of exercise, the airline determines, along with the manufacturer, when the delivery will occur, subject to production availability. Herein lies the problem. While the aircraft purchase price is fixed for the option, the delivery slot is not. In an environment of high demand for the specific aircraft product, other airlines may have acquired all the available near-term delivery slots. In this scenario the lead time for these option aircraft may be several years beyond the minimum manufacturing time. This increases the uncertainty surrounding the transaction for the customer and reduces the value of
the option. A customer may lose the opportunity to profitably operate an aircraft during this delay.

The primary drawback of American-style rolling options for aircraft purchase is the lack of time certainty. In periods of low demand for an aircraft, the rolling option is sufficient because there is little if any delay beyond the minimum manufacturing time between exercising the option and taking delivery. In periods of high demand, the rolling option is subject to the delivery queue which may extend well beyond the minimum manufacturing time. This forces a customer holding an option to wait for delivery and potentially miss strong revenue opportunities in the short term. Additionally, such a delivery delay, or even the fear of one, may force a customer to exercise an option preemptively, several years earlier than they would if there were no potential delivery lag. This decision is therefore made with less information and shifts more economic risk from the manufacturer back to the customer. A purchase option with time certainty would eliminate the need to risk the entire purchase price of the aircraft on that preemptive decision. Instead only the price of the option would be at risk.

There is very little published work evaluating purchase options in the airline industry. The negotiations between customer and manufacturer are often unique to the individual air carrier and are kept strictly private by both parties. The price of an aircraft may vary significantly from one carrier to another which affects the negotiation process. Those able to leverage the two major manufacturers, Boeing and Airbus, against one another can often put themselves in line for a good bargain. A negotiation between Spain's Iberia and the two primary manufacturers was chronicled in The Wall Street Journal in 2003 providing an unusual glimpse into the process. The bidding showdown between the two competing
manufacturers included dramatic price concessions, attractive financing packages, and even guarantees on maintenance costs and future resale values. Boeing argued its product was superior and cheaper to operate while Airbus' offering was cheaper and had more commonality with Iberia's existing fleet which was already composed of Airbus aircraft. With both manufacturers eager to fill their order book, the concessions made to Iberia and other customer airlines in recent years have been significant. [Michaels, 2003]

The premier article on the evaluation of the aircraft purchase options was written by John Stonier, Director of Airline Marketing of Airbus Industrie North America, Inc. and will be referenced frequently in this thesis. Stonier (1999) outlines methods, including purchase options, which increase flexibility for an airline. The drawbacks associated with different types of purchase options are explained as they relate to the industry cycles and manufacturing time. Several techniques for estimating the value of a purchase option are presented. These techniques include financial options theory, which is the traditional method for valuing real options such as an aircraft purchase option, as well as decision tree techniques. The drawbacks and limitations of these methods are also discussed. More will be mentioned about this article in chapter 2.

While significantly better than not having any purchase options, Rolling options have clear limitations in certain circumstances. In order to overcome these limitations airlines often preemptively finalize purchases in order to make sure they get the delivery timing they want. This can be very risky because it increases the lead time and the decision is based on less information. A complimentary, alternative type of purchase option may be able to limit the downside of the current rolling option technique.
1.2 The Alternative Purchase Option

The alternative purchasing option suggested here is intended to reconcile the lack of time certainty inherent in the rolling option. The new option is a European-style call option. When the option is purchased a specific delivery date is decided upon providing time certainty. Exactly 1 year before this delivery date the customer must decide whether or not to exercise the option with delivery taking place as scheduled. If they no longer wish to receive it, they may decline the option at which point the option expires. These types of options are intended to be used in large quantities, replacing rolling options, but may be used in smaller quantities in conjunction with firm orders and rolling options. A customer could purchase precisely as many options as they originally intend to exercise, or they could purchase a larger number with the intent to eventually decline a portion of them. This latter scenario would require a larger initial investment as more options are purchased, however it would allow the customer to maintain as much flexibility as possible to take advantage of a healthy economic environment.

Because this system would result in currently held delivery positions becoming newly available when options are declined, a queue of desiring customers, currently holding later delivery positions, would be created. Those customers at the top of the queue would have first refusal to newly available delivery slots. If a slot goes unclaimed by all customers in the queue it would then become available for a new order. By accepting an aircraft from the queue a customer would exchange an option held at a later time which would then return the later slot back to the queue. The goal of this queue process is to redistribute delivery positions to those customers who actually want them, rather than forcing delivery of expensive
aircraft onto airlines in potentially poor fiscal condition. It also gives customers in
the queue additional flexibility because they may have the opportunity to
exchange slots. This gives the fixed-term option some of the inherent flexibility
available to rolling options which may be exercised at anytime. However, owning
a fixed-term option still maintains complete time certainty for a customer.

The primary direct incentive for the manufacturer is the increased revenue from
the sale of options. If the fixed-term options are more valuable than the existing
rolling options then the sale of an equal number, or possibly a greater number
would increase revenue for the manufacturers. Additionally, if the added
flexibility of the fixed-term option creates stability for their customers, it may
protect future sales of aircraft to airlines who may otherwise struggle under the
burden of unwanted aircraft. The new option method and its impact on
manufacturers will be described in more detail in chapter 3.

1.3 Introduction to Real Options

In this section we will introduce the concept and basics of real options of which
aircraft purchase options are a type.

The owner of a real option has the right, but not the obligation to undertake a
business decision at a later date [Miller, 2005]. Typically, real options are used to
hedge the risks associated with large capital investments such as the introduction
of a new product line, the extension of a factory, or the purchase of a fleet of
vehicles. The purpose of a real option is to enable the owner to phase their
investment, and avoid sinking capital into a project which may ultimately be
unwanted or unprofitable. A certain amount of capital is necessary initially to obtain the option or begin the project. Further investment is delayed until a future point in time when more or better information can be obtained. If conditions are favorable at this future point, the option can be exercised thereby completing the investment. If conditions are unfavorable, the option can be declined, abandoning the investment with no further cost. The owner of an option can thus wait and judge industry or economic factors relevant to their investment before deciding whether to spend what is often a very significant amount of capital. The result is an increase in flexibility for the option owner, a reduction in risk associated with factors beyond their control, and the potential for improved reaction time because of their initial investment. These benefits are what lead entities to obtain real options and why they can be worth more than their initial cost.

To illustrate the value of a real option, consider a manufacturer with 2 major product lines that wishes to build a new facility. The company must decide how much land to purchase to build their new facility. It currently sells only 2 products but there is interest in adding a 3rd product at a future time. If enough land is purchased to build a 3rd production line at the time of the original investment, a real option has been created. This allows the company the option to launch their new product without the need to acquire land in the future. Acquiring the land at a later date may be time consuming or even impossible, potentially harming the company's competitiveness in the marketplace. Alternatively, the company may build the new assembly line at the same time as the first 2, allowing them to bring their new product to market very quickly after the decision to launch, however if the new product is never launched the investment in the third assembly line will be wasted. The real option allows its owner to avoid those potential negative scenarios at the cost of the additional land. This example was derived from a
similar example in “A Generalized Real Options Methodology for Evaluating Investments Under Uncertainty with Application to Air Transportation” by Bruno Miller, 2005.

The example above is a very straightforward one but the concept of applying a smaller investment to create flexibility is very common. Industries that require an expensive fleet of vehicles, such as mass transit agencies, freight companies, and airlines, use real options to build their fleets. For such entities, there is often an economy of scale in having many vehicles in a fleet that are identical. Standardization of operating procedures, training programs, maintenance activities, replacement and spare vehicles, flexibility in crew scheduling, and even customer familiarity are all reasons for having common fleet types. In these scenarios where multiple vehicles are to be purchased, a company will often stagger their investment using real options. They purchase an initial complement of vehicles and a further amount of options which can be exercised later in order to expand their fleet. Such options serve two primary functions. The first is to match the supply of their service with demand for that service. An airline with too many airplanes will not be able to fully utilize those assets, but an airline with too few airplanes will not be able to serve all of their potential customers. With options, an airline can acquire additional aircraft if and when their traffic levels warrant expansion, without being stuck with too many aircraft initially. The second primary function of an option for vehicle fleets is that they allow the holder to evaluate the product before committing to the entire quantity needed. If a transit agency orders 20 railcars with options for 100 more, it can evaluate the performance and economics of those 20 railcars against manufacturer performance claims and the transit agency's needs. If the vehicles turn out to be a poor fit for
the agency, or are mechanically unreliable, the agency may choose not to exercise its remaining options and pursue an alternative product.

The examples above are meant to illustrate why options are used. In reality there are industry factors which complicate the basic premise and reduce the value of the options. It is our intention to present an option method for the airline industry which will increase their value beyond the present implementation.

Discussion of real options began in 1977 with a mention by Stewart Myers in a paper on corporate borrowing. Myers claims, “many corporate assets, particularly growth opportunities, can be viewed as call options. The value of such ‘real options' depends on discretionary future investment by the firm” [Myers, 1977]. Companies with tradeable commodities such as those in the mining and the energy industries have made good use of them since that time as have companies in other industries that face situations similar to those described above. The following is a useful description from “A Generalized Real Options Methodology for Evaluating Investments Under Uncertainty with Application to Air Transportation” by Bruno Miller, 2005.

“Traditional real options analysis techniques are based on the theory of financial options. In financial options, the holder of the option has the right, but not the obligation, to purchase an asset (call option) or to sell an asset (put option). This asset is called the underlying asset and it is typically the stock upon which the option is written. Its value is given by the price of the stock, as determined by its valuation in the stock market. In a call option, the investment required to buy the underlying asset is called the strike
Airline options fall into the category of call options. [Stonier, 1999] Fixed-term options like those we are proposing in this thesis are known as European-style options. They have a specific date on which they may be exercised and expire at that time if they have not been exercised. American-style options, or rolling options as they are referred to in this thesis may be exercised at any point up to their expiration date, which may be quite distant. In either case, the negotiated purchase price of the aircraft, determined in the original contract, becomes the strike price required to exercise the option, and accept delivery.

1.4 Airline Fleet Planning

The fleet planning process is typically lengthy because of the high cost of aircraft and the length of time a new aircraft will be in service. Given the difficulties in successfully forecasting the economic climate 10-20 years into the future, airlines tend to be very cautious before committing to the investment. It is common for airlines to evaluate new products from the manufacturers even when there is no immediate need for new aircraft. This allows them to move more quickly when a need emerges.

There are two basic strategies for fleet planning. The first is a “top down” approach which evaluates options for a specific region or route. A demand forecast is used to determine the number of seats necessary to provide a certain
level of service to the market. Different aircraft models are evaluated within the forecast scenario for the market and operating realities so that the economics can be estimated. This should determine approximately how many of a given aircraft type would be needed to provide the desired level of service. [Belobaba, 2005]

The second fleet planning strategy is “bottom up”. This approach is much more detailed. Aircraft are evaluated on specific routes under forecast economic scenarios. The actions of competitors are included as they relate to the airline's market share and pricing power. This approach can provide more comprehensive evaluations, but it is time consuming and doesn't necessarily represent what will happen. It is difficult to predict the actions of competitors and such a detailed analysis may not be worth the effort given the speculative nature of many aspects of the process. [Belobaba, 2005]

Aside from the overall strategies there are several factors which affect fleet planning efforts. Certain routes require aircraft with specific requirements such as a certain range or payload capacity. For example, trans-pacific flights cannot be completed by the smaller aircraft common in US domestic service. While rare, airport characteristics such as runway length, altitude, and extreme temperatures may restrict the choice of aircraft to serve specific airports. This is particularly true of airports in very hot climates at high altitudes, or in areas where geographical factors limit runway length.

Each aircraft has its own financial characteristics which can affect the fleet planning process. In addition to the purchase price, the cost to operate an aircraft, measured in cost per available seat mile (CASM) varies. Two planes may be able to provide the same number of seats, and fly the same route, but at different costs.
Environmental constraints, including air pollution and noise pollution, can be important in some cases. Certain airports restrict older, noisier aircraft from operating into them. Marketing may be a factor as there are sometimes substantial differences between new aircraft and older models. In some cases, passengers may prefer a certain aircraft type which could be an advantage to an airline which is the first or only carrier to operate it. Finally, because many airlines are state-owned, a political decision to choose one model, or to purchase from one manufacturer over another may supersede the economic rationale. [Belobaba, 2005]

Once a model has been identified for a specific region or route, the quantity and timing of the fleet acquisition must be determined. The size of the order, including options, is likely determined using one of the approaches mentioned above. The number of options to acquire is likely a function of the confidence level in the forecast on which the fleet planning has been based. A confident forecast may result in more firm orders and fewer options. Timing of deliveries is a combination of when an airline wants to begin a new service, or when existing aircraft will be retired, and when the manufacturer is able to produce that aircraft.

The airline industry is very cyclical. Because of the lag between an order and delivery of an aircraft, airlines often believe they need to purchase aircraft in the trough of the cycle in order to take advantage of the upswing in the market. [Stonier, 1999] This notion can result in an airline paying for unwanted aircraft if the market does not rebound as expected by the time of the delivery date. This, combined with the ability to acquire additional units more quickly in the event that the market exceeds expectations, is the reason purchase options are an important tool for fleet planning in the airline industry. The time certainty available to
customers holding fixed-term options reduces the need to gamble during the trough of the industry cycle. Rather than make firm orders, they may instead acquire options which will put them in position to accept aircraft if the circumstances prove worthy while not being liable for the full purchase price in the event the market upswing does not take place. This contributes to the health of the individual carriers and collectively, the entire industry. By preventing unwanted aircraft from entering the market, a secondary effect is to reduce overcapacity. When too many seats are available for sale in the market, prices decline, and profitability declines along with it. US airlines have struggled in recent years because of overcapacity in the market. A stable and healthy customer base is theoretically very good for the manufacturer as well. [Gessing, 2005]

1.5 Thesis Objectives

Our primary objective in writing this thesis is to explore and evaluate an alternative type of aircraft purchasing option against the predominant existing option type in situations where the decision to acquire such purchase options has already been made. The alternative option type consists of a system of European-style call options, referred to here as fixed-term options, to reduce delivery uncertainty and transfer risk from the customer to the manufacturer. An array of these European-style options replaces, or compliments, the existing system of American-style rolling aircraft purchase options. Each fixed-term option is tied to a specific delivery date and requires the holder of the option to exercise or decline the option 1 year in advance of the specified delivery date. American-style options are subject to manufacturing availability and in times of high demand, an aircraft may not be available for several additional years. The proposed option system
would allow the customer to delay the final decision to acquire an aircraft, while the rolling option would force the decision to happen several years earlier to avoid delivery delay.

This methodology may improve upon the current approach by increasing the likelihood that deliveries are made to customers when they want them by transferring some risk of industry cycles to the manufacturer who may be better able to absorb such cycles due to their broad portfolio of customers. Under certain conditions the manufacturer would be able to increase revenues without incurring any additional costs. Customers would be able to transfer certain risks associated with regional industry cycles to the manufacturer for a reasonable financial payment. Compared with rolling options, a customer can make a decision to exercise an option later for the same delivery time, or accept delivery more quickly after exercising the option at the same time, in order to take advantage of a positive revenue environment as quickly as possible. It is ultimately in the interest of the manufacturer to help sustain industry profitability. Allowing customers a way to reduce their fleet liability in periods of economic hardship may result in stable customer who may in turn become repeat customers. The proposed purchasing model is compatible with the existing method allowing it to be used without large changes to current purchasing practices.

1.6 Thesis Contributions

By writing this thesis, we explore and demonstrate the value to both customer and manufacturer of modifying the structure of the purchase option system that is used in the commercial airline industry to provide a complimentary alternative. We do so by developing a simple model to simulate the option decision process for an
airline and the earning potential that results from that decision. By comparing the results of the model for the existing and fixed-term option structures we can estimate the relative value of the fixed-term option structure. We also suggest reasons why this option structure may be beneficial for manufacturers.

1.7 Thesis Overview

This thesis is divided into five chapters. A brief summary of aircraft purchase option practices in the airline industry is provided in chapter 2. In chapter 3, we will present the alternative purchasing model. We outline the structure of the fixed-term option system and how it compares with the current system including the potential benefits for the manufacturer. In chapter 4, we outline our model for examining the relative value of a fixed-term and a rolling option using a Monte Carlo simulation and discuss our results. Finally in chapter 5, we summarize our findings and suggest areas for further study.
Chapter 2

Literature Review

In this chapter we examine the different types of purchase options commonly used in commercial aircraft sales by presenting the basic structure of each option type. Several methods for determining the value of a purchase option are introduced and some limitations of such valuation techniques are discussed.

2.1 Real Options and Aircraft Purchasing

2.1.1 Simple Aircraft Purchase Options

The simplest type of aircraft purchase option is similar to a European-style call option. At a single and specific date, the customer must decide whether to accept delivery of a specified aircraft at a designated future point. If the option is exercised, the customer pays a price agreed to when the option was initially obtained. If the option is declined, it expires and no aircraft is delivered. A simple option gives the customer price certainty, meaning there is no risk of price
escalation. It also gives the customer delivery certainty. If the option is exercised, delivery will take place on a given date and penalties will be paid by the manufacturer to the customer if that date is not met.

### 2.1.2 Rolling Aircraft Purchase Options

A variation on the simple option is the rolling option which is similar to an American-style call option. Once a rolling option is acquired, the customer may exercise that option at any time until the option expires, which may be many years in the future. This type of option gives the customer price certainty like the simple option, however there is no delivery or time certainty. A delivery date is established when a rolling option is exercised rather than in advance. In situations of high demand for the product, the manufacturer may not have delivery positions available for 3 or 4 years. In situations of low demand, the customer may receive the aircraft as soon as possible, but this is unlikely to be less than the minimum manufacturing time for the aircraft. This is likely to be equivalent to the time between exercise and delivery of a simple option. In this regard there is no advantage for a rolling option and there may be a disadvantage in periods of high demand. The benefit for a rolling option over a simple option is the flexibility to exercise it at any time. Additionally, rolling options may not expire for many years. This is valuable because you aren't using up an option by deciding not to acquire an aircraft. They retain their value for a longer period. Rolling options are the standard industry practice today for large airlines. A recent Boeing 787 order for All Nippon Airways included 50 firm orders, with rolling options for 50 additional units. [Boeing 2, 2004]
2.1.3 Switching Options

Up until now we have discussed purchase options involving a single aircraft type. It has been assumed that the customer airline has chosen to acquire that specific type and only the mix of firm orders and options is to be determined. However, the type of aircraft can also be changed when the option is exercised. Switching options are simple options that allow the customer to change the aircraft type at the time they exercise an option. The major aircraft manufacturers have developed some of their products in groups, or families, with each group containing several different aircraft which are nearly identical except for capacity. These families typically start as one aircraft, referred to as the baseline model. The baseline's fuselage is then elongated, or shortened, to create essentially a similar aircraft but with more, or fewer, seats. These different models can then use the same spare parts, and be operated and maintained by the same personnel, without the need to retrain or re-certify labor groups. All aircraft in a family have the cockpit commonality allowing pilots to operate all family members with the same training. Flight crew ratings are also common throughout the aircraft family. Maintaining different aircraft types in a fleet adds costs for an airline, but this strategy of aircraft families allows for different-size aircraft without having too many fleet types. The different size planes can be used to better match the demand for seats in the markets in which they are used. Since all aircraft in a given family are built on the same assembly line, the manufacturer can assemble any of the types in the time between exercising the option and delivery. The result for the switching option holder is an additional choice (or choices) when deciding whether or not to exercise the option. Consider the example for an Airbus 320 switching option. In addition to the choice of declining the option, the customer has two, or more, future scenarios to examine. Suppose they estimate that an
Airbus 320 will generate $30M in its operational life and that an Airbus 321, a member of the same aircraft family, will generate $35M in its operational life. If the purchase price for the 320 is $30M and the 321 is $32M then the option will be switched and exercised for a 321. If the original option for the 320 had not been a switching option, the expected loss of money would have resulted in a declined option. [Stonier, 1999]

2.2 Valuation Approaches

2.2.1 Financial Options Theory

One approach for estimating the value of an aircraft purchase option is to use financial options theory. Aircraft purchase options are a type of real option, which have their basis in financial options theory. The Black-Scholes option pricing model, presented below, is the basis for all option pricing used today. [Stonier, 1999]

\[
\text{Option value} = SN(d) - Xe^{-r t} N(d - \sigma \sqrt{t})
\]

Where:

- \( S \) = share price today
- \( \sigma \) = volatility of the share price over time
- \( N(d) \) = proportion of shares required to replicate the option
- \( N(d - \sigma \sqrt{t}) \) = risk neutral probability that the call option will be exercised
- \( t \) = time to exercise or maturity
\[ X = \text{strike price} \]

\[ r = \text{risk-free discount rate} \]

The share price \( S \) is represented by the discounted present value of the cash flows generated by the aircraft over its life. One must be able to estimate this value, as well as the volatility \( \sigma \) of this value. The strike price \( X \) is the purchase price of the aircraft.

The Black-Scholes equation shows that the value of the option increases with both time to delivery and the volatility of the underlying asset value. The more time there is between acquiring the option and being forced to exercise it, and the more dramatic the potential swings in value, the more likely the aircraft are to be worth more than its purchase price. If on the other hand the aircraft is worth less than its purchase price, the option can be declined. A detailed description of using this methodology is beyond the scope of this chapter but more information can be found in the Stonier article cited in this thesis. [Stonier, 1999]

2.2.2 Decision Tree Analysis

Decision trees can be used to assess real options. Using a simple binomial decision tree, one can model 2 different potential outcomes for purchasing an aircraft. On one branch, there is a probability that the aircraft will be worth a certain amount, based on the analysis of its earning potential. On the other branch, there is the remaining probability that a different outcome, perhaps a negative one, occurs. An expected value of the purchase, positive or negative, can then be determined by combining the two branches and their respective likelihoods.
An option can be represented by determining the values after a given length of time subject to the ability to decline a negative value. If you pay a deposit, then re-evaluate your 2 outcomes one year later, you may then elect to decline the option. This would allow you to replace a negative outcome in the decision tree with the cost of declining the option. Since you have already paid for the option, there is no additional cost to decline it. When analyzing the expected value of the entire decision tree, including the option may make the expected value of the delayed purchase higher than the expected value were the aircraft to be bought today. If so, then this is the value of the option, created by the additional flexibility. More detailed examples are provided in the Stonier article referenced here. [Stonier, 1999] Stonier also discusses the ability to create risk-adjusted decision trees which account for varying costs of capital due to varying risks.

2.2.3 Complications

The valuation techniques available today for real options are based on financial theory and tradable securities in the financial world. There is no direct stock equivalent for large commercial aircraft meaning many assumptions must be made in the evaluation process. Often times the share price of the airline is used as a proxy since much of an airlines investment is in the form of aircraft. The time between exercising an option and delivery, the lead time, has no direct corollary in the financial markets that these techniques were created for. There is no delay before receiving the stock when exercising a stock option. A purchase option transfers the asset risk from the customer to the manufacturer, however once the option has been exercised, the risk returns to the customer despite them not yet having the asset. The longer the lead time, the more risk the customer bears and the less applicable the Black-Scholes and decision tree methods become.
The Black-Scholes equation method assumes that the stochastic element is a random walk whereas the airline industry has been shown to be cyclical, having periods of sustained growth or recession. Stonier (1999) suggests the use of a mean reverting model. Mean reverting models are discussed in Chapter 5. Another difficulty is the fact that all airlines have the ability to not make a decision regarding an aircraft purchase. Rather than pay for an option, they may wait and finalize the entire contract at a later point. They may elect to continue using their existing aircraft for another few years. This is referred to as the naturally occurring option to wait. Time resolves uncertainty regarding an airline’s future, therefore giving this natural option its value. However, many airlines will ignore this option and try to secure delivery slots by signing a contract in order to obtain favorable deals from the manufacturer or to prevent a rival from getting there first. There is a perception that such orders should occur at the trough of the economic cycle to ensure a strong position relative to one’s peers when the industry rebounds. [Stonier, 1999]

2.3 Literature Review Summary

There are several types of options available to airlines when purchasing commercial aircraft. Each option has its own characteristics and corresponding prices. There is no concrete method for determining the value of an option. Techniques involving financial options theory and decision trees are useful given sufficient information and educated assumptions, but there is little authoritative literature on the subject and as a result, the practice is under-utilized by airlines. "Although some airlines now intuitively realize the value of delivery flexibility,
they have not... explicitly quantified the value of this development. Additionally, airlines have tended to underestimate this value." [Stonier, 1999]
Chapter 3

Fixed-Term Options for Aircraft Purchase

The fundamental decision when placing an order involving purchase options under the current system is what mix of firm orders and options to secure. An order with too many firm orders can lock the customer into a large financial obligation which could be difficult to meet in the event of an industry downturn. An order with too many options may result in a delay in securing delivery of the additional planes desired in a period of industry strength. Our alternative option model seeks to reduce the uncertainty faced by the customer in exchange for an additional, yet modest, financial outlay. The reduced uncertainty for the customer is absorbed by the manufacturer, who is compensated financially for doing so and who may be able to deflect negative consequences of industry downturns more successfully than an individual customer.

It is important to note that this model will work in conjunction with the existing model. The approach most appropriate, be it fixed-term options, rolling options, or
no options, may be used given the industry status or outlook at the time of purchase. Firm orders will still require no additional cost beyond the purchase price. Some customers may be unwilling to pay for additional options. Our view is that the new option style primarily replaces the current options rather than any firm orders; however any firm orders replaced by fixed-term options will only further the transfer of risk from the customer to the manufacturer.

3.1 The Expiring Option

We are introducing a purchase option system where the manufacturers offer airlines the ability to purchase arrays of European-style fixed-term options in place of or in addition to the American-style rolling options they presently offer. During the negotiation process for the fixed-term type, the purchase price for each unit will be established and the number of firm orders determined. Each of these firm orders will be assigned a specific delivery date and presumably a manufacturing line number. Then, instead of agreeing to a specific number of American-style rolling options, which have no expiration date or delivery slot, an array of European-style fixed-term options would be purchased. Each of these options would require an initial payment and would guarantee the customer a specific delivery date and line number. Each option would mature a standard amount of time before delivery, such as 1 year. If the option is exercised, the customer takes delivery 1 year later and pays the original unit price agreed to in the order. The aircraft price is in addition to the amount paid for the option. If the option is declined the customer is liable for nothing beyond the purchase price of the option itself. The rights to that delivery slot revert to the manufacturer who is then free to sell or assign it to another customer. Regulations regarding the sale or transfer of
the options would need to be determined. If a customer were able to sell individual options to other airlines ahead of the maturity it might encourage customers to lock up early delivery spots they did not intend to keep for themselves, however this practice may interfere with the queue structure described below.

3.2 The Queue

For the manufacturer to be willing to offer this option structure to many customers it would need to feel comfortable that it could place an aircraft with another customer were an option to be declined. The manufacturer would then be selling the same number of units as if the declined option were a firm order. While there is little likelihood that every declined aircraft would be immediately reassigned, an organized structure could be created for the resale of aircraft delivery slots.

Take for example an airline from Asia that was not able to secure any delivery slots for 2007 and 2008 for the aircraft type in question and instead would begin taking delivery in 2009 and 2010. If they wished to receive aircraft earlier, they could be placed in the reassignment queue. In the event that options for delivery in 2007 and 2008 were declined by those airlines holding them, customers at the front of the queue, such as our Asian airline, would be offered those aircraft delivery slots from 2007 and 2008 as they became available. Queue customers receiving aircraft in this manner would still receive the 1 year of lead time before the delivery date. If for example the queue customer had a contract for 10 firm orders plus 20 fixed options, they would be able to reschedule either one of their firm slots in 2009/2010 or one of their later options to claim that aircraft. This rescheduling would thereby free a later delivery slot for another customer in the
queue or for a new sale. The ability to swap a later option or slot for an available
queue slot is a very important characteristic of the queue system. This function
mimics, albeit in a less flexible manner, the ability of the rolling option to be
exercised at any time. If an airline in the queue holds a later position, they have
the choice whether to accept a queue aircraft or do nothing and remain in their
original position. Every aircraft that falls to them in the queue increases the utility
of holding the original option; it increases the number of opportunities in which to
exercise their option, while guaranteeing the time certainty of the original delivery
date. The more times they have a queue choice, the more like a rolling option
their fixed-term option becomes. The secondary importance of this is that it may
allow the customer to purchase fewer fixed-term options. We have suggested that
more options should or could be purchased than a customer intends to use in order
to provide flexibility. The increased flexibility of the queue system just discussed
may allow that extra quantity of options purchased to be smaller. To facilitate
everything, the smooth and efficient operation of the queue would be important so
that the manufacturer isn't left with aircraft that have been built but which have no
destination customer and so that customers can adjust their delivery positions to
their liking with adequate lead time remaining.

Another important function of the queue would be to redistribute aircraft in the
event of regional differences in the economic health of the industry. Take our
Asian carrier example again. Assume that the majority of the fixed-term options
held for the 2007 and 2008 delivery years are held by North American carriers.
Significant industry-altering events can affect one region, or continent, much more
significantly than another. Additionally, carriers based elsewhere in the world who
serve the affected region more than their local competitors will also be unequally
affected. The Sudden Acute Respiratory Syndrome (SARS) epidemic and the 2001
terrorist attacks on New York were both examples of this. During the SARS epidemic, Asian carriers were hurt significantly as traffic levels decreased because travelers avoided Asia. Among US carriers, Northwest Airlines which serves many Asian destinations suffered extensively while other US carriers such as American Airlines, which has a minimal presence in Asia, suffered only slightly. [Maynard, 2003] The 2001 terrorist attacks significantly hurt the US airline market, while having a less pronounced affect on foreign carriers.

If an event or economic cycle were to predominantly affect the North American carriers during 2006, those customers holding options for 2007 and 2008 may not want those aircraft and elect to decline their options. The queued customer from Asia, elsewhere in the world, or perhaps even a North American carrier less affected by the event would be more likely to claim the delivery slot that would be now available. It is this global portfolio of customers that is available to the major aircraft manufacturers which makes the entire system possible. Both Boeing and Airbus have customers around the globe and by redistributing aircraft during economic cycles or crises, to customers unaffected or less affected by the economic downturn or crisis, the manufacturer is able to deflect the additional risk they assume by allowing the customer to reduce their individual risks.

3.3 The Manufacturer

Much of what has been discussed in this thesis has been from the perspective of the customer, the airline. In order for a new option system to be relevant there must be an incentive or at least the lack of disincentives for the manufacturer to offer the options. The most quantifiable benefit to the manufacturer is the direct
sales of the options. Our theory is that they are more valuable than rolling options under certain circumstances. If this were the case then a 1 for 1 replacement of rolling options for fixed-term options would yield an increase in revenue for the manufacturer. We have also suggested that a customer purchase more options than they intend to use in order to create flexibility for themselves. In this case, additional revenue from those unused options is obtained. The cost to the manufacturer for these sales is accepting additional risk. If many customers declined their options and no suitable queue customers could be found, the manufacturer could be left with unused delivery slots. This is not a desirable outcome for the manufacturer but we feel this risk is low for several reasons.

The current major manufacturers, Boeing and Airbus, each have global client portfolios. They sell aircraft to customers in every continent. The varying economic and airline industry health of different regions of the world allow for the manufacturers to take advantage of regional strengths when certain other regions are economically weak. An aircraft declined by a North American customer may be heavily sought after by an Asian or European customer. The second reason risk of unsold delivery slots is low is due to the fact that the new type of options may be offered only at the manufacturers discretion. Our suggestion of the fixed-term option system was made specifically for periods of high aircraft demand such as the launch of a new product like Boeing’s new 787 product. For periods of weak sales or for unpopular aircraft the manufacturer can elect whether or not to offer any type of option. The two option types are entirely complimentary allowing the manufacturer to offer neither, either, or both types in any given contract.

Customer loyalty is an important aspect of commercial aircraft sales. Because orders from major airlines can represent billions of dollars worth of goods, it is
important to keep customers happy and healthy to increase the likelihood of future sales. Keeping a customer happy may be reason enough to offer a different purchase option. If it prevents a large order from going to a rival manufacturer it may be worth accepting additional risk. For example, a purchase of 10 firm orders plus 30 fixed options may place risk on the manufacturer that those 30 options may go unused; however the value of the 10 firm aircraft, plus the income from the options themselves may outweigh the possible negative impact of 30 options being declined.

It is in the interest of the manufacturers to help ensure that their customer airlines remain in business. In a period of economic recession in an airlines’ geographic region, financial strain and bankruptcy can be very real possibilities. Part of this financial burden may be the capital payments for new aircraft that may no longer be desired. Enabling customers to structure their aircraft purchases so that they may decline options in the case of a recession may assist in the preservation of future aircraft sales that would not otherwise be possible due to a customer bankruptcy or liquidation.

Another potential benefit of the proposed option system may lie in the manufacturer’s ability to regulate and plan future production capacity. If the demand for aircraft outpaces production then it may be in the manufacturer’s interest to boost their production rate. Assuming the queue is functioning efficiently, the fixed-term option system provides more information about demand because all options have specific delivery slots; therefore the manufacturer knows when these aircraft will be built. In the rolling option system, manufacturers know how many outstanding options their customers hold, but they do not know if or
when their customers plan to exercise those options. Fixed-term options therefore provide time certainty for the manufacturer as well as the customer.

3.4 Lead Time

The time between the maturity of the option and the delivery date is called the option lead time. Determining the lead time is important for both the manufacturer and the customer. While it is generally true that shortening the lead time increases the value of the option because it reduces the uncertainty for the customer, as you get below 1 year of lead time there are diminishing, and even negative returns for an airline. An airline must potentially hire and train crew members and other personnel to operate and service an aircraft before it enters service. This hiring and training process takes several months. Airline schedules are made in advance and often are entered into global distribution systems as many as 330 days in advance. If an airline is going to fly a new aircraft on a certain route, it needs advance warning in order to assign it to the schedule, sell tickets, and properly staff that aircraft and those operations that support it. A lead time of less than 1 year would make this process difficult. The manufacturer has conflicting incentives to make the lead time both longer and shorter. As mentioned before, a shorter lead time makes an option worth more because it reduces uncertainty which is the fundamental value of the option. However, a longer lead time gives the manufacturer a longer period to reassign the aircraft in the event of a declined option. For our purposes we have assumed the lead time to be 1 year which we believe to be realistic for both the customer and manufacturer. This 1 year lead time is approximately equal to the manufacturing lead time. Until
advances are made to the manufacturing process, a lead time shorter than 1 year is unlikely. [Stonier, 1999]

Important for the discussion of lead time is product commonality. The more parts of an aircraft design that are tailored to an individual customer, the harder it is to place that aircraft with a different customer on short notice. If all airplanes were identical then the swapping of customers would be trivial. Commonality has also been identified as an important consideration when determining aircraft values on the second-hand market. Leasing companies would theoretically find it easier to place an aircraft with a new client if there were fewer major differences between models of the same family. This improves the aircraft's resale or second-hand value. If less work was required to recondition an aircraft for second-hand use, it would also make redistributing aircraft during economic downturns simpler and more cost effective.

Perhaps the most important factor affecting aircraft commonality at present is engine type. Airlines must invest capital in spare engines and replacement parts for each engine type they operate. Additionally they must employ, train, and certify mechanics to work on each engine type. Having the same aircraft type with 2 different engine types unnecessarily increases the quantity of spare parts and human capital that is necessary to maintain the fleet and is therefore avoided when possible. In response to this problem and to reduce the competition and associated risk for an engine maker, aircraft manufacturers are increasingly designing new aircraft lines with only one engine manufacturer. This assures that the engine maker is the supplier for 100% of all aircraft orders for that type. It also eliminates the problem of conflicting engine types on the 2nd hand market. As an alternative approach to the problem, the new Boeing 787 has been designed to
make multiple engines types interchangeable on the same airframe. Previously, such a change would have required extensive modifications to the aircraft and its control systems. [Boeing, 2004].

Even without these newly imposed monopolies, engine makers have global client portfolios making it more likely that a suitable owner could be found in our proposed queue system. Because of the trends in this area and the efforts of manufacturers to promote commonality throughout the aircraft, we have assumed that commonality concerns will not affect the operation of our proposed purchase option system.
Chapter 4

Exploring Fixed-Term Option Value

4.1 Introduction

A crucial element of the fixed-term option system presented here is that one must pay a price to acquire these options. This is an incentive for the manufacturer to offer options in the first place, giving the customer price certainty in the future, and in the fixed-term case, time certainty as well. How much one should be willing to pay for such an option is very much in question. It is impossible to know with certainty the value of the option at the time of purchase; however an educated estimate can be derived with appropriate sources of information.

In this chapter, we outline a way to compare the value of a single, fixed-term option with a similar rolling option, the prevailing industry standard. While it would be beneficial to put an absolute price tag on the fixed option, there are so many competing factors which determine such a price, that it is beyond the scope of this thesis. Instead we present a methodology for a relative comparison between
the two options types. It is presumed that the value of the standard rolling option is known to both customer and manufacturer and that information would be the basis for determining the value of a fixed-term option using our results.

Each type of option carries with it the value derived from price certainty. The price to be paid for the product, the strike price, is determined when the contract is signed, and carries through until the option expires. Thus neither option is superior in this regard; however in an environment of high-product demand, the fixed option carries the advantage of having either a later decision point for equivalent deliveries, or a shorter lead time for equivalent decision points.

To understand the benefit of a later decision for an equivalent delivery point, consider the following example. If an airline wishes to take delivery of an aircraft at year 5, their decision points for each option type will be different. If there is a long line of customers, the first available delivery position may be 3 years after an order. Thus the customer would have to exercise their rolling option at year 2 because a rolling option has no delivery certainty.

Alternatively, a fixed option would not need to be exercised until year 4 yet would still be delivered in year 5. This could be a tremendous advantage if something unexpected were to occur between years 2 and 4. Even if no major event took place, having those 2 additional years to evaluate demand and estimate future earning potential should lead to a more informed decision.

The other advantage of the fixed-term option is the shorter lead time before delivery. Suppose for example that demand was particularly strong today and the airline wishes to take advantage and order an aircraft for delivery as soon as
possible; perhaps to move before a competitor does. If they had fixed term options in place, it would allow for delivery roughly 1 year from now, because of the time certainty of delivery. A rolling option, if exercised today, may not result in a delivery until year 2, or 3, or perhaps even later. During the time between year 1 and the actual delivery date, the airline has lost the opportunity to use that aircraft in revenue service. It may also be losing the opportunity to open a new route, or challenge a competitor in a new market.

These are the primary advantages to the fixed-term option and the reason for their value. If they are indeed more valuable than rolling options, then it would be possible for manufacturers to charge more for them. However, it should be noted that a customer may need to acquire more fixed-term options than rolling options in order to ultimately acquire a fleet of a given size. Rolling options do not expire for many years, perhaps as many as 20; therefore they are not "used up" if no options are exercised. For example, instead of offering 10 rolling options, it may be possible to offer a package of 20 fixed options, of which only 10 may be used. After the 10th exercised option, the rest are canceled and those unused slots revert to the customer queue. Had there only been 10 fixed-term options to begin with, as soon as one was declined, it would reduce the total number of aircraft that could be acquired to 9. This uneven ratio of fixed-term options to rolling options necessary to build a future fleet is important to consider but for the purpose of our evaluation they will be compared one to one.
4.2 Modeling Process

In order to model the decision making process of an airline, we need to be able to predict the expected future earning potential of an aircraft. This expected future value becomes the basis for deciding whether to exercise an option for delivery. If you expect that you will make money operating the aircraft over its life, then you would exercise the option and take delivery of the airplane. If you expect to lose money operating the aircraft over its life, then you will defer the decision to a later point when new or better information suggests that operating the aircraft would be profitable. Depending on the specific details of your purchase option, delaying that decision may result in the expiration of the option.

4.3 The Financial Model

The foundation of our model is the annual growth of passenger enplanements in the US commercial airline market. Passenger enplanement totals for the 25 years following the deregulation of the US airline industry in 1978 were used to calculate growth rates over this period. 1978 was chosen as the starting point because the changes to the industry during deregulation made observed data prior to that point less useful for predicting future events. Any large shock to the industry, such as the terrorist attacks of September 11th, 2001 could have a similarly destructive effect on the value of data observed before that shock. Therefore one cannot always assume that there is a relatively infinite amount of data points when making decisions, even in an established industry such as commercial airlines.
The observed post-deregulation growth rates had a mean of approximately 3.5% per year with a standard deviation of approximately 6%. The growth rates were placed in a histogram and it was estimated that the sample was approximately normally distributed. Additionally, we have assumed that annual growth rates for this data are independent and therefore will be memory-less. We will begin with the description of the model for the fixed-term option.

Using the growth characteristics above we have used a normal distribution with mean of 3.5 and a standard deviation of 6.0 to generate new data points.

Table 4-1. Created growth rates

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth%</td>
<td>-0.5</td>
<td>14.7</td>
<td>5.2</td>
<td>5.3</td>
<td>11.5</td>
<td>-9.3</td>
<td>9.3</td>
<td>-0.4</td>
<td>2.4</td>
<td>-5.5</td>
<td>-5.2</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Table 4-1 contains the first 12 years of created growth rates for a sample simulation. The full data for this simulation begins in year 1 and extends to year 43 but is abbreviated here for simplicity in presentation. It is assumed that no data exists before year 1. This would be the case in an event such as the 1978 deregulation of the US airline industry where policy changes fundamentally altered the industry. Data prior to that point may exist but would not be relevant in predicting future events. Numbers in this and other tables in this section have been rounded for simplicity in presentation as well. Negative values indicate a decline in passenger enplanements in that year.

The fixed term option allows a customer holding this specific option to decide at year 10 whether or not to accept delivery at year 11. This is the only scenario
available for this type of option. Therefore the decision making process is based upon the data available up to year 10. These first 10 values for passenger enplanement growth are averaged, as shown in table 4-2, and used to project future growth using simple straight line estimation.

Table 4-2. Created growth rates to be averaged.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth%</td>
<td>-0.5</td>
<td>14.7</td>
<td>5.2</td>
<td>5.3</td>
<td>11.5</td>
<td>-9.3</td>
<td>9.3</td>
<td>-0.4</td>
<td>2.4</td>
<td>-5.5</td>
<td>-5.2</td>
<td>4.8</td>
</tr>
<tr>
<td>Average Growth%</td>
<td>3.3</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tbody>
</table>

The passenger enplanement growth data is combined with the baseline passenger total, in year 0, to create annual passenger totals for years 1 through 43. The baseline figure is determined using several assumptions. The aircraft being purchased will hold 150 passengers and fly 4 segments per day of 1200 miles per segment. The seating capacity of 150 is equivalent to the Boeing 737-800 or the Airbus 320-200, both very popular aircraft models. The 1200mi segments represent typical domestic flight lengths from a central US hub airport. In total if every seat were sold for each segment everyday of the year the aircraft could carry:

\[ 150 \times 4 \times 365 = 219,000 \text{ passengers} \]

The load factor for an aircraft is the actual passengers divided by this maximum total we’ve just calculated. Typical load factors for commercial airlines, averaged over their entire network, are approximately 80%. We have assumed that for the baseline year in each simulation, the load factor will be 50%. A 50% load factor would be considered a weak market, however it is intended to reflect a marginal or emerging market which has lower passenger totals now but has the potential for
growth over the life of an aircraft. In each simulation the decision to exercise the option takes place at least 8 years after the baseline point. In scenarios with strong growth, the load factor would then be closer to the current industry average. Including this 50% load factor the baseline passenger total for the fixed option simulations is:

\[ 219,000 \times 50\% = 109,500 \text{ passengers} \]

An additional consideration for load factor is that even if passenger demand grows extensively and exceeds the actual capacity of the aircraft, the airplane cannot carry more than the 219,000 maximum listed above. Also important is the daily variation in demand. If for example the annual demand was exactly 219,000 passengers, the same as the maximum capacity, it is nearly impossible that the demand would be exactly 600 passengers on each of 365 days. Instead it will vary. For example, on Monday there may be 500 passengers wishing to fly that route and 700 passengers on Tuesday. On Monday all 500 will be carried, however on Tuesday only 600 customers can fly because there are no more available seats. 100 customers won’t be able to buy tickets. When the load factor for the two days is averaged, the result is less than 100%. Because some of our simulations contain passenger totals that approach or exceed the capacity of the aircraft, we have limited the maximum load factor for any single year to 90%. Our data is at the annual level rather than the daily level. We chose to use this 90% cap as an estimation of the maximum sustainable load factor for an airline. Therefore in any single year, even with very high demand, the aircraft can only carry:

\[ 219,000 \times 90\% = 197,100 \text{ passengers} \]
Beginning with the passenger total from year 10, the average growth rate from the first 10 years is then applied to create passenger totals for years 11 through 43. The first few years of our sample simulation appear in table 4-3. The word passengers is abbreviated as pax.

**Table 3-3. Projected passenger volumes**

<table>
<thead>
<tr>
<th>Year</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pax</td>
<td>147313</td>
<td>152112</td>
<td>157068</td>
<td>162186</td>
<td>167470</td>
<td>172926</td>
<td>178560</td>
<td>184378</td>
<td>190385</td>
</tr>
</tbody>
</table>

Note that the value for year 10 is what was observed to be true based on the growth rates from the initial step. The values in years 11 through 18, and up to year 43 not shown here, all represent a 3.3% growth rate over the previous year. This 3.3% rate was calculated above in table 4-2. Due to the 90% load factor limit described above any value exceeding 197,100 would be reduced to 197,100. This completes the determination of passenger levels. The remaining steps use this annual passenger total and other assumptions to create a financial representation of the simulation.

In order to represent costs we will use a common industry statistic, cost per available seat mile, or CASM. This is the cost of flying 1 seat, 1 mile, regardless of whether or not it has a person in it. In our case it will represent all airline expenses except the purchase price of the aircraft. The purchase price of the aircraft is $30,000,000 to be paid in full in year 10. A CASM value of $0.10 was selected using data from [Clarke & Melconian, 2006] taking into account the 1200 mile stage length we had assumed. The aircraft in question is flying 150 seats over 1200 miles, 4 times per day, 365 days per year. At $0.10 per available seat mile the annual cost total is the following:
Total Annual Cost = 150 * 1200 * 4 * 365 * 0.10 = $26,280,000

Revenue for the airline is generated only via ticket sales. An average fare is multiplied by the total annual passengers in each simulation to produce total revenue.

Average Fare * Total Annual Passengers = Total Annual Revenue

The total annual cost is subtracted from the total annual revenue for each year to produce the annual net profit for each year. The net profit for each year is discounted by a net annual discount rate to produce the net present value (NPV), in year 10, of all future cash flows for the aircraft. This net discount rate is the difference between the nominal discount rate and the inflation rate.

Net Discount Rate = Nominal Discount Rate – Inflation Rate

The purchase price for the aircraft is subtracted from the sum of the NPV calculations for each year of the aircraft’s operating lifespan. We have assumed a 30 year lifespan. Summed together, the result is the expected profit over the life of the aircraft if the purchase option were to be exercised. If this result is positive, the aircraft is expected to produce a profit. If this result is negative, a loss is expected. This profit/loss result is the basis of the decision making logic for our model. Any expected profit greater than zero will cause the option to be exercised.
An option is therefore exercised based on what the airline predicts or expects to happen in the future. In order to evaluate the relative success of such a prediction one must know what actually happens to passenger volume in the 30 years in which the aircraft will be in use. We have done this by treating the original growth rate data we generated as the “true” outcome. Instead of using the average of the first 8/9/10 years of growth data to predict later years as was done to make the option decision, all of the original growth data is used and no averaging/extrapolating occurs. This results in different annual passenger totals for years 9/10/11 through 43, and therefore different overall profitability results. We have continued our assumptions about operating costs and fares and other inputs through this process.

**Table 4-4. Actual passenger volumes.**

<table>
<thead>
<tr>
<th>Year</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pax</td>
<td>147313</td>
<td>139720</td>
<td>146415</td>
<td>150566</td>
<td>151397</td>
<td>161141</td>
<td>152435</td>
<td>159168</td>
<td>162048</td>
</tr>
</tbody>
</table>

Comparing table 4-4 with table 4-3, the previous table contains passenger totals grown using the average growth rate from the first 10 years. Table 4-4 is the passenger totals as they were actually simulated to happen. Each year has a unique growth rate rather than the straight line projection used in table 4-3. It is this actual set of figures, and the resulting profitability based on them, that is the product of the overall simulation. The decision to exercise the option is based on the expected profit, but the results are the actual profit. In simulations where the option is not exercised, a result of zero is recorded instead because no airplane exists to make or lose money. While it is true that the result in this latter case is not zero because a price was paid to acquire this option, we set it equal to zero here for the purpose of comparison. The goal is to evaluate the relative value of the two option styles and therefore both types are valued at zero. A resulting
advantage of one option type over the other can then be altered based on specific option purchase price available to a customer. Once the final profitability figure is recorded the simulation is complete.

Our model contains 256 such simulations for the fixed-term option. Each simulation contains a unique set of growth rates based upon our original assumption of a normal distribution with stated mean and standard deviation. Some simulations result in the option not being exercised while others result in a purchase and resulting operational profit or loss. When the resulting profits for all 256 trials are averaged, the product is the expected value of possessing a single purchase option. The larger the number of simulations the less error there will be in this expected value. It would be useful to include more than 256 trials however we were limited to 256 because of computational capabilities.

The simulations of the rolling purchase option are structurally similar to the fixed-term option simulations we have described already. All the same outside data points and assumptions are the same, only the timing and structure of the option itself are different. The fixed-term option had a decision point at year 10 and a delivery point at year 11. That was the only choice. If the customer didn’t want the aircraft at year 10, the option expired. Rolling options do not expire and therefore have infinitely many decision points. For the purpose of comparison we had to simplify this structure. The rolling option we will analyze has up to 3 different possible decision points, in years 8, 9, or 10. Corresponding delivery dates for the 3 different decision points are at years 11, 12, and 13 respectively. The option will expire if not used in year 10. This option is similar in that one can receive the aircraft at the same time as the fixed option scenario or one can decide
to acquire the aircraft at the same point as the fixed option scenario. Table 4-5
summarizes the examined scenarios.

Table 4-5. Exercise to delivery matrix.

<table>
<thead>
<tr>
<th>Option</th>
<th>Exercise</th>
<th>Delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-term</td>
<td>Year 10</td>
<td>Year 11</td>
</tr>
<tr>
<td>Rolling</td>
<td>Year 8</td>
<td>Year 11</td>
</tr>
<tr>
<td></td>
<td>Year 9</td>
<td>Year 12</td>
</tr>
<tr>
<td></td>
<td>Year 10</td>
<td>Year 13</td>
</tr>
</tbody>
</table>

This change in option structure creates changes in the way the simulation works.
When making a decision for the year-8 option, the expected profitability of the
aircraft is based on only 8 years of growth rate data, rather than 10 years of data as
before.

Table 4-6. Rolling option growth rates to be averaged.

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth%</td>
<td>-0.5</td>
<td>14.7</td>
<td>5.2</td>
<td>5.3</td>
<td>11.5</td>
<td>-9.3</td>
<td>9.3</td>
<td>-0.4</td>
<td>2.4</td>
<td>-5.5</td>
<td>-5.2</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Average Growth% 4.5

Table 4-6 shows the new average growth rate based only on the first 8 data points.
This average growth rate is more than 1.0% higher than the one calculated before
using 10 years. When extrapolated 30+ years into the future, the lifespan of the
aircraft, significantly higher passenger totals will be expected. As before, a
decision to exercise the purchase option will be made based on the expected profit
over the 30 year lifespan of the aircraft. Like before, if the option is exercised, the
actual profit based on the original growth rates is recorded and the simulation
ends. In the year-8 option case the actual profit is identical to the fixed-term profit
because the lifespan of the aircraft, years 12-41, is the same. If the option is not
exercised nothing is recorded and an evaluation of the year-9 option begins. Like the year-8 option, the year-9 option uses a different number of data points to create an average growth rate. Years 1-9 are averaged and then extrapolated. The analysis takes place as before, but since delivery occurs one year later, the lifespan of the aircraft would be years 13-42. If the expected profit is greater than zero, the option is exercised and the actual profit is recorded, ending the simulation. If the option is not exercised the process is repeated for the year-10 option with incremental changes taking place as in the year-8 to year-9 transition. If after this final analysis the option is still not exercised the value of zero will be recorded and the option expires, ending the simulation. As in the fixed-term option analysis, there are 256 simulations.
Chart 4-1. An organizational chart depicting the simulation algorithm.
4.4 Discussion of Results

4.4.1 Quantity of Options Exercised

Our model consisted of 256 simulations. Of these 256 simulations the number of times the option was exercised was recorded for each option type. At virtually every data point analyzed, the rolling option was exercised more often than the fixed option. The only exceptions came on very low fares where a relatively small number of options were exercised. Data was collected beginning from the lowest fare at which any options were exercised. Slightly above this opening fare is the breakeven point around which the expected value of each option type is zero. As shown in figure 4-1, just above the opening fare, the rolling option was exercised in the range of 20%-40% more than the fixed option. As the fare increased, the rolling exercise total slowly declined until it settled at about 4% above the fixed option’s total. This pattern of behavior was similar over a range of net discount rates of 6% to 15%.

The consistently larger number of rolling options exercised can be attributed to the greater number of exercise points for such options. A fixed-term option must be exercised in year 10. If the forecast based on the data at that point is slightly negative the option will be declined. A rolling option has more opportunities for the historical data to improve. If a rolling option is not exercised after its first year, the additional data available a year later, or 2 years later may result in the option being exercised. The total increase in exercised options is then largely a
Figure 4-1. Ratio of options exercised using the rolling option to options exercised using the fixed-term option. Net discount rate = 6%.

4.4.2 Fixed Versus Rolling Options.

Figure 4-2 shows the difference in expected profit by average fare, for a range of net discount rates. This shows how much better (positive values) or worse (negative values) the fixed-term option is than the rolling option. The 4 series all have similar shapes. The discount rate does not appear to alter the characteristics of the model. The amplitude of the curve decreases as the discount rate increases. The higher discount rate dampens the relative advantage of either option type.
result of simulations where the decision sequence for the rolling option was Negative-Positive or Negative-Negative-Positive.

This also suggests an explanation for the greatest difference in the number of options exercised occurring near the break even point. Because the decision logic for the model is based purely on whether the forecasted NPV is above or below zero, near this break even point it is more likely that the forecast would change from below zero to above zero in the course of 1 or 2 years. If the initial forecasted profit is particularly negative it would take a more dramatic change in the average growth rate to change the sign of subsequent forecasts. Similarly if the initial forecast was very positive, the option would be exercised in both the fixed option and the rolling option cases.
Each series begins at zero. At this fare and below no options are exercised and therefore the expected profit of both option types is the same. As the fare increases the fixed option quickly reaches a peak of relative profitability. The peak advantage for the fixed option occurs when the option value is near its breakeven point. The option allows the customer to avoid some loss-making outcomes by making a decision based on more information. Avoiding the loss-making outcomes would make for a higher overall profit and therefore a higher expected profit.

Figure 4-2. Difference in expected profit between the fixed-term option and rolling option by fare. Each series represents a different net discount rate.

Beyond the initial fixed advantage peak, the difference between the two options decreases until at higher fares the rolling option becomes consistently superior. At higher fares, loss-making outcomes are much less likely; therefore the additional
aircraft purchased using the rolling option in the Negative-Positive and Negative-Negative-Negative-Positive scenarios help increase the rolling options expected profitability.

4.4.3 Increased Lead Time for Rolling Options

We have assumed a 3 year lead time between exercising a rolling option and receiving that aircraft. This delay was set arbitrarily but we wish to address the question of what impact this may have on our results. Figure 4-3 below shows the expected profits for rolling options with either a 3-year or a 4-year lead time. The expected profit for the 4-year option is consistently lower than its 3-year counterpart. The magnitude of the difference increases as the average fare increases. Because changing the lead time for the rolling option has no bearing on the fixed-term option, this lower expected profit for the 4-year option implies a greater advantage for the fixed-term option in this case.

These results are in line with our expectations. A longer lead time represents a longer delay in the event of positive economic circumstances. This is precisely what the fixed-term option was intended to prevent. It forces the decision to exercise the option to be made even earlier making the result less predictable and the decision less likely to be made correctly.
Figure 4-3. Expected profit for a rolling option by average fare. 4-year lead time and 3-year lead time series shown at a 12% net discount rate.
Chapter 5

Conclusions

5.1 Conclusions

It is important to remember that many assumptions and conditions were included in our study, however we believe the unambiguous nature of our results supports the concept we have proposed and warrants further study. We were limited to only 256 simulations. Additional simulations would strengthen our analysis. Passenger data was simulated at the annual level and included no seasonal or daily variation. A given average fare was constant throughout any given simulation therefore the model lacked the different fare environments which would normally affect an airline market. The rolling option we simulated did not have true freedom of exercise. We allowed for only annual decision points, and a relatively near term expiration date. A model with more frequent data points, for example one modeled on the month rather than the year, would allow for many more decision points to reflect to true flexibility of the rolling option. We have also assumed that queue airlines would be willing or able to make decisions on very
short notice to accept aircraft available to the queue. In reality, the decision to accept delivery may not be quick enough to ensure a timely progression through the queue. An improved analysis of this aspect of the system would be beneficial.

While the assumptions underlying our experiment might indeed limit the applicability of our results, it is clear that the fixed option method that has been described here is superior to the equivalent rolling option as described in our experiment. Scenarios where an acquired aircraft loses money are more likely to be avoided by using more data points to forecast the lifespan of the aircraft and periods of economic strength can be capitalized on more quickly by taking delivery with less lead time. Our results showed that the fixed-term options were exercised less frequently.

The expected result for a single option was as much as $2 million greater for the fixed option method and was consistently better than the rolling option method when fares were low. The expected result of the rolling option exceeded that of the fixed option at higher fares, but these fares and the resulting overall profitability of the aircraft would be difficult to obtain in practice. The fixed option method is most successful when the investment is close to the break even point. The combined longer observation period and shorter delivery time available with the fixed option allows the airline to both make a better measurement of the growth rate and respond much more quickly to that better measurement.

When the lead time was increased from 3 to 4 years, the expected profitability of the rolling option decreased. This made the relative advantage of the fixed-term option even greater and supports our suggestion that this delivery delay has a negative effect on the customer.
Our comparison assumed an equal acquisition price for the two option types. Given the results in favor of the fixed-term options it suggests that the manufacturer could charge a higher price for that type of option than for the rolling options presently offered. This would provide incentive for the manufacturer to provide this option to customers. Additionally we have suggested indirect motivations for the manufacturer including customer and industry stability.

5.2 Areas for further study

5.2.1 Manufacturer-Centric Analysis

An additional area of study we recommend would be a manufacturer-centric analysis of a fixed option and queue system like the one we have outlined in this thesis. How significant might the additional revenue generated by option sales be, and could the queue system suggested here be used to regulate or dictate production capacity for the manufacturer? It may be beneficial for a manufacturer to have a queue system where they could be confident that all aircraft would be successfully delivered to alternate carriers in the event of declined options. If the queue became sufficiently long it may encourage the manufacturer to increase production rates to better satisfy the demand for their product.
5.2.2 Mean Reverting Processes

Another approach to modeling the growth forecast aspect of the decision to exercise an aircraft option would be to use a mean reverting process. Assume the Arithmetic Ornstein-Uhlenbeck Process (AOUP), below, represents the behavior of the passenger enplanement data used in our experiment above.

\[ dx = \eta(m - x)dt + \sigma dz \]

The average annual growth rate of passenger traffic since deregulation tends toward an average of approximately 3.5 percent per year. There is a stochastic element which accounts for the fluctuation from this long term mean of 3.5 percent. Additionally there is a correction factor which pushes the value back towards the mean as it strays. Think of this as a spring; the farther you stretch a spring from its natural position the greater the force trying to return the spring to its natural state will be. This prevents individual values from straying too far from the mean for any extended period or in our spring example, from pulling the spring too far in either direction. Further details on the implementation of the mean reverting process can be found in Appendix A.
References


Appendix A

A.1 Estimating the Parameters of a Mean Reverting Process

There are several methods for estimating the parameters of a mean reverting process, one being the Dixit and Pindyck Model (DPM), applied to the AUOP below.

\[ dx = \eta(m - x)dt + \sigma dz \]

Where \( m \) is the expected value, \( \eta \) is the reversion factor, and \( \sigma \) is the standard deviation of the stochastic term. \( x \) represents your data, either actual or simulated. In order to determine these parameters one must begin with the following regression.

\[ x_t - x_{t-1} = a + bx_{t-1} + \varepsilon_t \]

This will yield the slope, intercept and standard error of the regression which can be used to find the 3 parameters of the mean reverting process using the following equations.

\[ m = -a / b \]

\[ \eta = -\ln(1 + b) \]
\[ \sigma = \sigma \sqrt{\frac{2 \ln(1+b)}{(1+b)^2 + 1}} \]

A.2 Simulating Data for a Mean Reverting Process

Once the 3 parameters have been determined from the method described above, it is possible to simulate a mean reverting process. By using the standard mean reverting equation below, with a starting value of \( x_0 \) and the identified parameters \( m \), \( \eta \), and \( \sigma \) new data points can be simulated.

\[ dx = \eta(m-x)dt + \sigma dz \]

This new data can be used to represent possible future outcomes. The process can be repeated many times for use with Monte Carlo simulation.

One issue with a model based on the mean reverting process described here is that one must successfully simulate data using the parameters obtained from the previous step. The value for \( \eta \) must fall between 0 and 2 in order to produce data that is reasonable. Outside this range for \( \eta \), the simulated data points will diverge from, rather than converge to the long term mean \( x \). Additionally, a small number of data points available to use in the maximum likelihood estimator may limit the viability of the model. If these issues can be resolved, or an alternative data source for the model can be found that can be represented successfully by a mean reverting process, we believe an algorithm similar to the one presented here would provide valuable insight into the relative value of the two option types.