Turbulent Transport of Inertial Aerosols

by

John A. Simo

Bachelor of Science in Mechanical Engineering,
University of Massachusetts at Amherst (1989)

Submitted to the Department of Mechanical Engineering
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This study examines the motion of micron-sized aerosols in a turbulent gas flow, in an effort to assess the effects of particle inertia on the statistical and spectral characteristics of the particle velocity field. Differences between the velocity statistics and power spectra of the gas phase and the aerosol phase are studied as a function of particle inertia. A turbulent, axisymmetric air jet seeded with a water aerosol is used as the experimental vehicle; properties are examined along the jet centerline.

A phase-doppler particle analyzer (PDPA) measures random-arrival time-series of particle velocity and particle diameter in the turbulent flow. The resulting data are sorted into particle size (inertia) bins at each downstream station; statistical and spectral analyses are then performed separately for each size class. The sub-inertial particles exhibit the classical statistical and spectral behavior of a turbulent jet. The centerline turbulence intensity asymptotically approaches 0.25 with increasing downstream distance. The small-particle spectra are close (but not equal) to Kolmogorov universal equilibrium scaling in the inertial subrange. A more accurate scheme is to scale the small-particle spectra with turbulent passive scalar mixing. An expression for downstream decay of the scalar dissipation rate is found.

Inertial effects on the particle statistics and spectra are found to depend on the relative magnitude of the particle relaxation time ($\tau$) and the local Kolmogorov time scale ($\tau_K$). Inertial particle mean velocity is higher than the gas-phase velocity within 45 diameters of the nozzle, but thereafter is identical to the flow velocity. Inertial-particle rms velocity is generally lower than the gas-phase rms velocity, with the notable exception particles having $\tau/\tau_K = 1$. For $\tau/\tau_K = 1$, the rms velocity of particles is slightly higher than the gas-phase rms velocity. For $\tau/\tau_K$ larger than unity, particle rms velocity decreases with particle inertia, as classical Lagrangian and Eulerian results predict. The variance caused by finite sample size in a given size or location bin is taken into account for both velocity statistics and power spectra. The present expression for power spectral variance appears to under-predict the magnitude of the variance. There appears to be a much stronger dependence on mean sampling rate.

A model is developed for the dropout periods that occur during DMA transfer. The highest sidelobe level is -13 db for the worst case scenario. Additional sidelobes unique to the dropout phenomenon are of no consequence.

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Title: G. N. Hatsopoulous Associate Professor of Mechanical Engineering
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<tr>
<td>a</td>
<td>Particle radius</td>
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<td>(a_0)</td>
<td>Particle radius at time = 0</td>
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<td>(B(\tau))</td>
<td>Bartlett window function</td>
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<td>(C^2)</td>
<td>The ratio of transverse velocity variance to streamwise velocity variance</td>
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<td>(d_e)</td>
<td>Distance that a particle travels before evaporating</td>
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<td>(d_p)</td>
<td>Particle diameter</td>
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<td>(d_{p_{\text{max}}})</td>
<td>Maximum diameter of a particle that does not slip, defined in Eqn. 3.4</td>
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<td>(E_{uu}(f))</td>
<td>Gas power spectral density, defined as the Fourier transform of the gas velocity autocorrelation</td>
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<tr>
<td>(f)</td>
<td>Frequency of the power spectrum</td>
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<td>(f_c)</td>
<td>Cutoff frequency based on the average interarrival time</td>
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<td>(F(\kappa\eta))</td>
<td>Kolmogorov universal spectrum function</td>
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<td>Modified universal spectrum function, (\alpha F(\kappa\eta))</td>
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<td>(i)</td>
<td>Index used in summations</td>
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<td>(j)</td>
<td>(\sqrt{-1})</td>
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<td>(k)</td>
<td>The length of time between dropouts</td>
</tr>
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<td>(L)</td>
<td>Number of spectral estimates in a smoothed spectrum</td>
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\( L_1 \) Diffuser length (0.350 m)

\( L_2 \) Nozzle length (0.080 m)

\( N \) A number of points (in a velocity bin or range of bins)

\( n \) Number of points in a file (usually \( n = 20,000 \))

\( N_e \) The minimum number of fringes required by the processor to validate a realization

\( N_f \) The number of fringes in the probe volume

\( Q \) The ratio of \( N_e \) to \( N_f \)

\( R_e \) Jet Reynolds number, \( U_0 D_0 / \nu_g \)

\( R_{e_p} \) Particle Reynolds number, \( d_p \nu_k / \nu \)

\( R_\lambda \) Microscale Reynolds number, defined as \( u'^2 \lambda / \nu \)

\( t \) The \( t \)-statistic for mean velocity associated with limited sample size

\( t_e \) Time for a particle to evaporate

\( T \) Elapsed time for a single data record

\( U \) Gas-phase mean velocity

\( U_0 \) Nozzle exit velocity of the jet

\( u' \) Gas-phase rms velocity, defined as the root-mean-square of the fluctuating component of the gas-phase velocity

\( U(t) \) The instantaneous three-dimensional vector gas-phase velocity at time \( t \), includes both mean and fluctuating components, \( U(t) = U + u(t) \)

\( u(t) \) The instantaneous three-dimensional fluctuating vector gas-phase velocity at time \( t \), \( u(t) = U(t) - U \)

\( V \) Particle mean velocity

\( V(t) \) The instantaneous total particle velocity at time \( t \), includes both mean and fluctuating components, \( V(t) = V + v(t) \)

\( v(t) \) The instantaneous fluctuating particle velocity at time \( t \), \( v(t) = V(t) - V \)
$V(t)$ The instantaneous three-dimensional vector particle velocity at time $t$, includes both mean and fluctuating components, $V(t) = V + v(t)$

$v(t)$ The instantaneous three-dimensional fluctuating vector particle velocity at time $t$, $v(t) = V(t) - V$

$V_i$ The instantaneous velocity of the $i$-th particle

$V_{\text{biased}}$ The biased particle mean velocity

$V_{\text{true}}$ The unbiased particle mean velocity

$v_K$ Kolmogorov velocity, $(v_k \varepsilon)^{1/4}$

$v'$ Particle rms velocity, defined as the root-mean-square of the fluctuating component of the particle velocity

$v'_{\text{biased}}$ The biased rms velocity

$v'_{\text{true}}$ The unbiased rms velocity

$w(f)$ Weighting function for smoothed spectral estimates

$x$ Axial distance from the nozzle

$z$ Number of smoothed points per decade (here, $z = 10$)

$Z(\Phi)$ Poisson distribution for $\Phi$
Greek Letters

\( \alpha \) Pao's (1965) constant for the energy spectrum
\( \chi^2 \) The chi-squared-statistic for rms velocity associated with limited sample size
\( \chi \) The scalar dissipation rate
\( \Delta t_i \) Interarrival time between the i-th and (i-1)-th particle
\( \Delta t_{avg} \) The average interarrival time
\( \varepsilon \) Turbulent kinetic energy dissipation rate
\( \Phi \) Poisson distribution parameter
\( \eta \) Kolmogorov length scale, \((v_s^3/\varepsilon)^{1/4}\)
\( \kappa \) Wavenumber, \(2\pi f/U\)
\( \lambda \) Sampling rate
\( \lambda_g \) The transverse dissipation length scale
\( \mu_0 \) Median droplet radius at \( t = 0 \)
\( \rho_p \) Particle density (996 kg/m\(^3\) at 300 K)
\( \rho_g \) Gas density (1.183 kg/m\(^3\) at 300 K)
\( \sigma_0 \) Variance of the log-normal distribution at \( t = 0 \)
\( \theta \) The length of time of a dropout
\( \tau \) Kolmogorov time scale, \((v_s/\varepsilon)^{1/2}\)
\( \tau_K \) Particle relaxation time, \(d_p^2 \rho_p/18 \mu_g\)
\( \tau_{max} \) Maximum correlation time of the discrete autocorrelation
\( \mu_g \) Dynamic viscosity of the gas phase (1.853\times10^{-5} at 300 K)
\( \nu_g \) Gas-phase kinematic viscosity (1.566\times10^{-5} at 300 K)
\( \Omega \) Poisson distribution parameter
\( \psi \) The number of dropouts during time \( T \)
\( \xi \) Logarithmic smoothing width
Chapter 1

Introduction

This study focuses on the small-scale physical processes which contribute to the transport of liquid and solid aerosols in a turbulent gas phase. Although the turbulence of the gas phase is fairly well-understood, little is known about the effects of turbulence on the motion of aerosols contained within the gas phase. It is of interest to learn more about this phenomenon, as this may aid our understanding of several large-scale processes such as atmospheric precipitation, air pollution, filtering, deposition, and fuel sprays, to name a few.

Particle inertia produces amplitude and phase differences between a particle's velocity and the velocity of a surrounding fluid when motion is unsteady. Theoretical treatments of such particle lag are classical for the low Reynolds numbers characteristic of aerosols (Fuchs, 1989), although refinement of the theory is ongoing (Maxey and Riley, 1983). Several authors have extended the theory to predict the Lagrangian velocity statistics and spectra of small particles in a turbulent flow (Hjelmfelt and Mockros, 1966; Levich, 1962; Ounis and Ahmadi, 1990). In general, all such theories predict a growing attenuation of a particle's motion as its inertia increases relative to the characteristic
frequencies of the flow.

However, experimental confirmation of the Lagrangian predictions are scarce, and still less is known, either theoretically or experimentally, about the Eulerian statistics and spectra of inertial particles. Modern instrumentation enables direct measurement of instantaneous particle velocity and diameter and so facilitates direct exploration of inertial influences on the motion of particles in a turbulent flow. The objective of the present research is to make such an investigation of the Eulerian motion of small aerosol particles.

Since a particle's behavior is determined by its unsteady motion relative to the surrounding turbulent air, we introduce an important distinction between inertial and sub-inertial particles. The former possess such significant mass that they are not able to follow either the linear accelerations or the curvilinear paths of the turbulent eddies; inertial forces overcome the viscous restraining forces of the medium and thus a particle will slip off of the fluid streamline. The sub-inertial particles, however, possess such small mass that they closely follow fluid streamlines and thus have negligible slip. Quantitative restrictions on size are developed in Chapter 3.

In this study, we seek to determine how a particle's inertia affects its Eulerian velocity statistics and turbulent power spectra relative to those of a well-defined and fixed gas-phase turbulence. The gas phase is identified with the behavior of the sub-inertial particles. The remaining, inertial particles are grouped into size bins of constant width. Their behavior is then compared to that of the sub-inertial particles; quantitative differences are identified and
qualitative explanations are provided. The aerosol phase here is water; the size range considered is between one and forty microns. Specifically, studies concentrate on finding how the behavior of very small particles compares to that of larger, more massive particles; an axisymmetric air jet provides the vehicle for the turbulence.

Historically, the dearth of work on the behavior of small particles in turbulent flows has been due, in part, to the absence of instruments that can effectively analyze these small particles. Particle size analyzers, such as the Sinclair-Phoenix (described in Jorgensen, 1983), have been used for quite some time. These optical systems, however, internally classify the size distributions of a provided sample; they are not suitable for in-situ measurements. Likewise, laser doppler velocimeters (LDV's) effectively measure the velocities of small particles, but they do not normally infer their sizes. Recently, however, researchers have modified LDV's to simultaneously record velocity and particle diameter for individual particles (Bachalo and Houser, 1984). These instruments (called phase Doppler particle analyzers, or PDPA's) obtain this information from scattered light that emanates from a particle passing through two intersecting laser beams; the information is stored for use in post-processing routines. This allows one to study not only the moments of velocity and turbulent power spectra, but also the arrival time histograms and velocity-diameter cross-correlations. And with knowledge of the probe volume and its cross-sectional area, the instrument can provide information on number density and mass flux.

Another important difference between this work and some of the earlier studies is due to the limitations of seeding and processing that are unique to
LDV's and PDPA's. Unlike hot-wire anemometers which provide a continuous velocity signal, PDPA's and LDV's use scattered light from individual particles to infer velocity. Thus, data collection is a discrete process and continuous records are not possible. When collecting these discrete samples (or, alternatively, individual realizations), faster particles are sampled more often than slower ones. This leads to velocity biasing, a concept examined in more detail in Chapter 3. Furthermore, the randomly-arriving character of the data requires different approaches to data reduction. Whereas ensemble averages are, for the most part, adequate for mean and rms velocity statistics, conventional Fourier transform routines must be carefully modified if one is to obtain meaningful power spectra. When power spectra information is coupled with information about the diameter of the particles, we are able to make a broad analysis of the behavior of inertial aerosols in turbulence, as provided by an axisymmetric air jet.

Generally, this work explores the velocity statistics and power spectra of gas and particle flow fields along a jet centerline. The third and fourth chapters deal with the data collection procedure and results of the experiments; the following chapter deals with the experimental apparatus that allowed the collection of the data. The final chapter provides a conclusion to this work; an attempt is made to fit the results into a larger logical framework and recommendations are made for the direction of future research.
Chapter 2

Experimental Apparatus

An axisymmetric, free turbulent air jet provides a well-defined gas-phase turbulent flow field. The air jet is seeded with a polydisperse water aerosol provided by a high pressure commercial sprayer, and mass loadings of aerosol are kept low enough to avoid altering the gas-phase flow. The particulate flow field is then mapped at six downstream stations in the jet using a laser doppler system. In this way, particulate motion can be examined in a known turbulent field.

2.1 The PDPA Operating Characteristics

2.1.1 The Optical System

The aerosol size and velocity distributions are measured using a Phase Doppler Particle Analyzer (PDPA), manufactured by Aerometrics, Inc. The PDPA system functionally consists of an optical system and a signal processing system. The optical system has a 200mm focal length output lens, a 160mm collimating lens, and a 500mm receiving lens. The beam intersection volume, modelled as an ellipsoid of parallel fringes, has a major diameter of 100 μm with a fringe spacing of 5 μm.
Each particle crossing the probe volume produces a doppler burst which is detected by photomultiplier tubes (PMT's) within the receiving optics. The software tags this as an attempt. Particle velocity is inferred from the product of doppler frequency and fringe spacing. The phase difference between doppler bursts from two PMT's is proportional to particle diameter. In order to measure the particle diameter, at least two PMT's are needed; our system uses three PMT's which provide two additional redundant measurements of phase and velocity. If phase and frequency information from the PMT's do not agree, the information is discarded and the software tags this as a rejection. If, however, redundant information between PMT's agree, then the software tags this as a validation, and the information is passed on to a buffer. The validation rate (defined as the ratio of validations to attempts) and its complement the rejection rate (defined as the ratio of rejections to attempts) are of importance later on this paper. Lastly, it is important to note that the scattered light intensity from a particle is proportional to the square of the particle's diameter. The 35:1 ratio of the maximum and minimum particle size is due to the 1200:1 ratio (61 dB) of maximum signal to noise ratio of the PMT's and the electronics. This phenomenon also contributes to the validation rate.

2.1.2 The Data Acquisition System

The PMT signals are fed to the signal processing system, digitized, and then put into a DMA (direct memory access) buffer. The signal processing system is several hardware boards and software programs contained within an IBM-compatible 80286 machine running at 12 MHz with a math coprocessor. The DMA buffer is 1.2 MB of extended memory. Each sample uses 16 bytes of buffer
space; the 16k buffer is filled after 1024 realizations. There is approximately a 25 ms wait for the buffer to be emptied, during which no data can be taken; however, the influence of this on the signal windowing is shown to be negligible in Chapter 3. The buffer is saved in binary format to mass storage devices, either a 20 MB hard disk or a 44 MB Bernoulli Box/Cartridge. Because each record contains over 2 MB of data (including ASCII files for valid samples and information for video graphics display), the records were usually stored on the Bernoulli Cartridges.

2.2 The Jet Rig

2.2.1 Aerosol Generators and Seeding

An essential element of the jet is the apparatus to introduce aerosols into the flow. The literature search yielded several possibilities for aerosol generation. These included a Berglund-Liu generator (1973); the apparatus of Schneider and Hendricks (1964); the generator of Lin, Eversole, and Campillo (1989); as well as the generator of Durst and Umhauer (1975). The Berglund-Liu generator produces a monodisperse aerosol in a fine stream; however, the particle density would be several orders of magnitude less than that desired for our studies. Furthermore, production of subinertial particles (less than 10 μm diameter) would require an orifice size of similar diameter. A geometry such as this is prone to clogging, even with the constant flow rate modification described by Berglund and Liu. The generator of Lin, Eversole, and Campillo as well as that of Schneider and Hendricks are even more susceptible to clogging. Not to be neglected is the fact that exceedingly close mechanical tolerances are required in the assembly of several parts of these devices.
The Durst-Umhauer generator (technically a compressed-air nebulizer) is a simple unit that utilizes the venturi principle: a high velocity jet flowing over a liquid will create a local vacuum and draw the liquid into the flow and atomize it into a polydisperse spray. Several generators of this type were characterized in a study, concurrent to this one, by Yoon (1991). The generator used in this experiment is similar in principle to these. However, rather than using the relatively low-pressure difference caused by the local vacuum, a high-pressure (40 to 60 psi) water jet is forced into the air flow. Through the action of strong viscous shear, the liquid is atomized and propelled forward. This commercial sprayer (made by Vortec, Inc., Model 1713) produces a dense polydisperse aerosol, approximately 0.3% of which is water; it assumed that this light loading does not significantly influence the gas phase.

2.2.2 Turbulence Management
To obtain an independently variable gas flow, the sprayer is contained within a much larger unit that creates a steady, controllable, and reproducible axisymmetric air jet. Furthermore, this arrangement acts to reduce a unique problem of high-pressure sprayers. In general, such nebulizers and sprayers atomize a liquid jet and provide a very strong velocity gradient that accelerates the atomized particles. Soon thereafter, the small particles decelerate while the larger particles maintain their velocity. The result is often an order of magnitude difference in velocity between the largest and smallest particles created by the sprayer. The present arrangement greatly
reduces this effect. A schematic drawing of the jet apparatus is shown in Fig. 1a.

The sprayer is located concentrically within a long 100 mm diameter tube to allow large particles a distance to slow down to the local gas velocity. The aerosol also passes through a narrow 50 mm section \( D_2 \) to facilitate turbulent mixing, after which it passes through a diffuser. The diffuser was designed to avoid stall following guidelines discussed by Blevins (1984). Essentially, to expand the flow from 50 mm \( (D_2) \) to 100 mm \( (D_1) \), a minimum distance of \( 7D_2 \) is required; thus, diffuser length \( (L_1) \) was about 350 mm through a half-angle of about 8°.

The flow then passes through a 250 mm-long turbulence calming section, which is of constant diameter and equal to 100 mm \( (D_1) \). Additional air from two high-static, multistage centrifugal blowers (made by Fasco, Inc., providing up to 7 cfm at 8 inH\(_2\)O) is added upstream of the sprayer to provide a high gas mass flux through an aerodynamic nozzle at the end. Also, honeycomb flow-straighteners reduce the turbulence level of this flow upstream of the aerosol generator. A nozzle contraction ratio \( (D_1/D_0) \) of 64 creates a maximum exit velocity of 52 m/s through a 13 mm diameter outlet \( (D_0) \). The nozzle strongly damps the turbulence that exists inside of the jet rig. The design of this nozzle conforms to specifications given by Morel (1975). Essentially, for \( D_1/D_0 \approx 4 \), the ratio of nozzle length \( (L_2) \) to \( D_1 \) should be near 0.75; a value of 0.8 was used.

The profile of the nozzle was numerically generated with two matched cubics. The nozzle itself was cut from a 125 mm diameter bar of high density polyethylene on an NC lathe to a tolerance of 250 \( \mu \)m. Note, in Fig. 1a, the existence of drainage containers. These collect the large quantities of water
that settle in the low points of the system.

Most of the jet apparatus upstream of the nozzle is designed with gas-phase turbulence reduction in mind. However, this design parameter inhibits the production of an effective seeding aerosol. The ideal aerosol would have a flat size histogram with equal densities of all particle sizes over the entire size range of interest (here, 1.4 μm to 50 μm). The narrow 50 mm section that induces strong turbulent mixing also produces strong local accelerations which propel larger particles into the surrounding walls; the severe contraction ratio of the nozzle, which reduces the turbulence intensity of the jet at the nozzle, also causes impaction of large particles. The result is that massive particles are preferentially removed from the flow and the size histogram is weighted towards the very lightest (sub-inertial) particles; this particle attrition process leads to a log-normal distribution of particle size, as is often the case (Jorgensen, 1983). These phenomena also reduce the density of the aerosol, yet fairly dense aerosols are required for the accurate realization of power spectra. The present design represents a compromise between the two competing factors of low turbulence intensity at the outlet and high particle density. Appendix C contains some data for the size distribution downstream of the nozzle.

2.3 The Traversing Platform

Mapping of the velocity field of the jet is provided by a two-axis laser traversing mechanism, shown in Fig. 1b. The transmitting and receiving optics are mounted on a 2' × 4' Newport optical table. The table sits inside of a
cradle which in turn is attached to two Thompson rail bearings. The optics/table/cradle assembly moves horizontally across a platform; six feet of horizontal travel is achieved. The platform is mounted on rotating frame which is supported and controlled through the action of a pneumatic air cylinder; approximately 18 inches of vertical motion is possible.
Chapter 3

Procedure

The experiments were organized into three functional groupings: mean and rms velocities, power spectra, and number density. In the discussion that follows, experimental procedures are the focus as well as the limitations that are placed on the data collection procedures and the assumptions that justify the validity of what is collected. The first section (mean and rms velocities) discusses particle slippage, velocity biasing, and the effects of limited sample size; the second section (power spectra calculations) covers limited sample size as well as the effects of windowing dropouts. In each circumstance, knowledge of well-documented errors discussed in the literature are combined with ad hoc assumptions to yield valid and reasoned procedures for the collection -- and reduction -- of data.

3.1 Mean and rms Velocities

3.1.1 Particle Slip

This section deals primarily with the comparison of the particle flow field to the gas flow field and the uncertainties that arise when one attempts to characterize random arrivals and turbulent particle motion. It is the aim of this research to identify inertially-induced trends that are more significant
than the errors caused by velocity bias, sizing uncertainty, and limited sample size.

The notation used throughout this paper is as follows: U is the gas-phase mean velocity; \( u' \) is the gas-phase fluctuating RMS velocity; V is the particle mean velocity; and \( v' \) is the particle RMS fluctuating velocity. The gas flow field must be characterized first so that experimental results can be compared to those in the literature. However, the PDPA measures particle velocities only; gas velocities are usually inferred from the velocities of the smallest particles in the flow. This assumes that these smallest particles follow the flow perfectly, from the largest down to the smallest scales of motion. Given our focus on inertial effects, it is necessary to explore the conditions under which this assumption is correct.

Two competing time scales are present: that of the particles and that of the gas. For a small liquid particle moving at low Reynolds number in air, the time of response to changes in the flow is characterized by the Stokes' Law particle relaxation time defined as 
\[ \tau = \frac{d_p^2 \rho_p}{18 \mu_g} \]
where \( d_p \) is the particle diameter, \( \rho_p \) is the particle density, and \( \mu_g \) is the dynamic viscosity of the gas phase (Fuchs, 1989; Maxey and Riley, 1983).

The fastest time scale of the turbulence is the Kolmogorov time scale, 
\[ \tau_K = \left( \frac{v_g}{\varepsilon} \right)^{1/2} \]
in which \( v_g \) is the gas kinematic viscosity and \( \varepsilon \) is the local turbulent kinetic energy dissipation rate of the gas in the jet. This time scale characterizes the small eddies in which the particles are embedded. We may estimate the streamwise variation of \( \tau_K \) in an axisymmetric jet using the correlation of Friehe, Van Atta, and Gibson (1971):
\[
\frac{D_0}{U_0^3} = 48(x/D_0)^{-4}
\] (3.1)

for \(x/D_0\) between 20 and 70 (this study is considers the region from 15 to 65).

This result applies for jet Reynolds numbers \(Re\) from 10,000 to 500,000, where \(Re = U_0 D_0 / \nu_g\), \(U_0\) is the mean velocity of the gas at the exit plane of the nozzle, and \(D_0\) is the diameter of the nozzle at the exit plane. Gas-phase and particle-phase properties at 300 K are used. A temperature error of 5°C results in a 3% error in the Reynolds number. Combining these relations, we find:

\[
\frac{\tau}{\tau_K} = 0.38 \left( \frac{X}{D_0} \right)^{-2} \left( \frac{d_p}{D_0} \right)^2 \left( \frac{\rho_p}{\rho_g} \right) \] (3.2)

A general criterion for no slip between the particles and the gas is that:

\[
\frac{\tau}{\tau_K} \ll 1
\] (3.3)

This was quantitatively interpreted as \(\tau/\tau_K \leq 0.01\); the extent to which this rather stringent criterion may be relaxed is discussed in Chapters 4 and 5. On this basis, the maximum sub-inertial particle diameter (which satisfies Eqn. 3.3) is found to be:

\[
d_{p, \text{max}} = 0.16 D_0 \left( \frac{X}{D_0} \right)^{1/2} \left( \frac{\rho_p}{\rho_g} \right)^{1/4} \] (3.4)

valid for \(x/D_0\) between 20 and 70. This is a very important equation which is referred to many times throughout the rest of this paper; it establishes a
criterion upon which gas-phase properties can be established. In this experiment, trials were run at three values of Re: 26,000, 33,000, and 43,000; the smallest measurable particles are 1.4 μm in diameter. Yet even at the lowest Reynolds number the particles do not accurately track the gas-phase flow until \( x/D_0 > 40 \). It might be imagined that the flow can be thoroughly characterized by using particles much smaller than those currently used. However, the degree of uncertainty in the particle size determination increases with decreasing diameter. This is due to certain geometrical features of the optical configuration and particle sizing technique. Sankar et al. (1988) indicate an error of up to \( \pm 1.0 \) μm in the diameter determination for particles less than 10 μm. Thus, there is some uncertainty in determining the gas-phase velocity statistics within 40 diameters of the nozzle.

A related consideration is particle size. The largest particles considered herein \( (d_p = 50 \) μm) have diameters less than or equal to the Kolmogorov length scale of the turbulence, viz. \( d_p \leq \eta \), where \( \eta = (\nu^3/\epsilon)^{1/4} \). The smallest particles \( (d_p = 1.4 \) μm) have \( d_p/\eta \leq 0.03 \) (for \( x/D_0 \geq 25 \)). Thus, while the largest particles may tend to penetrate the small eddies, the smallest particles are fully entrained by them. It follows that the drag on small particles is well-characterized by Stokes' Law, while for the larger particles, many other effects are present in the equation of motion (Maxey and Riley, 1983). However, if a particle Reynolds number is defined as \( \text{Re}_p = d_p v_K/v_g \), for \( v_K = (\nu^3/\epsilon)^{1/4} \) (the Kolmogorov velocity), then \( \text{Re}_p = d_p/\eta \leq 1 \) for all particles.

3.1.2 Velocity Biasing

In interpreting the velocity measurements, another issue which must be
addressed is that of velocity biasing, owing to the tendency for faster particles to be observed more frequently. Most investigators maintain that velocity bias errors are present to some degree. But, while some have claimed that biasing errors can be easily and routinely corrected for, others have found that evidence of biasing is inconclusive, contradictory, or non-existent.

Consider a probe volume at a fixed point in space that records $N$ particles, each crossing the probe volume with velocity $V_i$. Basically, faster moving particles are sampled more often than slower ones; a simple ensemble average of velocity realizations:

$$V_{\text{biased}} = \frac{1}{N} \sum_{i=1}^{N} V_i$$  \hspace{1cm} (3.5)

results in a biased average which is slightly higher than actual. Obviously, a steady laminar flow would not exhibit biasing; all the seeds passing through a given point will have the same velocity. Thus, one is led to believe that turbulence intensity is one of the parameters causing bias; indeed, Edwards (1979) indicated a correction of the form:

$$\frac{V_{\text{biased}}}{V_{\text{true}}} = 1 + \left(\frac{v'}{V_{\text{true}}}\right)^2$$  \hspace{1cm} (3.6)

where $v' = v_{\text{biased}} = v'_{\text{true}}$. These rms velocities are all equal because velocity biasing skews the velocity probability distribution so that only the median ($V$) is shifted; the standard deviation ($v'$) remains essentially unchanged. Edwards' estimate indicates a bias of $6\%$ along the jet centerline, where turbulence intensity is approximately $25\%$. More accurate corrections have been proposed. The original work of McLaughlin and Tiederman (1973) introduced
another factor, $C^2$, which is defined as:

$$C^2 = \frac{\text{transverse velocity variance}}{\text{streamwise velocity variance}} \quad (3.7)$$

Only in the case of purely one-dimensional "turbulence" does the transverse velocity variance vanish. However, this is not the case along a jet centerline in the self-similar region; Chevray and Tutu (1978) indicate a value of approximately 0.6 for $C^2$.

Other authors have identified additional parameters in evaluating velocity bias. Buchhave and George require evaluation of $Q$ for each realization, defined as:

$$Q = \frac{N_e}{N_f} \quad (3.8)$$

where $N_e$ is the minimum number of fringes required by the processor to validate a realization and $N_f$ is the number of fringes in the probe volume. Further work by Buchhave, George, and Lumley (1979) show that the true mean velocity is given by:

$$V_{\text{true}} = \frac{\sum_{i=1}^{N} V_i \Delta t_i}{\sum_{i=1}^{N} \Delta t_i} \quad (3.9)$$

where $\Delta t_i$ is the residence time of the i-th particle in the probe volume. The true rms velocity is also weighted by the residence time:
Barnett and Bentley (1974) claimed that mean velocity was biased in proportion to particle density; Durao and Whitelaw (1975) suggested that biasing is eliminated by a compensating effect. Bogard and Tiederman (1979) refute both of these assertions. Later, Edwards (1981) showed that statistical bias is a function of probability density, but only when the detector samples uniformly rather than randomly. Buchhave (1979) found that the McLaughlin-Tiederman correction applied to a counter processor matched the output from a tracker processor but differed from a hot-wire anemometer output. Stevenson and Thompson (1982) point out that this is a surprising result given that the calibration and interpretation of a hot-wire output is rather straightforward and usually accepted as standard.

Giel and Barnett (1979) were not able to obtain definite evidence that the predicted bias exists. They studied a turbulent axisymmetric jet with an LDV; a hot-wire anemometer and a pitot tube/pressure transducer combination were used as standards. In general, the uncorrected velocity averages were closer than the corrected ones to the standard. In fact, in studying the jet centerline, they found that mean velocity and turbulence intensity showed no evidence of bias. They also make the point that a "correction" routinely applied to data that is not biased will result in significant errors. In the light of the evidence, no biasing corrections were made to the velocity data in the experiments.
3.1.3 The Effects of Limited Sample Size

Another factor in this experiment is that of limited sample size. The particle size histograms are distributed in a log-normal fashion, with a median diameter less than 10 μm. So at a given location, the majority of the flow field is small (<10 μm) particles. There are fewer particles associated with the larger particle sizes. Since the number of samples (n) is finite, there is uncertainty in the velocity statistics; this includes mean velocity, rms velocity, and the turbulent power spectrum of the velocity. For mean velocities, Student's t-Distribution is used (McCuen, 1985), and estimates of 95% confidence intervals are obtained. The confidence envelopes are defined as:

\[ V - t_{a/2} \left( \frac{v'}{\sqrt{N}} \right) < V_{\text{expected}} < V + t_{a/2} \left( \frac{v'}{\sqrt{N}} \right) \] (3.11)

Similarly, finite sample sizes for particle rms velocities require the use of a \( \chi^2 \)-distribution to obtain 95% confidence intervals. These intervals are defined as:

\[ \frac{(n-1)v'^2}{\chi^2_{a/2}} < v'^{2}_{\text{expected}} < \frac{(n-1)v'^2}{\chi^2_{1-a/2}} \] (3.12)

3.1.4 Nondimensional Velocity Variance vs. the Nondimensional Relaxation Time Ratio

It is of interest to predict the form of \( v'^2 / u'^2 \) as a function of \( \tau / \tau_K \). The derivation is as follows. Given that \( V(t) \) is the particle instantaneous three-dimensional vector velocity at time \( t \), and given that \( U(t) \) is the gas-phase instantaneous three-dimensional vector velocity at time \( t \), we can write the
Lagrangian equation of particle motion as:

\[
\frac{dV(t)}{dt} = \frac{1}{\tau} [U(t) - V(t)]
\]  

(3.13)

The inertial attenuation of the Lagrangian power spectrum is (Hjelmfelt and Mockros, 1966):

\[
|H(f)|^2 = \frac{1}{1 + (2\pi f \tau)^2}
\]  

(3.14)

This form predicts strong attenuation of high frequencies and weak attenuation of low frequencies. For an oscillating Stokes flow at a single frequency, additional analysis shows:

\[
\frac{v^2}{u^2} = \frac{1}{1 + (2\pi f \tau)^2}
\]  

(3.15)

If the frequency of oscillation is taken as the frequency of the variance-producing scales, roughly:

\[
f_v = \frac{3.7}{R_h \tau_K}
\]  

(3.16)

then:

\[
\frac{v^2}{u^2} = \frac{1}{1 + \left(\frac{7.4\pi \tau}{R_h \tau_K}\right)^2}
\]  

(3.17)

A different approach is to start with the Eulerian equation of motion:
\[ \frac{\partial V(t)}{\partial t} + V(t) \cdot \nabla V(t) = \frac{1}{\tau} [U(t) - V(t)] \]  

(3.18)

The characteristic scales of the flow seen by the small scales are the Kolmogorov time scale, \( \tau_K \), and the Kolmogorov fluctuating velocity scale, \( v_K \). Assuming uniform mean flow, we may subtract the mean velocities \( U \) and \( V \) and expand the fluctuating particle velocity for small inertia as:

\[ v(t) = u(t) + c(t) \left( \frac{\tau}{\tau_K} \right) + O \left( \frac{\tau}{\tau_K} \right)^2 \]  

(3.19)

to find (Lienhard, 1991):

\[ \frac{c(t)}{v_K} = - \frac{\tau_K}{v_K} \left[ \frac{\partial U(t)}{\partial t} + U(t) \cdot \nabla U(t) \right] \]  

(3.20)

and so, for spatially homogeneous turbulence:

\[ v^2 = u^2 - \frac{2}{3} \tau_K \left( \frac{\tau}{\tau_K} \right) u(t) \cdot \frac{\partial U(t)}{\partial t} + O \left( \frac{\tau}{\tau_K} \right)^2 \]  

(3.21)

If the turbulence is homogeneous at the small scales, the velocity/acceleration correlation may be shown to equal \(-\varepsilon\), and we have:

\[ \frac{\nu^2}{u^2} = 1 + \frac{2}{3} \left( \frac{\varepsilon \tau_K}{u^2} \right) \left( \frac{\tau}{\tau_K} \right) + O \left( \frac{\tau}{\tau_K} \right)^2 \]  

(3.22a)

or:

\[ \frac{\nu^2}{u^2} = 1 + \frac{2}{3} \left( \frac{v_K^2}{u^2} \right) \left( \frac{\tau}{\tau_K} \right) + O \left( \frac{\tau}{\tau_K} \right)^2 \]  

(3.22b)
In the corresponding Lagrangian calculation, \( c(t) = -dU(t)/dt \) and the time average of \( u(t) \cdot (dU(t)/dt) = 0 \); inertial effects are second order in \( \tau/\tau_K \). Using the correlation for \( \varepsilon \) (Eqn. 3.1), we find:

\[
\frac{v_K^2}{u^2} = \frac{1}{R_\kappa} \left( 34.9 \frac{D_0}{x} \right) \quad (3.23)
\]

Therefore:

\[
\frac{v^2}{u^2} = 1 + 23.3 \frac{D_0}{R_\kappa} \left( \frac{\tau}{\tau_K} \right) + O \left( \frac{\tau}{\tau_K} \right)^2 \quad (3.24)
\]

### 3.2 Power Spectra

#### 3.2.1 The Slotting Technique

An important consideration in this experiment is the limitations imposed by random sampling. The data collected by the PDPA are referred to as "randomly-sampled" because the time differences between the occurrence of samples (the interarrival times, \( \Delta t_i \)) are basically Poisson-distributed. Fig. 11a illustrates a typical interarrival time histogram. Random sampling precludes the conventional process of obtaining the power spectra from the FFT algorithm which relies on uniformly-sampled data where the interarrival time is constant.

Saxena (1985) examined several methods used to obtain power spectra from
random data. The data can be made continuous by linear interpolation, then re-sampled to obtain uniform data; however, this technique is error-prone. Rather than ramping the time history between data points, random data can be re-sampled with a "sample and hold" technique, which creates a step-like time history (Adrian and Yao, 1985). However, this is likewise error-prone. Leneman and Lewis (1966) have shown that frequency resolution using these methods is limited to about one-third of \( f_c \), where:

\[
f_c = \frac{1}{2 \Delta t_{\text{avg}}}
\]  

(3.25a)

and:

\[
\Delta t_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} \Delta t_i
\]  

(3.25b)

The "slotting" technique of Gaster and Roberts (1975), which has smaller errors than these other methods, is therefore used.

Briefly, the slotting technique calculates a power spectrum from the discrete velocity autocorrelation. The specifics are as follows. The data records contain an arbitrary number of points (usually between 1000 and 10,000 points). This is in contrast to the uniformly-sampled case in which the number of points is a power of two. The data records then have the mean removed to avoid leakage from the spike at zero frequency. An autocorrelation is formed, with \( \tau \) as the lag time:

\[
R(\tau) = \frac{1}{N} \sum_{i=1}^{N} v(t) \cdot v(t + \tau)
\]  

(3.26)
where \( v(t) \) is the instantaneous fluctuating velocity. The maximum correlation time \( (\tau_{\text{max}}) \) is 0.05 seconds. The lag time axis is divided into 1024 bins, with equal width of 49 \( \mu \)s. The spectrum is found by applying the FFT algorithm with a triangular Bartlett window \( B(\tau) \) to the autocorrelation:

\[
E_{vv}(f) = 2 \int_0^{+\infty} B(\tau) R(\tau) e^{2\pi j f \tau} d\tau
\]

(3.27)

where \( B(0) = 1 \) and \( B(\tau_{\text{max}}) = 0 \). This is the one-sided power spectral density. The computer codes based on this algorithm are contained in Appendix E.

### 3.2.2 Spectral Variance and Logarithmic Smoothing

Logarithmic smoothing, as defined by Helland and Rosenblatt (1979), is used to reduce the variance of the spectral estimate and provide a more meaningful distribution of data that is always displayed on a logarithmic axis. A smoothed spectrum is defined as:

\[
E_{uu}^{\text{smoothed}}(f) = w(f) \sum_{f_1 < f < f_2} E_{uu}^{\text{raw}}(f)
\]

(3.28)

where:

\[
w(f) = \frac{\Delta f (\xi + 1)}{2f (\xi - 1)}
\]

(3.29a)

and:

\[
\xi = 10^{1/n}
\]

(3.29b)
where \( n \) is the number of smoothed points per decade (a value of 10 is used).

In this experiment:

\[
\Delta f = \frac{1}{2 \tau_{\text{max}}} = 10 \text{ Hz} \quad (3.29c)
\]

Smoothing is based on the statistical independence of spectral amplitudes of adjacent frequency components from the discrete Fourier transform. Helland and Rosenblatt show that smoothing and averaging reduce the variance of the spectral estimate. At a given location and Reynolds number, 15 time-series records are collected. Fifteen separate spectral estimates \((L = 15)\) are averaged to yield a single spectrum. The variance is correspondingly reduced by \(1/L\). Logarithmic smoothing further reduces the variance by averaging neighboring values. The resulting expression for spectral variance is:

\[
\text{var} \ E_{vv}(f) = \frac{1}{4 \ \tau_{\text{max}} \ L \ f} \left( \frac{\xi+1}{\xi-1} \right) E_{vv}(f) \quad (3.30)
\]

Mayo (1978), however, indicates that the slotting estimator has an intrinsic variance given by:

\[
\text{var} \ E_{vv}^2(f) = \frac{2}{3} \frac{\tau_{\text{max}}}{T} \left( E_{vw}(f) + \frac{v^2}{\lambda} \right)^2 \quad (3.31)
\]

where \( \lambda \), the average sampling frequency, is defined as \( n/T \). \( T \) is the total length of time of the record and \( n \) is the number of points in the sample (typically, \( n = 20,000 \)). The \( E_{vv}(f) \)-term establishes the low-frequency variance limit, and the \( v^2/\lambda \)-term establishes the high-frequency variance limit. This first term is necessary because increasingly high sample rates will
not reduce low frequency variance. A correct estimator for variance must include the effects described in both Eqns. 3.30 and 3.31. Thus, the resulting expression for total variance is:

\[
\text{var } E_{vv}(f) = \frac{1}{6 T L f} \left( E_{vv}(f) + \frac{\nu^2}{\lambda} \right) \left( \frac{\xi+1}{\xi-1} \right) 
\]  

(3.32)

A final consideration is in order. Although perfect Poisson sampling would give infinite frequency resolution (due to the infinitesimal time differences between some of the points) the actual instrument sampling rate is approximately limited to a 100 μs time gap or a 10,000 Hz limit. This corresponds to a "pseudo-Nyquist" frequency limit of 5000 Hz or so. Technically speaking, this is not a Nyquist limit and there is no Nyquist folding phenomenon. However, one cannot realistically obtain information on frequencies higher than the inverse of this 100 μs time gap.

3.2.3 Windowing and Dropouts

The PDPA system contains a 16k data buffer which is filled after 1024 samples have been collected. A peculiarity of the PDPA system is that there is a brief pause (25 ms duration) during which no data is taken; the buffer is emptied and prepared for more data. At the end of this pause, or "dropout," the buffer begins taking more data. This is functionally equivalent to windowing the data with a series of square windows, each window separated by a 25 ms dropout. In this section, this windowing phenomenon is modelled for a worst-case scenario so that its effect on the data can be determined. It is compared to the ideal situation of no dropouts.
Consider a velocity time-series $h(t)$ that is windowed two ways. The first window (shown in Fig. 2a) of time length $T$ has no dropouts; it is known (see Bendat and Piersol (1971), for example) that the square of the Fourier transform of this uncorrupted square window is:

$$|H_1(f)|^2 = \left(\frac{\sin \pi f T}{\pi f}\right)^2$$

(3.33)

where the subscript 1 denotes the no-dropout case. The second window contains dropouts and is characterized by several parameters as shown in Fig. 2b. The number of dropouts during the time $T$ is denoted by $\psi$. The dropout time is denoted by $\theta$ and is constant for a given processor (here, $\theta = 25$ ms). The parameter $k$ is defined as $T/\psi$ (valid for $k \gg \theta$); this is the width of the time window between dropouts. This second window is denoted by the subscript 2 and it is found that:

$$|H_2(f)|^2 = \left(\frac{1}{2\pi f}\right)^2 \left(\frac{2}{\sin \pi f (k+\theta)}\right)^2 \left[ 2\sin^2 \pi f \psi (k+\theta) + 2\sin^2 \pi f k - \sin^2 \pi f (k+\psi (k+\theta)) - \sin^2 \pi f (k-\psi (k+\theta)) \right]$$

(3.34)

The algebra is worked out in Appendix A. In the worst case scenario, the validation rate is near unity, data density and sampling rate are high, and the dropout duration is much greater than the average interarrival time:

$$\theta > \frac{1}{\lambda}$$

(3.35)

The highest sampling rate is about 1000 Hz. Thus, the buffer fills up with
about 1000 valid samples \((k = 1000 \text{ ms})\), following which there is a 25 ms dropout period \((\theta = 25 \text{ ms})\). Since \(T = \psi k\), the dropout phenomenon is a function of \(\psi\) only. Figs. 2c, 2d, and 2e compare the windows with and without dropouts for \(\psi = 3, 10,\) and 85, respectively. The PDPA has a DMA (direct memory access) buffer for 85,000 samples; \(\psi = 85\) corresponds to this maximum. It can be seen that there is very little error caused by dropouts for this worst-case scenario.

It is interesting to note that the major effect of dropouts is to cause the appearance of a secondary spectrum very far from the first primary lobe. In Fig. 2f, for \(\psi = 10\), additional harmonics begin to appear after the sidelobes of the uncorrupted window have decayed to a negligible level. In Fig. 2g, for \(\psi = 100\), these harmonics are very pronounced and appear to define a secondary spectrum of their own. Define the highest sidelobe level \((\text{HSL})\) as the ratio of the highest amplitude of a sidelobe of \(H_2(f)\) to \(H_1(0)\):

\[
\text{HSL} = 20 \log \left( \frac{H_2(f)_{\text{max}}^{\text{sidelobe}}}{H_1(0)} \right)
\]

\((3.36)\)

It is found that \(\text{HSL}\) is approximately -13 db for both cases. In general, dropouts caused by dead time have little effect on the validity of the spectral estimate.
Chapter 4

Results

4.1 Mean and rms Velocities

This section deals primarily with particle-phase and gas-phase velocities as a function of downstream distance \((x/D_0)\) and as a function of particle diameter \((d_p)\). We consider first the evolution of particle mean and rms velocities and then turn our attention to inertially-induced differences between gas-phase and particle-phase velocity statistics and spectra.

4.1.1 Velocities as a Function of Downstream Distance

It is necessary to determine the gas-phase velocity for comparison to the classical results. Since the minimum particle size is 1.4 \(\mu m\), Eqn. 3.4 establishes a criterion for distance beyond which gas-phase behavior can be assumed. Fig. 3a shows the mean velocity of 1.4 \(\mu m\) particles along the jet centerline as a function of downstream distance for the three Reynolds numbers. The 1.4 \(\mu m\) particles can be said to represent the gas-phase only for
the following ranges:

\[ \frac{x}{D_0} > 40 \quad \text{for} \quad Re = 26,000 \quad (4.1a) \]
\[ \frac{x}{D_0} > 48 \quad \text{for} \quad Re = 33,000 \quad (4.1b) \]
\[ \frac{x}{D_0} > 58 \quad \text{for} \quad Re = 43,000 \quad (4.1c) \]

The behavior of the centerline mean velocity is compared, in Fig. 3a, to the correlation for gas-phase centerline mean velocity given by Hinze (1975):

\[ \frac{U}{U_0} = \frac{5.9}{\frac{x}{D_0} - 0.5} \quad (4.2) \]

The agreement is very good for the ranges indicated above; the agreement is poorest near the nozzle. This is to be expected, as slip is greatest near the nozzle. The particle rms velocities can be compared to classical results also. It is known that within the self-similar region along a jet centerline:

\[ u' = 0.25 U \quad (4.3) \]

If this relation is substituted into Eqn. 4.1, then an expression for the centerline gas rms velocity is obtained:

\[ \frac{u'}{U_0} = \frac{1.5}{\frac{x}{D_0} - 0.5} \quad (4.4) \]

The particle rms velocities are compared to this in Fig. 3b. Again, the agreement is best far from the nozzle where slip is at a minimum. Finally, the particle turbulence intensities, defined as the ratio of rms velocities to mean
velocities, are shown in Fig. 3c, along with the asymptote at 0.25. Here, the disagreement is rather large (on the order of 20%) for all distances.

4.1.2 Velocities as a Function of Particle Diameter

The particle mean velocities are shown as a function of particle diameter in Figs. 4a through 4c for six different downstream locations ($15 < x/D_0 < 65$) and for the three Reynolds numbers. In all three cases, the profiles are flat for $x/D_0 \geq 45$.

Fig. 5 shows the 95% confidence envelopes for $Re = 43,000$. The smaller number of points associated with the larger sizes causes a greater uncertainty in the data (recall that the particle size histograms are weighted towards the smallest particles). Yet the trend of increasing velocity is of greater magnitude than the uncertainty envelopes, indicating that the particle mean velocity increase is a statistically significant trend. It can also be seen that the statistical confidence envelopes decrease with increasing $x/D_0$.

Figs. 4a through 4c indicate the ballistic character of the larger particles' mean velocities. This is most apparent for distances within 35 diameters of the nozzle. The upward trend in the particle mean velocity is explained by the following mechanism. After the particle laden gas flow exits the nozzle, the gas phase of the jet spreads as momentum is diffused. Sub-inertial particles and--to a lesser degree--small inertial particles slow down because they are able to quickly respond to the changes in the diverging flow field. However, the larger inertial particles maintain their momentum and continue on a
high-speed trajectory, responding more slowly to the changes in the gas-phase motion. Thus, large particles have higher mean velocities than smaller particles. The effect is most pronounced near the nozzle, diminishing farther away.

The behavior of particle rms velocity is shown in Figs. 6a, 6b, and 6c for $\text{Re} = 26,000$, 33,000, and 43,000, respectively; associated confidence envelopes for $x/D_0$ of 15, 25, 35 and 65 at $\text{Re} = 43,000$ are shown in Fig. 7. Once again, statistically significant trends are evident outside of the envelopes, and the confidence envelopes decrease with increasing $x/D_0$. However, these results are somewhat harder to decipher than the mean velocity results.

The large particles have significant inertia and are unable to follow the high frequency fluctuations of the turbulent flow, as indicated by a decrease in rms velocity for increasing particle size. However, the magnitude of this difference lessens farther downstream; the size of the smallest eddies grow and large particles are able to follow the larger eddy sizes. By the time the flow reaches 65 diameters from the nozzle, the rms velocity profiles are flat. Noteworthy aberrations occur near $x/D_0$ of 15 and 25. Here, rms velocities increase for $25 < d_p < 50 \mu m$.

Particle turbulence intensities (defined here as particle rms velocity $v'$ divided by particle mean velocity $V$) are plotted in Figs. 8a, 8b, and 8c. The variation is quite large, as much as 40% of the expected gas-phase 0.25 when $x/D_0 = 15$. Yet by the time the flow reaches 65 diameters, the particle turbulence intensity profiles are almost flat, in keeping with the results above.
4.1.3 Dimensionless Velocity Variance as a Function of the Dimensionless Relaxation Time Ratio

On the basis of the preceding figures, it would seem that the flow is adequately tracked by most particles beyond \( x/D_0 \) of 35 or so; this appears to indicate that Eqn. 4.1 imposes conditions that are far too stringent. Yet, a different approach to the data reduction indicates that this is not so. Fig. 9 shows the ratio of particle to gas velocity variance \((v')^2 \text{ divided by } u'^2\) as a function of the dimensionless relaxation time \((\tau/\tau_k)\) for four different values of \( R_\lambda \) (where \( R_\lambda \) is \( 1.26 \sqrt{Re} \), as determined in Appendix B). The data is sorted by Re and is included in Appendix D. The data for \( R_\lambda = 140 \) were acquired specifically to broaden the range of Reynolds numbers; they correspond to \( Re = 13,000 \) at \( x/D_0 = 25 \). For a set of particle velocity data to be meaningfully included in this graph, the gas-phase rms velocity \((u')\) must be accurately known. Once again, use of Eqn. 3.4 mandates the exclusion of the following data sets:

- \( x/D_0 < 25 \) for \( Re = 13,000 \) or \( R_\lambda = 140 \);
- \( x/D_0 < 40 \) for \( Re = 26,000 \) or \( R_\lambda = 202 \);
- \( x/D_0 < 48 \) for \( Re = 33,000 \) or \( R_\lambda = 229 \);
- \( x/D_0 < 58 \) for \( Re = 43,000 \) or \( R_\lambda = 261 \).

The data are scattered, but there is clearly a slight peaking trend for particle rms velocity near \( \tau/\tau_k = 1 \). It appears, therefore, that a particle will be slightly energized when its relaxation time is approximately equal to that of the local Kolmogorov timescale. The peak value is about 3% for \( R_\lambda = 140 \) and somewhat lower for higher \( R_\lambda \). Figs. 10a through 10d illustrate the curves for individual Reynolds numbers with the associated 95% confidence envelopes, and Figs. 10e through 10h show the curves with smoothing. Note that Eqn. 3.17 (the Lagrangian result) does not predict any energization. Eqn. 3.24 predicts a maximum energization of 0.7% for \( R_\lambda = 140, \tau/\tau_k = 1, \) and \( x/D_0 = 25 \), as opposed to the value of 3% that was observed at somewhat lower \( \tau/\tau_k \). It is
possible that spatial biasing (Maxey, 1987) may be a factor in this discrepancy. A possible mechanism for energization is as follows.

In regions of small-scale motion, only very small particles will follow rapid accelerations of turbulent straining motions. Conversely, particles having significant inertia can overcome the small-scale viscous motions that are responsible for dissipating small eddies. It is postulated that there exist intermediate-size particles which are accelerated by the straining motions, but which are less affected by associated viscous damping. In Eqn. 3.21, this effect appears as convection of the particle slip velocity by the gas-phase turbulence. The present experimental evidence indicates that this intermediate size is a function of the dissipation rate and the scale of the smallest eddies; in a nondimensional formulation, this is where \( \tau / \tau_k = 1 \). Related effects may include the predicted tendency for small inertia particles to collect in regions of high strain rate or low vorticity (Maxey, 1982).

4.2 Power Spectra

4.2.1 Interarrival Times

It was said in Chapter 3 that the interarrival-time histogram is approximately Poisson-distributed. Fig. 11a shows this histogram for \( \text{Re} = 33,000 \) and \( x/D_0 = 25 \). Fig. 11b shows the interarrival times (\( \Delta t_i \)) for each occurrence (i). The average sampling rate is somewhere near 750 Hz, the minimum interarrival time of 100 \( \mu \)s is evident, and the dropout period of 25 ms is visible. Because the validation rate was about 75%, the dropouts occur for every 750
counts rather than every 1000 counts. The Poisson distribution:

\[ Z(\Omega) = \frac{\Omega^\phi e^{-\Omega}}{\phi!} \]

is shown in Fig. 11c for \( \Omega = 1, 3 \) and 5.

4.2.2 Power Spectra for Several Downstream Distances

Particle-phase turbulent power spectra were estimated for six downstream distances \( (15 \leq x/D_0 \leq 65) \) at the three Reynolds numbers. Figs. 12 illustrate the curves for \( \text{Re} = 26,000 \). Figs. 13 illustrate the curves for \( \text{Re} = 33,000 \). Figs. 14 illustrate the curves for \( \text{Re} = 43,000 \). Several comments can be made about the spectra. First of all, the shape of the curves agrees well with the theory. A flat region representative of the large eddies is evident at low frequencies. This is followed by an inertial subrange with a -5/3 slope, similar to that found by Gibson and Schwarz (1963), Chevray and Tutu (1978), and others. The final region at high frequencies is a "noise floor" that prevents further resolution. Although perfect Poisson sampling would give unlimited frequency resolution, a noise floor of practical significance exists because of limited average sampling frequency and the minimum time between samples (100 \( \mu \text{s} \)) that the instrument can resolve (as discussed at the end of Section 3.2.1).

It is expected that the spectral estimate of the turbulent kinetic energy should yield a smooth shape, as is the case for hot-wire results. The results using a PDPA are not smooth; the spectral estimates contain fluctuations, particularly
for high frequencies, large particle sizes, few estimates per averaged spectrum (small L), and large downstream distances. I use the term *spectral distortion* in this paper to describe the smoothness or quality of a spectral estimate. Spectra with low distortion are shown in Fig. 15a. These are identical to those in Fig. 13b; they represent the average of 15 spectral estimates (L = 15). Spectra with a high level of distortion are shown in Fig. 15b; these spectra represent a single spectral estimate (L = 1). The effect of distortion for a single curve is shown in Figs. 15c and 15d for the 1 to 5 μm size range; also shown are the associated confidence envelopes as defined in Eqn. 3.32.

Given this new terminology, it is instructive to consider the effects of mean sampling rate, as shown in Tables 1 and 2, on the distortion of the spectra. In general, sampling rate decreases as downstream distance increases (due to jet spreading). As sampling rate goes down, spectral distortion increases. The point at which the spectral estimate begins to distort increases from approximately 1000 Hz for high sampling rate to approximately 100 Hz for low sampling rate. Furthermore, since the size histograms are weighted towards the smallest particles, the sampling rate decreases with increasing particle size. It can be seen that the spectral estimates are the least distorted for the smallest particles at any given downstream station. The most notable exceptions to these trends occur for x/D_0 = 15 at Re = 33,000 and 43,000 (Figs. 13a and 14a, respectively). Here, particle speed and particle density are so high that the PDPA cannot validate many of the particles; this leads to a correspondingly small sample rate and therefore low spectral coherence.
Distortion at high frequencies makes it hard to distinguish the separate power spectrum curves for the various size bins; yet at low frequencies, distortion is small enough so that a trend is apparent: there appears to be a trend of decreasing power associated with increasing particle size. These results are, for the most part, common to all spectra measured in this study. This low-frequency spectral behavior is somewhat counter-intuitive. It is expected that large and small particles alike are able to follow the largest scales of motion. Thus, one would expect close agreement between all curves at low frequencies. However, the opposite trend is evident: the spectra begin to coincide only at the higher frequencies. Another surprise is the "crossover" effect: The largest-particle spectrum (for the size bin that includes the 35 μm particles) has a higher power spectral density than the second-largest-particle spectrum. The largest-particle spectrum appears to crossover the others.

4.2.3 Power Spectral Variance
Are in fact these spectral estimates accurate, and not just the result of random fluctuations? Figs. 16a through 16f show the estimated variance (as determined in Eqn. 3.32) associated with the six spectra at x/D₀ = 35 and Re = 26,000 (Fig. 12c). The spectral behavior appears to be statistically significant even when taking into account the confidence envelopes. Figs. 15c and 15d illustrate the effect of L on the confidence envelopes. The comments above about sampling rate appear to be validated in these figures: the best estimates are for low frequencies, small particles, and small x/D₀.

Note, however, that the magnitude of the spectral distortion is of greater magnitude than the bounds of the confidence envelopes. This would seem to
indicate that Eqn. 3.20 under-predicts the magnitude of spectral variance, or, at least, sampling rate is much more important in the estimate of spectral variance.

4.2.4 Sampling Rate Restrictions
The spectral estimates are limited by the effects of finite sampling rate. Table 1 lists the overall sampling rate for the data and Table 2 provides a breakdown by particle size for Re = 33,000. Because many sampling rates are so low, some spectra are of questionable validity. It appears that spectra remain free of appreciable distortion up to about five or six times their mean sample rate. Since we are interested in frequencies at least up to 100 Hz, then spectra with \( \lambda \) less than 20 Hz are probably not too useful. Spectra from Fig. 13 with \( \lambda > 17 \) are plotted in Fig. 18. Spectral distortion is not evident in the remaining curves for frequencies less than 5\( \lambda \) (except for \( x/D_0 = 15 \), which was an exceptional case described above). Furthermore, there is no crossover effect; the remaining curves at each downstream station coincide rather closely.

Fig. 17 illustrates some dissipation spectra (\( fE_{vv} \) vs. \( f \)). In both graphs, it can be seen that maximum dissipation occurs at 100 Hz for the smallest particles. The largest particles dissipate their energy at around 50 Hz. A low signal-to-noise ratio in this experiment obscures the dissipation scale beyond 150 Hz. These dissipation spectra are useful because they indicate a frequency range within which the spectral estimate is of high quality. The area under a given power spectral density curve (\( E_{vv} \) vs. \( f \)) is equal to the particle velocity
variance \((v^2)\) and this provides an independent check on the validity of the spectral estimates. Although spectral noise is very high in Figs. 17a and 17b, we can extrapolate a line with a -2/3 slope from the peak of the dissipation spectra and in doing so imagine what the "true" shape of the dissipation curves is. Ad hoc assumptions were made in this fashion to evaluate the area under the power spectral density curves. Tables 1 and 2 provide a tabulation of these quantities. It seems that the area under the power spectral density curves only provides an order-of-magnitude estimate of the fluctuating rms velocity.

4.2.5 Power Spectral Behavior for the Smallest Particle Sizes

A useful subset of Figs. 12, 13, and 14 is the power spectra for the smallest particle size range. In light of Figs. 9, 10, and 11 above, the smallest particle size range is close (within a few percent) to the gas-phase power spectral density, \(E_{uu}\). Fig. 19a illustrates the six spectra for \(Re = 26,000\); Fig. 19b illustrates the six spectra for \(Re = 33,000\); and Fig. 19c illustrates the six spectra for \(Re = 43,000\). These same spectra, in Fig. 20, are expressed in Kolmogorov universal equilibrium form:

\[
F(\kappa \eta) = \frac{E_{uv}(\kappa \eta)}{\nu_g^{5/4} \varepsilon^{1/4}}
\]  

(4.5)

where \(\kappa = 2\pi f/U\) is the wavenumber, and \(\eta\) is the Kolmogorov length scale. Although the shape of the curves agrees well with theory, these universal equilibrium spectra do not collapse onto a single curve; in fact, they exhibit an
apparent increase in energy with increasing downstream distance.

These spectra are compared to the expected high-Reynolds-number spectrum predicted by Pao (1965) which would characterize the gas-phase turbulence. It is quite possible that the largest particles in the size range are slipping enough to cause this deviation from Pao's expected spectrum, yet it may also be possible that the spectra do not scale with this universal Kolmogorov form.

The particle velocity flow field may scale with the spectrum associated with turbulent passive scalar mixing rather than that of turbulent kinetic energy. This is because the suspended (non-slipping) aerosol is, in some respects, like a scalar which tags the fluid leaving the jet nozzle. Gibson and Schwarz (1963) indicate that, for a passive scalar field, there are four regions of the turbulent mixing spectrum (as opposed to three for the turbulent kinetic energy spectrum): 1) a flat region at low wavenumbers (slope of zero) common to all finite Reynolds-number flows, 2) an inertial subrange region (slope of -5/3) at intermediate wavenumbers, 3) a unique viscous-convective region (slope of -1) at higher wavenumbers, and 4) the final diffusive roll-off region at the highest wavenumbers. The transition from inertial subrange to the viscous-convective region occurs for $k\eta$ on the order of 0.1. Therefore, Fig. 20 also displays the departure from the inertial subrange due to the possible existence of the viscous-convective region. The agreement is slightly better, although low spectral coherence prevents a truly exact comparison.

If we assume that the spectra scale with the turbulent passive scalar mixing spectra:
then all we need is the expression for scalar dissipation rate $\chi$. However, since $\chi$ depends on the magnitude of the "diffusivity of droplets in air," we are not able to form an explicit expression for $\chi$. However, the ratio of velocity to scalar dissipation rate ($\varepsilon/\chi = \alpha$) is obtainable by multiplying $F(\kappa\eta)$ by $\alpha$ so that the new spectra ($F_2(\kappa\eta)$) coincide with Pao's curve in the inertial subrange. The scaled spectra, defined as $\alpha F(\kappa\eta) = F_2(\kappa\eta)$, are shown in Fig. 21. The corresponding values of $\alpha$ are plotted in Fig. 22. Since $\alpha$ decays as a $-1.2$ power of $x/D_0$ and $\varepsilon$ decays as a $-4$ power of $x/D_0$, the scalar dissipation rate for the particle field must decay as a $-2.8$ power of $x/D_0$. 

$$F_2(\kappa\eta) = \frac{E_{vv}(\kappa\eta) \varepsilon^{3/4}}{v_g^{5/4} \chi}$$ (4.6)
Chapter 5

Conclusions and Recommendations

5.1 Theoretical Findings
Direct measurements of Eulerian velocity statistics and spectra have been made as a function of aerosol particle inertia in an independently-controlled turbulent flow. Time-series of particle velocity and diameter were measured under various flow conditions and the size-sorted statistics and random-arrival spectra were calculated. Confidence intervals were obtained for all measured quantities to ensure that the observed behavior is statistically significant. Mass loadings of aerosol were kept sufficiently low that particle influence on the gas-phase turbulence is negligible. Results are presented in terms of the ratio of particle relaxation time to the local Kolmogorov time scale, $\tau/\tau_k$.

Inertia clearly attenuates the particle motion at large $\tau/\tau_k (> 1)$, in agreement with the expectations of classical Lagrangian theory. The form of that attenuation, however, is surprising. For particles of low inertia, ($\tau/\tau_k = 0.01$), mean and rms velocities are very close to those of the gas-phase turbulence, and particle velocity spectra satisfy classical Kolmogorov scaling in the near
field of the jet. However, for particles of somewhat larger inertia, an energization of a few percent occurs, showing a maximum for $\tau/\tau_k = 1$. This may or may not be a major consideration for the general researcher, especially considering that velocity biasing, particle sizing ambiguity, and limited sample size may cause errors many times greater than this. The magnitude of this energization is somewhat smaller than the Eulerian analysis predicts. At this same relative frequency, a Lagrangian Stokes' flow estimate of $v'^2/u'^2$ would predict no energization and, for larger $\tau/\tau_k$, a much faster attenuation of $v'^2$. Measurements suggest, then, that a strict criterion for subinertial particles would be $\tau/\tau_k = 0.01$, and that inertial attenuation grows more slowly than might otherwise be expected.

Additional surprises are found in the Eulerian velocity spectra. As particle inertia increases, the low-frequency part of the spectrum drops below the Kolmogorov universal spectrum of the subinertial particles, rather than commencing at higher frequencies first as Lagrangian theory predicts. The matter is complicated by the fact that there are fewer particles associated with the larger inertial particles; thus the sampling rate is lower and both spectral variance and distortion are higher. However, if one considers only those size ranges for which $\lambda > 20$ Hz, then this effect is not visible. Clearly, fuller experimental resolution of the large particles and high wavenumbers is desirable in exploring this trend, but the observed behavior lies well above the established spectral variance. It is worth saying that Eqn. 3.32 in its present form under-predicts the magnitude of the variance; future work should address this problem. Difficulties arise in the spectral estimate as the sample rate decreases, either downstream or for less numerous particle sizes; higher number densities are quite desirable in diminishing the magnitude of
these problems.

Several possible mechanisms may account for the disagreements with Lagrangian theory. One is the presence of convective effects in the Eulerian correlations, which are absent in the Lagrangian forms. The second is the spatial bias of inertial-particle number density in nonuniform flows which has been described by Maxey (1987). Future efforts should consider these theoretical issues as well as further experimental efforts at improved spectral resolution, variation of $R_\lambda$ and other flow conditions, and the contributions of other parameters beyond $\tau/\tau_k$.

5.2 Equipment

Several recommendations are in order for the equipment problem. A wider range of nozzle Reynolds numbers must be achieved, so as to achieve greater separation between large and small scales (a parameter characterized by $R_\lambda$). This is also necessary because higher jet velocities will lessen the spatial effects of time-dependent aerosol evaporation. Another aim should be to construct a higher velocity, lower turbulence intensity jet using a modified approach to turbulence management.

One of the most pressing problems faced in this experiment is that of evaporation. It is conceivable that many different substances other than water can be used for the aerosol phase. However, this introduces the possibility of merely trading one set of problems for another. Oils and glycerols will resist evaporation not only in the jet but also in the laboratory.
in which the experiments are held. Fluorocarbons present both immediate and long-term environmental hazards, as well as requiring substantial ventilation. Solid particles, dusts, glass beads, and more exotic formulations (such as polystyrene latex suspensions) are entirely free from evaporative restrictions but likewise pose similar environmental hazards.

Of course, most of these problems can be alleviated by enclosing the jet. Once again, though, new problems arise. A free jet is no longer "free" when it is enclosed in such a small space that recirculation changes the character of entrainment. A small enclosure would also cause fouling of the optical access windows due to deposition. The enclosure can be made much larger than the largest length scales of the jet, but this would require that the focal lengths of both the transmitting and receiving optics be proportionally increased. This will reduce the ability of the optical system to resolve very dense particle fields that are so essential in many aspects of the experiment.

Future efforts should also investigate the properties of several aerosol generation methods. The ideal outcome would involve coupling theoretical knowledge of the instabilities and competing forces associated with fluid break-up with the practical construction of aerosol generation systems. Perhaps several different generators, operating concurrently over several different size ranges, will produce a robust aerosol with a wide range of sizes that would both follow the smallest scales of motion (with small particles) and yet provide robust particles (with large diameters) which would resist evaporation and thus provide information about the flow regions far downstream.
Finally, there should be an investigation of the possibility of replacing the present 10 milliwatt Helium-Neon laser with a high-powered five watt Argon-ion laser in the PDPA optics. This will increase the doppler burst signal-to-noise ratio by roughly a factor of 300, deducing two and one half decades from the present spectral noise floor.
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Dimensionless particle rms velocity

Dimensionless time ratio, $\tau/\tau_K$
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<td>348</td>
<td>6.40</td>
<td>2.56</td>
</tr>
<tr>
<td>35</td>
<td>362</td>
<td>3.76</td>
<td>2.06</td>
</tr>
<tr>
<td>45</td>
<td>119</td>
<td>2.16</td>
<td>1.48</td>
</tr>
<tr>
<td>55</td>
<td>34</td>
<td>1.39</td>
<td>0.56</td>
</tr>
<tr>
<td>65</td>
<td>45</td>
<td>1.10</td>
<td>0.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x/D_0$</th>
<th>Overall Sample Rate λ (Hz)</th>
<th>$v^2$</th>
<th>$\int E_{vv} df$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>150</td>
<td>46.37</td>
<td>12.20</td>
</tr>
<tr>
<td>25</td>
<td>374</td>
<td>14.44</td>
<td>9.75</td>
</tr>
<tr>
<td>35</td>
<td>331</td>
<td>6.76</td>
<td>4.03</td>
</tr>
<tr>
<td>45</td>
<td>196</td>
<td>4.49</td>
<td>3.18</td>
</tr>
<tr>
<td>55</td>
<td>65</td>
<td>3.28</td>
<td>2.45</td>
</tr>
<tr>
<td>65</td>
<td>79</td>
<td>2.34</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Table 1. Sample Rates (λ) and Velocity Variance ($v^2$ and $\int E_{vv} df$) for the Smallest Particles at All Reynolds Numbers (Re) and Downstream Distances ($x/D_0$)
For \( x/D_0 = 15 \):

<table>
<thead>
<tr>
<th>Particle Size Range (μm)</th>
<th>Sample Rate ( \lambda ) (Hz)</th>
<th>( v'^2 )</th>
<th>( \int E_{vv} df )</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 - 05</td>
<td>176</td>
<td>13.10</td>
<td>3.52</td>
</tr>
<tr>
<td>05 - 10</td>
<td>88</td>
<td>14.82</td>
<td>4.03</td>
</tr>
<tr>
<td>10 - 15</td>
<td>35</td>
<td>13.76</td>
<td>3.12</td>
</tr>
<tr>
<td>15 - 20</td>
<td>21</td>
<td>12.25</td>
<td>2.71</td>
</tr>
<tr>
<td>20 - 25</td>
<td>11</td>
<td>11.15</td>
<td>1.92</td>
</tr>
<tr>
<td>25 - 30</td>
<td>7</td>
<td>12.04</td>
<td>1.34</td>
</tr>
<tr>
<td>30 - 35</td>
<td>4</td>
<td>13.03</td>
<td>3.08</td>
</tr>
</tbody>
</table>

For \( x/D_0 = 25 \):

<table>
<thead>
<tr>
<th>Particle Size Range (μm)</th>
<th>Sample Rate ( \lambda ) (Hz)</th>
<th>( v'^2 )</th>
<th>( \int E_{vv} df )</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 - 05</td>
<td>174</td>
<td>6.40</td>
<td>2.56</td>
</tr>
<tr>
<td>05 - 10</td>
<td>87</td>
<td>6.65</td>
<td>2.99</td>
</tr>
<tr>
<td>10 - 15</td>
<td>35</td>
<td>6.76</td>
<td>2.99</td>
</tr>
<tr>
<td>15 - 20</td>
<td>21</td>
<td>6.40</td>
<td>2.62</td>
</tr>
<tr>
<td>20 - 25</td>
<td>10</td>
<td>6.30</td>
<td>5.90</td>
</tr>
<tr>
<td>25 - 30</td>
<td>7</td>
<td>5.90</td>
<td>1.18</td>
</tr>
<tr>
<td>30 - 35</td>
<td>3</td>
<td>5.57</td>
<td>1.63</td>
</tr>
</tbody>
</table>

For \( x/D_0 = 35 \):

<table>
<thead>
<tr>
<th>Particle Size Range (μm)</th>
<th>Sample Rate ( \lambda ) (Hz)</th>
<th>( v'^2 )</th>
<th>( \int E_{vv} df )</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 - 06</td>
<td>181</td>
<td>3.76</td>
<td>2.06</td>
</tr>
<tr>
<td>06 - 11</td>
<td>91</td>
<td>3.84</td>
<td>2.22</td>
</tr>
<tr>
<td>11 - 16</td>
<td>36</td>
<td>3.72</td>
<td>2.14</td>
</tr>
<tr>
<td>16 - 21</td>
<td>22</td>
<td>3.68</td>
<td>2.05</td>
</tr>
<tr>
<td>21 - 26</td>
<td>11</td>
<td>3.39</td>
<td>1.00</td>
</tr>
<tr>
<td>26 - 35</td>
<td>7</td>
<td>2.99</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Table 2. A Comparison of the Two Velocity Variances (\( v'^2 \) and \( \int E_{vv} df \)) for All Particle Sizes and Downstream Distances (\( x/D_0 \)) at \( \text{Re} = 33,000 \)
For $x/D_0 = 45$:

<table>
<thead>
<tr>
<th>Particle Size Range (µm)</th>
<th>Sample Rate $\lambda$ (Hz)</th>
<th>$v'^2$</th>
<th>$\int E_{vv} \text{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 - 07</td>
<td>60</td>
<td>2.16</td>
<td>1.48</td>
</tr>
<tr>
<td>07 - 12</td>
<td>30</td>
<td>2.22</td>
<td>1.48</td>
</tr>
<tr>
<td>12 - 17</td>
<td>12</td>
<td>2.19</td>
<td>1.45</td>
</tr>
<tr>
<td>17 - 22</td>
<td>7</td>
<td>2.10</td>
<td>0.70</td>
</tr>
<tr>
<td>22 - 27</td>
<td>4</td>
<td>2.09</td>
<td>0.40</td>
</tr>
<tr>
<td>24 - 35</td>
<td>2</td>
<td>2.07</td>
<td>0.82</td>
</tr>
</tbody>
</table>

For $x/D_0 = 55$:

<table>
<thead>
<tr>
<th>Particle Size Range (µm)</th>
<th>Sample Rate $\lambda$ (Hz)</th>
<th>$v'^2$</th>
<th>$\int E_{vv} \text{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 - 09</td>
<td>17</td>
<td>1.39</td>
<td>0.56</td>
</tr>
<tr>
<td>09 - 14</td>
<td>8</td>
<td>1.39</td>
<td>0.82</td>
</tr>
<tr>
<td>14 - 19</td>
<td>3</td>
<td>1.37</td>
<td>0.99</td>
</tr>
<tr>
<td>19 - 24</td>
<td>2</td>
<td>1.34</td>
<td>0.33</td>
</tr>
<tr>
<td>24 - 29</td>
<td>1</td>
<td>1.39</td>
<td>0.13</td>
</tr>
<tr>
<td>29 - 35</td>
<td>1</td>
<td>1.37</td>
<td>0.22</td>
</tr>
</tbody>
</table>

For $x/D_0 = 65$:

<table>
<thead>
<tr>
<th>Particle Size Range (µm)</th>
<th>Sample Rate $\lambda$ (Hz)</th>
<th>$v'^2$</th>
<th>$\int E_{vv} \text{df}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>01 - 11</td>
<td>23</td>
<td>1.10</td>
<td>0.80</td>
</tr>
<tr>
<td>11 - 16</td>
<td>12</td>
<td>1.10</td>
<td>0.71</td>
</tr>
<tr>
<td>16 - 21</td>
<td>5</td>
<td>1.12</td>
<td>0.55</td>
</tr>
<tr>
<td>21 - 26</td>
<td>3</td>
<td>1.12</td>
<td>0.28</td>
</tr>
<tr>
<td>26 - 36</td>
<td>1</td>
<td>1.08</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 2 (cont.). A Comparison of the Two Velocity Variances ($v'^2$ and $\int E_{vv} \text{df}$) for All Particle Sizes and Downstream Distances ($x/D_0$) at Re = 33,000
Appendix A

The Effect of Windowing with Dropouts

As shown in Chapter 3, there are two functions of interest. The derivation of the first function is given in Bendat and Piersol (1971). The derivation of the second function is given here. This function is the Fourier transform of a window with dropouts as characterized in Fig. 2b; in the following, it is simply called $H(f)$. Given a time series $h(t)$, define the Fourier transform of it as:

$$H(f) = F(h(t)) \quad (A.1)$$

which is:

$$H(f) = \int_{-\infty}^{+\infty} h(t) e^{2\pi jft} \, dt \quad (A.2)$$
where \( h(t) \) is defined for \( \psi \) dropouts as:

\[
\begin{align*}
  h(t) &= 0 & t < 0 \\
  & = 1 & 0 < t < t_{c1} \\
  & = 0 & t_{c1} < t < t_{c1} + \theta \\
  & = 1 & t_{c1} + \theta < t < t_{c2} \\
  & \vdots & \quad \vdots \\
  & = 1 & t_{c(\psi - 1)} + \theta < t < t_{\psi} \\
  & = 0 & t > T
\end{align*}
\]

Therefore:

\[
H(f) = \sum_{i=1}^{\psi} H_i(f) = \sum_{i=1}^{\psi} \int_{t_{c(i-1)} + \theta}^{t_{ci}} e^{2\pi jft} \, dt
\]

(A.4)

For \( i \geq 1 \):

\[
t_{ci} = ik + (i-1)\theta
\]

(A.5)

so that:

\[
H(f) = \left( \frac{-i}{2\pi f} \right) \sum_{i=1}^{\psi} \left( e^{2\pi jf(ik + (i-1)\theta)} - e^{2\pi jf(i-1)(k+\theta)} \right)
\]

(A.6)
After some manipulation:

\[
H(f) = \left(\frac{-i}{2\pi f}\right) \left(1 - e^{-2\pi j f k}\right) \left(e^{-2\pi j f \theta}\right) \sum_{i=1}^{\psi} e^{2\pi j f (k+\theta)i}
\]  \hspace{1cm} (A.7)

Using the following identity:

\[
\sum_{i=1}^{\psi} x^i = \frac{x(1-x^\psi)}{1-x}
\]  \hspace{1cm} (A.8)

it follows that:

\[
H(f) = \left(\frac{-i}{2\pi f}\right) e^{2\pi j f k} \left(1 - e^{-2\pi j f k}\right) \left(1 - e^{2\pi j f (k+\theta)\psi}\right) \left(1 - e^{2\pi j f (k+\theta)}\right)
\]  \hspace{1cm} (A.9)

The quantity of interest here is \(H^2(f)\), defined as:

\[
|H(f)|^2 = H(f) \cdot H^*(f)
\]  \hspace{1cm} (A.10)

where \(H^*(f)\) is the complex conjugate of \(H(f)\). Making use of the following trigonometric identity:

\[
1 - \cos 2\alpha = 2 \sin^2 \alpha
\]  \hspace{1cm} (A.11)
it is found that:

\[ |H(f)|^2 = \left( \frac{1}{2\pi f} \right)^2 \left( \frac{2}{\sin \pi f(k+\theta)} \right)^2 \left[ 2\sin^2 \pi f \psi(k+\theta) + 2\sin^2 \pi f k - \sin^2 \pi f(k+\psi(k+\theta)) - \sin^2 \pi f(k-\psi(k+\theta)) \right] \] (A.12)

which is Eqn. 3.34.
Appendix B

The Reynolds Number Relations

The relation between the jet Reynolds number and the microscale Reynolds number was given in Chapter 4 as:

\[ R_\lambda = 1.26 \sqrt{Re} \quad (B.1) \]

The derivation is as follows. Hinze (1975) gives the definition of the microscale Reynolds number as:

\[ R_\lambda = \frac{u' \lambda_g}{v_g} \quad (B.2) \]

where \( \lambda_g \) is the transverse dissipation length scale, also defined in Hinze. \( \lambda_g \) is found implicitly by equating the two relations for the turbulent kinetic energy dissipation rate \( \varepsilon \):

\[ \varepsilon = \frac{48 U_0^3}{D_0 (x/D_0)^4} \quad (B.3) \]
which is Eqn. 3.1, and:

\[ \varepsilon = 15 \nu_{e} \left( \frac{u'}{\lambda_{e}} \right)^{2} \]  \hspace{1cm} (B.4)

which comes from Hinze. Furthermore, along a jet centerline, it is known that:

\[ \frac{u'}{U} = 0.25 \]  \hspace{1cm} (B.5)

and from Eqn. 4.1:

\[ \frac{U}{U_{0}} = \frac{5.9}{\frac{x}{D_{0}} - 0.5} \]  \hspace{1cm} (B.6)

Combining Eqns. B.2 through B.6 yields:

\[ R_{x} = 1.26 \sqrt{Re} \]  \hspace{1cm} (B.7)
Appendix C

Evaporation and Number Density

Attempts were made at mapping the number density field of the jet in an effort to compare it to known results for concentration fields. However, the results are rather disappointing due to the phenomenon of evaporation. The evaporation problem is quite profound for high-velocity water aerosols issuing into quiescent air at room temperature and average (50% or so) relative humidity. The jet is almost completely evaporated when $x/D_0 = 100$. More importantly, most of the particles in the jet, and therefore the major contributors to the number density, are quite small—less than 10 $\mu$m in diameter. These smallest particles, although contributing very little to the mass of the jet, evaporate the soonest.

An attempt was made to quantify the evaporation phenomenon. This analysis provides a qualitative result which predicts the form of the downstream size histograms; quantitative results are not presently possible. Work by Davies (1978) provides a starting point. Davies found that the rate of change of the surface area of an evaporating droplet is constant:

$$\frac{da^2}{dt} = c_1 \quad \text{(C.1)}$$
where \( a \) is the droplet radius, and where \( c_1 \) depends on the diffusion coefficient of water vapor in air, the density of the particle, the relative humidity of the air, and the saturation vapor pressure of water near the drop surface. Accurate measurements of these properties are not easily realized in the laboratory. From this it follows that:

\[
a^2 = c_1 t + c_2
\]  

(C.2)

Define the radius at \( t = 0 \) as the initial radius \( a_0 \). Then \( c_2 = a_0^2 \). The time required for the particle to evaporate is \( t_e \), defined as:

\[
t_e = \frac{d_e}{V}
\]  

(C.3)

where \( d_e \) is the distance that a particle travels before evaporating and \( V \) is the particle mean velocity. Thus \( c_1 = -a_0^2/t_e \). Note that the average distance a particle travels as it is carried in the jet (\( d \)) is not the same as the the downstream centerline distance (\( x \)). Because a particle is subject to turbulent chaotic motions as well as recirculation, the distance \( d \) is several times greater than \( x \). This distance \( d \) is gross average Lagrangian particle motion, an expression for which is not presently available. If one assumes that the form of this downstream Lagrangian particle mean velocity decay is:

\[
V = \frac{c_3}{d}
\]  

(C.4)

then the expression for downstream decay of the droplet radius is:
The size histogram of the aerosol at \( t = 0 \) (and therefore \( d = 0 \)) is distributed in an approximately log-normal fashion. Define the probability distribution of this size histogram at \( t = 0 \) as:

\[
P(a_0) = \frac{1}{\sqrt{2\pi} a_0 \sigma_0} \exp \left[ -\frac{1}{2} \left( \ln a_0 - \mu_0 \right)^2 \right]
\]

where \( \mu_0 \) is the median droplet radius at \( t = 0 \) and \( \sigma_0 \) is the variance of the distribution at \( t = 0 \). The general probability distribution is a function of Lagrangian distance \( d \) as well as particle radius \( a \) and can be expressed as:

\[
P(d, a) = \left[ a_0^2 - c_4 d^2 \right] \left[ P(a) \right] \left[ S\left( a_0^2 - c_4 d^2 \right) \right]
\]

where:

\[
S(x) = \begin{cases} 
0 & \text{for } x \leq 0 \\
1 & \text{for } x > 0 
\end{cases}
\]

The function \( P(d, a) \) is plotted against \( d_p \) in Fig. 23a for several values of the parameter \( c_4 \). Fig. 23b shows an experimentally measured size histogram from Simo and Lienhard (1990), taken at \( Re = 13,000 \) for several downstream distances, using a Durst-Umhauer (1975) generator and a different jet.
apparatus. Note that in Fig. 23b, $x/r_0 = 2x/D_0$. The agreement is close and the shape of the decaying histograms is similar. Thus, in a qualitative sense, Eqn. C.8 gives the correct form of the probability distribution of particle size in a particle field where evaporation is present.

Lastly, it is important to note certain limitations imposed by the instrumentation that prevent an accurate measurement of number density. A primary concern is the limited range of resolution of the particle sizes. The instrument maintains a 35:1 diameter ratio; in this experiment, $1.4 \mu m < d_p < 50 \mu m$. It is not possible to analyze particles which may exist outside of this range. Another limitation is the finite probe volume size. At high number densities (essential for high spectral coherence), multiple particles may exist within the probe volume and the validation rate is correspondingly low. This effect occurs near the nozzle where particle density is the highest.
Appendix D

Tables of Dimensionless Velocity Variance \((v'^2/u'^2)\) Versus the Dimensionless Relaxation Time Ratio \((\tau/\tau_K)\)

This appendix contains the raw data for \(v'^2/u'^2\) versus \(\tau/\tau_K\).
For \( R_\kappa = 140:\)

<table>
<thead>
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<th>( \tau/\tau_\kappa )</th>
<th>( \nu^2/\mu^2 )</th>
</tr>
</thead>
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<tr>
<td>0.004104</td>
<td>1.000000</td>
</tr>
<tr>
<td>0.011752</td>
<td>1.012492</td>
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<tr>
<td>0.023334</td>
<td>1.013272</td>
</tr>
<tr>
<td>0.038850</td>
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<td>0.058299</td>
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</tr>
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<td>0.081682</td>
<td>1.028664</td>
</tr>
<tr>
<td>0.109295</td>
<td>1.032233</td>
</tr>
<tr>
<td>0.140585</td>
<td>1.020410</td>
</tr>
<tr>
<td>0.175808</td>
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<td>0.214965</td>
<td>1.036025</td>
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<td>0.258055</td>
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</tr>
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</tr>
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<td>0.356037</td>
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</tr>
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</tr>
<tr>
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</tbody>
</table>
For $R_s = 202$:

<table>
<thead>
<tr>
<th>$\tau/\tau_K$</th>
<th>$v'z^2/u^2$</th>
<th>$\tau/\tau_K$</th>
<th>$v'z^2/u^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.009248</td>
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<td>1.383490</td>
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</tr>
<tr>
<td>0.013816</td>
<td>1.000000</td>
<td>1.434578</td>
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</tr>
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<td>1.511293</td>
<td>1.006638</td>
</tr>
<tr>
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Appendix E

Spectral Codes

There are three codes contained in this appendix. The codes are COMB.PAS (written in Borland's Turbo Pascal), SLOT.FOR (written in Microsoft's Fortran), and AVERAGE.PAS (again, Borland's Turbo Pascal). They are executed in that order. They operate on the data structures formed by the PDPA software published by Aerometrics, Inc. The output of these files is ASCII, usually in tabular form for plotting. The plotting package used for these files (indeed, for most of the graphs in this work) is Easyplot (© Massachusetts Institute of Technology). Where noted, the Numerical Recipes routines are from the book by the same name.
Program Comb (Input, Output);

Uses Crt;

Const [Global Constants]
    NumFiles  = 15;
    Date      = 'DEC17';
    MaxSize   = 35;
    Spectral_Bin_Width = 5;

Written by: John A. Simo, 1990

This program is very similar to SORTDATA.PAS; it is a refined version of FASTSORT.PAS. This program is used for sorting many RAWDAT.TXT files according to particle size. Essentially, it breaks up the raw file into several smaller files, each one containing the time/velocity info for a given size range. Currently the maximum number of spectral bins is 11. The size ranges of the comb are set in the CONST declaration section.

The program sorts the data by particle size and then writes the sorted file to disk, in the form:

   Time[i]    Velocity[i]

If necessary, change these lines:

   NumFiles = ......
   Date     = ......

Procedure Comb_1;

Var
    RAW,
    PDP,
    FL,
    W1, W2, W3, W4,
    W5, W6, W7, W8,
    W9, W10, W11 : Text;
    i,
    k,
    Num_Spec_Bins,
    Bin,
    j : Integer;
    SubSize,
    Time,
Diameter,
Velocity : Real;
dummy1, dummy3 : String[2];
dummy2 : String[40];
dummy4 : Array[1..11] of String[6];
dummy5 : Array[1..11] of String[10];
dummy6 : Array[1..11] of String[40];
npts : Array[1..NumFiles,1..11] of Integer;

Begin (Procedure Comb_1)

ClrScr;
Writeln ('Enter the sub-inertial-limit particle diameter in microns. ');
Writeln ('This is the size above which particles no longer accurately');
Writeln ('track the flow field. The information for this is provided');
Writeln ('by the code STOPTIME.PAS. Note that Subsize may be zero');
Writeln ('close to the nozzle, particularly for high nozzle exit');
Writeln ('velocities. ');
Writeln;
Write ('Enter Subsize: ');
Readln (SubSize);
If Subsize = 0 Then Subsize := Spectral_Bin_Width;
Writeln;

Assign (PDP, 'E:\PDPDATA\THESIS\' + Date + '\RUN01\PDPDATA.TXT');
Reset (PDP);
Close (PDP);

Assign (FL, 'D:\CODES\' + Date + 'FILELIST');
Rewrite (FL);

Num_Spec_Bins := Trunc((MaxSize - SubSize)/Spectral_Bin_Width) + 1;
If Num_Spec_Bins > 11 Then Num_Spec_Bins := 11;

For j := 1 to NumFiles do Begin
  Str(j, dummy1);
  If ((j >= 1) AND (j < 10))
    Then dummy1 := '0' + dummy1;
  dummy2 := 'E:\PDPDATA\THESIS' + Date + '\RUN'
    + dummy1 + 'RAWDAT.TXT';
  Assign (RAW, dummy2);
  Reset (RAW);
  Writeln ('File being sorted is ', dummy2);

  For i := 1 to Num_Spec_Bins do Begin
    Str (i, dummy3);
    If ((i >= 1) AND (i < 10))
      Then dummy3 := '0' + dummy3;
    dummy4[i] := dummy1 + '.' + dummy3;

End;
dummy5[i] := 'RUN' + dummy4[i];
dummy6[i] := 'D:\CODES\' + Date + \n' + dummy5[i];

Case i of 
  1 : Begin 
    Assign (W1, dummy6[i]);
    Rewrite (W1);
  End;
  2 : Begin 
    Assign (W2, dummy6[i]);
    Rewrite (W2);
  End;
  3 : Begin 
    Assign (W3, dummy6[i]);
    Rewrite (W3);
  End;
  4 : Begin 
    Assign (W4, dummy6[i]);
    Rewrite (W4);
  End;
  5 : Begin 
    Assign (W5, dummy6[i]);
    Rewrite (W5);
  End;
  6 : Begin 
    Assign (W6, dummy6[i]);
    Rewrite (W6);
  End;
  7 : Begin 
    Assign (W7, dummy6[i]);
    Rewrite (W7);
  End;
  8 : Begin 
    Assign (W8, dummy6[i]);
    Rewrite (W8);
  End;
  9 : Begin 
    Assign (W9, dummy6[i]);
    Rewrite (W9);
  End;
  10: Begin 
    Assign (W10, dummy6[i]);
    Rewrite (W10);
  End;
  11: Begin 
    Assign (W11, dummy6[i]);
    Rewrite (W11);
  End;
End; (Case i)
End; (For i := 1 to Num_Spec_Bins)
For \( k := 1 \) to NumFiles do
    For \( i := 1 \) to Num_Spec_Bins do
        npts\( [k,i] \) := 0;
    i := 0;

Readln (RAW);
While NOT (EOF(RAW)) do Begin
    i := i + 1;
    Readln (RAW, Diameter, Velocity, Time);
    If ((Time > 0.000001) AND (Time < 1000)) Then Begin
        If Diameter <= SubSize
            Then Begin
                Bin := 1;
            End
        Else If (Trunc((Diameter-SubSize)/Spectral_Bin_Width)+2) > Num_Spec_Bins
            Then Begin
                Bin := Num_Spec_Bins;
            End
        Else Begin
            Bin := Trunc((Diameter-SubSize)/Spectral_Bin_Width)+2;
        End;
        Case Bin of
            1: Writeln (W1, Time:12:6, Velocity:12:6);
            2: Writeln (W2, Time:12:6, Velocity:12:6);
            3: Writeln (W3, Time:12:6, Velocity:12:6);
            4: Writeln (W4, Time:12:6, Velocity:12:6);
            5: Writeln (W5, Time:12:6, Velocity:12:6);
            6: Writeln (W6, Time:12:6, Velocity:12:6);
            7: Writeln (W7, Time:12:6, Velocity:12:6);
            8: Writeln (W8, Time:12:6, Velocity:12:6);
            9: Writeln (W9, Time:12:6, Velocity:12:6);
            10: Writeln (W10, Time:12:6, Velocity:12:6);
            11: Writeln (W11, Time:12:6, Velocity:12:6);
        End; (Case Bin)
    End; (If Time)
End; (While NOT)

For \( i := 1 \) to Num_Spec_Bins do Begin
    Writeln (FL, dummy5[i]);
    Writeln (FL, 'SPEC' + dummy4[i]);
    Writeln (FL, npts[j,i]);
End;

For \( i := 1 \) to Num_Spec_Bins do Case i of
1: Close (W1);
2: Close (W2);
3: Close (W3);
4: Close (W4);
5: Close (W5);
6: Close (W6);
7: Close (W7);
8: Close (W8);
9: Close (W9);
10: Close (W10);
11: Close (W11);
End; {Case i}

Close (RAW);
End; {For j := 1 to NumFiles}
Close (FL);

Writeln;
Writeln;
Writeln ('You are now ready to use the Fortran program SLOT.');
Writeln ('Make sure that SLOT.EXE is in the D\CODES directory');
Writeln ('You must be in the directory: 
' + 'D:\CODES' + \Date + '\'>
Writeln ('when you type SLOT. Enter a value of: 
' + '(NumFiles * Num_Spec_Bins):10);
Writeln ('at the prompt that asks for the number of loops.');
Repeat Until Keypressed;

End; {Procedure Comb_1}

**********************************************************************
Begin {Main}
  Comb_1;
End. {Main}
program slot

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Written by: John H. Lienhard V, 1990

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
This is a specific version of SLSM. This program assumes:
nslots=1024, tamx=0.05, smoothing into 10 intervals per decade

Calculate uniform correlation function and power spectrum from randomly
sampled data. Current time series array sizes up to 21,000 points.

Revision: November 19, 1990

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
implicit real*8 (a-h,o-z)
real*8 t(21000),x(21000),sum(8192)
real y(1024),f(1024),rdf
integer nsum(1024)
character ans
character*32 namel,name2

c
common /smth/ y,f

data twopi /6.28318531/

Bartlett window function

w(k) = 1.0 - float(k)/float(nslots)

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
open (unit=70, file='FILELIST', status='old')

write(5,1)
1 format(5x,'Input number of loops: ')$
read(5,*), iloop

do 200 ibig = 1, iloop

write(5,1)
1 format(5x,'Input file name: ')$
read(70,2), namel
2 format(a)
   open(unit=1,file=namel,status='unknown')
   write(5,69), name1
69 format(5x, 'Working on file ', a)

write(5,3)
3 format(5x,'Output file name: ')$
read(70,4) name2
4 format(a)
   open(unit=2,file=name2,status='unknown')

    write(5,7)
7 format(5x,'Enter N_points: ')$
   read(70,*) npts

    write(5,8)
8 format(5x'Enter nslots, tau_max: '$)
   read(5,*) nslots,tamx

    nslots=1024
    tamx=0.05
    dtau=tamx/float(nslots)
    dta2=dtau/2.
    taub=tamx-dtau/2.
    fs=1/dtau
    df=fs/(2.*nslots)
    fny=fs/2.
    rdf=df

    write(5,9) dtau,fs,nslots,fny,rdf
9 format(/5x,'dtau = 'e14.7,5x,'f_sample = 'e11.4,3x,
  'nslots = 'i5/5x'Nyquist frequency = 'f8.2,5x,
  'Frequency resolution = 'f7.3)

    do 11 k=1,npts
      read(1,*) t(k),x(k)
10    format(e14.7,3x,e14.7)
11    continue

    do 20 k=1,nslots
      sum(k)=0.
      nsum(k)=0
    20    continue

    sum1=0.
    sum2=0.
    do 30 k=1,npts
      sum1=sum1+x(k)
30    continue
    sum1=sum1/npts

    do 35 k=1,npts
      x(k)=x(k)-sum1
    35    continue
    sum2=sum2+x(k)**x(k)
    do 35 k=1,npts
      x(k)=x(k)-sum1
    35    continue
    sum2=sum2/x(k)**x(k)
    cO0=sum2/fs
    write(5,999) cO
999 format(5x'c0 = 'e14.7/)
c
    do 50 k=2,npts
        km=k-1
        do 40 l=1,km
            j=km-l+l
            tau=t(k)-t(j)
            if ((tau.le.dta2)) go to 40
            if (tau.ge.taub) go to 50
            i=(tau+dta2)/dtau
            sum(i)=sum(i)+x(k)*x(j)
            nsum(i)=nsum(i)+1
        40 continue
    50 continue
    c
    do 60 k=1,nslots
        if (nsum(k).ne.0) sum(k)=sum(k)/nsum(k)
    c        t(k)=dtau*(k-1)
        y(k+l)=w(k)*sum(k)
    60 continue
    y(1)=sum2
    c
    c nslots must be a power of two; isign > 0 forward; isign <0
    c and multiply result by 2/nslots backward.
    c
    isign=1
    call cosft(y,nslots,isign)
    c
    do 70 k=1,nslots
    c    Recall: df=fs/(2.*nslots-1.); fs=1./dtau
    c
    Factor of 2. in df is tied up in doubling of nslot.
    c    Multiply y(k) by 4. to convert from cosine transform to
    c    two-sided spectral density (2 for cosine to FT, 2 for one-sided
    c    to two-sided).
    c    Divide y(k) by fs to yield psd, not periodogram.
    c    Subtraction of sum2, as by Mayo (1978), is necessary to account
    c    folding FFT from two-sided to one-sided.
    c
    f(k)=df*(k-1)
    y(k)=4.*y(k)/fs - 2.*c0
    c
    70 continue
    c
    write(5,71)
    c    format(5x,'Smooth spectrum (y/n): '$)
    c    read(5,75) ans
    75 format(al)
    c    if ((ans.eq.'y').or.(ans.eq.'Y')) then
    c        call smooth(rdf,nslots)
    c    elseif((ans.ne.'n').and.(ans.ne.'N')) then
write(5,77)
77 format(/5x,'Invalid response. Answer y or n: 'S)
go to 72
endif
do 90 k=1,nslots
    write(2,85) f(k),y(k)
85 format(e14.7,3x,e14.7)
90 continue
close(unit=1)
close(unit=2)
c200 continue
close(unit=70)
stop '
end
subroutine smooth(df,nslots)
Logarithmically smooth the spectrum into n intervals per decade
real y(1024),f(1024)
common /smth/ y,f
write(5,1)
1 format(/5x'Number of intervals per decade: 'S)
read(5,2) nint
2 format(i4)
nint=10
rint=1./float(nint)
xi=(10.)**rint
wf=0.5*df*(xi+1.)/(xi-1.)

Find the endpoints of the interval
f0max=df*(nslots-1)
mm=log10(f0max)
nm=(log10(f0max)-mm)*nint-1
k0=1./(1.-xi**(-0.5)) + 1
do 5 k=2,k0
    f(k-1)=f(k)
y(k-1)=y(k)
5 continue
f0=(k0-1)*df
m0=log10(f0)
n0=(log10(f0)-m0)*nint + 1
if (mO.lt.0) then
    nO=nint+nO-1
    mO=mO-1
endif

kkO=kO
  k=kO

do 30 m=m0,mm

do 20 n=1,10
    if ((m.eq.m0).and.(n.lt.n0)) go to 20
    if ((m.eq.mm).and.(n.gt.nm)) go to 30
    sum=0.0
    f1=(10.)**(m+n*rint)
    f2=f1*xi

do 10 l=k0,nslots
    if (f(l).lt.fl) go to 10
    if (f(l).gt.f2) then
        kk0=l
        go to 11
    endif
    sum=sum+y(l)
10 continue

11  fave=0.5*(f1+f2)
  w=wf/fave
  f(k)=fave
  y(k)=w*sum
  k=k+1
  k0=kk0

20 continue
30 continue
nslots=k-1

return
end

*******************************************************************************

SUBROUTINE COSFT(Y,N,ISIGN)

This subroutine is from Numerical Recipes.

REAL*8 WR, WI, WPR, WPI, WTEMP, THETA
DIMENSION Y(N)
THETA=3.14159265358979D0/DBLE(N)
WR=1.0D0
WI=0.0D0
WPR=-2.0D0*DSIN(0.5D0*THETA)**2
WPI=DSIN(THETA)
SUM=Y(1)
M=N/2
DO 11 J=1,M
WTEMP=WR
WR=WR*WPR-WI*WPI+WR
WI=WI*WPR+WTEMP*WPI+W1
Y1=0.5*(Y(J+1)+Y(N-J+1))
Y2=(Y(J+1)-Y(N-J+1))
Y(J+1)=Y1-WI*Y2
Y(N-J+1)=Y1+WI*Y2
SUM=SUM+WR*Y2
11 CONTINUE
CALL REALFT(Y,M,+1)
Y(2)=SUM
DO 12 J=4,N,2
SUM=SUM+Y(J)
Y(J)=SUM
12 CONTINUE
IF (ISIGN.EQ.-1) THEN
EVEN=Y(1)
ODD=Y(2)
DO 13 I=3,N-1,2
EVEN=EVEN+Y(I)
ODD=ODD+Y(I+1)
13 CONTINUE
ENFO=2.0*(EVEN-ODD)
SUMO=Y(1)-ENFO
SUME=(2.0*ODD/FLOAT(N))-SUMO
Y(1)=0.5*ENFO
Y(2)=Y(2)-SUME
DO 14 I=3,N-1,2
Y(I)=Y(I)-SUMO
Y(I+1)=Y(I+1)-SUME
14 CONTINUE
ENDIF
RETURN
END

C

C SUBROUTINE REALFT(DATA,N,ISIGN)
C
C This subroutine is from Numerical Recipes.
C
REAL*8 WR,WI,WPR,WPI,WTEMP,THETA
DIMENSION DATA(2*N)
THETA=6.28318530717959D0/2.0D0/DBLE(N)
WR=1.0D0
WI=0.0D0
C1=0.5
IF (ISIGN.EQ.1) THEN
   C2=-0.5
   CALL FOUR1(DATA,N,+1)
   DATA(2*N+1)=DATA(1)
   DATA(2*N+2)=DATA(2)
ELSE
   C2=0.5
   THETA=-THETA
   DATA(2*N+1)=DATA(2)
   DATA(2*N+2)=0.0
   DATA(2)=0.0
ENDIF
WPR=-2.0D0*DSIN(0.5D0*THETA)**2
WPI=DSIN(THETA)
N2P3=2*N+3
DO 11 I=1,N/2+1
   I1=2*I-1
   I2=I1+1
   I3=N2P3-I2
   I4=I3+1
   WRS=SNGL(WR)
   WIS=SNGL(WI)
   H1R=C1*(DATA(I1)+DATA(I3))
   H1I=C1*(DATA(I2)-DATA(I4))
   H2R=-C2*(DATA(I2)+DATA(I4))
   H2I=C2*(DATA(I1)-DATA(I3))
   DATA(I1)=H1R+WRS*H2R-WIS*H2I
   DATA(I2)=H1I+WRS*H2I+WIS*H2R
   DATA(I3)=H1R-WRS*H2R+WIS*H2I
   DATA(I4)=-H1I+WRS*H2I+WIS*H2R
   WTEMP=WR
   WR=WR*WPR-WI*WPI+WR
   WI=WI*WPR+WTEMP*WPI+WI
11 CONTINUE
IF (ISIGN.EQ.1) THEN
   DATA(2)=DATA(2*N+1)
ELSE
   CALL FOUR1(DATA,N,-1)
ENDIF
RETURN
END

C **************************************************************
C SUBROUTINE FOUR1(DATA,NN,ISIGN)
C This subroutine is from Numerical Recipes.
C
REAL*8 WR, WI, WPR, WPI, WTEMP, THETA
DIMENSION DATA(2*NN)
N=2*NN
J=1
DO 11 I=1,N,2
IF(J.GT.I)THEN
  TEMPR=DATA(J)
  TEMPI=DATA(J+1)
  DATA(J)=DATA(I)
  DATA(J+1)=DATA(I+1)
  DATA(I)=TEMPR
  DATA(I+1)=TEMPI
ENDIF
M=N/2
1 IF ((M.GE.2).AND.(J.GT.M)) THEN
  J=J-M
  M=M/2
  GO TO 1
ENDIF
J=J+M
11 CONTINUE
MMAX=2
2 IF (N.GT.MMAX) THEN
  ISTEP=2*MMAX
  THETA=6.28318530717959D0/(ISIGN*MMAX)
  WPR=-2.DO*DSIN(0.5DO*THETA)**2
  WPI=DSIN(THETA)
  WR=1.DO
  WI=0.DO
  DO 13 M=1,MMAX,2
  DO 12 I=M,N,ISTEP
    J=I+MMAX
    TEMPR=SNGL(WR)*DATA(J)-SNGL(WR)*DATA(J+1)
    TEMPI=SNGL(WR)*DATA(J+1)+SNGL(WI)*DATA(J)
    DATA(J)=DATA(I)-TEMPR
    DATA(J+1)=DATA(I+1)-TEMPI
    DATA(I)=DATA(I)+TEMPR
    DATA(I+1)=DATA(I+1)+TEMPI
  12 CONTINUE
  WTEMP=WR
  WR=WR*WPR-WI*WPI+WR
  WI=WI*WPR+WTEMP*WPI+WI
  13 CONTINUE
  MMAX=ISTEP
  GO TO 2
ENDIF
RETURN
END
Program Average (Input, Output);

Uses CRT, Printer;

Const (Global)
   Name = 'DEC27';
   NumFiles = 14;
   Num_Dia_Bins = 40;
   Subsize = 9;
   Num_Spec_Bins = 6;
   Spec_Bin_Width = 5;
   Max_Spec_Size = 35;
   TauMax = 0.05;
   Nozzle_Emit_Runtime = 400.0;  [157.4, 64.1, 18.74]

Var (Global)
   Station_Identifier : String[2];
   Gas_RMS_Vel,
   Validations,
   Avg_Runtime : Real;
   Reply : Char;

[*********************************************]
Procedure Smooth_Velocities;
[*********************************************]

Written by: John A. Simo, 1990

This program assumes that a maximum of twenty PDPA runs are being averaged (Num_Files).
It also assumes that up to 50 of the diameter bins will be examined (Num_Dia_Bins), and that all of these have data. The program will halt and display a message if it finds diameter bins with no data; it will advise you to use a smaller number for Num_Dia_Bins.

The program reads the PDPADATA.TXT files and finds the number density fields, the mean velocity fields, and the RMS velocity fields. It also finds the information to plot corrected particle count versus diameter. The program averages this information for the twenty runs. It then writes to four files:

1) the particle mean velocity vs. diameter, VMEA*
2) the particle RMS velocity vs. diameter, VRMS*
3a) the particle turbulence intensity (v'/V) VTUR*
3b) the non-dimensional RMS velocity (v'/U) VTUR*
4) the particle diameter vs. count (or probability) DIAM*

where the * is the Filename_Identifier input by the user. Furthermore, information for chi-squared error envelopes is also written to the files.
Finally, the program computes and displays, on the screen, the 20-file average of the number density, run time, gas mean velocity, gas RMS velocity, and the gas turbulence intensity, as well as the count versus diameter info. This info should be written down or printed using printscreen; it is useful for comparing the experimental and the predicted (i.e., Hinze) quantities.

Notes: The corrected count is used for obtaining a size histogram to compare to the evaporation theory. Since the size histogram is normalized by the total number of particles, however, it is more correctly called a probability distribution. The uncorrected count is used to properly weight the velocity bins for averaging.

If you need to average less than twenty runs change the variable NumFiles. Likewise for Num_Dia_Bins less than 50.
Adjust Nozle_Exit_Runtime for each data series.
Make sure that the printer is ON.

Var
Filename_Write_Begin_Vmea, [Mean velocity]
Filename_Write_Begin_Vrms, [RMS velocity]
Filename_Write_Begin_Vtur, [Turbulence Intensity]
Filename_Write_Begin_Diam : String[70]; [Diameter Histograms]
Filename_Read_Begin : String[70];
Filename_Read_End : String[70];
Legend_Text : String[40];
dummy1 : String[2];
Bin, i, j : Integer;
Corr_Count_Sum,
Total_Corr_Count,
Uncorr_Count_Sum,
Total_Uncorr_Count,
Attempts,
Attempts_Sum,
u : Longint;
Full_Name : String[70];
ND : String[40];
R, W1, W2, W3, W4 : Text;
ND_Label : String[20];
Runtime_Label : String[9];
Number_Density,
Runtime : Array[1..NumFiles] of Real;
Number_Density_Sum,
Turb_Inten,
Vel_Mean_Sum,
Vel_RMS_Sum,
Nondimensional_Mean_Vel,
Nondimensional_RMS_Vel,
Temp,
Runtime_Sum,
Gas_Mean_Vel,
T_dist,
T_dist_lo, T_dist_hi,
chi2_lo, chi2_hi,
Integral_Area : Real;
Diameter : Array[1..Num_Dia_Bins] of Real;
Velocity_Mean,
Velocity_RMS : Array[1..NumFiles, 1..Num_Dia_Bins] of Single;
Corr_Count,
Uncorr_Count : Array[1..NumFiles, 1..Num_Dia_Bins] of Longint;

Begin (Procedure Smooth_Velocities)
Filename_Read_Begin := 'E:\PDPDATA\THESIS\' + Name + '\RUN';
Filename_Read_End := '\PDPDATA.TXT';
ClrScr;
Write ('Enter the downstream distance for the Easyplot legend : ');
Readln (LegendText);
Writeln;

Writeln ('Enter a two-digit filename identifier for the DOS file corresponding to the station number of the traverse: ');
Readln (StationIdentifier);
Writeln;

Writeln ('Files will go to D:\CODES\', Name, '\SAVE...');
Writeln ('They will have a ', StationIdentifier, '.extension.');
Writeln ('Hit CTRL-BREAK to abort.');
Writeln ('Hit RETURN if this is okay.');
Readln;

Filename_Write_Begin_Vmea := 'D:\CODES\' + Name + '\SAVE\VMEA.';
Filename_Write_Begin_Vrms := 'D:\CODES\' + Name + '\SAVE\VRMS.';
Filename_Write_Begin_Vtur := 'D:\CODES\' + Name + '\SAVE\VTUR.';
Filename_Write_Begin_Diam := 'D:\CODES\' + Name + '\SAVE\DIAM.';
Assign (W1, (Filename_Write_Begin_Vmea + Station_Identifier));
Assign (W2, (Filename_Write_Begin_Vrms + Station_Identifier));
Assign (W3, (Filename_Write_Begin_Vtur + Station_Identifier));
Assign (W4, (Filename_Write_Begin_Diam + Station_Identifier));
Rewrite (W1);
Rewrite (W2);
Rewrite (W3);
Rewrite (W4);
Writeln (W1, '/sm OFF');
Writeln (W1, '/td xy');
Writeln (W1, '/et x "Particle diameter \(d_p\) (\text{mm})"');
Writeln (W1, '/et y "Particle mean velocity \(V\) (\text{m/s})"');

Writeln (W2, '/sm OFF');
Writeln (W2, '/td xy');
Writeln (W2, '/et x "Particle diameter \(d_p\) (\text{mm})"');
Writeln (W2, '/et y "Particle rms velocity \(v\) (\text{m/s})"');

Writeln (W3, '/sm OFF');
Writeln (W3, '/td xy');
Writeln (W3, '/et x "Particle diameter \(d_p\) (\text{mm})"');
Writeln (W3, '/et y "Particle turbulence intensity, \(v/V\)"');
Writeln (W3, '/at y 0.25 ".25"');
Writeln (W3, '/ogs y m');
Writeln (W3, '/ogs x n');
Writeln (W3, '/og ON');

Writeln (W4, '/sm OFF');
Writeln (W4, '/et x "Particle diameter \(d_p\) (\text{mm})"');
Writeln (W4, '/et y "Probability"');

Total_Corr_Count := 0;  \{Total_Corr_Count contains the total number of particles -- corrected count -- for all diameter bins and for all twenty files\}
Total_Uncorr_Count := 0;
Number_Density_Sum := 0;
Runtime_Sum := 0;

Attempts_Sum := 0;
For i := 1 to NumFiles do Begin
  Str (i, dummy1);
  If ((i >= 1) AND (i < 10)) Then dummy1 := '0' + dummy1;
  Full_Name := Filename_ReadBegin + dummy1 + Filename_ReadEnd;
  Writeln ('Reading: ', Full_Name);
  Assign (R, Full_Name);
  Reset (R);
  For j := 1 to 4 do
    Readln (R);
  Readln (R, NDLabel, Attempts);
  Attempts_Sum := Attempts_Sum + Attempts;
  Readln (R);
  Readln (R);
  Readln (R, Runtime_Label, Runtime[i]);
Runtime_Sum := Runtime_Sum + Runtime[i];

For j := 9 to 14 do
  Readln (R);

Readln (R, ND_Label, Number_Density[i]);
Number_Density_Sum := Number_Density_Sum + Number_Density[i];

For j := 1 to 109 do
  Readln (R);

For j := 1 to Num_Dia_Bins do Begin
  Readln (R, Bin, Diameter[j], Corr_Count[i, j]);
  Total_Corr_Count := Total_Corr_Count + Corr_Count[i, j];
End;

For j := 1 to (53 - Num_Dia_Bins) do
  Readln (R);

For j := 1 to Num_Dia_Bins do Begin
  Read (R, Diameter[j], Velocity_Mean[i, j]);
  Readln (R, Velocity_RMS[i, j], Uncorr_Count[i, j]);
  Total_Uncorr_Count := Total_Uncorr_Count + Uncorr_Count[i, j];
  If Uncorr_Count[i, j] < 1 then Begin
    Writeln;
    Writeln ('V_rms value of zero at bin # ', j:3);
    Writeln ('Use a smaller number of bins. ');
    Write ('Hit CTRL-BREAK, change Num_Dia_Bins, ');
    Writeln ('and restart. ');
    Readln;
  End;
End;
Close (R);
End;

ClrScr;
Writeln;
Writeln ('***********************************************************');
Writeln (' D:\CODES\', Name,
  'SAVExxxxxx\', Station_Identifier);
Writeln (' For station ', Station_Identifier);
Writeln (' x/D_O = ', Legend_Text);
Writeln (' NumFiles = ', NumFiles:4);
Writeln ('***********************************************************');
Write ('Average of the ', NumFiles, ' number densities ');
Write ('is: ', (Number_Density_Sum / NumFiles):8:1);
Writeln (' particles per cc.');
Avg_Runtime := Runtime_Sum / NumFiles;
Write ('Average of the run times is: ', Avg_Runtime:8:2);
Writeln ('seconds.');
Integral_Area := 0;
(The average size histogram is formed and written to a file
like DIAM06 -- for the sixth station from the nozzle, for example.
The corrected count in each bin is normalized by the
total number of particles -- Total_Corr_Count -- to yield what
is properly called a probability distribution. The sum of the
individual probability values is close to unity.
Furthermore, the runs after station 01 (the nozzle exit plane)
are normalized by the the runtime fraction --
how long the sampling time for runs after station 01 compare to the
runtime for station 01. Integral_Area compares the areas under the
size histogram (or size probability distribution).
)

For j := 1 to Num Dia Bins do Begin
    Vel_Mean_Sum := 0;
    Vel_RMS_Sum := 0;
    Corr_Count_Sum := 0; {For a given diameter bin -- like all
particles of 10.64 micron size -- Corr_Count_Sum holds the sum
of the corrected number of particles for all five files. Used
in the evaporation theory (DIAM.xx) file; used also for finding
the average sampling frequency and the spectral variance.}
    Uncorr_Count_Sum := 0; {Same type of addition as above. This
quantity is used to properly weight the velocity bins for
averaging.}
    For i := 1 to NumFiles do Begin
        Corr_Count_Sum := Corr_Count_Sum + Corr_Count[i, j];
        Uncorr_Count_Sum := Uncorr_Count_Sum + Uncorr_Count[i, j];
        Vel_Mean_Sum := Vel_Mean_Sum + Uncorr_Count[i, j]
            * Velocity_Mean[i, j];
        Vel_RMS_Sum := Vel_RMS_Sum + Uncorr_Count[i, j]
            * Velocity_RMS[i, j];
    End;
{Generate the average mean velocity for bin j}
    Vel_Mean_Sum := Vel_Mean_Sum / Uncorr_Count_Sum;
{Generate the average RMS velocity for bin j}
    Vel_RMS_Sum := Vel_RMS_Sum / Uncorr_Count_Sum;
{Generate the average turbulence intensity for bin j}
    Turb_Inten := Vel_RMS_Sum / Vel_Mean_Sum;
If j = 1 Then Begin
    Gas_Mean_Vel := Vel_Mean_Sum;
    Write ('The gas mean velocity is: ', Vel_Mean_Sum:8:2);
    Writeln (' meters per second.');
Gas_RMS_Vel := Vel_RMS_Sum;
Write ('The gas RMS velocity is: ', Vel_RMS_Sum:8:2);
Writeln (' meters per second.');

Write ('The gas turbulence intensity is: ');
Writeln (Turb_Inten:8:4);
End;

{Generate the non-dimensional mean velocity}
Nondimensional_Mean_Vel := Vel_Mean_Sum / Gas_Mean_Vel;

{Generate the non-dimensional rms velocity}
Nondimensional_RMS_Vel := Vel_RMSSum / Gas_RMS_Vel;

nu := Uncorr_Count_Sum - 1;
If nu > 10000 Then Begin
  chi2_lo := 0.98;
  chi2_hi := 1.01;
  T_dist := 1.96;
End;
If (nu <= 10000) AND (nu > 3000) Then Begin
  chi2_lo := 0.96;
  chi2_hi := 1.03;
  T_dist := 1.96;
End;
If (nu <= 3000) AND (nu > 1500) Then Begin
  chi2_lo := 0.94;
  chi2_hi := 1.07;
  T_dist := 1.96;
End;
If (nu <= 1500) AND (nu > 900) Then Begin
  chi2_lo := 0.92;
  chi2_hi := 1.09;
  T_dist := 1.96;
End;
If (nu <= 900) AND (nu > 195) Then Begin
  chi2_lo := 0.80;
  chi2_hi := 1.15;
  T_dist := 1.96;
End;
If (nu <= 195) AND (nu > 48) Then Begin
  chi2_lo := 0.72;
  chi2_hi := 1.33;
  T_dist := 2.00;
End;
If (nu <= 48) AND (nu > 22) Then Begin
  chi2_lo := 0.55;
  chi2_hi := 1.58;
  T_dist := 2.05;
End;
If (nu <= 22) AND (nu > 12) Then Begin
chi2_lo := 0.43;
chi2_hi := 1.80;
T_dist := 2.13;
End;
If nu = 12 Then Begin
  chi2_lo := 0.37;
  chi2_hi := 1.94;
  T_dist := 2.18;
End;
If nu = 11 Then Begin
  chi2_lo := 0.34;
  chi2_hi := 1.99;
  T_dist := 2.20;
End;
If nu = 10 Then Begin
  chi2_lo := 0.32;
  chi2_hi := 2.05;
  T_dist := 2.23;
End;
If nu <= 9 Then Begin
  chi2_lo := 0.30;
  chi2_hi := 2.11;
  T_dist := 2.26;
End;

T_dist_lo := Vel_Mean_Sum - T_dist * Vel_RMS_Sum / sqrt (nu);
T_dist_hi := Vel_Mean_Sum + T_dist * Vel_RMS_Sum / sqrt (nu);
chi2_lo := Vel_RMS_Sum / sqrt(chi2_lo);
chi2_hi := Vel_RMS_Sum / sqrt(chi2_hi);

Write (W1, Diameter[j]:8:2, Vel_Mean_Sum:8:2);
WriteLn (W1, T_dist_hi:10:4, T_dist_lo:10:4,
  Nondimensional_Mean_Vel:10:2);
Write (W2, Diameter[j]:8:2, Vel_RMS_Sum:10:4);
WriteLn (W2, chi2_lo: 10:4, chi2_hi:10:4,
  Nondimensional_RMS_Vel:10:2);
Write (W3, Diameter[j]:8:2, Turb_Inten:10:4);
WriteLn (W3, (VelRMS_Sum/Gas_Mean_Vel):10:4);

{Generate the size histogram for bin j}
Temp := Corr_Count_Sum / Total_Corr_Count;
Temp := Temp * Nozzle_Exit_Runtime / (Runtime_Sum/NumFiles);
WriteLn (W4, Diameter[j]:8:2, Temp:12:6);
Integral_Area := Integral_Area + Temp;
End; {For j := 1 to Num_Dia_Bins}
WriteLn ('Integral_Area = ', Integral_Area:10:4);

WriteLn (W1);
WriteLn (W2);
Writeln (W3);
Writeln (W4);
Writeln (W1, '//It "', Legend_Text, '" 2');
Writeln (W2, '//lt "', LegendText, '" 2');
Writeln (W3, '/It "', Legend_Text, '" 2');
Writeln (W3, '//It "', Legend_Text, '" 3');
Writeln (W4, '//It "', Legend_Text, '" 2');
Close (W1);
Close (W2);
Close (W3);
Close (W4);

Validations := Total_Uncorr_Count / NumFiles;
Writeln ('Average number of validations per file: ',
    Round(Validations):7);
Writeln ('Average sampling frequency: ',
    Round(Validations/Avg_Runtime):7, ' Hz');
Writeln ('Percent validations: ',
    Round(Validations * NumFiles * 100 / Attempts_Sum):6, ' %');
Writeln ('***********************************************');
Writeln;
Writeln ('Use PRINTSCREEN to get a hardcopy.');
Writeln ('Hit RETURN when finished.');
Readln;
End; [Procedure Smooth_Velocities]

[***********************************************************************]
Procedure Smooth_Spectra;
***********************************************************************

Written by: John A. Simo, 1990
***********************************************************************

This program read several power spectra files (with extensions such as '.01'
and '.02') and performs an arithmetic average of them. The averaged file
contains a smoothed spectra which is written to a file with a '.SMA'
extension. Also written to the bottom of the file is a double-slash
legend text identifier for EASYPLOT graphing.
***********************************************************************

Before running, change the following:
    File_Name_Read
    File_Name_Write
Use Ctrl QA for speed.
***********************************************************************

Var
    R, W : Text;
    temp  : Real;
    File_Name_Read,
    File_Name_Write : String[40];
Legend_Text : String[15];
dummy1, dummy2,
dummy3, dummy4 : String[2];
i, j, k,
temp1, temp2,
NumPts : Integer;
PSD : Array [1..NumFiles, 1..Num_Spec_Bins,
1..33] of Real;
Freq, PSD_Sum : Array [1..33] of Real;
Ch : Char;

************************************************************************

Function Spectral_Variance_Hi (y, x : Real) : Real;

Const
  zeta = 1.259;

Var
  temp1, [variance from Rosenblatt and Helland]
  temp2, [variance from Mayo]
  Mean_Sampling_Frequency : Real;

Begin
  Mean_Sampling_Frequency := Validations / Avg_Runtime;
  temp1 := sqrt((1/(2 * x * 2 * TauMax * NumFiles))
     * (zeta + 1) / (zeta - 1));
  temp2 := sqrt(2 * TauMax / (3 * Avg_Runtime))
     * (y + (Gas_RMS_Vel / Mean_Sampling_Frequency));
  Spectral_Variance_Hi := y + (temp1 * temp2);
End;

************************************************************************

Function Spectral_Variance_Lo (y, x : Real) : Real;

Const
  zeta = 1.259;

Var
  temp1, [variance from Rosenblatt and Helland]
  temp2, [variance from Mayo]
  Mean_Sampling_Frequency : Real;

Begin
  Mean_Sampling_Frequency := Validations / Avg_Runtime;
  temp1 := sqrt((1/(2 * x * 2 * TauMax * NumFiles))
     * (zeta + 1) / (zeta - 1));
  temp2 := sqrt(2 * TauMax / (3 * Avg_Runtime))
     * (y + (Gas_RMS_Vel / Mean_Sampling_Frequency));
  Spectral_Variance_Lo := y - (temp1 * temp2);
End;

************************************************************************

Begin (Smooth_Spectra)
  Writeln;
  Writeln;
  For i := 1 to NumFiles do Begin
    Str (i, dummy1);
    If ((i >= 1) AND (i < 10)) Then dummy1 := '0' + dummy1;
    Writeln ('Reading SPEC', dummy1);
    For j := 1 to Num_Spec_Bins do Begin
      Str (j, dummy2);
      If ((j >= 1) AND (j < 10)) Then dummy2 := '0' + dummy2;
      File_Name_Read := 'D:\CODES\' + Name + '\SPEC' + dummy1 + '.' + dummy2;
      Assign (R, File_Name_Read);
      Reset (R);
      k := 0;
      While NOT(EOF(R)) do Begin
        k := k + 1;
        Readln (R, Freq[k], PSD[i, j, k]);
      End;
      Close (R);
    End; (For j := 1 to NumSpecBins)
  End; (For i := 1 to NumFiles)
  NumPts := k;

  Writeln ('Averaging');
  For j := 1 to Num_Spec_Bins do Begin
    Str (j, dummy2);
    If ((j >= 1) AND (j < 10)) Then dummy2 := '0' + dummy2;
    File_Name_Write := 'D:\CODES\' + Name + '\SAVE\SPEC' + dummy2 + '.' + Station_Identifier;
    Assign (W, File_Name_Write);
    Rewrite (W);
    Writeln (W, '/sm OFF');
    Writeln (W, '/td xy');
    Writeln (W, '/ol x ON');
    Writeln (W, '/or x 10 10000');
    Writeln (W, '/ol y ON');
    Writeln (W, '/or y 1e-5 1e-1');
    Writeln (W, '/et x "Frequency (Hz)"');
    Writeln (W, '/et y "E_u_u (m^2/s^2Hz)"');
    For k := 1 to NumPts do Begin
      PSD_Sum[k] := 0;
    End;
  End; (For j := 1 to NumSpecBins)
For i := 1 to NumFiles do
  PSD_Sum[k] := PSD_Sum[k] + PSD[i, j, k];
  PSD_Sum[k] := PSD_Sum[k] / NumFiles;

If PSD_Sum[k] >= 0 Then
  If (PSD_Sum[k] > 1E-6) Then Begin
    Write (W, Freq[k]:12:1,
    PSD_Sum[k]:12:7,
    (Spectral_Variance_Hi(PSD_Sum[k],
    Freq[k])):12:7);
    If (Spectral_Variance_Lo(PSD_Sum[k],
    Freq[k]) < 1E-6) Then Writeln (W, ' 0.0000001')
    Else Writeln (W, (SpectralVariance_Lo
    (PSD_Sum[k],Freq[k])): 12:7);
  End;
End; (For k)

Writeln (W);

If j = 1 Then Begin
  Str (Subsize, dummy3);
  If ((Subsize >= 1) AND (Subsize < 10))
    Then dummy3 := '0' + dummy3;
  Legend_Text := '01 to ' + dummy3 + 'mm';
End;

If ((j > 1) AND (j < Num_Spec_Bins)) Then Begin
  temp1 := ((j-2) * Spec_Bin_Width) + Subsize;
  Str (temp1, dummy3);
  If ((temp1 >= 1) AND (temp1 < 10))
    Then dummy3 := '0' + dummy3;

  temp2 := ((j-1) * Spec_Bin_Width) + Subsize;
  Str (temp2, dummy4);
  If ((temp2 >= 1) AND (temp2 < 10))
    Then dummy4 := '0' + dummy4;

  Legend_Text := dummy3 + ' to ' + dummy4 + 'mm';
End;

If (j = Num_Spec_Bins) Then Begin
  temp1 := ((j-2) * Spec_Bin_Width) + Subsize;
  Str (temp1, dummy4);
  If ((temp1 >= 1) AND (temp1 < 10))
    Then dummy4 := '0' + dummy4;

  Str (Max_Spec_Size, dummy3);
If ((Max_Spec_Size >= 1) AND (Max_Spec_Size < 10))
    Then dummy3 := '0' + dummy3;
    Legend_Text := dummy4 + ' to ' + dummy3 + '\text{mm}';
End;

Writeln (W, '/it ', Legend_Text, '/it "2');
Close (W);

End; {For j := 1 to Num_Spec_Bins}
End; {Smooth_Spectra}

{******************************************************************************************}

Begin {Program Average}
    Smooth_Velocities;

    Writeln;
    Write ('Include spectral data/RAWDAT.TXT files (Y/N)? '); Readln (Reply);
    If ((Reply = 'Y') OR (Reply = 'y')) Then Smooth_Spectra;
End. {Program Average}